PCA-based Linear Parameter Varying Control of SCR Aftertreatment Systems

Mona Meisami-Azad[†], Javad Mohammadpour[†], Karolos M. Grigoriadis[†],
Michael P. Harold[‡], Matthew A. Franchek[†]

Abstract— Hydrocarbons, carbon monoxide, and other polluting emissions produced by diesel engines are usually much lower than those by gasoline engines. However, higher combustion temperature in diesel engines cause substantially larger percentage of nitrogen oxides (NO_x) emissions. Selective catalytic reduction (SCR) is a well proven technology for reducing NO_x emissions from automotive sources and in particular, heavy-duty diesel engines. In this paper, we develop a quasi linear parameter varying (qLPV) model to capture the nonlinearities in the dynamics of the ammonia SCR system with varying catalyst surface temperature. To effectively enable the use of LMI-based control design methods, the number of LPV parameters in the qLPV model is then reduced by using the principal component analysis (PCA) technique. An LPV feedback/feedforward controller is designed for the qLPV model with reduced number of scheduling parameters. The designed full-order controller is further simplified to a first-order transfer function with parameter-varying gain and pole. Finally, simulation results illustrate the high conversion efficiency with minimum ammonia slip of the closed-loop SCR system using the parameter-varying control law.

I. INTRODUCTION

The lean burn conditions of diesel combustion that yield improved efficiency produce exhaust gas containing an excess of oxygen (up to 10%). While net-oxidizing exhaust enables the comparatively straightforward oxidation of hydrocarbons and carbon monoxide (CO) on precious metal catalysts, it complicates the chemical reduction of nitrogen oxides (NO_x) to N_2. This scenario has led to the vigorous development of technologies for NO_x reduction to meet the stringent NO_x exhaust limit mandated by the EPA. The NO_x emissions are one of the main air pollutants responsible for ozone depletion and photochemical smog formation causing severe respiratory problems to humans. Selective catalytic reduction (SCR) is a well proven technology used in power generation for more than 30 years. Stationary power generation involves very slow variation of operating conditions, allowing simple open-loop controllers to efficiently tackle the task of ACR control. However, automobile engines work in a broad envelope of fast varying conditions, necessitating the use of advanced SCR control techniques.

The SCR systems operates as follows: A urea injector, driven by a command signal from the controller, pumps the mixture of compressed air and aqueous urea solution into the exhaust stream through a nozzle. Ammonia (NH_3) and carbon dioxide are formed as a result of urea decomposition and HNCO hydrolysis in the exhaust pipe [5]. The mixture of ammonia, CO_2, remaining urea, and the exhaust then enters the SCR catalyst, where NH_3 reacts with NO_x from the exhaust producing pure nitrogen and water. A sensor placed at the catalyst outlet measures concentration of the unreacted NO_x and supplies this information to the controller, thereby closing the loop. The control problem consists of achieving the appropriate regulation of the urea injection to minimize NO_x emissions without significant ammonia slip.

There have been some efforts on lumped parameter modeling of the SCR reactions in the literature. These simplified models are appropriate for model-based control, since they reduce the complexity of the design. Tronconi and Forzatti [2] develop one- and two-dimensional steady-state isothermal models of SCR for different geometries of the catalyst. Upadhyay and Van Nieuwstadt [8] derive a lumped parameter model of SCR by first assuming that the catalyst behaves as an isothermal continuously stirred tank reactor (ICSTR) and neglecting mass transfer, and next using the method of weighted residuals. The reaction mechanism involves the DeNO$_x$ reaction, adsorption/desorption of NH_3 and NH_3 oxidation. This model considers the reduction of only NO by ammonia. Since, in Fe-zeolite catalyst NO_2 based reactions are highly favored, and NO_2 is more toxic compared to NO, Devarakonda *et al.* [12] present a set of ordinary differential equations (ODEs) to model the SCR reactions considering both fast SCR reaction involving NO_2 and the standard SCR reaction.

