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Adaptive Failure Compensation of Hysteric Actuators in Controlling Uncertain Nonlinear Systems

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***Abstract*— Hysteresis nonlinearity exists in many physical actuators and actuator failures seem inevitable in practice. However, there is still no result available to compensate for failures of hysteric actuators in the design of controllers based on adaptive approaches. In this paper, we address such a problem by considering controlling a class of unknown nonlinear systems with multiple hysteric actuators. Two schemes are presented to design control signals for these actuators. Both schemes can accommodate uncertain patterns, values and time of actuator failures, in addition to system parametric uncertainties. It is shown that the designed controllers can compensate for failure and hysteresis effects of the actuators in the sense that system stability and tracking performance are maintained no matter whether this is any actuator failure or not.**

I. INTRODUCTION

In practice, hysteresis nonlinearity exists in many physical actuators. However, it is often ignored in the design and analysis of control systems for simplicity. Nevertheless, In the context of adaptive control, several schemes based on adaptive control approaches have been proposed, see for examples in [1] - [7].In [3] and [4] a dynamic differential function was defined to pattern a backlash-like hysteresis. By approximating the hysteresis with a disturbance-like term, an adaptive control law was designed to stabilize the overall system. In [2] and [3], an assumption that all the system parameters must be in a known bounded set was imposed. This assumption was removed in [4] by using backstepping design approaches. The result was extended to decentralized adaptive control systems in [5]. In [6]and[7] an adaptive output feedback control law was designed to ensure the stability of system by constructing the inverse of backlash.

On the other hand, actuator failures seem inevitable in practice especially in control systems. Such failures which may lead to instability or even catastrophic accidents are often uncertain in time, value and pattern. To address the actuator failure compensation problem, some adaptive design methods have also been proposed see for example [8]-[15]. In [10] adaptive control laws were designed for nonlinear systems with the backstepping techniques. In [11] an output feedback adaptive control law was proposed for a class of nonlinear systems with actuator failures and in [12] an adaptive control scheme was developed for a class of MIMO systems with unknown actuator failures. By using pre-filters, the previously required relative degree condition was relaxed for linear systems in [13] and nonlinear systems in [14]. In [15] an adaptive control scheme based on prescribed performance bound was proposed to guarantee transient performance.

However, there is still no result available to compensate for failures of hysteric actuators based on adaptive approaches. In this paper, we address such a problem by considering controlling a class of unknown nonlinear systems. With adaptive approaches, available actuation redundancy is usually utilized to realize control objectives as shown in [8] and [9]. Therefore we will consider multiple hysteric actuators which means the nonlinear system is a multi-input system. Actuator failures considered here include both partial loss of effectiveness and total loss of effectiveness as in [15]. Note that actuator failures are uncertain in patterns, values and time. Thus the designed control signals for the actuators should accommodate such uncertainties in addition to system parametric uncertainties. Also they should be able to compensate for failure and hysteresis effects of the actuators and maintain system stability as well as tracking performance. In this paper, we will present two schemes to design the required controls by using backstepping techniques. In the first scheme, a discontinuous function $sign(\cdot)$ is included in the control law, which may result in chattering. To avoid this phenomenon, a series of smooth functions are used in the second scheme to approximate function $sign(\cdot)$, similar to [4]. It is shown that system stability is guaranteed with both schemes. Perfect tracking is achieved by using the first scheme, while the tracking error can be ensured within an arbitrarily given small bound with the second scheme.

The rest of the paper is outlined as follows. In Section 2, problem is formulated followed by presenting system and actuator models. Control schemes together with systems performance analysis are given in Section 3. A simple simulation example is used to illustrate the effectiveness of the two schemes in Section 4 and finally the paper is concluded in Section 5.

II. PROBLEM STATEMENT

For illustrating our design ideas, the following class of nonlinear systems with uncertain parameters, similar to those

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in [3] and [4], is considered. The system model is given as

$$x^{(n)}+\sum_{i=1}^{r}a_iY_i(x(t),\dot{x}(t),\cdots,x^{(n-1)}(t))=\sum_{i=1}^{m}b_iu_i+\bar{d}(t) \quad (1)$$

where Y_i are known continuous functions and $\bar{d}(t)$ represents bounded external disturbances with unknown bound, m is the number of actuators and n is the known order of the system, $a_j(j=1,2,\cdots,r), b_i(i=1,2,\cdots,m)$ are unknown parameters, $u_j(j=1,2,\cdots,m)$ are inputs to the system. x is its output and let $y=x$. System (1) can be rewritten as

