2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Robust Control of Predator-Prey-Hunter Systems

Juan C. Cockburn, Ken McLoud and Jeffrey Wagner

Abstract

We explore the use of robust control techniques to manage conflicting economic and ecological goals and constraints that naturally arise in many predator-prey-hunter systems. In particular, we focus on a general predator-prey-hunter system in which both the predator and prey may be harvested. Our goal is to design and test a controller that can maintain populations at prescribed target levels in the presence of uncertainty in the system dynamics and noisy measurements, while rejecting external disturbances. The dynamic equations of the model are based upon Brown et al. (2005, Marine Resource Economics), suitably modified for control systems design. Given this specification, we define a robust control problem and simulate its properties using the set of parameters for which performance of the closed-loop system is worst. One of the main advantages of using a robust control approach is that it provides deterministic tools for analysis of sensitivities and robustness to multiple uncertain parameters and affords the designer a framework to balance competing objectives. Simulations of the controlled system show promising results, with policy implications not only for managing existing predator-prey-hunter systems, but also for the planning of sustainable reintroduction efforts involving large, mammalian carnivores.

I. Introduction

The successful reintroduction of large, mammalian carnivores, and the sustainable management of the ecosystems they inhabit, depends upon a careful integration of several ecological, economic and social factors. Such carnivores (we focus upon wolves in this paper) provide use value in helping to maintain balance in predator-prey relationships within local ecosystems, and they provide non-use value to citizens who seek opportunities to view wolves or who derive utility from their mere existence. The carnivore's presence, however, lowers the population of recreationally harvested ungulates (such as elk) and they occasionally cause external damage (e.g., prey on livestock outside official reintroduction program boundaries). Active management of the predator-prey-hunter relationship following species reintroduction efforts is therefore of social value. However, as Eberhardt *et al.* (2003), Fieberg and Jenkins (2005) and others note, managers must proceed with an imperfect, evolving understanding of the ecosystem's structure and of economic valuations collected from stakeholders.

Both ecological and economic literatures speak to the challenges in modeling the changes in ecosystem structure that one may expect from the reintroduction of large mammalian carnivores under uncertainty. The ecological literature focuses primarily upon measuring the carnivore's ability, as a keystone species, to maintain stability in ecosystem dynamics, and therefore guard against trophic cascades. For instance, multiple studies (Ripple *et al.* (2001), Beschta (2003), Ripple and Beschta (2003), White *et al.* (2003), and White and Garrott (2005)) find that predator reintroduction in the US/Canadian Rocky Mountain West region altered ungulate behavior (feeding and migration behavior) in a manner that seems to have enhanced the sustainability of local ecosystems (e.g., by enabling saplings along riverbanks that were overgrazed by elk in the absence of wolves to instead mature, thereby providing shade and erosion control that reduces stress on other species). In the absence of large mammalian carnivores, ungulates must be regulated by costly professional culling and fertility control methods, and if outside national parks, by recreational hunting. Bradford and Hobbs (2008) simulate the efficacy of the first two strategies in controlling the elk population in Rocky Mountain National Park (where recreational hunting is not permitted); White and Garrott (2005, 150) discuss multiple studies of the impact of recreational hunting on elk populations, noting that there is disagreement in the literature regarding the magnitude of the impact. Eberhardt *et al.* (2003, 782) note that recreational elk hunting may turn out to be the driver of the elk-wolf system. Varley and Boyce (2005) present the standard ecological model of wolf reintroduction efforts, entitled WOLF 6, based upon adaptive management principles. Additional features of the WOLF 6 model are discussed below.

