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A Discrete-Time Parameter Estimation Based Adaptive Actuator Failure Compensation Control Scheme

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Abstract— **This paper studies discrete-time adaptive failure compensation control of systems with uncertain actuator failures, using an indirect adaptive control method. A discrete-time model of a continuous-time linear system with actuator failures is derived and its key features are clarified. A new discrete-time adaptive actuator failure compensation control scheme is developed, which consists of a total parametrization of the system with parameter and failure uncertainties, a stable adaptive parameter estimation algorithm, and an on-line design procedure for feedback control. This work represents a new design of direct adaptive compensation of uncertain actuator failures, using an indirect adaptive control method. Such an adaptive design ensures desired closed-loop system stability and asymptotic tracking properties despite uncertain actuator failures. Simulation results are presented to show the desired adaptive actuator failure compensation performance.**

Keywords: Actuator failure, indirect adaptive control, discrete-time design, parameter estimation, tracking.

I. Introduction

Faults such as actuator failures are undesirable for control system performance. It may bring loss of performance and even cause catastrophic accidents. Research on fault-tolerant control (FTC) systems has recently received extensive attention. A fault-tolerant control system is capable of automatically compensating for the effects of faults and maintaining the control system performance at some desired level. A recent survey paper [1] provides an extensive review of reconfigurable FTC schemes. Different failure compensation designs have been developed including multiple-model [2], probabilistic [3], sliding-mode [4], detection and tolerant designs [5], adaptive designs [6]. Such specifically designed control systems have not only robust stability but also tracking performance guarantees.

To handle uncertain failure time, patterns and values, adaptive control designs, which are effective for controlling systems with unknown parameters, are widely used to build failure compensation control schemes [7], [8], [9], [10], [11]. In [9], the disturbance attenuation performance of an adaptive FTC system is addressed. In [10], the robust FTC problem for a class of singular systems subject to both time-varying state-dependent nonlinear perturbation and actuator fault is investigated. In [11], a passive FTC design approach is developed for a class of linear discrete-time systems with matching and unmatching uncertainties based on the state feedback, which uses analytical redundancy and does not need duplicated actuators.

In this paper we address the fault tolerant control problem, using a new method: indirect adaptive control based direct adaptive actuator failure compensation, as compared with our recent work [6] which uses a direct adaptive control method. Such an adaptive actuator failure compensation control scheme consists of a total parametrization of the system with parameter and failure uncertainties, a stable adaptive parameter estimation algorithm, and an online design procedure for feedback control. Such a fault tolerant control method is developed in a discrete-time system framework. We will first derive the discrete-time model of a continuous-time system with actuator failures, and then develop a discrete-time adaptive actuator failure compensation control scheme which can be applied to the continuous-time system. The development of adaptive control scheme is based on adaptive estimation of the unknown system parameters and uncertain actuator failure. We will show how to develop an adaptive failure control law using adaptive estimates of plant and failure parameters, to maintain closed-loop signal boundedness and achieve asymptotic output tracking when both the system and failure parameters and patterns are unknown.

This paper is organized as follows. In Section II, we formulate the control problem. In Section III, we derive the discrete-time model of a continuous-time system with actuator failures and clarify some key feature of such a discrete-time model. In Section IV, we design an indirect adaptive control based direct adaptive actuator failure compensation scheme, by estimating the parameters of system and failures and calculating the controller parameters on line. In Section V, we present simulation results to verify the desired adaptive failure compensation performance.

II. Problem Statement

In this section, we formulate the control problem: discrete-time adaptive control of systems with uncertain actuator failures. We first present the continuous-time system model with actuator failures, and then derive its discrete-time version for which the control problem is formulated, with several key technical issues to be addressed.

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A. System with Actuator failures

To formulate the actuator failure compensation problem, consider a linear time-invariant system with actuator failures:

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Cx(t) \tag{1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{1\times n}$ are unknown constant parameter matrices, the state vector $x(t) \in R^n$ is not available for measurement, $y(t) \in R$ is the system output, and $u(t) = [u_1(t), \ldots, u_m(t)]^T \in R^m$ is the system input whose components may fail during system operation. When the system is in failure-free operation, the input signal $u(t)$ is equal to a designed control signal $v(t)$. When an actuator fails, the corresponding component of $v(t)$ cannot reach the corresponding component of the input signal $u(t)$.

One type of practical actuator failures [6] are modeled as

$$u_j(t) = \bar{u}_j(t) = \bar{u}_{j0} + \sum_{l=1}^{n_j} \bar{u}_{jl} f_{jl}(t),\ t \geq t_j \tag{2}$$

where $j \in \{1, 2, \cdots, m\}$, t_j is the unknown failure time instant, $\bar{u}_{j0}$ and $\bar{u}_{jl}$ are some unknown constants, $f_{jl}(t)$, $l = 1, \ldots, n_j$ are known bounded signals, and $n_j \geq 1$.

