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Adaptive Iterative Learning Control for Uncertain Delay Systems Based on Model Matching Technique

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***Abstract*— In this paper, an adaptive iterative learning control(ILC) scheme is proposed for trajectory tracking of uncertain delay systems based on model matching technique. The reference model is a delay system operating over in a finite time interval. An iterative model matching controller is designed and an iteration domain adaptive law is chosen to estimate the unknown parameters. It shows that the model matching technique can be applied in a straightforward method to ILC problem. A simulation example is included to illustrate the designed scheme.**

I. Introduction

In practical control problems, many tasks are repetitive and to track the given whole trajectory completely in a specified time interval, such as robotic manipulators, chemical batch processes, vehicles and man-machine systems. Iterative learning control is just a technique to control such systems so that a perfect tracking over a finite time-interval is achieved [1]–[3]. In the early works, the contraction mapping approach was mainly used to prove the convergence of the iterative process [4], [5]. But this method had some limitations, such as resetting conditions, global Lipschitz condition for systems. In the 1990s, combination of ILC with adaptive control was employed to control more complex systems [6]–[9], which was based on Lyapunov-like theory. In [6], it was shown that some standard Lyapunov adaptive design can be modified in a straightforward manner to ILC problems. And in [9], a unified adaptive iterative learning control framework was given for uncertain nonlinear systems. These results demonstrate the effectiveness of the adaptive ILC scheme.

In particular, researches have paid attention to ILC of time delay systems. For example, in [10], ILC with smith time delay compensator for batch process was investigated, sufficient conditions were given to guarantee the convergence of the tracking error. In [11], LaSalle-Razumikhin theorem and backstepping method were used to design an adaptive feedback controller for a class of nonlinear time-delay systems. For uncertain nonlinear systems with state delays, a PID iterative learning control algorithm was proposed and convergence conditions were given in [12]. However, few attempts have been made to the adaptive ILC of time delay systems.

The model matching technique is first proposed in the book [13], it is to determine a controller so that the closed-loop transfer function coincides exactly with the reference model transfer function. A model matching controller was designed for single-input single-output(SISO) delay systems in [14] and the result was extended to adaptive control scheme design in [15]. Comparing the definition of model matching and the tracking objective of ILC problems, we infer that the model matching technique can be used to solve ILC problems. Therefore, we apply this technique to ILC problem of delay systems in this paper. Iterative learning model matching controller is designed determinately for nominal systems first. Then iterative learning adaptive law is chosen to estimate the unknown parameters of the controller. To the best of our knowledge, it is the first result about ILC for delay systems based on model matching technique.

This paper is organized as follows. Section 2 is problem statement. In Section 3, the adaptive ILC scheme is designed. In section 4, a simulation example is given to illustrate the designed scheme. The last section is a conclusion of this paper.

II. Problem statement

Consider the time delay systems described by

$$y_k(s) = \frac{gr(s)}{p(s)} e^{-Ls} u_k(s) \tag{1}$$

where $y_k(s)$ and $u_k(s)$ are the output and the input of the system, respectively. g is gain, L is the known constant time delay, $r(s)$ and $p(s)$ are monic prime polynomials, their degrees are m and n, respectively. Denote $\partial[r(s)] = m, \partial[p(s)] = n$, they satisfy $0 \leq m \leq n-1$.

The reference model is in the form of

$$y_m(s) = t_m(s)\upsilon(s) \tag{2}$$

where $y_m(s)$ is output, $\upsilon(s)$ is a bounded reference input, and the transfer function is

$$t_m(s) = \frac{g_m r_m(s)}{p_m(s)} e^{-Ls} \tag{3}$$

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where g_m is gain, $r_m(s)$ and $p_m(s)$ are monic prime polynomials, $\partial[r_m(s)] = m_d, \partial[p_m(s)] = n_d$. Without loss of generality, $p_m(s)$ is a stable polynomial.

We assume that all the system parameters are unknown, the objective is to design an adaptive ILC scheme for plant (1) to track the whole trajectory (2) completely in the special time interval $[0, T]$. To this end, The following assumptions are needed.

Assumption 1: $r(s)$ is asymptotically stable.

Assumption 2: $n_d - m_d \geq n - m$.