On the economic side of the literature, Tu and Wilman (1992), Ströble and Wacker (1995), Brown et al. (2005), and Hoekstra and van den Bergh (2005) present general economic predator-prey models that capture many important elements of the predator reintroduction dynamic under uncertainty. Tu and Wilman (1992) analyze the stability properties of a predator-prey equilibrium model (featuring wolves and elk) that takes into account self-limiting density effects, minimum viable population levels, and harvesting of both the predator and prey. They show how the stability of the equilibrium decreases if there is parametric uncertainty regarding the intrinsic growth rates for one or both species. While their paper does shed light on the stability properties of the species' population dynamics, those dynamics are

J.C. Cockburn is with the Dept. of Computer Engineering, RIT, Rochester, NY 14623, USA juan.cockburn@rit.edu

K. McLoud is with the Dept. of Mechanical Engineering, RIT, Rochester, NY 14623, USA kbm3899@rit.edu

J. Wagner is with the Dept. of Economics, RIT, Rochester, NY 14623, USA jeffrey.wagner@rit.edu

not modeled in their paper within a larger private or social welfare maximization framework. Ströble and Wacker (1995) present a model in which a manager chooses harvest rates for both the predator and prey in order to maximize social welfare. The flows of both species thus enter the maximization objective, but the stocks of the species do not. While they are able to solve for optimal rates of harvest, they note (p. 79) that stability analysis of their four-dimensional system of differential equations (governing population growth and harvest rates for each of the two species) does not readily yield interpretable results. Thus, they assume a steady-state predator population and analyze the stability of the system in which only the prey is harvested.

Brown et al. (2005) present a predator-prey model in which a manager seeks to maximize an infinite stream of profit from the harvest of two fish species in Lake Victoria: perch (predator) and dagaa (prey). As in the Ströble and Wacker (1995) model, Brown et al. enable harvesting of both species and potential existence or ecosystem value of the species do not enter the maximization routine. They derive equilibrium rates of harvest for each species; conduct a comparative static analysis of the steady-state predator-prey equilibrium (pp. 228-229); and conclude with some simulations of the model based upon the best available data. Hoekstra and van den Bergh (2005) build upon Tu and Wilman (1992) and others by substituting existence value of the predator for the harvest value of the predator. They present an optimal control model in which harvesting of the prey is controlled and harvesting of the predator is illegal in order to maximize a stream of social welfare that is therefore a function of the harvesting benefits from the prey and ecosystem benefits from the predator.

Rondeau (2001) and Dyar and Wagner (2003) also present models related to the uncertainties involved in species reintroduction programs; however, their models do not capture the predator-prey relationship. Rondeau (2001) presents an optimal control model in which a wildlife manager seeks to maximize a discounted stream of net benefits from a single species (white–tailed deer). The net benefits comprise three components: non-consumptive benefits, damage the species causes (e.g., automobile accidents in urban areas, in the case of white-tailed deer), and net benefits from harvesting the species. His model demonstrates the importance of planning for the future management of reintroduced species, on economic efficiency grounds, and he shows that while the model is estimated using data on white-tailed deer control, the lessons learned extend to the case of wolf reintroduction. Dyar and Wagner (2003) focus upon the uncertainty that the Department of Interior (DOI) faces over liability for harm caused by reintroduced large mammalian carnivores (such as wolves). They show that such concern can alter the types of management strategies that DOI takes in a manner that probably yields spillover ecological benefits (i.e., the theoretical model suggests that DOI should favor placing more land (buying easement rights) between wolves and ranchers and local communities in order to reduce harm over placing radio collars on wolves or erecting engineered barriers).

While all of the above ecological and economic models shed important light on the topic, all of the above models proceed under the assumption that we are either certain of the model's correctness (and that the model will not change over time) or that there is uncertainty only over a relatively small subset of parameters of the model (as opposed to uncertainty over more fundamental aspects of the model). Since Varley and Boyce (2005) and White and Garrot (2005), among others, show that different models of the wolf-elk-hunter relationship yield significantly different results, there is a need to consider more robust modeling alternatives. Norton and Reckhow (2008) advocate employing robust control methods in our particular wolf-elk-hunter context of interest, and there is relatively recent and growing interest in robust control theory in the economics literature (Janssen *et al.* (2004, 2007), Brock and Durlauf (2005), and Gonzalez (2008), for instance).