The input signal $u_i(t)$ can then be described by

$$u_i(t) = v_i(t) + \sigma_i(t)(\bar{u}_i(t) - v_i(t)),\ i = 1, 2, \cdots, m, \tag{3}$$

where $\sigma_i = 1$ if the ith actuator has failed, i.e., $u_i(t) = \bar{u}_i(t)$, since $t_i \leq t$ and $\sigma_i = 0$ otherwise, and $v_i(t)$ are components of $v(t)$. The key feature of our control problem is that it is unknown in the design of the control signal $v(t)$ that when, how much and which actuators fail.

B. Control Problem

Our goal is to design a discrete-time adaptive control scheme for the continuous-time system (1) with uncertain actuator failures (2), that is, to generate the applied input signal $v(t)$ in the piecewise constant form:

$$v(t) = v(kT),\ t \in [kT, (k+1)T), \tag{4}$$

$T > 0,\ k = 0, 1, 2, \ldots$, subject to the actuator failures (3).

The control objective is to design the discrete-time signal $v(kT)$ to ensure that, despite the presence of uncertain actuator failures characterized by all possible σ belonging to a set Σ of interest, the closed-loop system signals are bounded and the tracking error $y(kT) - y_m(kT)$ converges to a small residual set, for a given reference output signal $y_m(kT)$ from a reference model system

$$y_m(kT) = W_m(z)[r](kT), \tag{5}$$

where $W_m(z)$ is a stable transfer function and $r(kT)$ is a bounded signal.

Some related technical issues include

- Derivation of a discrete-time system model
- Analysis of a discrete-time model with failures
- Design of an adaptive failure compensation scheme
- Evaluation of adaptive system performance.

A discrete-time model of the system with actuator failures is crucial for adaptive failure compensation design. Such a model has some unique features which are yet to be derived. For example, the effect of actuator failures on model discretization needs to be specified, because the sampling can only be done on the applied control input signal $v(t)$ but not on the actuator failure signals $\bar{u}_j(t)$ whose effect on a discrete-time model should be taken into account when a control scheme is designed.

III. Development of Discrete-Time System Model

Modern control systems implement their control laws using digital computers which calculate desired control signals in digital form. Digital control systems have a number of advantages, such as easy to build, flexible to change, and less sensitive to noise and environmental variations. It is desirable to develop adaptive failure compensation schemes in discrete time, based on discrete-time models of continuous-time systems, for digital control implementation.

Then we derive the discrete-time version of system (1) with actuator failure (2). Letting $t = (k+1)T$ and $t_0 = kT$, defining the fictitious vector signal $\hat{u}(t) = \bar{u}(kT)$ and the fictitious matrix signal $\hat{\sigma}(t) = \sigma(kT)$, $t \in [kT, (k+1)T)$, and introducing the error signals $\tilde{u}(t) = \bar{u}(t) - \hat{u}(t)$, $\tilde{\sigma}(t) = \sigma(t) - \hat{\sigma}(t)$, and the residual signals

$$\delta_\sigma = -\int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} B\tilde{\sigma}(\tau)v(\tau)d\tau \tag{6}$$

$$\delta_{\bar{u}} = \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} B(\sigma(\tau)\bar{u}(\tau) - \hat{\sigma}(\tau)\hat{u}(\tau))d\tau, \tag{7}$$

we express the solution of (1) as

$$\begin{aligned} x((k+1)T) &= \delta_\sigma(kT) + \delta_{\bar{u}}(kT) + e^{AT}x(kT) \\ &+ \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} Bd\tau(I_m - \sigma(kT))v(kT) \\ &+ \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} Bd\tau\sigma(kT)\bar{u}(kT). \end{aligned} \tag{8}$$

So we have the following discretization result:

Proposition 1: *The discrete-time version of the system (1) with actuator failures (2) is*

$$\begin{aligned} x((k+1)T) &= A_d x(kT) + B_d(I_m - \sigma(kT))v(kT) \\ &\quad + B_d\sigma(kT)\bar{u}(kT) + \delta_\sigma(kT) + \delta_{\bar{u}}(kT) \\ y(kT) &= Cx(kT) \end{aligned} \tag{9}$$

with a finite number impulse signal $\delta_\sigma(kT)$ and an error signal $\delta_{\bar{u}}(kT)$ which approaches to 0 as T approaches 0, where $A_d = e^{AT}$ and $B_d = \int_0^T e^{A\tau}Bd\tau$.

