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A Unified Control Architecture for Navigation of Nonholonomic Systems

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Abstract— **The paper presents a unified control architecture for motion planning and navigation of constrained systems. It provides a systematic approach for planning any motion that may be specified by equations of algebraic or differential constraints. It is based upon one dynamic control model for constrained systems, which is not sensitive to the constraint kind and order. The preplanned reference motion may be executed by nonlinear control algorithms.**

I. Introduction

Constrained nonholonomic systems require nonlinear control methods since their linearized control models are usually not controllable [1]. A nonlinear control design process consists of three basic steps: model building, a controller design and its implementation. Usually modeling and control design are related to constraints on a system.

In the paper we consider control oriented dynamic modeling and a controller design for constrained systems. We show that in the modeling step, we may obtain a unified dynamic model suitable for designing controllers despite of the kind and order of constraints imposed on a system.

The first motivation for this research is that mechanical systems are subjected to material and non-material constraints. The latter ones are task, control and design based, and they may be specified by differential equations of high order. We refer to them as programmed constraints [2,3]. They are non-material since they may be put by a designer like a trajectory to follow, which is specified by an algebraic equation but is not treated as a constraint in control setting [1,4]. The trajectory is either given a priori or by a motion planner and next it is passed to a controller [5-7]. An industrial manipulator, holonomic by its nature, may become nonholonomic when constraints are imposed upon its motion properties [9]. A space vehicle is nonholonomic due to the conservation of its angular momentum. Also, a leader-follower system that consists of a couple of robots is a noholonomic system dedicated to navigate towards task based missions, which are not treated as constraints on motion [10-13]. Then, there was no unified constraint formulation for control applications. An exception is the second order nonholonomic constraint due to an unactuated degree of freedom [8].

Secondly, a control framework that incorporates a system dynamics, i.e. model-based, is developed on traditional two-level tracking control architecture for nonholonomic systems. The lower control level operates within a kinematic model to stabilize a system motion to a desired trajectory. The upper control level uses a dynamic model and stabilizes feedback obtained on the lower control level [12,14]. The underlying dynamics is based on the Lagrange approach, so first order nonholonomic constraints may be merged into it.

Finally, latest results in modeling constrained systems showed that material and programmed constraints might be presented in a unified constraint formulation suitable to control design [9,15]. This is in contrast to classical analytical mechanics that offers methods of the generation of dynamic models of systems with first and second order nonholonomic constraints [16]. Constraints on motion specified by equations of high order could not be included into these dynamic models. The Lagrange approach is used the most often for model building in control. It is not suitable then due to the constraint order it may incorporate and the reduction procedure that has to be performed. Thus, there were neither systematic nor unified approaches to modeling systems with constraints of order higher than one.

A unification of modeling constrained systems, in both kinematic and dynamics settings are presented in the paper. It yields the generalized programmed motion equations (GPME) that may capture systems with high order constraints [2]. It results in a unified control oriented models of constrained systems and design of a new control strategy.

II. A Unified Specification of Constrained Systems

A. Sources of Constraints on Control Systems

In mechanics, a type of a nonholonomic constraint arises from a condition of rolling without slipping. It is first order and of the material type. For space vehicles, a first order nonholonomic constraint results from the conservation of the angular momentum but it is referred to as the conservation law not as a kinematic constraint [16,17]. In control, there are more constraint sources. A wheeled vehicle undergoes motion constraints that depend on its design, its interaction with the environment, control design and task specifications [1,4]. They are not treated as constraints. However, they may be regarded as non-material constraints, i.e. in control setting types of constraints may be as follows [3,15]:

1. Material constraints [16].
2. Conservation laws [16,17].
3. Design constraints – they may arise from bounded linear and angular velocities, the lateral acceleration, or from a bounded trajectory curvature for wheeled vehicles [1,4].

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4. Control constraints – they arise mostly from the limitation of a number of control inputs [8,15].
5. Programmed constraints – they may arise from task and requirement specifications put by designers [1,4,9,15].