The purpose of our paper is to introduce the preliminary design and simulation of a controller for general predator-prey-hunter systems based on robust control theory. We therefore assume that socially optimal rates of predator (wolf) and prey (elk) populations and harvest rates have been determined using an economic model along the lines of Ströble and Wacker (1995) or Brown et al. (2005), and we focus upon the problem of keeping *actual* population rates "near" the *optimal* rates prescribed by such models.

This type of problem is analogous to keeping an airline flight close to its optimal path, given inherent uncertainties and stochasticities involved in the flight's *actual* journey. Another example comes from the process control industry where a "golden recipe" dictates the optimal sequence of operations required to obtain a product of a desired quality. The *actual* replication of the recipe is subject to stochasticities (accuracy of sensors, environmental changes, equipment aging) and scientific uncertainty regarding the theoretical properties of the product.

The dynamic model used in this work is a continuous time second order, parameter dependent, state space model inspired by the dynamic model of Brown et al. (2005). The two states are the size of the wolf population (predator) and the elk population (prey). While we could have used a more detailed model, for example WOLF6, the focus of our work is to highlight the potential that robust control offers to real-time management in face of uncertainty in both system dynamics and changes in the environment. All of the parameters in our model are assumed to be uncertain (in a deterministic way). It is worth noting that parametric uncertainty can be used to represent nonlinear phenomena at the expense of conservatism. These uncertainties allow the designer to accommodate for partially known dynamics and inaccurate population estimates. The controller is designed using a robust control approach that minimizes the effect of worst case exogenous disturbances on certain carefully chosen output error signals in the presence of uncertainty in the dynamic model (Doyle (1985)). The controller accepts population estimates as inputs and produces control signals prescribing the number of wolf and elk hunting tags to issue per time period[1]. Simulations using the preliminary controller show promising results. Population levels quickly converge to their target values for a wide range of initial conditions and parameter values. Populations also rapidly converge to target levels after deviations caused by seasonal disturbances such as harsh winters or birth seasons. In addition to demonstrating a way to handle significant model and parametric uncertainty in this particular wolf-elk-hunter, predator-prey relationship, our paper introduces the DK iteration method to the economics literature where various other types of problems may be fruitfully explored with this method.

II. The Predator-Prey-Hunter Model

The main objective of this paper is to explore the use of robust control techniques to manage conflicting economic goals and ecological constraints that naturally arise in many predator-prey-hunter systems. In particular, we will focus on a wolf-elk-hunter system in which both the predator and prey can be harvested. The standard approach (described in the previous section) in the environmental/ecological economics literature is one of optimal control, either under the assumption of model and parametric certainty or under the assumption that there is uncertainty over a relatively small subset of model parameters. Given significant model and parameter uncertainty in ecological, economic systems, however, we focus instead upon the problem of designing a mechanism that will maintain any particular population levels of interacting species sufficiently near their target levels. The target levels may not be "optimal" in the traditional, global sense we would like (and that we could derive if we were certain of the model and its dynamics). However, the robust control approach is based on the premise that some inefficiency of this type is tolerable if it leads to greater certainty that the selected population levels are sustainable.

The control system designed in this paper takes two measurements for feedback: the current wolf and elk populations. The controller then generates two control signals, corresponding to the rate at which wolf hunting tags (in tags per day) and elk hunting tags should be issued. The objective of the controller is to maintain the elk and wolf populations at prescribed target values in the presence of uncertainty in the system dynamics and noisy measurements while rejecting external disturbances, such as harsh winters and disease.

Several assumptions will be made to make the problem tractable. First, we assume that the elk and wolf populations can be sensed in "real time". In practice these measurements may come from estimates; so we are ignoring the dynamics of the estimators. However, we take into account the uncertainty in these estimates by introducing a noise signal in the feedback path. While many methods currently exist to obtain these estimates, the frequency at which these estimates are made would have to be increased for this controller to be applied in practice. Second, we assume that the elk-wolf-hunter system spans a large enough geographic area that the net migration of wolves and elk into and out of the region can be considered essentially zero. Most conventional conservation policies (such as hunting seasons and bag limits) are applied over wildlife management zones specifically chosen so that this assumption is not too restrictive. A third assumption lies in the realm of public policy. In order for this controller to be implemented, public officials will need to relinquish control of conservation policy to the controller. Easing this transition will be the fact that officials will still be able to choose the target elk and wolf populations. However, one of the more challenging aspects of implementing such a controller would be the break from traditional hunting practices such as hunting only in the fall season or hunting primarily males.