Remark 1: From this derivation, we see that the discretized dynamic model of a continuous-time system with actuator failures consists of a regular controlled part (A_d, $B_d(I_m - \sigma(kT))$), a failure-related part $B_d\sigma(kT)\bar{u}(kT)$, and some modeling error terms $\delta_\sigma(kT)$ (which is nonzero only for some finite number of time instants k, less than the number of time instants at which actuator failures occur) and $\delta_{\bar{u}}(kT)$ (which is bounded and becomes smaller when the sampling time interval T becomes smaller). The parametrizable uncertainties are from $(A_d, B_d(I_m - \sigma(kT)))$ and $B_d\sigma(kT)\bar{u}(kT)$ for $\bar{u}(t)$ given in (2), which can be handled by an adaptive design (which in this paper is an indirect adaptive control based failure compensation design), and the essential unparametrizable part is $\delta_{\bar{u}}(kT)$ which can be handled by a robust adaptive control law. This motivates our discrete-time adaptive actuator failure compensation design to be developed in the rest of this paper. □

For discrete-time control of a continuous-time system with actuator failures, we will base our control design and analysis on the system model (9), that is, we will synthesize the signal $v(kT)$ to achieve the desired control objective, despite unknown plant parameter and unknown failures.

To proceed, we first express the system (9) in the z-domain. Since $\sigma(kT)$ is a piecewise constant matrix (it changes values only when a new failure occurs), we consider a piecewise z-transform of the system (9) with $\sigma(kT) = \sigma$ (as a piecewise constant matrix) and $x(0) = 0$ as

$$\begin{aligned} y(z) \;&=\; C(zI_n - A_d)^{-1}B_d((I_m - \sigma)v(z) + \sigma\bar{u}(z)) \\ &+\; C(zI_n - A_d)^{-1}(\delta_\sigma(z) + \delta_{\bar{u}}(z)). \end{aligned} \qquad (10)$$

Letting $G(z) = C(zI_n - A_d)^{-1}B_d$ and $G_d(z) = C(zI_n - A_d)^{-1}$, we simplify the presentation of (10) as

$$\begin{aligned} y(z) \;&=\; G(z)((I_m - \sigma)v(z) + \sigma\bar{u}(z)) \\ &+\; G_d(z)(\delta_\sigma(z) + \delta_{\bar{u}}(z)). \end{aligned} \qquad (11)$$

The transfer function matrix $G(z)$ has the form

$$G(z) = \frac{Z(z)}{P(z)} = \frac{\left[\begin{array}{cccc} Z_1(z) & Z_2(z) & \cdots & Z_m(z) \end{array}\right]}{P(z)}, \quad (12)$$

where $P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0$, $Z_i(z) = b_{i\,n-1}z^{n-1} + \cdots + b_{i1}z + b_{i0}$, and $G_d(z)$ is the transfer function from the modeling error $\delta_\sigma(z) + \delta_{\bar{u}}(z)$, which can be expressed as $G_d(z) = Z_d(z)/P(z)$, for some polynomial vector $Z_d(z)$.

For the rest of the paper, to simplify the notation, we will drop "T" in the discrete time variable "kT", to only use "k". We also use $G(z)[u](k)$ or $P(z)[y](k)$ to represent the output of $G(z)$ or $P(z)$ as an operator with input u or y.

IV. Adaptive Control Design

In this section, we design an indirect adaptive control scheme by estimating the parameter of the system (11), in order that all closed-loop signals are bounded and the plant output $y(k)$ tracks a given reference output $y_m(k)$ as close as possible. We first express the parametrized model of the system (11), then develop a robust adaptive algorithm to update estimates of the parameters , obtain the structure and parameters of the controller.

To handle some additional issues caused by the redundant actuators $u_i(k)$, $i = 1, 2, \cdots, m$ (for example, if they are not properly managed, two actuators may be against to each other if being letting free), a proportional actuation scheme

$$v_i(k) = \alpha_i v_0(k),\ \alpha_i > 0,\ i = 2, 3, \cdots, m, \qquad (13)$$

is an appropriate choice for actuators with similar physical characteristics, where $v_0(k)$ is a feedback control signal to be designed. We will design the adaptive actuator failure compensation control scheme for v_0, based on indirect model reference adaptive control. To design such a scheme, we first make the following assumptions:

(A.1): For all failure patterns σ in the failure pattern set Σ under consideration (which are characterized by some $j = j_1, j_2, \ldots, j_p$, with $\{j_1, j_2, \ldots, j_p\} \subset \{1, 2, \ldots, m\}$ and $p = 0, 1, \ldots, q < m$ for some $q > 0$, such that the jth actuator fails: $u_j(k) = \bar{u}_j(k)$), the polynomial $Z_a(z) = \sum_{j\neq j_1,j_2,\ldots,j_p} \alpha_j Z_j(z) = b_{n-n^*}z^{n-n^*} + \cdots + b_1z + b_0$ has its degree equal to $n - n^*$ for some $n^* > 0$, its leading coefficient is always positive (or always negative), and its zeros are all stable, that is, all its zeros are in $|z| < 1$. For all $j = j_1, j_2, \ldots, j_p$, $Z_j(z)$ have their degree equal to $n - n^*$.