Throughout this paper, we will use the $\mathcal{L}_{pe}$ norm defined as following

$$\|x(t)\|_{pe} \triangleq \begin{cases} \left(\int_0^t \|x(\tau)\|^p d\tau\right)^{\frac{1}{p}} & if \quad p \in [0, \infty) \\ \sup\limits_{0\leq\tau\leq t} \|x(\tau)\| & if \quad p = \infty \end{cases} \tag{4}$$

where $\|x\|$ denotes any norm of x, and t belongs to the finite interval $[0, T]$. We say that $x \in \mathcal{L}_{pe}$ when $\|x(t)\|_{pe}$ exists($i.e.$, when $\|x\|_{pe}$ is finite).

III. ADAPTIVE ILC DESIGN

For the convenience of controller design, we transform the reference model into a new form first. Introduce any monic stable polynomials $r^*(s), p^*(s)$, and $\partial[r^*(s)] = m$, $\partial[p^*(s)] = n$. Then the reference model can be rewritten as

$$y_m(s) = t^*(s)\bar{v}(s) \tag{5}$$

where

$$t^*(s) = \frac{g_m r^*(s)}{p^*(s)} e^{-Ls}, \bar{v}(s) = \frac{r_m(s)p^*(s)}{r^*(s)p_m(s)} v(s) \tag{6}$$

From Assumption 2, it is known that each $r_m(s)p^*(s)/r^*(s)p_m(s)$ is proper and stable. Consider $\bar{v}(s)$ as the new input dynamic. Then $t^*(s)$ becomes the transfer function of the new reference model (5). As a result of this transformation, the objective can be viewed as to design an ILC so that the transfer function of the closed-loop from $\bar{v}(s)$ to $y_k(s)$ coincides with $t^*(s)$ in the time interval $[0, T]$. In the following controller design procedure, we will consider the case where $p(s)$ and $r^*(s)$ have single or distinct roots.

By employing virtual precompensators

$$\frac{r^*(s)p(s) - gr(s)p^*(s)}{r^*(s)p(s)} = \sum_{k=1}^{m+n} \frac{\beta^k}{s - z^k} + 1 - g, \tag{7}$$

where z^k are roots of $r^*(s)$ for $k = 1, \cdots, m$ and roots of $p(s)$ for $k = m+1, \cdots, m+n$, respectively. Define the polynomial $\phi(s)$ satisfying the equations

$$\frac{r^*(s)p(s) - \phi(s)}{r^*(s)p(s)} = \sum_{k=1}^{m+n} \frac{\beta^k e^{Lz^k}}{s - z^k} + 1 - g \tag{8}$$

Choose any monic stable polynomial $\tau(s)$ with degree $n - m - 1$, define a polynomial equation by

$$k(s)p(s) + gh(s)r(s) = g\tau(s)r^*(s)p(s) - \tau(s)\phi(s) \tag{9}$$

where $k(s)$ and $h(s)$ are unknown polynomials. Obviously, the equation has solutions $k(s)$ and $h(s)$, and $\partial[k(s)] \leq n - 2$, $\partial[h(s)] \leq n - 1$.

Using equations (7) and (8), we can calculate the following integrator

$$\begin{aligned} &\int_{-L}^{0} \sum_{k=1}^{m+n} \beta^k e^{-\sigma z^k} u_k(s) e^{\sigma s} d\sigma \\ &= \sum_{k=1}^{m+n} \frac{\beta^k}{s - z^k} u_k(s) - \sum_{k=1}^{m+n} \frac{\beta^k e^{Lz^k}}{s - z^k} u_k(s) e^{-Ls} \\ &= \left\{ \sum_{k=1}^{m+n} \frac{\beta^k}{s - z^k} + 1 - g \right\} u_k(s) \\ &\quad - \left\{ \sum_{k=1}^{m+n} \frac{\beta^k e^{Lz^k}}{s - z^k} + 1 - g \right\} u_k(s) e^{-Ls} \\ &\quad -(1-g)u_k(s) + (1-g)u_k(s)e^{-Ls} \\ &= -\frac{gr(s)p^*(s)}{r^*(s)p(s)} u_k(s) + \frac{\phi(s)}{r^*(s)p(s)} u_k(s) e^{-Ls} \\ &\quad + gu_k(s) - gu_k(s)e^{-Ls} \end{aligned} \tag{10}$$