Model-based control and optimization of the SCR system have been the focus of few recent published papers. Upadhyay and Van Nieuwstadt [10] present a model-based control strategy using sliding mode observer-based control. The nonlinear observer is designed using the measured NO_x concentration downstream the SCR catalyst for estimation of the surface coverage fraction and ammonia slip concentration. A similar control design method which incorporates both the NO and NO_2 conversion efficiency in addition to the ammonia slip is proposed in [12]. Schar et al. [11] use a similar model as in [8] and design a model-based feedforward controller to limit the ammonia slip and a PI feedback controller for disturbance rejection purposes. Chi and Dacosta [9] present a more advanced SCR model where the catalyst channel is discretized axially and radially. A simplified first-order model of the system is then used in [9] for control design purposes, where the parameters are estimated in real-time using a model reference adaptive

[†]Mechanical Engineering Department, University of Houston, TX 77204. Corresponding author's email address: mmeisami@mail.uh.edu
[‡]Chemical and Biomolecular Engineering Department, University of Houston, TX 77204

controller. In [14] a PID-nonlinear model predictive control method is presented for a urea-SCR system.

Principal component analysis is an effective method to reduce the dimensionality of a data set consisting of a large number of interrelated variables, while retaining as much the data variation as possible [1]. This technique has been recently extended to LPV models in [13] to reduce the number of scheduling parameters. We take advantage of this method for a lumped parameter SCR model where the temperature of the catalyst is varying.

In the present paper, we use a third-order nonlinear model developed in [8] and design a reduced-order gain-scheduled LPV controller to maximize the NO conversion efficiency and minimize the NH$_3$ slip. The simulation results of this paper illustrate a comparison between the performance of the developed output feedback controller to that of a static state feedback and also of sliding mode control, where the latter methods require design of an observer for state estimation.

II. SCR SYSTEM LUMPED-PARAMETER MODEL AND ITS QUASI-LPV REPRESENTATION

The key requirement of an SCR catalyst is to selectively reduce NO$_x$ to N$_2$ in the presence of ammonia (NH$_3$). Typical SCR washcoats contain base metals such as Cu, Fe, and zeolites that store ammonia to enable NO$_x$ reduction. Ammonia is obtained by the thermal decomposition and hydrolysis of externally supplied aqueous urea. The two key reactions involved in the SCR NO$_x$ reduction process are as follows:

$$4NH_3 + 4NO + O_2 \longrightarrow 4N_2 + 6H_2O$$
$$4NH_3 + 2NO + 2NO_2 \longrightarrow 4N_2 + 6H_2O.$$
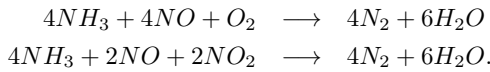
The second reaction, which involves both NO and NO$_2$ at equimolar amounts, is much faster than the first reaction with only NO. The NO$_x$ conversion is improved by the presence of NO$_2$ except at high temperatures,where ammonia oxidation limits NO$_x$ conversion.

The lumped-parameter model we use in the present paper was developed in [8], in which the NO and NH$_3$ concentrations, as well as ammonia surface coverage fraction are three states of the resulting differential equations. The ammonia coverage fraction is defined as the ratio of number of stored sites filled with NH$_3$ to the total number of storage sites in the catalyst. The model below is associated with the first reaction described above. The complete model is as follows

$$\begin{bmatrix} \dot{C}_{NO} \\ \dot{\theta} \\ \dot{C}_{NH_3} \end{bmatrix} = \begin{bmatrix} f_1(C_{NO}, \theta, C_{NH_3}) \\ f_2(C_{NO}, \theta, C_{NH_3}) \\ f_3(C_{NO}, \theta, C_{NH_3}) \end{bmatrix} + \begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ \frac{F}{V} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{NO} \\ \theta \\ C_{NH_3} \end{bmatrix} \qquad (1)$$

$$f_1 = -C_{NO}(\Theta_{SC} R_{red} \theta + \frac{F}{V}) + R_{ox} \Theta_{SC} \theta$$

$$f_2 = -\theta(R_{ads} C_{NH3} + R_{des} + R_{red} C_{NO} + R_{ox}) + R_{ads} C_{NH3}$$

$$f_3 = -C_{NH3} \left(\Theta_{SC} R_{ads}(1 - \theta) + \frac{F}{V} \right) + \Theta_{SC} R_{des} \theta$$

where C_{NO} and C_{NH_3} are concentrations of NO and NH$_3$ in mole/m^3, respectively. The reaction rate for reaction i is represented by $R_i = k_i exp(\frac{-E_i}{RT})$, $i = ads, des, red, ox$ where E_i is the activation energy and k_i is the pre-exponential term for the corresponding reaction. The parameter Θ_{SC} is the maximum ammonia storage capacity, and θ is the surface coverage fraction. The range of variation for θ is between 0 and 1. The variables F, V, R, and T represent the constant flow rate through catalyst, catalyst volume, universal gas constant, and catalyst surface temperature, respectively. The input u is the concentration of ammonia entering the catalyst and is the only controllable variable. The input d is the NO concentration upstream the catalyst treated as an external disturbance. The only measurement available is the concentration of NO downstream the catalyst denoted by y.