Then, we define a set $\{k_1, k_2, \ldots, k_q\} \subset \{1, 2, \ldots, m\}$ such that for all possible failure indices $j_1, j_2, \ldots, j_p$, $p = 0, 1, \ldots, q$, we have $\{j_1, j_2, \ldots, j_p\} \subset \{k_1, k_2, \ldots, k_q\}$. We also assume

(A.2): The set $\{k_1, k_2, \ldots, k_q\}$ is known.

A. System Parametrization

The first key issue in adaptive parameter estimation is parametrization of the system model. We will develop the parametrized model of the system (11) in this subsection. We rewrite the system (11) as

$$\begin{aligned} &\;(z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0)[y](k) \\ =&\;(b_{n-n^*}z^{n-n^*} + \cdots + b_1z + b_0)[v_0](k) \\ +&\sum_{i=k_1,k_2,\ldots,k_q}(b_{i\,n-n^*}z^{n-n^*} + \cdots + b_{i1}z + b_{i0})[\bar{u}_i](k) \\ +&\;Z_d(z)[\delta_\sigma + \delta_{\bar{u}}](k). \end{aligned} \qquad (14)$$

For the failure-related part, we have

$$\begin{aligned} &Z_i(z)[\bar{u}_i](k) = (b_{i\,n-n^*}z^{n-n^*} + \cdots + b_{i0})[\bar{u}_i](k) \\ =&\;\theta_{bi}^{*T}b(z)(\bar{\theta}_{i0}^* + \bar{\theta}_i^{*T}f_i(k)) = \bar{\theta}_{bi0}^* + \bar{\theta}_{bi}^{*T}B_i(z)[f_i](k), \end{aligned}$$

where $\theta_{bi}^* = [b_{i0}, b_{i1}, \ldots, b_{i\,n-n^*}]^T \in R^{n-n^*+1}$, $b(z) = [1, z, \ldots, z^{n-n^*}]^T$, $\bar{\theta}_{i0}^* = \bar{u}_{i0}$, $\bar{\theta}_i^* = [\bar{u}_{i1}, \ldots, \bar{u}_{i\,n_i}]^T \in R^{n_i}$, $f_i(k) = [f_{i1}(k), f_{i2}(k), \ldots, f_{in_i}(k)]^T \in R^{n_i}$, $\bar{\theta}_{bi0}^* = (b_{in-n^*} + \cdots + b_{i0})\bar{\theta}_{i0}^*$, $\bar{\theta}_{bi}^* = \theta_{bi}^* \otimes \bar{\theta}_i^* \in R^{(n-n^*+1)n_i}$ with $\otimes$ being the Kronecker product, and $B_i(z) = [I_{n_i}, zI_{n_i}, z^2I_{n_i}, \ldots, z^{n-n^*}I_{n_i}]^T$.

Then, we choose a monic stable polynomial $\Lambda_e(z) = z^n + \lambda_{en-1}z^{n-1} + \cdots + \lambda_{e1}z + \lambda_{e0}$ and operate both sides of (14) by $\frac{1}{\Lambda_e(z)}$. To parametrize the failure-free parts, we introduce the parameter vectors

$$\theta_a^* = [\lambda_{e0} - a_0, \lambda_{e1} - a_1, \ldots, \lambda_{en-1} - a_{n-1}]^T \in R^n, \quad (15)$$

$$\theta_b^* = [b_0, b_1, \ldots, b_{n-n^*}]^T \in R^{n-n^*+1}, \tag{16}$$

the vector $\bar{a}(z) = [1, z, z^2, \ldots, z^{n-1}]^T$, and the term $\delta(k) = \frac{Z_d(z)}{\Lambda_e(z)}[\delta_\sigma + \delta_{\bar{u}}](k)$, where $\delta_\sigma(k)$ and $\delta_{\bar{u}}(k)$ are bounded, $\Lambda_e(z)$ is stable, so $\delta(k)$ is bounded. The system (14) can be expressed as

$$\begin{aligned} y(k) \ &= \ \theta_a^{*T}\phi_a(k) + \theta_b^{*T}\phi_b(k) \\ &+ \sum_{i=k_1,k_2,\ldots,k_q} (\bar{\theta}_{bi0}^* + \bar{\theta}_{bi}^{*T}\phi_{fi}(k)) + \delta(k), \end{aligned} \tag{17}$$

where $\phi_a(k) = \frac{\bar{a}(z)}{\Lambda_e(z)}[y](k)$, $\phi_b(k) = \frac{b(z)}{\Lambda_e(z)}[v_0](k)$, $\phi_{fi}(k) = \frac{B_i(z)}{\Lambda_e(z)}[f_i](k)$. Introducing the overall parameter vector