Multiplying polynomial equation (9) by $\frac{1}{\tau(s)r^*(s)p(s)}$, we obtain

$$\frac{\phi(s)}{r^*(s)p(s)} = -\frac{k(s)}{\tau(s)r^*(s)} - \frac{gh(s)r(s)}{\tau(s)r^*(s)p(s)} + g \tag{11}$$

Substituting (11) into (10), we have

$$\begin{aligned} &\int_{-L}^{0} \sum_{k=1}^{m+n} \beta^k e^{-\sigma z^k} u_k(s) e^{\sigma s} d\sigma \\ &= -\frac{gr(s)p^*(s)}{r^*(s)p(s)} u_k(s) - \frac{k(s)}{\tau(s)r^*(s)} u_k(s) e^{-Ls} \\ &\quad -\frac{gh(s)r(s)}{\tau(s)r^*(s)p(s)} u_k(s) e^{-Ls} + gu_k(s)e^{-Ls} \\ &\quad + gu_k(s) - gu_k(s)e^{-Ls} \\ &= -\frac{gr(s)p^*(s)}{r^*(s)p(s)} u_k(s) - \frac{k(s)}{\tau(s)r^*(s)} u_k(s) e^{-Ls} \\ &\quad -\frac{gh(s)r(s)}{\tau(s)r^*(s)p(s)} u_k(s) e^{-Ls} + gu_k(s) \end{aligned} \tag{12}$$

It is noticed that

$$\frac{gh(s)r(s)}{\tau(s)r^*(s)p(s)} u_k(s) e^{-Ls} = \frac{h(s)}{\tau(s)r^*(s)} y_k(s) \tag{13}$$

Therefore, (12) can be rewritten as

$$\begin{aligned} &\int_{-L}^{0} \sum_{k=1}^{m+n} \beta^k e^{-\sigma z^k} u_k(s) e^{\sigma s} d\sigma \\ &= -\frac{gr(s)p^*(s)}{r^*(s)p(s)} u_k(s) - \frac{k(s)}{\tau(s)r^*(s)} u_k(s) e^{-Ls} \\ &\quad -\frac{h(s)}{\tau(s)r^*(s)} y_k(s) + gu_k(s) \end{aligned} \tag{14}$$

Choose the ILC controller by

$$\begin{aligned} u_k = \frac{1}{g} \Bigg\{ &\frac{k(s)}{\tau(s)r^*(s)} u_k(s) e^{-Ls} + \frac{h(s)}{\tau(s)r^*(s)} y_k(s) \\ &+ \int_{-L}^{0} \sum_{k=1}^{m+n} \beta^k e^{-\sigma z^k} u_k(s) e^{\sigma s} d\sigma + g_m \bar{v}(s) \Bigg\} \end{aligned} \tag{15}$$

we can get

$$\frac{gr(s)p^*(s)}{r^*(s)p(s)}u_k(s) = g_m\bar{v}(s) \tag{16}$$

which implies that

$$\frac{gr(s)}{p(s)}u_k(s)e^{-Ls} = \frac{r^*(s)}{p^*(s)}g_m\bar{v}(s)e^{-Ls} \tag{17}$$

that is

$$y_k(s) = y_m(s) \tag{18}$$

Therefore, the controller (15) can guarantee the output tracking in the interval $[0, T]$ when the system (1) is known exactly. For systems with unknown parameters, the iterative adaptive law is needed to estimate the unknown parameters of the controller. Then the controller and the adaptive law consist the adaptive ILC scheme.

To design the adaptive ILC scheme, we should find the parametric representation of the controller (15) first. Because the degrees of polynomials $k(s)$ and $h(s)$ satisfy $\partial[k(s)] \leq n-2$, $\partial[h(s)] \leq n-1$, they can be written as

$$\begin{aligned} k(s) &= k_{n-2}s^{n-2} + k_{n-3}s^{n-3} + \cdots + k_0, \\ h(s) &= h_{n-1}s^{n-1} + h_{n-2}s^{n-2} + \cdots + h_0 \end{aligned} \tag{19}$$