The above system can be transformed to a quasi-LPV (qLPV) form [6] with x as the state vector as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \left[\begin{array}{cc} -\Theta_{SC} R_{red} x_2 - \frac{F}{V} & R_{ox} \Theta_{SC} \\ -R_{red} x_2 & -(R_{ox} + R_{des}) \\ 0 & \Theta_{SC} R_{des} \end{array} \right.$$
$$\left. \begin{array}{c} 0 \\ R_{ads}(1 - x_2) \\ -\Theta_{SC} R_{ads}(1 - x_2) - \frac{F}{V} \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} +$$
$$\begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ \frac{F}{V} \end{bmatrix} u. \qquad (2)$$

As observed, the system matrix A is affinely dependent on the second state, *i.e.*, surface coverage fraction θ, and therefore θ is considered as the scheduling parameter in the above LPV model provided that it is known in real-time. In the state-space representation above, the system matrix A can be represented as $A(\theta) = A_0 + \theta A_1$ and the rest of the system matrices are parameter-independent.

III. LPV CONTROL DESIGN APPROACH FOR SCR SYSTEM

In the SCR system described above, the only measurement available is the NO concentration downstream the catalyst. The control design objective is to maximize the NO conversion efficiency and minimize the ammonia slip, as well as the amount of urea or ammonia to be injected. For the design purposes of this paper, we consider ammonia as the control input; however, it is noted that in an actual SCR system, the concentration of urea is the control variable. We present the proposed control design method with the objective of minimizing the H_∞ norm from the disturbance input (NO concentration upstream the SCR catalyst) to an appropriately defined controlled output, which is a weighted combination of NO concentration downstream the catalyst and ammonia slip. The control design method combines a feedforward control law with a feedback action, where feedback control gains are determined using a dynamic output feedback control law which will be further simplified to achieve a reduced-order controller. The controller is designed to minimize the H_∞ norm of the closed-loop system as the performance

measure. Observability and controllability of the system (1) are discussed in [10]. The model introduced in (2) can be rewritten as

$$\dot{x} = A(\theta)x + B_1 d + B_2 u$$
$$y = C_2 x \qquad (3)$$
$$z = C_1 x.$$

by defining appropriate state matrices $A(\theta)$, B_1, B_2, C_1 and C_2.

A. LPV Output Feedback Control Design Approach

We next consider the design of a gain-scheduled output feedback controller represented with the following state-space formulation

$$\dot{x}_k = A_k(\theta)x_k + B_k(\theta)y \qquad (4)$$
$$u = C_k(\theta)x_k + D_k(\theta)y.$$

As discussed earlier, the surface coverage fraction θ is assumed to be the scheduling parameter. However, the ratio θ is not actually measurable and must be estimated in real-time. In this paper, we estimate θ using its steady-state value to prevent the use of a full-state observer. From the first equation in (1), we have the following at steady-state:

$$-C_{NO}(\Theta_{SC}R_{red}\theta + \frac{F}{V}) + R_{ox}\Theta_{SC}\theta + \frac{F}{V}d = 0.$$

Therefore, θ can be estimated by

$$\hat{\theta} = \frac{\frac{F}{V}(C_{NO} - d)}{-C_{NO}\Theta_{SC}R_{red} + R_{ox}\Theta_{SC}} \qquad (5)$$

which implies that the knowledge of inlet and outlet NO concentrations is sufficient to estimate θ at each time instant.