$$\theta^* = [\theta_a^{*T}, \theta_b^{*T}, \bar{\theta}_{bk_10}^*, \bar{\theta}_{bk_1}^{*T}, \ldots, \bar{\theta}_{bk_q0}^*, \bar{\theta}_{bk_q}^{*T}]^T \tag{18}$$

and the associated regressor vector

$$\phi(k) = [\phi_a^T(k), \phi_b^T(k), 1, \phi_{fk_1}^T(k), \ldots, 1, \phi_{fk_q}^T(k)]^T,$$

the parametric model of (11) can be express as

$$y(k) = \theta^{*T}\phi(k) + \delta(k). \tag{19}$$

Based on this parametrized system model, an adaptive parameter estimation algorithm can be derived to obtain the adaptive estimates $\theta_a(k)$, $\theta_b(k)$, $\bar{\theta}_{bi0}(k)$ and $\bar{\theta}_{bi}(k)$ of the unknown parameters θ_a^*, θ_b^*, $\bar{\theta}_{bi0}^*$ and $\bar{\theta}_{bi}^*$, $i = k_1, k_2, \ldots, k_q$.

B. Robust Parameter Estimation

Because of the existence of the modeling errors $\delta(k)$, we use robust adaptive laws to update estimates of the parameter. For the parameterized model (19), let $\theta(k)$ be the estimate of θ^* and define the estimation error

$$\epsilon(k) = \theta^T(k)\phi(k) - y(k), \tag{20}$$

which can be expressed as

$$\epsilon(k) = \widetilde{\theta}^T(k)\phi(k) - \delta(k), \quad \widetilde{\theta}(k) = \theta(k) - \theta^*. \tag{21}$$

To handle the modeling errors $\delta(k)$, we modify the standard gradient algorithm. The modified algorithm is

$$\theta(k+1) = \theta(k) - \frac{\Gamma\epsilon(k)\phi(k)}{m^2(k)} + f(k), \tag{22}$$

where $0 < \Gamma = \Gamma^T < 2I_{n\theta}$, $n\theta$ is the dimension of θ^*,

$$m(k) = \sqrt{\kappa + \phi^T(k)\phi(k)}, \ \kappa > 0, \tag{23}$$

and $f(k)$ is a modification term for robustness with respect to the modeling errors $\delta(k)$. We use parameter projection for the modification term $f(k)$, which uses the knowledge of the parameter region $[\theta_j^a, \theta_j^b]$, $j = 1, 2, \ldots, n_\theta$, such that $\theta_j^* \in [\theta_j^a, \theta_j^b]$, $j = 1, 2, \ldots, n_\theta$, for $\theta^* = [\theta_1^*, \theta_2^*, \ldots, \theta_{n_\theta}^*]^T$.

For parameter projection, a suitable choice of Γ in (22) is

$$\Gamma = \mathrm{diag}\{\gamma_1, \ldots, \gamma_{n_\theta}\}, \ 0 < \gamma_j < 2, \ j = 1, 2, \ldots, n_\theta.$$

Denote $\theta_j(k)$, $f_j(k)$, and $g_j(k)$ as the jth components of $\theta(k)$, $f(k)$ and

$$g(k) = -\frac{\Gamma\phi(k)\epsilon(k)}{m^2(k)}, \tag{24}$$

respectively, for $j = 1, 2, \ldots, n_\theta$, choose the initial estimates as $\theta_j(0) \in [\theta_j^a, \theta_j^b]$, and set the projection function components as

$$f_j(k) = \begin{cases} 0, & \text{if } \theta_j(k) + g_j(k) \in [\theta_j^a, \theta_j^b] \\ \theta_j^b - \theta_j(k) - g_j(k), & \text{if } \theta_j(k) + g_j(k) > \theta_j^b \\ \theta_j^a - \theta_j(k) - g_j(k), & \text{if } \theta_j(k) + g_j(k) < \theta_j^a. \end{cases}$$

The adaptive law has the desired properties:

Lemma 1: The adaptive algorithm (22) guarantees

(i) $\theta(k)$ and $\frac{\epsilon(k)}{m(k)}$ are bounded;
(ii) $\frac{\epsilon(k)}{m(k)}$ and $\theta(k+1) - \theta(k)$ satisfy

$$\sum_{k=k_1}^{k_2} \frac{\epsilon^2(k)}{m^2(k)} \le c_1 + c_2 \sum_{k=k_1}^{k_2} \frac{\delta^2(k)}{m^2(k)} \tag{26}$$

$$\sum_{k=k_1}^{k_2} \|\theta(k+1) - \theta(k)\|_2^2 \le c_3 + c_4 \sum_{k=k_1}^{k_2} \frac{\delta^2(k)}{m^2(k)} \tag{27}$$

$\forall k_2 \ge k_1 \ge 0$, for some constants $c_i > 0$, $i = 1, 2, 3, 4$.