$$\begin{aligned}&\dot{x}_l=x_{l+1} \quad (l=1,2\cdots n-1)\\&\dot{x}_n=a^TY+\sum_{i=1}^{m}b_iu_i+\bar{d}(t)\\&y=x_1\end{aligned} \quad (2)$$

where

$$x_1=x, x_2=\dot{x},\ldots,x_n=x^{(n-1)}$$

$$a=[-a_1,-a_2,\cdots,-a_r]^T, Y=[Y_1,Y_2,\cdots,Y_r]$$

We now consider the ith hysteric actuator which may fail during its operation. It exhibits backlash-like hysteresis behavior denoted as $v_i=B(u_{ci})$ with u_{ci} being the control signal to be designed and v_i as the output during its normal operation, i.e. $u_i=v_i$ if it does not fail. As in [3] [4] [5], the following hysteresis model is studied here,

$$\frac{dv_i}{dt}=\alpha_i\left|\frac{du_{ci}}{dt}\right|(c_iu_{ci}-v_i)+B_{i1}\frac{du_{ci}}{dt} \quad (3)$$

where α_i, c_i and B_{i1} are constants. $c_i>0$ and $c_i>B_{i1}$. According to the analysis in [3], it can be solved as

$$\begin{aligned}v_i &= c_iu_{ci}+\bar{d}_{1i}(u_{ci}) \quad (4)\\ \bar{d}_{1i}(u_{ci}) &= (v_i-c_iu_{ci}(0))e^{-\alpha_i(u_{ci}-u_{ci}(0))sign(\dot{u}_{ci})}\\ &\quad +e^{-\alpha_iu_{ci}sign(\dot{u}_{ci})}\int_{u_{ci0}}^{u_{ci}}(B_{i1}-c_i)e^{\alpha_i\xi sign(\dot{u}_{ci})}d\xi\end{aligned}$$

where $\bar{d}_{1i}$ is bounded as shown in [3].

As in [15], the failure of the ith actuator at time instant t_{if} can be modelled as follows

$$\begin{aligned}&u_i=\rho_iv_i+u_{ki}, \quad (\forall t\geq t_{if})\\&\rho_iu_{ki}=0\end{aligned} \quad (5)$$

where $0\leq\rho_i\leq1$, u_{ki} and t_{if} are unknown constants. When the actuator works normally, the constant $\rho_i=1$ which implies $u_{ki}=0$ and thus $u_i=v_i$. Other cases are discussed as follows

- $0<\rho_i<1$,
  It indicates $u_i=\rho_iv_i$. The ith actuator is called partial loss of effectiveness.
- $\rho_i=0$,
  It indicates $u_i=u_{ki}$. The ith actuator is called total loss of effectiveness.

Then the ith hysteric actuator can be modeled as

$$\begin{aligned}&u_i=\rho_iB(u_{ci})+u_{ki}, \quad (\forall t\geq t_{if})\\&\rho_iu_{ki}=0\end{aligned} \quad (6)$$

From (4) and (6), we have

$$\begin{aligned}&u_i=\rho_ic_iu_{ci}+u_{ki}+\bar{d}_i, \quad (\forall t\geq t_{if})\\&\rho_iu_{ki}=0\end{aligned} \quad (7)$$

where $\bar{d}_i=\rho_i\bar{d}_{1i}(u_{ci})$. Because $\bar{d}_{1i}(u_{ci})$ is bounded and ρ_i is a constant, so $\bar{d}_i=\rho_i\bar{d}_{1i}(u_{ci})$ is bounded.

Now the system (2) can be re-written as follows

$$\begin{aligned}&\dot{x}_l=x_{l+1} \quad (l=1,2\cdots n-1)\\&\dot{x}_n=a^TY+\sum_{i=1}^{m}b_i(\rho_ic_iu_{ci}+u_{ki})+d(t)\\&y=x_1\end{aligned} \quad (8)$$

where $d(t)=\sum_{i=1}^{m}b_i\bar{d}_i+\bar{d}(t)$ is bounded with a unknown bound D. We will propose an adaptive law to estimate its bound D as in [4].

To design adaptive controllers, the following assumptions are made.

Assumption 1: The number of actuators with total loss of effectiveness is up to $m-1$. Also any actuator can change only from normal to partial failure or total failure.

Remark 1 Assumption 1 is a basic assumption required in adaptive failure compensation as explained in [10] [15]. Note that all actuators are allowed to have partial loss of effectiveness simultaneously.

Remark 2 Note that any actuator can become faulty at a uncertain time instant t_{if}, but only fails once. Because the number of total actuators is m, so t_f is finite and no new failure will occur after t_f.