Define the parameter signals by

$$\begin{aligned} \theta &= \frac{1}{g}\left[k_{n-2}, \cdots, k_0, h_{n-1}, h_0, g_m\right]^T, \\ \lambda(\sigma) &= \sum_{k=1}^{m+n} \beta^k e^{-\sigma z^k} \end{aligned} \tag{20}$$

and vector signal by

$$\begin{aligned} \omega_k(t) = \Bigg[&\frac{p^{n-2}}{\tau(p)r^*(p)}u_k(t-L), \cdots, \\ &\frac{1}{\tau(p)r^*(p)}u_k(t-L), \frac{p^{n-1}}{\tau(p)r^*(p)}y_k(t), \\ &\cdots, \frac{1}{\tau(p)r^*(p)}y_k(t), \bar{v}(t)\Bigg]^T \end{aligned} \tag{21}$$

Then the designed ILC controller (15) can be represented by

$$u_k(t) = \hat{\theta}_k^T(t)\omega(t) + \int_{-L}^{0}\hat{\lambda}_k(t,\sigma)u_k(t+\sigma)d\sigma \tag{22}$$

where $\hat{\theta}_k(t)$ and $\hat{\lambda}_k(t,\sigma)$ are the estimates of the real parameters θ and $\lambda(\sigma)$, respectively.

Define tracking error $e_k(t)$ by

$$e_k(t) = y_k(t) - y_m(t) \tag{23}$$

The following theorem gives the dynamic parametric representation of the tracking error.

Theorem 1. The tracking error can be represented by

$$\begin{aligned} e_k(t) = g&\frac{r^*(p)}{p^*(p)}q^{-L}\Big\{\tilde{\theta}_k(t)\omega_k(t-L) \\ &+ \int_{-L}^{0}\tilde{\lambda}_k(t,\sigma)u_k(t+\sigma)d\sigma\Big\} \end{aligned} \tag{24}$$

where q^{-L} is a delay operator, *i.e.*, $q^{-L}u_k(t) = u_k(t-L)$.
Proof. From (14), we have

$$\begin{aligned} &\frac{gr(p)p^*(p)}{r^*(p)p(p)}u_k(t) \\ &= gu_k(t) - \frac{k(p)}{\tau(p)r^*(p)}u_k(t-L) - \frac{h(p)}{\tau(p)r^*(p)}y_k(t) \\ &\quad - \int_{-L}^{0}\sum_{k=1}^{m+n}\beta^k e^{-\sigma z^k}u_k(t+\sigma)d\sigma \end{aligned} \tag{25}$$

In view of ILC controller (15) and its parametric representation (22), the above equation can be represented by

$$\begin{aligned} &\frac{p^*(p)}{r^*(p)}y_k(t+L) \\ &= g\Big\{\hat{\theta}_k^T(t)\omega_k(t) + \int_{-L}^{0}\hat{\lambda}_k(t,\sigma)u_k(t+\sigma)d\sigma\Big\} \\ &\quad -g\Big\{\theta_k^T(t)\omega_k(t) + \int_{-L}^{0}\lambda_k(t,\sigma)u_k(t+\sigma)d\sigma\Big\} \\ &\quad +g_m\bar{v}(t) \\ &= g\Big\{\tilde{\theta}_k^T(t)\omega_k(t) + \int_{-L}^{0}\tilde{\lambda}_k(t,\sigma)u_k(t+\sigma)d\sigma\Big\} + g_m\bar{v}(t) \end{aligned} \tag{26}$$

Because the polynomials $r^*(p)$ and $p^*(p)$ are stable, we have

$$\begin{aligned} &y_k(t) - g_m\frac{r^*(p)}{p^*(p)}q^{-L}\bar{v}(t) \\ &= \frac{gr^*(p)}{p^*(p)}q^{-L}\Big\{\tilde{\theta}_k^T(t)\omega_k(t) + \int_{-L}^{0}\tilde{\lambda}_k(t,\sigma)u_k(t+\sigma)d\sigma\Big\} \end{aligned} \tag{27}$$

It implies that the parametric representation of tracking error is (24). Thus, the proof is completed.