With those desired parameter estimation properties, next we develop the adaptive actuator failure compensation control scheme using the adaptive parameter estimates.

C. Adaptive Control Law

We use the adaptive controller structure for the system (11) under the actuation scheme (13):

$$v_0(k) = \theta_1^T\omega_1(k) + \theta_2^T\omega_2(k) + \theta_3 y_m(k+n^*) + \theta_4(k), \tag{28}$$

where $\theta_1 \in R^{n-1}$, $\theta_2 \in R^n$, and $\theta_3 \in R$ are parameters, $\theta_4(k) \in R$ is a signal to be chosen for compensation of the actuation error, $\omega_1(k) = a_\lambda(z)[v_0](k)$, $a_\lambda(z) = [z^{-n+1}, \cdots, z^{-1}]^T$, $\omega_2(k) = b_\lambda(z)[y](k)$, and $b_\lambda(z) = [z^{-n+1}, \cdots, z^{-1}, 1]^T$.

The parameters θ_1, θ_2, and θ_3 satisfy the design equation:

$$\theta_1^T a_\lambda(z)\hat{P}(z) + \theta_2^T b_\lambda(z)\hat{Z}_a(z) = \hat{P}(z) - \theta_3\hat{Z}_a(z)z^{n^*}, \tag{29}$$

where $\hat{P}(z)$ and $\hat{Z}_a(z)$ are the estimates of $P(z)$ and $Z_a(z)$. This equation always has a solution and the solution is unique if $\hat{P}(z)$ and $\hat{Z}_a(z)$ are co-prime.

The nominal version of the compensation signal $\theta_4(k)$ is

$$\theta_4^*(k) = -\sum_{i=k_1,k_2,\ldots,k_q} \frac{P_a(z)Z_i(z)}{\Lambda(z)}[\bar{u}](k), \tag{30}$$

where $\Lambda = z^{n-1}$, the polynomial $P_a(z)$ of degree n^*-1 is calculated from

$$1 - \theta_1^T a_\lambda(z) = z^{-n+1}Z_a(z)P_a(z) \tag{31}$$

whose existence is ensured by the matching equation (29). Denoting $P_a(z) = p_{n^*-1}z^{n^*-1} + \cdots + p_1 z + p_0$, in view of (15), we obtain

$$\begin{aligned} & P_a(z)Z_i(z)[\bar{u}_i](k) \\ = \ & (p_{n^*-1}z^{n^*-1} + \cdots + p_1 z + p_0)[\bar{\theta}_{bi0}^* + \bar{\theta}_{bi}^{*T}B_i(z)f_i](k) \\ = \ & \theta_{4i0}^* + (p_{n^*-1}z^{n^*-1} + \cdots + p_1 z + p_0)[\bar{\theta}_{bi}^{*T}f_{iB}](k), \end{aligned} \tag{32}$$

where $\theta^*_{4i0} = (p_{n^*-1} + \cdots + p_1 + p_0)\bar{\theta}^*_{bi0}$, and $f_{iB}(k) = B_i(z)[f_i](k) = [f_i^T(k), z[f_i^T](k), \ldots, z^{n-n^*}[f_i^T](k)]^T$. Hence the final expression of $\theta^*_4(k)$ is

$$\theta^*_4(k) = -\sum_{i=k_1,k_2,\ldots,k_q} \left(\theta^*_{4i0} + \sum_{l=1}^{n_i}\sum_{k=0}^{n-1} \theta^*_{4ikl} \frac{z^k}{\Lambda(z)}[f_{il}](k) \right)$$

where θ^*_{4ikl} depends on the parameter p_i and the parameter in $\bar{\theta}^*_{bi}$, which are difficult to be written in general form, but they can be readily derived for a special case. For example, for $n^* = 1$, $P_a(z) = p_0$, $\sum_{k=0}^{n-1} \theta^*_{4ikl} \frac{z^k}{\Lambda(z)}[f_{il}](k) = p_0 \bar{\theta}^{*T}_{b2} \frac{1}{\Lambda(z)}$ $[f_{il}(k), z f_{il}(k), \cdots, z^{n-1} f_{il}(k)]^T$. This parametrizes $\theta^*_4(k)$ in terms of the elements of θ^* in (18). So the parameter $\theta_4(k)$ in (28) can be obtained as

$$\theta_4(k) = -\sum_{i=k_1,\ldots,k_q} \left(\theta_{4i0} + \sum_{l=1}^{n_i}\sum_{k=0}^{n-1} \theta_{4ikl} \frac{z^k}{\Lambda(z)}[f_{il}](k) \right) \quad (33)$$

where θ_{4i0} and θ_{4ikl} are the estimates of θ^*_{4i0} and θ^*_{4ikl} respectively.