Regarding the system to be controlled, we have

Assumption 2: $b_i\neq0$. and the sign of b_i is known.

Assumption 3: Reference signal $y_r(t)$ and its i-order ($i=1,2,\ldots,n-1$) derivatives are known and bounded.

Then our control problem is to propose adaptive control laws for the above described system and actuators such that

- The closed loop system is globally stable.
- The system output y will asymptotically track reference signal y_r, or at least $lim_{t\to\infty}|y-y_r|=\delta_1$ for any pre-specified nonzero constant δ_1.

III. DESIGN OF CONTROLLERS

To carry out the design of control law and adaptive law, we first make coordinate changes by following backstepping technology.

$$\begin{aligned}&z_1=x_1-y_r\\&z_i=x_i-\alpha_{i-1}-y_r^{(i-1)}, (i=2,\ldots,n)\end{aligned} \quad (9)$$

where variable z_1 represents the tracking error and α_{i-1} ($i=2,\ldots,n$) is the virtual control in step $i-1$.

Two schemes will be proposed to solve the control problem.

A. Scheme A

By following the backstepping approaches, we have the following recursive design steps.

1) Control Design: Step 1: From (8) and (9) the derivative of tracking error can be written as

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = z_2 + \alpha_1 \quad (10)$$

where α_1 is the virtual control. With the consideration of Lyapunov function $\overline{V}_1 = \frac{1}{2}z_1^2$ and by choosing $\alpha_1 = -l_1 z_1 (l_1 > 0)$, the derivative of $\bar{V}_1$ is

$$\dot{\overline{V}}_1 = -l_1 z_1^2 + z_1 z_2 \quad (11)$$

Step $i (i = 2, \ldots, n-1)$: In this step, we consider following Lyapunov function $\overline{V}_i$.

$$\overline{V}_i = \overline{V}_{i-1} + \frac{1}{2}z_i^2 \quad (12)$$

By choosing the virtual control α_i as

$$\alpha_i = -l_i z_i - z_{i-1} + \dot{\alpha}_{i-1} \quad (13)$$

where l_i is a positive design parameter, the derivative of $\overline{V}_i$ along with (9) and (13) is given by

$$\dot{\overline{V}}_i = -\sum_{k=1}^{i} l_k z_k^2 + z_i z_{i+1} \quad (14)$$

Step n: From (8) and (9) the derivation of z_n can be expressed as follows

$$\dot{z}_n = a^T Y + \sum_{i=1}^{m} b_i(\rho_i c_i u_{ci} + u_{ki}) + d(t) - \dot{\alpha}_{n-1} - y_r^{(n)} \quad (15)$$

Note that the input to the plant is generated by the term $\sum_{i=1}^{m} b_i(\rho_i c_i u_{ci} + u_{ki})$ where $u_{ci} (i = 1, 2, \cdots, m)$ is to be designed for the ith actuator. Unlike the standard backstepping approach, the following virtual control α is designed at this step

$$\alpha = -l_n z_n - z_{n-1} - \hat{a}^T Y + \dot{\alpha}_{n-1} + y_r^{(n)} - sign(z_n)\hat{D} \quad (16)$$

This virtual control together with the control law and parameter update laws below is obtained based on the control-Lyapunov function approach which will become clear in the stability analysis of the next subsection.

If knowing the system parameters and failures, we could choose the control law as

$$u_{ci} = sign(b_i)\kappa^T \omega \quad (17)$$

where κ is a desired parametric vector and ω is a known vector to be specified in the stability analysis later in (24) and (28). Both are $m+1$ dimensional vectors denoted as

$$\kappa = (\kappa_1, \kappa_{21}, \ldots, \kappa_{2m})^T, \omega = (\omega_1, \omega_{21}, \ldots, \omega_{2m})^T \quad (18)$$

Because κ is unknown owing to unknown system parameters and failures, it is replaced by its estimate $\hat{\kappa}$ and the control law is designed as

Control Law:

$$u_{ci} = sign(b_i)\hat{\kappa}^T \omega \quad (19)$$

Update Laws:

$$\dot{\hat{D}} = \eta |z_n|; \dot{\hat{\kappa}} = -\Gamma_\kappa \omega z_n; \dot{\hat{a}} = \Gamma_a z_n Y \quad (20)$$

where $\hat{a}$ and $\hat{D}$ are estimates of a and D where D is the upper bound of disturbance, η is a positive constant and Γ_a, Γ_κ are positive definite matrices.