B. Control Oriented Constraint Formulation

The idea is to develop a unified constraint formulation, which may include the constraint types listed above, and a unified dynamic model of a system with such constraints. The constraint formulation is proposed to be [2,15]

$$B(t,q,\dot{q},\dots,q^{(p-1)})q^{(p)} + s(t,q,\dot{q},\dots,q^{(p-1)}) = 0\,, \tag{1}$$

where p is the constraint order, q - n-vector of generalized coordinates, B - full rank $(k\times n)$ matrix, $n>k$ and s - k-vector. We assume that (1) are linear in p–th order derivative of coordinates or we can transform them to this form. They may specify both material and non-material constraints since the type of a constraint equation does not influence the generation of equations of motion of a system subjected to it. For $p=0$ we get a configuration constraint, which may be material and specify a constant distance between link ends or be a programmed constraint on a trajectory. When $p=1$ a constraint equation may be material and specify a condition of rolling without slipping. However, it may arise from the conservation law or be a programmed constraint on a desired velocity. Material constraints are of orders $p=0$ or $p=1$, the equation of the conservation law is of order $p=1$, and constraint equations for $p>1$ are of the non-material type.

Definition 1: The equations of the constraints (1) are completely nonholonomic if they cannot be integrated, i.e. cannot be presented as equations of a lower order in coordinates.

If we can integrate (1) $(p-1)$ or less times, they are partially integrable. If (1) can be integrated completely, they are holonomic. We assume that (1) are completely nonholonomic. Definition 1 extends the definition of completely nonholonomic first order and second order constraints [8,17]. Necessary and sufficient integrability conditions for differential equations of arbitrary order such as (1) are formulated in [18].

The unified constraints (1) can be presented in the standard state-space control form [17].

III. A Reference Model of a Constrained System

A. A Dynamic Reference Model

A unified dynamic model of a system with the constraints (1) is derived using the GPME applying the algorithm [19].

Algorithm

Assume that (1) may be solved, at least locally, with respect to a vector $q_\beta^{(p)}$ of dependent coordinates, i.e.

$$q_\beta^{(p)} = g_\beta^{(p)}(t,q,\dot{q},\dots,q_\mu^{(p)}) \tag{2}$$

and $q=(q_\beta,q_\mu), q_\beta\in R^k, q_\mu \in R^{n-k}$. The selection is due to a designer, e.g. with respect to control inputs.

1. Construct a function P_p such that

$$P_p = \tfrac{1}{p}\left[T^{(p)} - (p+1)T_0^{(p)}\right] \tag{3}$$

and T is the kinetic energy of an unconstrained system, $T^{(p)}$ is its p-th order time derivative, and $T_0^{(p)} = \sum_{\sigma=1}^{n}\frac{\partial T}{\partial q_\sigma}q_\sigma^{(p)}$.

2. Construct a function R_p such that

$$R_p = P_p - \sum_{\sigma=1}^{n} Q_\sigma q_\sigma^{(p)} = R_p(t,q,\dot{q},\dots,q_\mu^{(p)},q_\beta^{(p)},q^{(p+1)}) \tag{4}$$

3. Construct R_p^* where equations (2) replace $q_\beta^{(p)}$

$$R_p^* = R_p^*(t,q,\dot{q},\dots,q_\mu^{(p)},q^{(p+1)}). \tag{5}$$

4. Assuming that components of external forces satisfy $\partial Q_\sigma / \partial q_\sigma^{(p)} = 0$, the generalized programmed motion equations of a system with the constraints (1) are

$$\frac{\partial R_p^*}{\partial q_\mu^{(p)}} = \frac{\partial R_p}{\partial q_\mu^{(p)}} + \sum_{\beta=1}^{k}\frac{\partial R_p}{\partial q_\beta^{(p)}}\frac{\partial g_\beta^{(p)}}{\partial q_\mu^{(p)}} = 0. \quad \mu=k+1,\dots,n \tag{6}$$

Equations (6) and (1) admit the following properties.

Property 1: Equations (6) are $(n-k)$ second order differential equations and together with (1) can be presented as [2,9]

$$\begin{aligned} &M(q)\ddot{q} + V(q,\dot{q}) + D(q) = Q(t,q,\dot{q}),\\ &B(t,q,\dot{q},\dots,q^{(p-1)})q^{(p)} + s(t,q,\dot{q},\dots,q^{(p-1)}) = 0, \end{aligned} \tag{7}$$

where $M(q)$ is a $(n-k)\times n$ inertia matrix, $V(q,\dot{q})$ is a $(n-k)$-velocity dependent vector, $D(q)$ is a $(n-k)$-vector of gravity forces, and $Q(t,q,\dot{q})$ is a $(n-k)$-vector of external forces. Equations (7) are a unified constrained dynamic model.

2: Equations (7) are in the reduced-state form; constraint reaction forces are eliminated in the derivation.

Property 3: Dynamic models of systems with constraints of order $p=1$, i.e. Lagrange's based, transformed to the reduced-state form are peculiar cases of (7) [14,16,17].