We use the basic characterization of the gain-scheduled control design with guaranteed stability and H_∞ performance as described in [4] for the qLPV model represented by (2). Since, the dependence on the scheduling parameter θ is affine the synthesis LMIs need to be solved only at the two corners (*i.e.*, maximum and minimum allowable quantities for θ). For simplicity we consider constant basis functions for Lyapunov functions $R(.)$ and $S(.)$ and the auxiliary controller matrices $\hat{A}_k$, $\hat{B}_k$, $\hat{C}_k$, and D_k in the corresponding synthesis LMIs presented in [4]. Therefore, we have a finite number of decision variables to optimize. Taking into account the above structure for the decision variables, the only parameter-dependent controller matrix is A_k characterized by

$$A_k(\theta) = A_{k_0} + \hat{\theta}A_{k_1}. \qquad (6)$$

In the controller representation (4), we force the matrix D_k to be zero. The obtained controller is a full-order one; however, it was observed that the dominant pole of this system is constant for a fixed temperature and over the range of variation of the LPV parameter θ. Therefore, we determine a reduced-order controller that has the same DC gain as
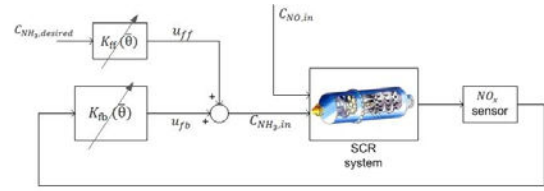

Fig. 1.   Schematic of the LPV control strategy for urea-SCR system

the original full-order controller. We found that the reduced-order controller is a first-order transfer function and since D_k is zero, it can be represented by

$$K(s) = \frac{\alpha(\hat{\theta})}{s + p} \qquad (7)$$

where p is the dominant pole of the designed controller. In order to have equal DC gains for both full-order and reduced-order controllers, the following relation holds

$$\frac{\alpha(\hat{\theta})}{p} = -C_k A_k^{-1}(\hat{\theta})B_k \qquad (8)$$

which indicates that the knowledge of p provides the parameter α scheduled based on $\hat{\theta}$.

To improve the steady-state performance of the closed-loop system, a feedforward term is added to the feedback control output (as shown in Figure 1) by keeping the ammonia slip at a desired level at steady-state using the following law

$$u_{ff} = \frac{V}{F}[C_{NH3,desired}\left(\Theta_{SC}R_{ads}(1-\hat{\theta}) + \frac{F}{V}\right)$$
$$-\Theta_{SC}R_{des}\hat{\theta}] \qquad (9)$$

where $C_{NH3,desired}$ is the desired ammonia slip and $\hat{\theta}$ is the real-time estimate of the LPV parameter using (5).

B. Temperature Effect on Reduced-Order Control Design

In the previous section, the catalyst temperature was assumed to remain constant in the control design process. In this section, we modify the control design method designed earlier to account for variability of the exhaust temperature as a result of different speeds and loads. It has been reported that there is a slight difference between the exhaust temperature and the catalyst downstream temperature [12], and the catalyst operating temperature can be considered as the average of these two temperatures.

As discussed in the previous section, the dominant pole of the full-order LPV output feedback controller (at a fixed operating point) was found out to be a function of temperature over the range of variation of the parameter $\hat{\theta}$. Therefore, the reduced-order controller will be in the following form

$$K(s) = \frac{\alpha(T,\hat{\theta})}{s + p(T)} \qquad (10)$$

and the DC gain of the controller is calculated as

$$\frac{\alpha(T,\hat{\theta})}{p(T)} = -C_k A_k^{-1}(\hat{\theta})B_k \qquad (11)$$

It is found out that $-C_k A_k^{-1}(\hat{\theta})B_k$ has an approximately affine dependency on the scheduling parameter $\hat{\theta}$. Therefore, we have

$$-C_k A_k^{-1}(\hat{\theta})B_k = g_1(T)\hat{\theta} + g_2(T) \qquad (12)$$

in which the line characteristic changes with respect to the operating temperature, and $p(T)$, $g_1(T)$ and $g_2(T)$ are mappings that are determined offline. We found out that these parameters can be estimated using polynomials of 2nd and 3rd order. The algorithm uses the exhaust gas temperature to calculate the controller dominant pole. Further simulation studies revealed that this dependence is logarithmic, *i.e.*,