2) Stability analysis: We now analyze the stability of the closed loop system with the control law and update laws in (19) and (20). Suppose that all actuators are normal in time interval $[T_0, T_1)$. Consider the following Lyapunov function

$$V_0 = \overline{V}_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{a}^T \Gamma_a^{-1} \tilde{a} + \sum_{i=1}^{m} \frac{c_i |b_i|}{2} \tilde{\kappa}^T \Gamma_\kappa^{-1} \tilde{\kappa} + \frac{1}{2\eta}\tilde{D}^2 \quad (21)$$

where $\tilde{a} = a - \hat{a}, \tilde{\kappa} = \kappa - \hat{\kappa}$ and $\tilde{D} = D - \hat{D}$. Since no actuator fails in time interval $[T_0, T_1)$, $\rho_i = 1, u_{ki} = 0 (i = 1, 2 \ldots, m)$. Then (15) can be rewritten as

$$\dot{z}_n = a^T Y + \sum_{i=1}^{m} b_i c_i u_{ci} + d(t) - \dot{\alpha}_{n-1} - y_r^{(n)} \quad (22)$$

Vectors κ and ω are chosen such that

$$\sum_{i=1}^{m} c_i |b_i| \kappa^T \omega = \alpha \quad (23)$$

This gives

$$\kappa_1 = \frac{1}{\sum_{i=1}^{m} c_i |b_i|}; \kappa_{2i} = 0; \omega_1 = \alpha; \omega_{2i} = 1 (i = 1, \ldots, m) \quad (24)$$

With (16) (21) (22) (23) and update laws (20), we have

$$\begin{aligned} \dot{V}_0 &= \dot{\overline{V}}_{n-1} + z_n \dot{z}_n - \tilde{a}^T \Gamma_a^{-1} \dot{\hat{a}} - \sum_{i=1}^{m} c_i |b_i| \tilde{\kappa}^T \Gamma_\kappa^{-1} \dot{\hat{\kappa}} - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} \\ &\leq -\sum_{k=1}^{n} l_k z_k^2 - \tilde{a}\Gamma_a^{-1}(\dot{\hat{a}} - \Gamma_a z_n Y) - \sum_{i=1}^{m} c_i |b_i| \tilde{\kappa}^T \Gamma_\kappa^{-1}(\dot{\hat{\kappa}} \\ &\quad + \Gamma_\kappa \omega z_n) - \frac{1}{\eta}\tilde{D}(\dot{\hat{D}} - \eta |z_n|) \\ &\leq -\sum_{k=1}^{n} l_k z_k^2 \end{aligned} \quad (25)$$

It is clear that V_0 is non increasing. So we have $V_0(T_1^-) \leq V_0(T_0)$. From (21) and (25) it can be concluded that all signals z_i, $\tilde{a}$, $\tilde{D}$, $\tilde{\kappa}$ are bounded in the time interval $[T_0, T_1)$.

Now assume that there are $p_1 (1 \leq p_1 < m)$ actuators are faulty from time instant T_1 and in time interval (T_1, T_2) no new normal actuator fails. Let set Q_T denote the actuators of total failure. It indicates that if $i \in Q_T$, then $\rho_i = 0$. Using the set $\bar{Q}_T$ to represent the remaining actuators, then $0 < \rho_i \leq 1$ if $i \in \bar{Q}_T$. It is clear that $Q_T \cup \bar{Q}_T = \{1, 2, \ldots, m\}$.

In the time interval (T_1, T_2), the Lyapunov function is chosen as

$$\begin{aligned} V_1 &= \overline{V}_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{a}^T \Gamma_a^{-1} \tilde{a} + \sum_{i \in \bar{Q}_T} \frac{\rho_i c_i |b_i|}{2} \tilde{\kappa}^T \Gamma_\kappa^{-1} \tilde{\kappa} \\ &\quad + \frac{1}{2\eta}\tilde{D}^2 \end{aligned} \quad (26)$$

Now κ and ω should be chosen to satisfy

$$\sum_{i \in \bar{Q}_T} b_i c_i \rho_i u_{ci} = \alpha - \sum_{i \in Q_T} b_i u_{ki} \quad (27)$$

By fixing ω the same as in (24), κ can be chosen as

$$\kappa_1 = \frac{1}{\sum_{i \in \bar{Q}_T} \rho_i c_i |b_i|}; \kappa_{2i} = 0, (i \in \bar{Q}_T); \kappa_{2i} = \frac{-b_i u_{ki}}{\sum_{i \in \bar{Q}_T} \rho_i c_i |b_i|} (i \in Q_T) \quad (28)$$