From a control systems point of view, a dynamic model of the elk-wolf-hunter system that captures the essential behavior that we want to control will suffice. This is in contrast with models used for forecasting populations. The dynamic equations of the predator-prey-hunter system used in this work are based on Brown et al. (2005), suitably modified for control systems design. The nonlinear dynamic equations are

$$\dot{x_1} = r_1 x_1 \left(1 - \frac{x_1}{q_1}\right) + \alpha\, x_1 x_2 - k_1 h_1(t) + d_1(t) \qquad (1)$$

$$\dot{x_2} = r_2 x_2 \left(1 - \frac{x_2}{q_2}\right) - \beta\, x_2 x_1 - k_2 h_2(t) + d_2(t) \qquad (2)$$

where x_1 is the size of the predator (wolf) population and x_2 is the size of the prey (elk) population. The inputs to the system are the rates of hunting tags for predator, h_1, and prey, h_2, respectively. The parameters r_1 and r_2 represent the intrinsic growth rate of the predator and prey, respectively; q_1 and q_2 are the carrying capacities of predator and prey. Alpha and beta are the interaction parameters. Our model also includes the factors k_1 and k_2 to model the rates of success of hunters, and introduced external disturbances d_1 and d_2 to represent unmodeled environmental effects such as harsh winters and disease.

All the parameters of the model are assumed uncertain and given in Table I. Nominal values and bounds on the parameters are chosen to reflect prior knowledge of the system.

Our main objective is to design a controller that maintains desired predator-prey populations, X_1 and X_2, in the presence of uncertainty in the model dynamics and the environment. Therefore, the nonlinear equations (1) and (2) will be linearized around a desired equilibrium (including states and inputs). At equilibrium it is assumed that there are no external disturbances, *i.e.*, $d_1 = d_2 = 0$. Under this

[1]Thus, in contrast with Hoekstra and van den Bergh's (2005) model that features recreational hunting of prey but not the predator, our model enables recreational hunting of both the prey and the predator. A particularly timely example of such an arrangement is in the state of Idaho, wherein controlled hunting of both elk and wolves is permitted (beginning in September 2009).

Description	Nominal	Range
growth rate (pred.), r_1	0.15	$[0.04, 0.30]$
growth rate (prey), r_2	0.20	$[0.1, 0.40]$
carrying capacity (pred.), q_1	1500	$[500, 2000]$
carrying capacity (prey), q_2	10000	$[6000, 12000]$
interaction coefficient, α	5E$-$7	[1E$-$6, 7E$-$7]
efficiency (pred.) , β	5E$-$5	[1E$-$5, 7E$-$5]
hunting success (pred.) , k_1	0.80	$[0.6, 1.0]$
hunting success (prey) , k_2	0.80	$[0.6, 1.0]$

TABLE I
MODEL PARAMETERS AND UNCERTAINTY RANGES

assumption the equilibrium harvesting rates are

$$H_1 = \frac{X_1}{k_1}\left(r_1 - \frac{r_1}{q_1}X_1 + \alpha X_2\right) \quad (3)$$

$$H_2 = \frac{X_2}{k_2}\left(r_2 - \frac{r_2}{q_2}X_2 - \beta X_1\right) \quad (4)$$

The state-space equations of the system linearized about the equilibrium $(X_1, X_2; H_1, H_2)$ are

$$\begin{aligned}
\delta\dot{x}_1 &= (r_1 - 2\frac{r_1}{q_1}X_1 + \alpha X_2)\delta x_1 + \alpha X_1\,\delta x_2 - k_1\delta h_1 + d_1 \\
\delta\dot{x}_2 &= -\beta X_2\,\delta x_1 + (r_2 - 2\frac{r_2}{q_2}X_2 - \beta X_1)\delta x_2 - k_1\delta h_1 + d_2 \\
\delta y_1 &= \delta x_1 + n_1 \\
\delta y_2 &= \delta x_2 + n_2
\end{aligned}$$

where δ denotes deviation from equilibrium. Note also that we added the noise signal n_1 and n_2 to model the uncertainty in sensing the populations. In the sequel we will let x_i, y_i and h_i denote both the original variables and the deviation variables. The meaning should be clear from the context.