Because $\frac{gr^*(p)}{p^*(p)}q^{-L}$ is not passive, we need to use an augmented error to design adaptive law for unknown parameters. Define a signal by

$$\begin{aligned} \eta_k(t) = &\Big\{\tilde{\theta}_k^T(t)\bar{\omega}_k(t) + \int_{-L}^{0}\tilde{\lambda}_k(t,\sigma)\bar{u}_k(t+\sigma)d\sigma\Big\} \\ &-\frac{r^*(p)}{p^*(p)}q^{-L}\Big\{\tilde{\theta}_k^T(t)\omega_k(t) \\ &+\int_{-L}^{0}\tilde{\lambda}_k(t,\sigma)u_k(t+\sigma)d\sigma\Big\} \end{aligned} \tag{28}$$

where

$$\bar{\omega}_k(t) = \frac{r^*(p)}{p^*(p)}q^{-L}\omega_k(t), \bar{u}_k(t) = \frac{r^*(p)}{p^*(p)}q^{-L}u_k(t) \tag{29}$$

Define the augmented error by

$$\begin{aligned} \varepsilon_k(t) = &e_k(t) + \hat{g}_k(t)\eta_k(t) \\ = &g\Big\{\tilde{\theta}_k^T(t)\bar{\omega}_k(t) + \int_{-L}^{0}\tilde{\lambda}_k(t,\sigma)\bar{u}_k(t+\sigma)d\sigma\Big\} \\ &+\tilde{g}_k(t)\eta_k(t) \end{aligned} \tag{30}$$

where $\hat{g}_k(t)$ is the estimate of g, and $\tilde{g}_k(t) = \hat{g}_k(t) - g$.

Define the signal vector by

$$\Omega_k(t) = [\bar{\omega}_k(t), \sup_{-L\leq\sigma\leq 0}\bar{u}_k(t+\sigma), \eta_k(t)] \tag{31}$$

Choose the following adaptive law

$$\begin{aligned}\hat{g}_k(t) &= \hat{g}_{k-1}(t) - \tau_g \frac{\eta_k(t)}{1+\|\Omega_k(t)\|^2}\varepsilon_k(t),\\ \hat{\theta}_k(t) &= \hat{\theta}_{k-1}(t) - \tau_\theta \frac{\bar{\omega}_k(t)}{1+\|\Omega_k(t)\|^2}\varepsilon_k(t),\\ \hat{\lambda}_k(t,\sigma) &= \hat{\lambda}_{k-1}(t,\sigma) - \tau_\lambda \frac{\bar{u}_k(t+\sigma)}{1+\|\Omega_k(t)\|^2}\varepsilon_k(t)\end{aligned} \tag{32}$$

where $\tau_g, \tau_\theta, \tau_\lambda$ are some positive constants to be chosen.

The ILC adaptive control scheme consists of the the controller (22) and adaptive law (32). The adaptive law in the equation (32) is used to estimates the unknown parameters of the controller (22). The following theorem gives the main result of this paper.

Theorem 2. Consider the system (1), the controller (22) and adaptive law (32), we set $\hat{g}_{-1}(t) = c$, where c is a finite non-zero constant, $\hat{\theta}_{-1}(t) = 0, \hat{\lambda}_{-1}(t,\sigma) = 0$ and $\hat{g}_k(0) = \hat{g}_{k-1}(T), \hat{\theta}_k(0) = \hat{\theta}_{k-1}(T), \hat{\lambda}_k(0,\sigma) = \hat{\lambda}_{k-1}(T,\sigma)$, then we have $\tilde{g}_k(t), \tilde{\theta}_k(t), \tilde{\lambda}_k(t,\sigma), \omega_k(t), u_k(t) \in \mathcal{L}_{2e}$ for all $k \in \mathbb{Z}_+$ and $t \in [0,T]$, and $\lim_{k\to\infty} e_k(t) = 0$ for $\forall t \in [0,T]$.

Proof. Without loss of generality, g is assumed to be positive. Consider the following Lyapunov-like function

$$\begin{aligned}V_k(t) &= \frac{1}{2\tau_g}\int_0^t \tilde{g}_k^2(\tau)d\tau + \frac{g}{2\tau_\theta}\int_0^t \tilde{\theta}_k^T(\tau)\tilde{\theta}_k(\tau)d\tau\\ &+\frac{g}{2\tau_\lambda}\int_0^t\int_{-L}^0 \tilde{\lambda}_k^2(\tau,\sigma)d\sigma d\tau\end{aligned} \tag{33}$$