From (16),(20),(26),(27) and (28), we can obtain

$$\begin{aligned}\dot{V}_1 &\le -\sum_{k=1}^{n-1} l_k z_k^2 + z_n z_{n-1} + z_n(a^T Y + \sum_{i\in\bar{Q}_T} b_i c_i \rho_i u_{ci} \\ &+ \sum_{i\in Q_T} b_i u_{ki} + d(t) - \dot{\alpha}_{n-1} - y_r^{(n)}) - \tilde{a}^T\Gamma_a^{-1}\dot{\hat{a}} \\ &- \sum_{i\in\bar{Q}_T} \rho_i c_i |b_i| \tilde{\kappa}^T \Gamma_\kappa \dot{\hat{\kappa}} - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} \le -\sum_{k=1}^{n} l_k z_k^2 \quad (29)\end{aligned}$$

With this, $V_1(T_2^-) \le V_1(T_1^+)$. Comparing (21) with (26), we can see that the difference between $V_1(T_1^+)$ and $V_0(T_1^-)$ is only the coefficients in front of $\tilde{\kappa}^T\Gamma_\kappa\tilde{\kappa}$ of the second last term. Since the possible jumping of κ is bounded, $V_1(T_1^+)$ and thus $V_1(T_2^-)$ are bounded.

According to Assumption 1, there is a finite instant T_f such that in the time interval (T_f, ∞), there is no more actuator failure. So by using the same argument as above we can ensure $V_1(T_1^+)$,$V_2(T_2^+)$ ...,$V_f(T_f^+)$ bounded in time interval (T_1,T_2),(T_2,T_3),$\cdots$,(T_f,∞). Also for $t \in (T_f,\infty)$, it can be shown that $V_f(t) \le V_f(T_f^+)$. Then it can be concluded that z_i, $\tilde{a}$, $\tilde{D}$, $\tilde{\kappa}$ are all bounded over $[0,\infty]$. Now we are at the position to state our first result in the following theorem.

Theorem 1. Consider the adaptive system consisting of nonlinear plant (1), m hysteric actuators modeled in (3) with unknown failures described by (5)and the adaptive controller designed using the control law (19) and the update laws (20). Under Assumptions 1 to 3, the system is globally stable in the sense that all the signals are bounded. In addition, asymptotic tracking is achieved, i.e. $lim_{t\to\infty}(y - y_r) = 0$.

Proof: As analyzed above, signals z_i, $\tilde{a}$, $\tilde{D}$ and $\tilde{\kappa}$ are bounded. Then all the virtual control $\alpha_i, i = 1,2,\cdots,n-1$, α and states $x_i, i = 1,2,\cdots,n$ are bounded. From the control law (19), u_{ci} is ensured bounded. Then, by applying the Lasalle-Yoshizzawa theorem, it follows that $lim_{t\to\infty}(y - y_r) = 0$.

B. Control scheme B

In scheme A, a discontinuous function $sign(z_n)$ exists in α and thus the control law is discontinuous. This may result in chattering. In this subsection, we will give an alternative control scheme to avoid this phenomenon by using a series of smooth functions to approximate the function $sign(\cdot)$. Similar to [4], these functions, denoted as $sg_i(z_i), i = 1,2,\ldots,n$ are defined as

$$sg_i(z_i) = \begin{cases} \frac{z_i}{|z_i|}, & |z_i| \ge \delta_i \\ \frac{z_i}{\left(\delta_i^2 - z_i^2\right)^{n-i+2} + |z_i|}, & |z_i| < \delta_i \end{cases} \quad (30)$$

where δ_i is a positive design parameter. It can be shown that $sg_i(z_i)$ is $(n-i+2)$th order differentiable. Another function $f_i(z_i)$ is defined as

$$f_i(z_i) = \begin{cases} 1, & |z_i| \ge \delta_i \\ 0, & |z_i| < \delta_i \end{cases} \quad (31)$$

For simple notation, we will use f_i to denote $f_i(z_i)$.

The following properties which are useful to our analysis can be shown

$$(f_i(z_i))^2 = f_i(z_i), f_i(z_i)(sg_i(z_i))^2 = f_i(z_i) \quad (32)$$

$$sign(z_i) f_i(z_i) = sg_i(z_i) f_i(z_i) \quad (33)$$

1) Control design: For simplicity, only the first, second and final steps are illustrated in details.

Step 1: From(8) and (9) the derivative of tracking error can be rewritten as

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = z_2 + \alpha_1 \quad (34)$$

where α_1 is the virtual control in this step.