III. THE ROBUST CONTROL PROBLEM

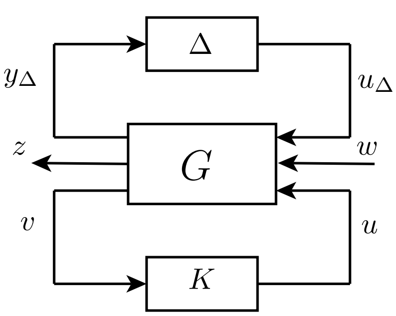

Fig. 1. The standard robust control problem

In this section we will give an overview of the robust control problem formulation and solution. Figure 1 describes the standard robust control problem setup. For more details see, *e.g.*, Skogestad and Postlethwaite (2007). The vector $\boldsymbol{w}$ represents all exogenous inputs to the system (*e.g.*, severity of the weather, behavior of species not specifically modeled and noise in the measurements) and $\boldsymbol{z}$ is a vector of all "error" signals (in our case, deviations of populations from their targets). G is a transfer function that represents a "generalized plant" that includes the system dynamics as well as models of the exogenous signals and performance weights used to penalize certain sensitivity functions, and Δ is an block diagonal operator that models the effect and location of uncertainties in the dynamic model. The objective is to design a controller K such that the closed loop system is stable for all possible perturbations Δ in a prescribed set $\boldsymbol{\Delta}$ and for all possible exogenous inputs $\boldsymbol{w}$ in a prescribed set, and minimizes a certain norm of the error $\boldsymbol{z}$.

If the signals $\boldsymbol{w}$ and $\boldsymbol{z}$ are assumed to be square integrable, *i.e.*, have finite energy, and all the operators are linear-time invariant, the problem becomes

$$\inf_{K,\Delta\in\boldsymbol{\Delta},||w||_2\leq 1} ||\boldsymbol{z}||_2 = \inf_{K,\widetilde{\Delta}\in\widetilde{\boldsymbol{\Delta}}}, \mu\left(T_{zw}(G,K)\right).$$

In the control literature this is called the μ-synthesis problem (Doyle (1995)); its solution requires the minimization of the *structured singular value* (called μ) of the transfer function from $\boldsymbol{w}$ to $\boldsymbol{z}$, T_{zw}. The controller will achieve robust performance if the $\hat{\mu} < 1$.

The computation of the structured singular value is in general intractable (NP hard); therefore, in practice an upper bound for the structured singular value, $\hat{\mu}$, is minimized, leading to the following problem

$$\inf_{K,D\in\widetilde{\boldsymbol{\Delta}}}, \hat{\mu}\left(T_{zw}(G,K)\right) = \inf_{K,D\in\widetilde{\boldsymbol{\Delta}}}, \left\|D\,T_{zw}(G,K)\,D^{-1}\right\|_\infty,$$

where D are scaling systems that commute with the structure of an augmented set of perturbations $\tilde{\Delta}$. Since the perturbation block has real and complex blocks this is a mixed μ problem. An approximate solution can be found using the DK iteration approach (or more precisely a $D-G-K$ iteration) where the variables D and K are fixed, alternatively, and a corresponding optimization problem in each variable is solved. The result is a linear time invariant robust controller K.

A critical step in the formulation of a robust control problem is the choice of dynamic weights or filters to scale signals, shape their spectra, and trade-off conflicting objectives. These weights must be chosen to be stable with a stable inverse. If the weights are not chosen properly the robust control problem may not have a solution. A block diagram showing where these dynamic weights are placed is shown in Figure 2.