$$\log p(T) = \gamma_0 + \gamma_1 T + \gamma_2 T^2. \qquad (13)$$

C. PCA-based LPV Control Design

The principal component analysis (PCA) of typical scheduling trajectories is a method to reduce the number of parameters of qLPV models for the synthesis of LPV gain scheduling controllers. This method enables a systematic trade-off between the reduced number of parameters and the desired accuracy [13]. In this section, we briefly describe the process involved and present the results of applying the PCA method to the SCR system model discussed before where the scheduling parameters $\hat{\theta}$ and T are estimated and measured, respectively (*i.e.*, $\rho = \begin{bmatrix} \hat{\theta} & T \end{bmatrix}^T$). For the varying temperature case, the system matrices can be parameterized as a function of a time-varying parameter vector $\delta(t)$ that depends on the vector of scheduling parameter vector $\rho(t)$ as follows

$$\delta_1 = R_{red}\hat{\theta} \qquad \delta_2 = R_{ox} \qquad \delta_3 = R_{des}$$
$$\delta_4 = R_{ads} \qquad \delta_5 = R_{ads}\hat{\theta} \qquad (14)$$

By such parameter vector selection, the system matrix A in (2) becomes

$$A = \begin{bmatrix} -\frac{F}{V} - \Theta_{sc}\delta_1 & \Theta_{sc}\delta_2 & 0 \\ -\delta_1 & -\delta_3 - \delta_2 & \delta_4 - \delta_5 \\ 0 & \Theta_{sc}\delta_3 & -\Theta_{sc}\delta_4 + \Theta_{sc}\delta_5 - \frac{F}{V} \end{bmatrix}$$

which is now affine in terms of $\delta_i's$. Assume that $\delta = \begin{bmatrix} \delta_1 & ... & \delta_p \end{bmatrix}^T$, where $p = 5$ in our case. We generate typical data for the scheduling signals and construct the data matrix

$$E = \begin{bmatrix} \delta(0), & ... & ,\delta((N-1)T_s) \end{bmatrix}$$

where T_s is the sampling time. This data matrix is then normalized to achieve scaled zero mean values shown by E^n. Next, a singular value decomposition (SVD) is performed on E^n as follows:

$$E^n = \begin{bmatrix} U_m & U_k \end{bmatrix} \begin{bmatrix} \Sigma_m & 0 & 0 \\ 0 & \Sigma_k & 0 \end{bmatrix} \begin{bmatrix} V_m^T \\ V_k^T \end{bmatrix}.$$

The basic idea behind the PCA is that if the data are correlated some singular values are small compared to the others [1]. Therefore, smaller singular values can be neglected and the reduced parameter vector $\phi(t) = q(\rho(t)) \in \mathbb{R}^m$

where $m < p$ can be reconstructed from the data only corresponding to the m largest singular values as

$$\hat{E}^n = U_m \Sigma_m V_m^T \approx E^n$$

where $\hat{E}^n$ is an approximation of the original data matrix. For quantifying this approximation accuracy the fraction of total variation is defined as

$$\upsilon = \frac{\sum_{i=1}^m \sigma_i^2}{\sum_{i=1}^p \sigma_i^2} \qquad (15)$$

where υ close to 1 shows a better accuracy. In (15), σ_i is the ith singular value of the data matrix E^n .Note that there is a trade-off between the accuracy and model complexity. Keeping a higher number of singular values leads to better accuracy but not necessarily an appropriate parameter reduction. The bounds on $\phi(t)$ are determined as follows

$$\underline{\phi}_i = \min_j q_i(\rho(jT_s)), \qquad \bar{\phi}_i = \max_j q_i(\rho(jT_s))$$

IV. SIMULATION RESULTS

In order to meet the dual goals of maximizing NO conversion efficiency and minimizing ammonia slip, we chose $z = a_1x_1 + a_3x_3$ as the control output in (3). The feedforward part of the output feedback controller is designed to keep the ammonia slip around 10 ppm in all of the performed simulations. The emission data corresponds to an EPA Urban Dynamometer Driving Schedule (UDDS) which has been developed for chassis dynamometer testing of heavy-duty vehicles. The basic parameters of the cycle are duration for 1060 seconds, distance of 5.55 miles, average speed of 18.86 mile/h, and maximum speed of 58 mile/h.