By considering the following Lyapunov function

$$\overline{V}_1 = \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} f_1 \quad (35)$$

α_1 can be designed as

$$\alpha_1 = -(l_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) - (\delta_2 + 1) sg_1(z_1) \quad (36)$$

where l_1 is a positive constant.

Such a virtual control together with (32) - (35) and the following inequality

$$\begin{aligned}&(|z_1| - \delta_1)^n f_1 sg_1(z_1)(z_2 - (\delta_2 + 1) sg_1(z_1)) f_1 \\ \le\ &(|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 \quad (37)\end{aligned}$$

yields

$$\dot{\overline{V}}_1 \le -(l_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} f_1 + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 \quad (38)$$

Step 2: The derivative of z_2 is computed as

$$\dot{z}_2 = z_3 + \alpha_2 - \dot{\alpha}_1 \quad (39)$$

The virtual control α_2 can be designed as

$$\alpha_2 = -(l_2 + \frac{5}{4})(|z_2| - \delta_2)^{n-1} sg_2(z_2) + \dot{\alpha}_1 - (\delta_3 + 1) sg_2(z_2) \quad (40)$$

where l_2 is a positive constant, based on the following Lyapunov function

$$\overline{V}_2 = \overline{V}_1 + \frac{1}{n}(|z_2| - \delta_2)^n f_2 \quad (41)$$

With (38), (39) and (40), the derivative of $\overline{V}_2$ is given by

$$\begin{aligned}\dot{\overline{V}}_2 \le\ & -\sum_{i=1}^{2} l_i (|z_i| - \delta_i)^{2(n-i+1)} f_i - \frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + (|z_1| \\ & -\delta_1)^n (|z_2| - \delta_2 - 1) f_1 - (|z_2| - \delta_2)^{2(n-1)} f_2 + (|z_2| \\ & -\delta_2)^{n-1}(|z_3| - (\delta_3 + 1)) f_2 - \frac{f_2}{4}(|z_2| - \delta_2)^{2(n-1)} \quad (42)\end{aligned}$$

Similar to [4], we can get

$$\begin{aligned}&-\frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 \\ &-(|z_2| - \delta_2)^{2(n-1)} f_2 \le 0 \quad (43)\end{aligned}$$

Then we have

$$\dot{\overline{V}}_2 \le -\sum_{i=1}^{2} l_i(|z_i|-\delta_i)^{2(n-i+1)} f_i - \frac{1}{4}(|z_2|-\delta_2)^{2(n-1)} f_2 + (|z_2|-\delta_2)^{n-1}(|z_3|-(\delta_3+1)) f_2 \quad (44)$$

Step $i(i=3,4,\ldots,n-1)$:
From the Lyapunove function

$$\overline{V}_i = \overline{V}_{i-1} + \frac{1}{n-i+2}(|z_i|-\delta_i)^{n-i+2} f_i \quad (45)$$

α_i can be designed as

$$\alpha_i = -(l_i+\frac{5}{4})(|z_i|-\delta_i)^{n-i+1} sg_i(z_i) - (\delta_{i+1}+1) sg_i(z_i) + \dot{\alpha}_{i-1} \quad (46)$$

With this virtual control, we have

$$\dot{\overline{V}}_i \le -\sum_{k=1}^{i} l_k(|z_k|-\delta_k)^{2(n-k+1)} f_k - \frac{1}{4}(|z_i|-\delta_i)^{2(n-i+1)} f_i + (|z_i|-\delta_i)^{(n-i+1)}(|z_{i+1}|-\delta_{i+1}-1) f_i \quad (47)$$

Step n: From the system model, the derivative of z_n is

$$\dot{z_n} = a^T Y + \sum_{i=1}^{m} b_i(\rho_i c_i u_{ci} + u_{ki}) + d(t) - \dot{\alpha}_{n-1} - y_r^{(n)} \quad (48)$$

Similar to Scheme A with control-Lyapunov functions, the virtual control α is designed as

$$\alpha = -l_n sg_n(|z_n|-\delta_n) - sg_n(|z_n|-\delta_n) - \hat{a}^T Y + \dot{\alpha}_{n-1} + y_r^{(n)} - sg_n(z_n)\hat{D} \quad (49)$$

and the control law and parameter update laws are given below.