First, we show that the sequence V_k is non-increasing with respect to k. The difference of V_k is

$$\begin{aligned}\triangle V_k &= V_k - V_{k-1}\\ &= \frac{1}{2\tau_g}\int_0^t (\tilde{g}_k^2(\tau) - \tilde{g}_{k-1}^2(\tau))d\tau\\ &+\frac{g}{2\tau_\theta}\int_0^t (\tilde{\theta}_k^2(\tau) - \tilde{\theta}_{k-1}^2(\tau))d\tau\\ &+\frac{g}{2\tau_\lambda}\int_0^t\int_{-L}^0 (\tilde{\lambda}_k^2(\tau,\sigma) - \tilde{\lambda}_{k-1}^2(\tau,\sigma))d\sigma d\tau\\ &= -\frac{1}{2\tau_g}\int_0^t \bar{g}_k^2(\tau)d\tau + \frac{1}{\tau_g}\int_0^t \bar{g}_k(\tau)\tilde{g}_k(\tau)d\tau\\ &-\frac{g}{2\tau_\theta}\int_0^t \bar{\theta}_k^T(\tau)\bar{\theta}_k(\tau)d\tau + \frac{g}{\tau_\theta}\int_0^t \bar{\theta}_k^T(\tau)\tilde{\theta}_k(\tau)d\tau\\ &-\frac{g}{2\tau_\lambda}\int_0^t\int_{-L}^0 \bar{\lambda}_k^2(\tau,\sigma)d\sigma d\tau\\ &+\frac{g}{\tau_\lambda}\int_0^t\int_{-L}^0 \bar{\lambda}_k(\tau,\sigma)\tilde{\lambda}_k(\tau,\sigma)d\sigma d\tau\\ &\le \frac{1}{\tau_g}\int_0^t \bar{g}_k(\tau)\tilde{g}_k(\tau)d\tau + \frac{g}{\tau_\theta}\int_0^t \bar{\theta}_k^T(\tau)\tilde{\theta}_k(\tau)d\tau\\ &+\frac{g}{\tau_\lambda}\int_0^t\int_{-L}^0 \bar{\lambda}_k(\tau,\sigma)\tilde{\lambda}_k(\tau,\sigma)d\sigma d\tau\end{aligned} \tag{34}$$

where

$$\begin{aligned}&\bar{g}_k(t) = \tilde{g}_k(t) - \tilde{g}_{k-1}(t), \bar{\theta}_k(t) = \tilde{\theta}_k(t) - \tilde{\theta}_{k-1}(t),\\ &\bar{\lambda}_k(t) = \tilde{\lambda}_k(t,\sigma) - \tilde{\lambda}_{k-1}(t,\sigma)\end{aligned} \tag{35}$$

Noting the definition of the augmented error in the equation (30), it is known that substituting (32) into (34) yields

$$\begin{aligned}\triangle V_k &\le -\frac{1}{\tau_g}\int_0^t \tau_g \frac{\eta_k(\tau)\tilde{g}_k(\tau)}{1+\|\Omega_k(\tau)\|^2}\varepsilon_k(\tau)d\tau\\ &-\frac{g}{\tau_\theta}\int_0^t \tau_\theta \frac{\bar{\omega}_k^T(\tau)\tilde{\theta}_k(\tau)}{1+\|\Omega_k(\tau)\|^2}\varepsilon_k(\tau)d\tau\\ &-\frac{g}{\tau_\lambda}\int_0^t\int_{-L}^0 \tau_\lambda \frac{\bar{u}(\tau+\sigma)\tilde{\lambda}_k(\tau,\sigma)}{1+\|\Omega_k(\tau)\|^2}\varepsilon_k(\tau)d\sigma d\tau\\ &\le -\int_0^t \frac{1}{1+\|\Omega_k(\tau)\|^2}\varepsilon_k^2(\tau)d\tau\\ &\le 0\end{aligned} \tag{36}$$

Therefore, V_k is a non-increasing sequence for k. If $V_0(t)$ is bounded for $t \in [0,T]$, then V_k is bounded for all $k \in \mathbb{Z}_+$ and $\forall t \in [0,T]$. Now, we start to prove $V_0(t)$ is bounded for $t \in [0,T]$. From (33), it is known that

$$\begin{aligned}V_0(t) &= \frac{1}{2\tau_g}\int_0^t \tilde{g}_0^2(\tau)d\tau + \frac{g}{2\tau_\theta}\int_0^t \tilde{\theta}_0^T(\tau)\tilde{\theta}_0(\tau)d\tau\\ &+\frac{g}{2\tau_\lambda}\int_0^t\int_{-L}^0 \tilde{\lambda}_0^2(\tau,\sigma)d\sigma d\tau\end{aligned} \tag{37}$$