To show the effectiveness of the design method of Section III-B for the temperature-varying case we compare the performance of the reduced-order LPV controller of Section III-A with that presented in Section III-B for the example considered above, with a varying temperature profile shown in Figure 2. The First controller we design is scheduled only based on $\hat{\theta}$ and the second one is scheduled based on both $\hat{\theta}$ and the exhaust temperature T_e. The results are shown in Figures 3 and 4. The NO conversion efficiencies for the first and second cases are calculated to be 30% and 87%, respectively. Also, using a feedforward term similar to (9) but temperature-dependent, we are able to keep the ammonia slippage around our desired steady-sate value of 10 ppm using a gain-scheduling control method. The control inputs corresponding to the two control methods are shown in Figure 5. As Figure 5 demonstrates, for the temperature-varying case the LPV controller scheduled based on only $\hat{\theta}$ results in a control action (injected ammonia) with a large number of switchings between low ammonia and high ammonia levels. However, the LPV controller scheduled on both $\hat{\theta}$ and T leads to a smooth control action due to the consideration of varying temperature in the reduced-order controller dynamics.

For the SCR quasi-LPV model with varying temperature, there are five scheduling parameters. Using the PCA technique described in Section III-C, we are able to reduce the
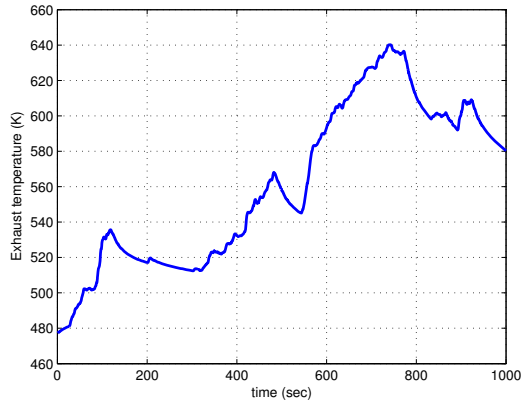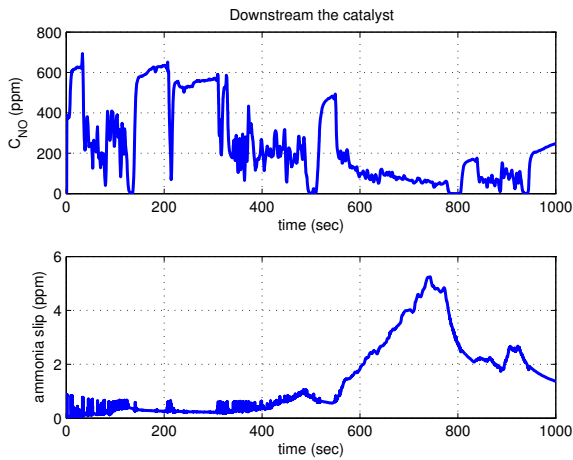
Fig. 2. Temperature profile

Fig. 3. NO concentration downstream the catalyst and ammonia slip for LPV control scheduled only on $\hat{\theta}$.
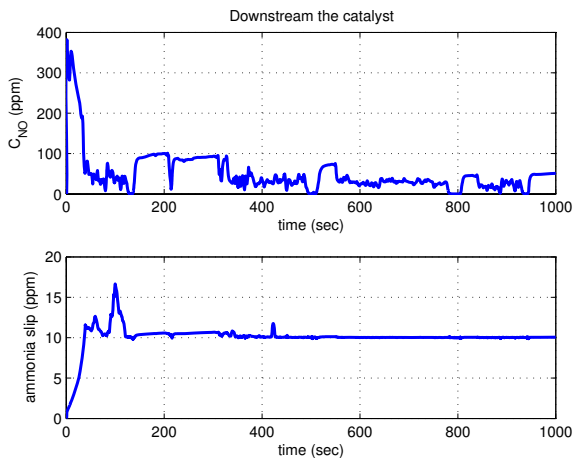

Fig. 4. NO concentration downstream the catalyst and ammonia slip for LPV control scheduled on both $\hat{\theta}$ and T.
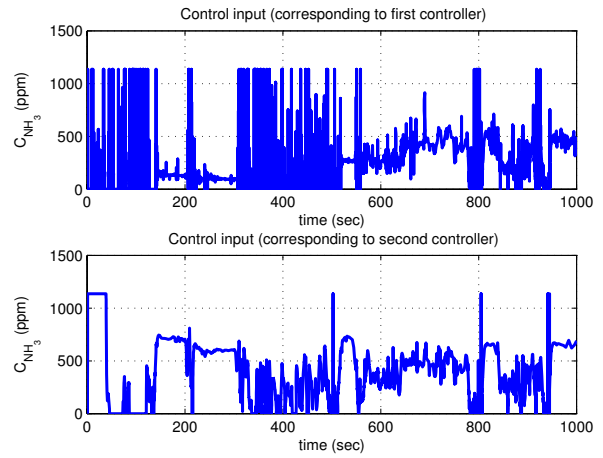