Control Law: With κ and ω of (18) specified in (23), (24) and (27), (28), the control u_{ci} is given by

$$u_{ci} = sign(b_i)\hat{\kappa}^T \omega \quad (50)$$

Update laws:

$$\dot{\hat{D}} = \eta(|z_n|-\delta_n) f_n; \quad \dot{\hat{\kappa}} = -\Gamma_\kappa(|z_n|-\delta_n) f_n sg_n(z_n)\omega$$
$$\dot{\hat{a}} = \Gamma_a Y(|z_n|-\delta_n) f_n sg_n(z_n) \quad (51)$$

where $\hat{D}, \hat{\kappa}, \hat{a}$ are estimations of D, κ, a, η is a positive constant and Γ_a, Γ_κ are positive define matrices.

2) Stability analysis: Similar to Scheme A, assume all actuators are normal in time interval $[T_0, T_1)$. The Lyapunov function considered is

$$V_0 = \sum_{i=1}^{n} \frac{1}{n-i+2}(|z_i|-\delta_i)^{n-i+2} f_i + \sum_{i=1}^{m} \frac{c_i|b_i|}{2}\tilde{\kappa}^T \Gamma_\kappa^{-1}\tilde{\kappa} + \frac{1}{2}\tilde{a}^T \Gamma_a^{-1}\tilde{a} + \frac{1}{2\eta}\tilde{D}^2 \quad (52)$$

As all actuators are normal, then

$$\rho_i = 1, u_{ki} = 0, (i=1,2,\ldots,m) \quad (53)$$

Choosing κ and ω satisfying

$$\sum_{i=1}^{m} c_i|b_i|\kappa^T \omega = \alpha \quad (54)$$

we have

$$\dot{V}_0 \le -\sum_{i=1}^{n} l_i(|z_i|-\delta_i)^{2(n-i+1)} f_i - \frac{1}{\eta}\tilde{D}(\dot{\hat{D}} - \eta(|z_n|-\delta_n) f_n) - \sum_{i=1}^{m} c_i|b_i|\tilde{\kappa}^T \Gamma_\kappa^{-1}(\dot{\hat{\kappa}} + \Gamma_\kappa(|z_n|-\delta_n) f_n sg_n(z_n)\omega) - \tilde{a}^T \Gamma_a^{-1}(\dot{\hat{a}} - (|z_n|-\delta_n) f_n sg_n(z_n)\Gamma_a Y) + \Xi \quad (55)$$

where

$$\Xi = -\frac{1}{4}(|z_{n-1}|-\delta_{n-1})^4 f_{n-1} + (|z_{n-1}|-\delta_{n-1})^2 (|z_n|-\delta_n-1) f_{n-1} - (|z_n|-\delta_n)^2 f_n \quad (56)$$

Similar to (43), we can get $\Xi \le 0$. Then from the update laws in (51),

$$\dot{V}_0 \le -\sum_{i=1}^{n} l_i(|z_i|-\delta_i)^{2(n-i+1)} f_i \quad (57)$$

So V_0 is non increasing in time interval $[T_0, T_1)$.

Now suppose that there are $p_1(p_1 < m)$ actuators faulty from time instant T_1 and there is no new failure occuring in time interval (T_1, T_2). Consider the Lyapunov function

$$V_1 = \sum_{i=1}^{n} \frac{1}{n-i+2}(|z_i|-\delta_i)^{n-i+2} f_i + \sum_{i\in\bar{Q}_T} \frac{\rho_i c_i|b_i|}{2}\tilde{\kappa}^T \Gamma_\kappa^{-1}\tilde{\kappa} + \frac{1}{2}\tilde{a}^T \Gamma_a^{-1}\tilde{a} + \frac{1}{2\eta}\tilde{D}^2 \quad (58)$$

From (27) and (28),

$$\dot{V}_1 \le -\sum_{i=1}^{n-1} l_i(|z_i|-\delta_i)^{2(n-i+1)} f_i - \frac{1}{4}(|z_{n-1}|-\delta_{n-1})^4 f_{n-1} + (|z_{n-1}|-\delta_{n-1})^2(|z_n|-\delta_n-1) f_{n-1} - \tilde{a}^T \Gamma_a^{-1}\dot{\hat{a}} + (|z_n|-\delta_n) f_n sg_n(z_n)[a^T Y + \sum_{i=1}^{m} b_i(\rho_i c_i u_{ci} + u_{ki}) + d(t) - \dot{\alpha}_{n-1} - y_r^{(n)}] - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} - \sum_{i=1}^{m} c_i|b_i|\tilde{\kappa}^T \Gamma_\kappa \dot{\hat{\kappa}} \quad (59)$$

Then with (50) and (51), we can get

$$\dot{V}_1 \le -\sum_{i=1}^{n} l_i(|z_i|-\delta_i)^{2(n-i+1)} f_i \quad (60)$$

By following similar analysis in Scheme A, the boundedness of z_i, $\tilde{a}$, $\tilde{D}$, $\tilde{\kappa}$ can be established and we have the following results.