Its derivative is

$$\begin{aligned}\dot{V}_0(t) &= \frac{1}{2\tau_g}\tilde{g}_0^2(t) - \frac{1}{2\tau_g}\tilde{g}_0^2(0) + \frac{g}{2\tau_\theta}\tilde{\theta}_0^2(t) - \frac{g}{2\tau_\theta}\tilde{\theta}_0^2(0)\\ &+\frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_0^2(\tau,\sigma)d\sigma - \frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_0^2(0,\sigma)d\sigma\\ &= \frac{1}{2\tau_g}\tilde{g}_0^2(t) + \frac{g}{2\tau_\theta}\tilde{\theta}_0^2(t) + \frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_0^2(t,\sigma)d\sigma\\ &= \frac{1}{2\tau_g}\tilde{g}_0^2(t) - \frac{1}{2\tau_g}\tilde{g}_{-1}^2(t) + \frac{1}{2\tau_g}\tilde{g}_{-1}^2(t)\\ &+\frac{g}{2\tau_\theta}\tilde{\theta}_0^2(t) - \frac{g}{2\tau_\theta}\tilde{\theta}_{-1}^2(t) + \frac{g}{2\tau_\theta}\tilde{\theta}_{-1}^2(t)\\ &+\frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_0^2(\tau,\sigma)d\sigma - \frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_{-1}^2(\tau,\sigma)d\sigma\\ &+\frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_{-1}^2(\tau,\sigma)d\sigma\end{aligned} \tag{38}$$

From (36), we have

$$\begin{aligned}&\frac{1}{2\tau_g}\tilde{g}_0^2(t) - \frac{1}{2\tau_g}\tilde{g}_{-1}^2(t) + \frac{g}{2\tau_\theta}\tilde{\theta}_0^2(t) - \frac{g}{2\tau_\theta}\tilde{\theta}_{-1}^2(t)\\ &+\frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_0^2(\tau,\sigma)d\sigma - \frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_{-1}^2(\tau,\sigma)d\sigma \le 0\end{aligned} \tag{39}$$

Therefore, the equation (38) satisfies

$$\begin{aligned}\dot{V}_0(t) &\le \frac{1}{2\tau_g}\tilde{g}_{-1}^2(t) + \frac{g}{2\tau_\theta}\tilde{\theta}_{-1}^2(t) + \frac{g}{2\tau_\lambda}\int_{-L}^0 \tilde{\lambda}_{-1}^2(\tau,\sigma)d\sigma\\ &\le \frac{1}{2\tau_g}(\hat{g}_{-1}(t) - g)^2 + \frac{g}{2\tau_\theta}(\hat{\theta}_{-1}(t) - \theta)^2\\ &+\frac{g}{2\tau_\lambda}\int_{-L}^0 (\hat{\lambda}_{-1}(\tau,\sigma) - \lambda(\sigma))^2 d\sigma\end{aligned} \tag{40}$$

By choosing $\hat{g}_{-1}(t) = c$, where c is a finite non-zero constant, and $\hat{\theta}_{-1}(t) = 0, \hat{\lambda}_{-1}(t,\sigma) = 0$, we have

$$\dot{V}_0(t) \le \frac{1}{2\tau_g}(c-g)^2 + \frac{g}{2\tau_\theta}\theta^2 + \frac{g}{2\tau_\lambda}\int_{-L}^0 \lambda(\sigma)^2 d\sigma \tag{41}$$

It is noting that $(c-g)^2, g\theta^2, g\int_{-L}^{0}\lambda(\sigma)^2 d\sigma$ are bounded, it follows that $\dot{V}_0(t)$ is bounded for $t\in[0,T]$. Thus, $V_0(t)$ is uniformly continuous and bounded for $t\in[0,T]$. Then we can conclude that $V_k(t)$ is bounded for $k\in\mathbb{Z}_+$ and $t\in[0,T]$. Consequently, $\tilde{g}_k(t), \tilde{\theta}_k(t), \tilde{\lambda}_k(t,\sigma), u_k(t)\in\mathcal{L}_{2e}$ for all $k\in\mathbb{Z}_+$ and $t\in[0,T]$