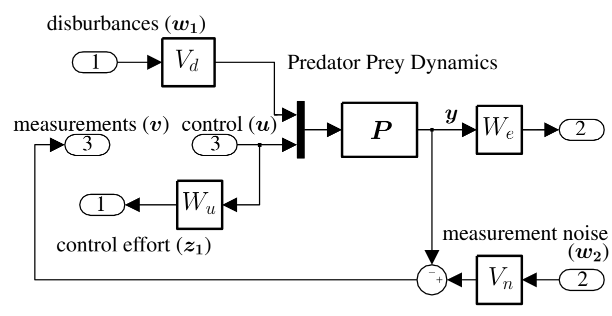

Fig. 2. Shaping Filters and Dynamic Weights for Robust Design

The filters V_d and V_n are used to scale and shape the spectrum of disturbances and noise. V_d is a biproper low pass filter with low frequency gain of 2, high frequency gain of 0.001 and crossover frequency of $0.5/\pi$ cycles/month. V_n is a biproper filter with low frequency gain of 0.1, high frequency gain of 10 and crossover frequency of $5/\pi$ cycles/month. This high pass filter models errors in the population estimates without significantly introducing a constant bias. The tracking error weight W_e is designed to control the allowable tracking error for the target population. It must be large at frequencies where the tracking error should be small. For tracking constant target populations, it was chosen as a biproper low pass filter with low frequency gain of 2, crossover of $0.05/\pi$ cycles/month and a high frequency gain of 0.001. The low frequency gain of this filter determines the achievable steady-state tracking error. Finally the control effort weight W_u is used to limit the bandwidth and aggressiveness of the control signal. It is a biproper low pass filter with low frequency gain of 0.001, crossover of $1/\pi$ cycles/month and a high frequency gain of 2.

IV. RESULTS

The problem was solved via DK iterations using MATLAB/Simulink with the *robust control toolbox*. The DK iterations yield a linear state-space controller of 26^{th} order. While it is possible to reduce the order of the controller, at the expense of loss of performance, we deemed it unnecessary since the controller we propose can easily run on a standard microcomputer. We also performed a robustness analysis of the closed loop system that includes finding the sensitivity of the closed loop system to changes in uncertain parameters and the set of parameters (*e.g.*, the block perturbation Δ) that causes the *worst* performance. (this can be done using **`wcgain`** in the *Robust Control Toolbox*.) The results are summarized in Table II

Description	Sensitivity	Worst Case
growth rate (pred.), r_1	3%	0.30
growth rate (prey), r_2	2%	0.10
carrying capacity (pred.), q_1	100%	2000
carrying capacity (prey), q_2	1%	10000
interaction coefficient, α	0%	7E−7
efficiency (pred.) , β	2%	7E−5
hunting success (pred.) , k_1	4%	0.6
hunting success (prey) , k_2	2%	0.6

TABLE II

CLOSED LOOP SENSITIVITIES AND WORST CASE PARAMETERS

Sensitivity analysis revealed that the controller rendered the closed loop mostly insensitive to all but one parameter, the carrying capacity of predators (q_1). Indeed, the sensitivity to changes in the carrying capacity of predators is 100%, *e.g.*, increasing q_1 by 25% will decrease the *stability margin* by the same amount. With this controller, the predator carrying capacity parameter can exert a disproportionately strong role in the closed loop performance, suggesting that the economic impact of research focused upon this parameter would be relatively great. It is important to note that this is a property of the controlled system. By choosing different filters and weights the designer can, in principle, adjust the closed loop sensitivities. Note also that these results are valid only for the particular operating point chosen.