The adaptive design procedure can be summarized as

- derive the parametrized model (17)
- obtain the adaptive parameter estimates of the model
- solve the design equations (29) and (31) for θ_i, $i = 1, 2, 3$, and $\hat{P}_a(z)$
- calculate the parameters of θ_4 in (33)
- construct the control law (28) for the actuation scheme (13) for the system (9).

The control law ensures stability of an adaptive control system and the following properties can be proves.

Theorem 1: The adaptive controller (28) with adaptive laws (22), applied to the system (9) with actuator failures (2), ensures closed-loop signals boundness and

$$\sum_{k=k_1}^{k_2} (y(k) - y_m(k))^2 \le c_1 + c_2 \sum_{k=k_1}^{k_2} \delta^2(k), \quad (34)$$

$\forall k_2 \ge k_1 \ge 0$ for some constants $c_1 > 0, c_2 > 0$.

Its proof can be derived using discrete-time robust adaptive control theory [12] (due to space limit, it is omitted).

V. Simulation Study

In this section, we present simulation results to verify the desired performance of the developed adaptive actuator failure compensation system. In simulations, we use the linearized lateral dynamics model of a Boeing 747 airplane [13] as the controlled plant. The aircraft model [13] is modified with two augmented actuation vectors $b_2 u_2$ and $b_3 u_3$, for the study of actuator failure compensation. We first calculate discrete-time model of the aircraft system, then simulate the discrete-time adaptive control system.

A. A Linear Aircraft System Model

The linearized lateral dynamics of Boeing 747 with two augmented actuation vectors can be described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t),\ y(t) = x_2(t) = y_r(t) \\ x(t) &= [\beta, y_r, p, \phi]^T,\ B = [b_1, b_2, b_3] \end{aligned} \quad (35)$$

where β is the side-slip angle, y_r is the yaw rate, p is the roll rate, ϕ is the roll angle, y is the system output, which is the yaw rate in this case, and u is the control input vector, which contains three control signals: $u = [u_1, u_2, u_3]^T$ to represent three rudder servos' angles: δ_{r1}, δ_{r2}, δ_{r3}, from a three-piece rudder with needed redundancy for achieving failure compensation in the presence of actuator failures (the no-redundancy case [13] is with for $u_1 = \delta_r$, the single rudder servo angle, while $u_2 = u_3 = 0$).

According the data of horizontal flight 40,000 ft and nominal forward speed 774 ft/sec (Mach 0.8), which provided in [13], we obtain the discrete-time model of the Boeing 747 lateral-perturbation dynamics with sampling time $T = 0.1s$

$$A_d = \begin{bmatrix} 0.9902 & -0.09852 & 0.008162 & 0.004133 \\ 0.05968 & 0.9855 & -0.002846 & 0.000124 \\ -0.2956 & 0.05251 & 0.9533 & -0.00062 \\ -0.0147 & 0.0104 & 0.09766 & 1 \end{bmatrix},$$

$$b_{d1} = \begin{bmatrix} 0.003138 & -0.04718 & 0.01369 & 0.0005229 \end{bmatrix}^T,$$

$$b_{d2} = \begin{bmatrix} 0.00355 & -0.04966 & 0.01818 & 0.000741 \end{bmatrix}^T,$$

$$b_{d3} = \begin{bmatrix} 0.002022 & -0.0298 & 0.008972 & 0.0003466 \end{bmatrix}^T$$

where b_2 and b_3 are the augmented actuation vectors for actuator failure compensation.

B. Conditions and Results of Simulation

In this subsection, we verify the design conditions for the above plant model, give the simulation conditions, and present the simulation results.

1) **Failure and Design Conditions:** For the plant with three actuators, that is, $u = [u_1, u_2, u_3]^T \in R^3$, there can be four possible failure patterns, that are failure-free, failure of u_1, failure of u_2, and failure of u_1 and u_2. We consider the first, second, and fourth cases, describe the three cases in detail as follows:

(i) failure-free: $u_1(k) = u_2(k) = u_3(k) = v_0(k)$, for $k < 500$;
(ii) failure of u_1: $u_1(k) = 0.3\text{rad}$, $u_2(k) = u_3(k) = v_0(k)$, for $k \ge 500$; and
(iii) failure of u_1 and u_2: $u_1(k) = 0.3\text{rad}$, $u_2(k) = 0.1\sin(0.1k)\text{rad}$, $u_3(k) = v_0(k)$, for $k \ge 1000$.