From (34) and (36), we have

$$\begin{aligned} V_k &= V_0 + \sum_{j=1}^{k}(V_j - V_{j-1}) \\ &= V_0 + \sum_{j=1}^{k}\Delta V_j \\ &\le V_0 - \sum_{j=1}^{k}\int_0^t \frac{1}{1+\|\Omega_j(\tau)\|^2}\varepsilon_j^2(\tau)d\tau \end{aligned} \tag{42}$$

It follows that

$$\sum_{j=1}^{k}\int_0^t \frac{1}{1+\|\Omega_j(\tau)\|^2}\varepsilon_j^2(\tau)d\tau \le V_0 - V_k \tag{43}$$

Since V_k is a non-increasing sequence for k, then we have

$$\sum_{j=1}^{k}\int_0^t \frac{1}{1+\|\Omega_j(\tau)\|^2}\varepsilon_j^2(\tau)d\tau \le V_0 \tag{44}$$

V_0 is bounded so that we can obtain

$$\lim_{k\longrightarrow\infty}\frac{1}{1+\|\Omega_k(t)\|^2}\varepsilon_k^2(t) = 0 \tag{45}$$

If $\Omega_k(t)$ is uniformly bounded, then we can get $\lim_{k\longrightarrow\infty}\varepsilon_k(t)=0$. If $\Omega_k(t)$ is not uniformly bounded, then (45) implies that the increasing rate of $\varepsilon_k(t)$ is lower than $\Omega_k(t)$. However, from (30), we known that if $\lim_{k\longrightarrow\infty}\varepsilon_k(t)\neq 0$, then the increasing rate of $\varepsilon_k(t)$ is the same as that of $\Omega_k(t)$. This is a contradiction. Therefore, we get $\lim_{k\longrightarrow\infty}\varepsilon_k(t)=0$. From (24) and (30), we can obtain $\lim_{k\longrightarrow\infty}e_k(t)=0$. The proof is completed.

IV. Simulation Example

Consider the delay system

$$y_k(s) = g\frac{s+a}{s^2+bs+c}e^{-Ls}u_k(s) \tag{46}$$

where system parameters g, a, b, c are unknown constants, $L=2$ is the known time delay. The objective is to make y_k track the output of the reference model

$$y_m(s) = \frac{0.5}{s^2+7s+1}e^{-Ls}\upsilon(s) \tag{47}$$

over the time interval $[0,10]$, the reference input υ is $sin(t)$. When the control plant parameters are $g=2, a=1, b=3, c=1$, apply the designed adaptive ILC scheme which consists of the controller (15) and the iteration domain adaptive law (32) to the system (46), choose the stable polynomials

$$r^*(s) = s+0.5, \quad p^*(s) = s+5s+1, \quad \tau(s)=1 \tag{48}$$

and the parameters $\tau_g=0.5, \tau_\theta=0.6, \tau_\lambda=0.8$, simulation result is shown in Fig.1. It shows that the tracking error convergent to zero. The designed adaptive ILC scheme can achieve the object.

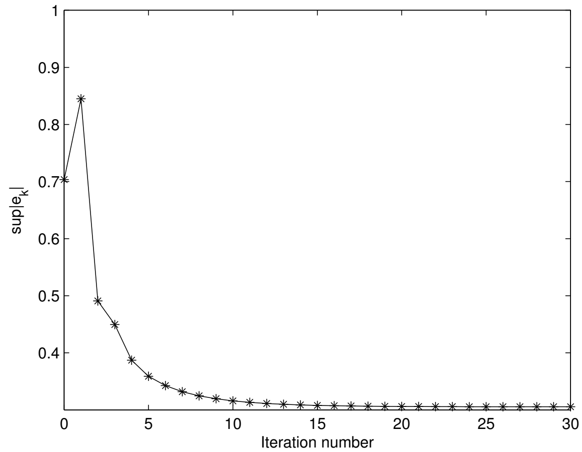

Fig. 1: Sup-norm of the tracking error versus the iteration number.

V. Conclusion

In this paper, an adaptive ILC scheme is proposed for a class of SISO time delay systems with unknown parameters. Model matching technique is used to obtain the controller structure and an iteration domain adaptive law is designed to estimate the unknown parameters of the controller. From this result, we concluded that the model matching technique can be applied in a straightforward method to ILC problem.

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