To validate the controller, simulations of the closed loop system were conducted with the nonlinear dynamic equations and the worst-case parameters. To use the linear controller with the nonlinear plant, the measurement and control signals were properly biased. The measurement bias is just an offset to the desired target populations, while the control bias is an estimate of the hunting tags required for the equilibrium given by equations (3) and (4). Note that this would require exact knowledge of the parameters describing the dynamics of the predator-prey system. This is not realistic since our best a priori knowledge is given by the nominal parameters. Therefore, we used the nominal parameters to calculate this bias, that is, the harvesting rates of the predator and prey to achieve the desired target populations. In addition, saturation limits were added at the input of the plant to avoid “negative” harvesting rates, *e.g.*, releasing individuals into the wild. In practice a low threshold should be used to warn the manager of dangerously low stocks. The simulation results for a period of 5 years are shown in Figure 3. After a transient period, the controller was able to keep target populations close to the desired (reference) values of 5000 (for elk) and 250 (for wolves). Note there is a small but noticeable non-zero “steady-state” tracking error. This is partially due to the fact that equilibrium harvesting rates were calculated using the nominal values of the parameters, while the nonlinear dynamics of the predator-prey-hunter system are based upon the worst-case set of parameters. The steady-state error can be reduced to zero by changing the tracking error filter W_e to enforce integral action, if desired. Finally, it should be clear that the results obtained are only local. In practice several controller would be designed at different operating points and their robustness analyzed.

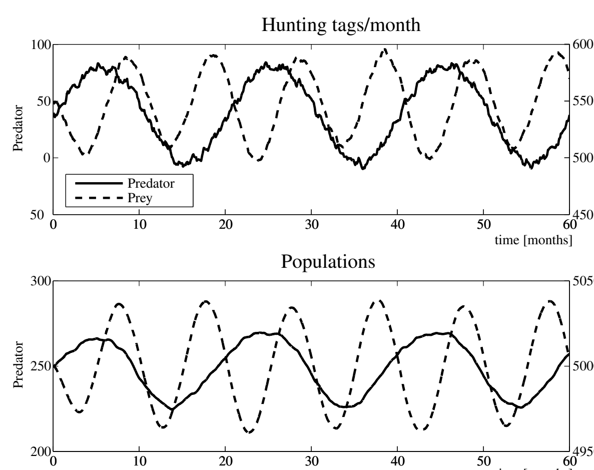

Fig. 3. Evolution of the controlled predator-prey-hunter system

V. CONCLUSION

Simulations suggest that the robust controller designed via DK iterations is able to maintain its performance (set-point tracking) in the face of a wide range of uncertainties and disturbances. While the target populations for the competing species may or may not be at globally optimal levels, the advantage of this robust control approach to controlling predator-prey-hunter systems is that it allows one to explicitly take into account uncertainty in the system dynamics and the environment. These results are promising but further research has to be done to make it practical. First, the model can be revised to include more states to distinguish cows from bulls, and calves from sexually mature adults. Second, the model could allow for a more realistic separation of "antlered" versus "antlerless" elk hunting tags. Third, the wolf component of the model could be improved to account for the number of packs. This is important because wolf social structure features only one breeding pair per pack; thus, the number of packs has a dramatic impact on the reproduction rate. While the objective of the controller presented in this paper is to minimize the effect of worst-case disturbances to the target populations of wolves and elk in the presence of uncertainty in a dynamic model, we anticipate generalizing this framework such that the controller minimizes the effect of worst-case disturbances to a social welfare function that accounts for both stock and flow benefits and costs of both predator (wolves) and prey (elk) management. Such benefits and costs include those from recreational hunting opportunities for both predator and prey.

Finally, it should be noted that the application of this control problem is not without ethical concerns. In natural ecosystems, populations are determined by a complex balance between the ecosystem's resources and the dynamics of the predator and prey species. We humans often view these systems as less than ideal. However, this balance drives the process of natural selection and certainly has complex, unmodeled interconnections with other systems. The application of this controller essentially removes the regulation of population levels from nature and delivers them to politicians. It should be noted, however, that politicians already regulate the size of game populations and have done so for many decades. Robust control strategies simply allow them to accomplish this goal in a more effective manner.

VI. REFERENCES

Beschta, R. L. (2003). Cottonwoods, elk, and wolves in the Lamar Valley of Yellowstone National Park. *Ecological Applications* 13(5): 1295-1309.