Theorem 2: Consider the adaptive system consisting of nonlinear plant (1), m hysteric actuators modeled in (3) with unknown failures described by (5)and the adaptive controller designed based on the control law (50) and the update laws (51). Under Assumptions 1 to 3, all the signals are globally bounded. In addition, the tracking error approaches δ_1 asymptotically, i.e. $lim_{t\to\infty}|y-y_r| = \delta_1$.

Proof: The results follow from similar proof of Theorem 1.

Remark 3 The tracking error bound δ_1 is a user design parameter that can be chosen arbitrarily small.

IV. SIMULATION STUDIES

In this section, we illustrate the two design schemes by applying them to the following simple system

$$\dot{x} = a^T Y + b_1 u_1(t) + b_2 u_2(t) \tag{61}$$

where $u_1(t)$ and $u_2(t)$ are the outputs of two hysteric actuators. The known function $Y = x^2$. The actual parameters value are $\theta = 2$ and $b_1 = b_2 = 1$, but unknown to designers. The reference signal is $sin(t)$.

The backlash-like hysteresis is described by (3). The actural value of unknown parameters are selected as $\alpha_2 = \alpha_1 = 1$, $c_2 = c_1 = 3.1635$ and $B_{21} = B_{11} = 0.345$. In our simulation studies, $l_1 = 10$, $\Gamma_a = 1, \eta = 1$ and Γ_κ is a 3×3 identity matrix. The initial value are chosen as follows: $z(0) = 0.5$, $u_1(0) = u_2(0) = 0$, $\hat{a}(0) = 0$, $\hat{\kappa}(0) = 0$, $\hat{D}(0) = 0$. Suppose at $t = 60$ second, the actuator output u_1 is stuck at an unknown value 5.

Figures 1-2 show the tracking error, actuator outputs $u_1(t)$ and $u_2(t)$ when Scheme A is applied, while the respective results of applying Scheme B are given in Figures 3-4.

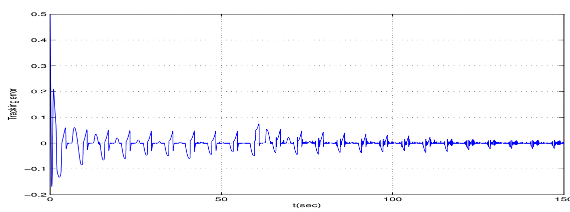

Fig. 1. Scheme A: Tracking error

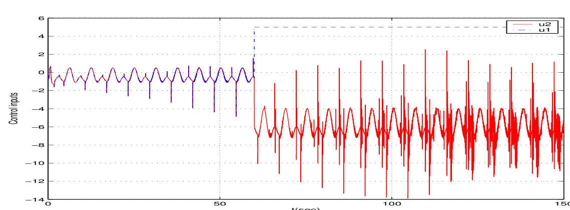

Fig. 2. Scheme A: Control inputs

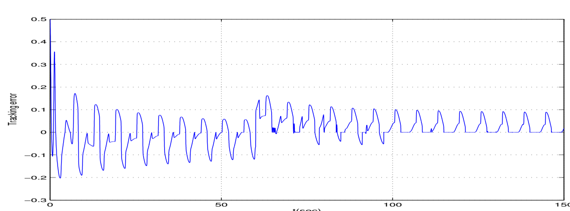

Fig. 3. Scheme B: Tracking error

From the results, it is observed that both schemes work well. Scheme B gives smoother control signals, but scarifies tracking performance.

V. CONCLUSIONS

Two different adaptive state feedback schemes are proposed to compensate for uncertain failures of hysteric actuators in controlling a class of unknown nonlinear systems.

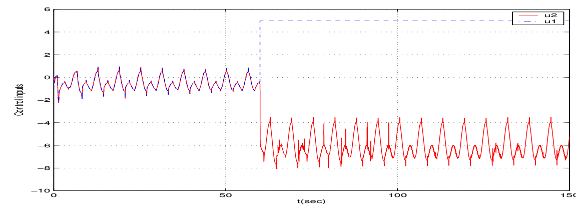

Fig. 4. Scheme B: Control inputs

System stability and output tracking performance can be ensured by these schemes. The second scheme provides smoother control signals to avoid possible chattering brought by the first scheme, but scarifies tracking performance. Our simulation results also verify the established theoretical results.

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