Fig. 5. Injected ammonia concentration

number of parameters to only one or two parameters. Our studies showed that the first principal component captures 63% and the first two principal components capture 93% of the model dynamics. Due to the relatively good accuracy and lower complexity, the model with one parameter is used to for controller design purposes. The nonlinear model is controlled by an LPV H_∞ output feedback controller that is scheduled on the reduced parameter vector $\phi(t)$. Since the model is affine in terms of ϕ, taking advantage of the multi-convexity property, the controller synthesis LMIs hold for all $\phi_i \in [\underline{\phi}_i, \bar{\phi}_i]$ if they hold at only the vertices [3]. The parameter reduction makes the computations tractable in the LPV design process because the number of LMIs to be solved depends exponentially on the number of LPV parameters. In addition, the PCA method results in a model that is affine in the reduced parameter space.

Simulation results demonstrate that the designed full-order controller can be reduced to a first-order parameter-varying one without drastically sacrificing performance. The simulation results are shown in Figures 6, 7 and 8. The Performance of the reduced-order controller gain scheduled based on $\hat{\theta}$ and T is compared with that of the open-loop case where constant amount of ammonia is injected (320 ppm). By making use of a gain-scheduled feedforward control law, we are able to keep the NH$_3$ slip around 10 ppm as before. However, this is not possible using open-loop control. Therefore, the constant amount of injected ammonia to the open-loop SCR system is adjusted so that the mean value of NH$_3$ slip remains around 10 ppm. The conversion efficiency for open-loop and closed-loop systems is calculated to be 88% and 74%, respectively. The injected ammonia is 17.82 mole/m^3 for the closed-loop controlled system and 14.06 mole/m^3 for the system with a fixed input.

V. CONCLUDING REMARKS

In this paper we proposed an LPV control design method for ammonia-selective catalytic reduction (SCR) aftertreatment system. The designed output feedback controller used
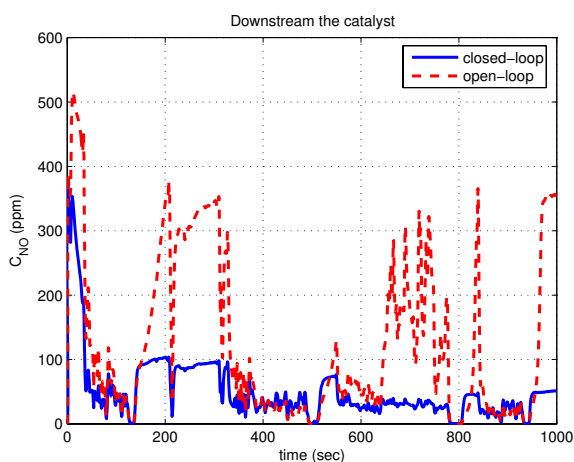
Fig. 6. NO concentration downstream the catalyst for open-loop and closed-loop controller
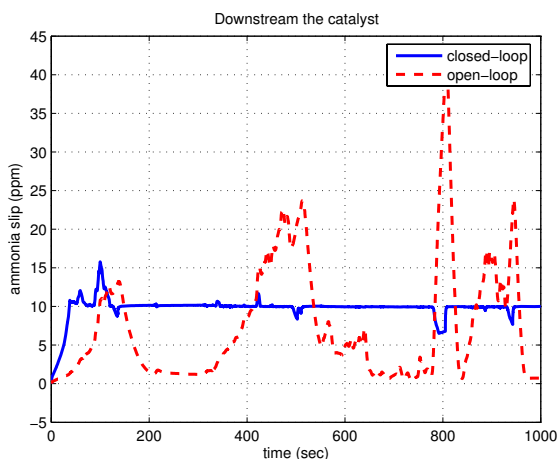