Bradford, John B.; Hobbs, N. Thompson (2008). Regulating overabundant ungulate populations: an example for elk in Rocky Mountain National Park, Colorado. *Journal of Environmental Management* 86(3): 520-528.

Brock, William A.; Durlauf, Steven N. (2005). Local robustness analysis: theory and applications. *Journal of Economic Dynamics & Control* 29(11): 2067-2092.

Brown, Gardner; Berger, Brett; Ikiara, Moses (2005). A predator-prey model with an application to Lake Victoria Fisheries. *Marine Resource Economics* 20(3): 221-247.

Doyle, J.C. (1985) Structured Uncertainty in Control Systems Design. *Proc. 24 IEEE Conference on Decision and Control*, December: 260-265.

Dyar, Julie A.; Wagner, Jeffrey (2003). Uncertainty and species recovery program design. *Journal of Environmental Economics and Management* 45(2S): 505-522.

Eberhardt, L. L.; Garrott, R. A.; Smith, D. W.; White, P. J.; Peterson, R. O. (2003). Assessing the impact of wolves on ungulate prey. *Ecological Applications* 13(3): 776-783.

Fieberg, John; Jenkins, Kurt J. (2005). Assessing uncertainty in ecological systems using global sensitivity analysis: a case example of simulated wolf reintroduction effects on elk. *Ecological Modelling* 187(2-3): 259-280.

Gonzalez, Fidel (2008). Precautionary principle and robustness for a stock pollutant with multiplicative risk. *Environmental and Resource Economics* 41(1): 25-47.

Hoekstra, Jeljer; van den Bergh, Jeroen C. J. M. (2005). Harvesting and conservation in a predator-prey system. *Journal of Economic Dynamics & Control* 29(6): 1097-1120.

Janssen, Marco A.; Anderies, John M.; Walker, Brian M. (2004). Robust strategies for managing rangelands with multiple stable attractors. *Journal of Environmental Economics and Management* 47(1): 140-162.

Janssen, Marco A.; Anderies, John M. (2007). Robustness trade-offs in social-ecological systems. *International Journal of the Commons* 1(1): 43-65.

Norton, J.; Reckhow, K. (2008). Modeling and monitoring environmental outcomes in adaptive management. In *Environmental Modeling, Software and Decision Support*, edited by Anthony J. Jakeman, Alexey A. Voinov, Andrea E. Rizzoli, and Serena H. Chen, Elsevier, pp. 181-202.

Ripple, William J.; Beschta, Robert L. (2003). Wolf reintroduction, predation risk and cottonwood recovery in Yellowstone National Park. *Forest Ecology and Management* 184(1-3): 299-313.

Ripple, William J.; Larsen, Eric J.; Renkin, Roy A.; Smith, Douglas W. (2001). Trophic cascades among wolves, elk and aspen on Yellowstone National Park's northern range. *Biological Conservation* 102(3): 227-234.

Rondeau, Daniel (2001). On the way back from the brink. *Journal of Environmental Economics and Management* 42(2): 156-182.

Skogestad, Sigurd; Postlethwaite, Ian (2007). Multivariable Feedback Control, 2nd Edition, John Wiley and Sons.

Ströble, Wolfgang J.; Wacker, Holger (1995). The economics of harvesting predator-prey systems. *Journal of Economics* 61(1): 65-81.

Tu, Pierre N. V.; Wilman, Elizabeth A. (1992). A generalized predator-prey model: uncertainty and management. *Journal of Environmental Economics and Management* 23(2): 123-138.

Varley, N.; Boyce, M. S. (2005). Adaptive management for reintroductions: updating a wolf recovery model for Yellowstone National Park. *Ecological Modelling* 193(3-4): 315-339.

White, C.; Feller, M. C.; Bayley, S. (2003). Predation risk and the functional response of elk-aspen herbivory. *Forest Ecology and Management* 181(1-2): 77-97.

White, P. J.; Garrott, R. A. (2005). Yellowstone's ungulates after wolves—expectations, realizations, and predictions. *Biological Conservation* 125(2): 141-152.