B. A Kinematic Reference Model

When the number of both material and programmed, or only programmed constraints is $k<n$, the program is partly specified. When $n=k$, i.e. B is a full rank $(n\times n)$ matrix the program is fully specified. Then, instead of the unified dynamics (7), the constraints (1) become a unified kinematic reference model, i.e.

$$B(t,q,\dot{q},\dots,q^{(p-1)})q^{(p)} + s(t,q,\dot{q},\dots,q^{(p-1)}) = 0\,. \tag{8}$$

When the unified kinematics (8) fully specifies motion, it has to be verified by analyzing its solutions if the constraints are eligible for a system, i.e. if it is capable of reaching desired positions, velocities and accelerations to follow programmed constraints, and if they do not violate any material constraint.

IV. A Unified Control Strategy for Tracking Predefined Motions

A. Constrained Motion Planning

For control purposes we introduce definitions.

Definition 2 [19]: The unified dynamic model (7) is a reference dynamic model for a constrained motion, shortly the reference dynamics.

It is the extension of models reported in [14] which apply only to holonomic and first order nonholonomic systems.

Definition 3: The unified kinematic model (8) is a reference kinematic model for a constrained motion, shortly the reference kinematics.

It is the extension of kinematic models applied to control, e.g. reported in [1,17].

The reference model, either dynamic or kinematic, may be employed to plan motion according to the constraints on a system. The selection of the scheme of the generation of the reference motion depends upon the constraints on a system.

Definition 4: Constrained motion planning for a system subjected to the constraints (1) consists in finding time histories of programmed positions $q_p(t)$ and their time derivatives in motion consistent with the constraints.

Specifically, trajectory planning consists in obtaining a solution $q_p(t)$ of (7) or (8), in which a programmed constraint equation is algebraic.

B. Constrained Motion Navigation

Originally, the reference dynamics (7) is employed to design the model reference tracking control strategy for programmed motion, shortly - the strategy for programmed motion tracking [19,20]. It may be extended to encompass the reference kinematics (8).

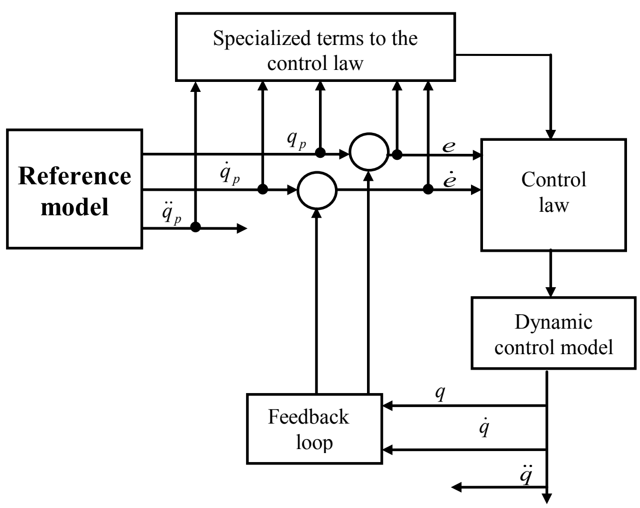

Fig.1. Architecture of the model reference tracking control strategy for programmed motion.

The control goal is as follows: *Given a programmed motion specified by the constraints (1) and the system reference dynamics (7) or kinematics (8), design a feedback controller to track the desired programmed motion.*

Architecture of the strategy, presented in Fig. 1, is based upon two models: the reference dynamics (7) or kinematics (8), whose outputs are inputs to a tracking controller, and the unified dynamic control model

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q) = \tau_p, \qquad B_1(q)\dot{q} = 0. \tag{9}$$

Equations (9) are the GPME for *p=1*. They consist of *(n-k)* equations of motion and *k* equations of material constrains and conservation laws. The matrix *M(q)* is then $(n\text{-}k)\times n$ and $B_1(q)$ is a full rank $(k\times n)$ matrix. Since the constraints are linear first order, $V(q,\dot{q})$ is replaced by $C(q,\dot{q})\dot{q}$, which quantifies effects of Coriolis and centripetal forces. Other forces can be added to the left-hand side of (9).

The following properties of (9) can be derived from properties 1-3.

Property 4: The unified dynamic control model (9) is equivalent to the reduced-state Lagrange equations [17].