Fig. 7. NH₃ concentration downstream the catalyst for open-loop and closed-loop controller
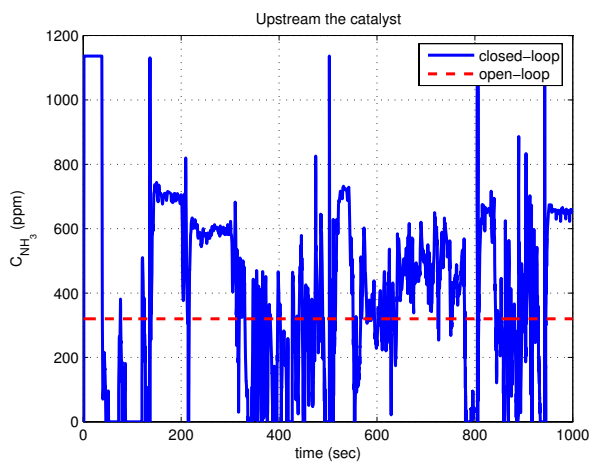

Fig. 8. Injected ammonia concentration (control input)

the concentration of NO upstream and downstream the catalyst. Eliminating the fast modes in the full-order LPV output feedback controller resulted in a simple first-order transfer function gain-scheduled as a function of the LPV parameter (the surface coverage fraction at steady-state). Inspired by the gain-scheduling control strategy, we also investigated the impact of exhaust temperature on the structure of the reduced-order controller as an additional LPV parameter. To deal with the complexity of the LPV model due to the large number of gain-scheduling parameters, we used PCA as a powerful data reduction method that works based on singular value decomposition of the collected data matrix. The use of PCA not only led to a lower number of LPV parameters, but also provided an affine model (in terms of the reduced LPV parameters) that was significantly easier to use in the control design process. The authors are currently implementing the proposed controllers on a lab-bench reactor to validate the effectiveness of the design.

VI. ACKNOWLEDGEMENT

REFERENCES

[1] I. T. Jolliffe, *Principal component analysis*, Springer series in statistics, second edition 2002.
[2] E. Tronconi and P. Forzatti, "Adequacy of lumped parameter models for SCR reactors with monolith structure," in *Proc. of AIche*, vol. 38, no. 2, 1992.
[3] P. Gahinet, P. Apkarian, and M. Chilali, "Affine Parameter-Dependent Lyapunov Functions and Real Parametric Uncertainty," *IEEE Trans. Automatic Control*, vol. 41, no. 3, pp. 436-442, 1996.
[4] P. Apkarian and R.J. Adams, "Advanced Gain Scheduling Techniques for Uncertain Systems," *IEEE Trans. Control Systems Technology*, vol. 6, no. 1, pp. 21-32, 1998.
[5] M. Koebel, M. Elsener, and M. Kleemann, "Urea-SCR: a promising technique to reduce NOx emissions from automotive diesel engines," *Catalysis Today 59*, pp. 335-345, 2000.
[6] W. J. Rugh and J.S. Shamma, "Research on gain scheduling," *Automatica*, 36, pp. 1401-1425, 2000.
[7] H. K. Khalil, *Nonlinear Systems*, Prentice Hall, Upper Saddle River, NJ, third edition, 2002.
[8] D. Upahhyay and M. Van Nieuwstadt, "Modeling of a urea SCR catalyst with automotive applications," in *Proc. of ASME IMECE*, New Orleans, Louisiana, 2002.
[9] J. N. Chi and H. F. M. Dacosta, "Modeling and control of a urea-SCR aftertreatment system," *SAE International*, 2005.
[10] D. Upahhyay and M. Van Nieuwstadt, "Model based analysis and control design of a urea-SCR deNOx aftertreatment system," *ASME Journal of Dynamic Systems , Measurement, and Control*, vol. 128, n. 3, pp. 737-741, 2006.
[11] C. M. Schar, C. H. Onder, and H. P. Geering, "Control of an SCR catalytic converter system for a mobile heavy-duty application," *IEEE Trans. Control Systems Technology*, vol. 14, no 4, pp. 641-653, 2006.
[12] M. Devarakonda, G. Parker, J. H. Johnson, V. Strots, and S. Santhanam, "Model-based estimation and control system development in a urea-SCR aftertreatment system," *SAE International*, 2008.
[13] A. Kwiatkowski and H. Werner, "PCA-based parameter set mappings for LPV models with fewer parameters and less overbounding," *IEEE Trans. Control Systems Technology*, vol. 16, no. 4, 2008.
[14] M. F. Hsieh and J. Wang, "Computationally-efficient nonlinear model predictive control of SCR ammonia surface coverage," in *Proc. of ASME DSCC*, Hollywood, CA, Oct. 2009.