With the Assumption (A.1), we choose $\alpha_j = 1$, $j = 1, 2, 3$, and the polynomial $Z_a(z)$ is

$$Z_a(z) = \begin{cases} -0.1267z^3 + 0.3731z^2 - 0.3667z + 0.1202, \\ \qquad \text{for no failure} \\ -0.0795z^3 + 0.2341z^2 - 0.2301z + 0.0754, \\ \qquad \text{for failure of } u_1 \\ -0.0298z^3 + 0.08781z^2 - 0.08629z + 0.02828, \\ \qquad \text{for failures of } u_1 \text{ and } u_2. \end{cases}$$

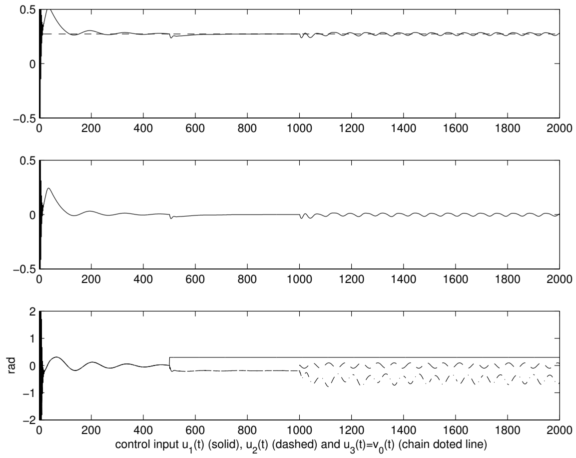

Fig. 1. System response with robust adaptive law ($r(k) = 0.3$).

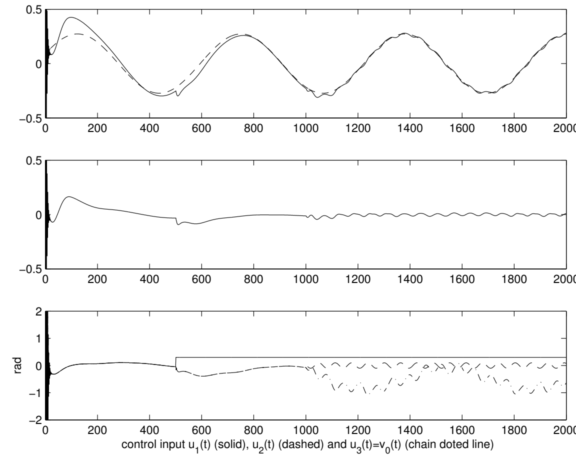

Fig. 2. System response with robust adaptive law ($r(k) = 0.3sin(0.01k)$).

From the expressions of $Z_a(z)$, we have

(i) the degree of $Z_a(z)$ is 3, that is, $n^* = 4 - 3 = 1$;

(ii) the leading coefficients of $Z_a(z)$ are all negative; and

(iii) the zeros of $Z_a(z)$ are all stable.

Thus, Assumptions (A.1) is satisfied. Moreover, the set $\{k_1, k_2\} = \{1, 2\}$ is known, to satisfy Assumptions (A.2).

2) **Simulation Results:** For simulation, we used: $\Gamma = 0.75I$, $\kappa = 0.75$, $W_m(z) = \frac{1}{z}$, $\Lambda_e(z) = (z + 0.5)^4$, $\Lambda(z) = z^3$, $y(0) = 1$, $y_m(0) = 0$, and $\theta(0) = 90\%\theta^* = [0.0255, -0.0777, 0.0790, -0.0268, -0.7883, 3.8756, -3.8672, 5.3361, 0, 0, 0, 0, 0]^T$(the initial parameters θ_a and θ_b are chosen as $90\%\theta_a^*$ and $90\%\theta_b^*$ respectively, to simulate a case in which we know some information about the system model in a specific application). The initial failure parameter estimates are chosen as zero, because the system is failure-free at the beginning. The reference input is chosen as a step signal and a sine signal, that is, $r(k) = 0.3$ and $r(k) = 0.3\sin(0.01k)$.

The simulation results are shown in Figures 1 and 2, respectively for the adaptive system response with $r(k) = 0.3$ and with $r(k) = 0.3\sin(0.01k)$. These Figures verify the desired system performance: the closed-loop system are stable, the tracking error $e(k) = y(k) - y_m(k)$, with some transient values at the actuator failure time instant, becomes smaller as the time advances.

VI. Conclusions

In this paper, we have developed a new discrete-time framework of adaptive actuator failure compensation for systems with uncertain actuator failures, using an indirect adaptive control method. We derived a discrete-time model of continuous-time systems with actuator failures, and developed an adaptive actuator failure compensation scheme based on the derived discrete-time model which captures the essential system model features. The developed adaptive failure compensation scheme is a new design of direct failure compensation control which does not use explicit failure detection. Due to the presence of discrete-time actuator failure modeling errors, robust adaptive laws should be used for adaptive parameter estimation. Simulation results verified desired system stability and tracking properties.

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