Property 5: The unified dynamic control model (9) can be presented in a standard control form by reusing the constraint equations presented as $\dot{q} = G(q)\dot{q}_1$, where partition of q is $q = (q_1, q_2)$, $q_1 \in R^{n-k}$ and $q_2 \in R^k$ are vectors of independent and dependent coordinates, respectively. Columns of the matrix *G(q)* span the right null space of $B_1(q)$. It is a $(n\times m)$ matrix, $m=n\text{-}k$, and has the form

$$G = \begin{bmatrix} I_{(m\times m)} \\ -B_{12}^{-1}(q)B_{11}(q) \end{bmatrix},$$

where I is a $(m\times m)$ identity matrix, $B_{12}^{-1}(q)B_{11}(q)$ is a locally smooth $(k\times m)$ matrix function. The matrix $B_1(q)$ is expressed as $B_1 = [B_{11}(q), B_{12}(q)]$, $B_{11}(q)$ is a $k\times(n\text{-}k)$ matrix function, and $B_{12}(q)$ is a $(k\times k)$ locally nonsingular matrix function. Elimination of second order derivatives of dependent coordinates from the first of equations (9) yields

$$\overline{M}_c(q)\ddot{q}_1 + \overline{C}_c(q,\dot{q}_1)\dot{q}_1 + \overline{D}_c(q) = \tau_p, \qquad \dot{q} = G(q)\dot{q}_1, \tag{10}$$

where $\overline{M}_c(q) = M(q)G(q)$, $\overline{D}_c(q) = D(q)$, $\overline{C}_c(q,\dot{q}_1) = M(q)\dot{G}(q) + C(q,\dot{q})G(q)$.

Equations (10) are exactly the reduced-state Lagrange equations of a nonholonomic system.

Property 6: $\overline{M}_c(q)$ is a symmetric, positive definite $(n\text{-}k)\times(n\text{-}k)$ matrix [14,17].

Property 7: There exists a static state feedback $U(\dot{q}_1, q, u)$: $R^m\times R^n\times R^m \to R^m$ such that the dynamics (10) can be transformed to the state-space control form. Indeed, introduce a new state variable $x = (q,\dot{q}_1) = (x_1, x_2)$, $x_1 \in R^n$, $x_2 \in R^m$, for which (10) takes the form

$$\overline{M}_c(x_1)\dot{x}_2 + \overline{C}_c(x_1,\dot{x}_1)x_2 + \overline{D}_c(x_1) = \tau_p, \qquad \dot{x}_1 = G(x_1)x_2. \tag{11}$$

Selecting $U(x_2, x_1, u)$: $R^m\times R^n\times R^m \to R^m$ as $\overline{M}_c(x_1)u + \overline{C}_c(x_1,\dot{x}_1)x_2 + \overline{D}_c(x_1) = \tau_p$, (11) yields

$$\dot{x}_1 = G(x_1)x_2, \qquad \dot{x}_2 = u, \tag{12}$$

which is a desirable state-space control form. The controlled variable x_2 is usually a vector of controlled velocities.

Property 8: Based on properties 4-7, all theoretical control results obtained for the Lagrange based control dynamics can be applied to the unified control dynamics (7).

The strategy is not sensitive to the constraint order. This is in contrast to current control design approaches, in which each constraint type requires a control strategy modification.

The strategy may be specialized in two ways. Firstly, it is applicable to systems with completely known or uncertain dynamics [20]. Secondly, different control laws may be employed to it, i.e. we may switch between controllers to ensure a desired tracking control precision. The block of “specialized terms to the control law” reflects these specializations. The modular strategy architecture enables replacing the reference dynamics (7) by the reference kinematics (8). The strategy is developed for tracking but it may be applied to more general tasks, e.g. to navigation robot formations [21].

Main advantages of the strategy are as follows:

- The reference dynamics (7) captures high order nonholonomic constraints on systems and enables planning any programmed motion.
- It extends trajectory tracking to programmed motion tracking.
- The separation of programmed constraints from others results in the unified dynamic control model (9) equivalent to models actually used in control theory.
- The equivalence of (9) and the Lagrange based models promotes adaptation of existing control algorithms even these dedicated to holonomic systems.
- It uses one dynamic control model (9) to a system subjected to the constraints (1).
- A library of reference models for different tasks can be generated off-line and stored in a computer.

V. Examples – Constrained Motion Navigation

A. Material and Task Based Constraints

Consider a two-wheeled robot whose kinematics is equivalent to that of a unicycle. Let φ be the heading angle of the wheel, measured from the axis x and θ - rotation angle due to rolling. Coordinates of the wheel contact point with the ground are (x,y). Nonholonomic material constraints due to rolling the wheels without slipping on a plane surface are

$$\dot{x} - r\dot{\theta}\cos\varphi = 0,\;\; \dot{y} - r\dot{\theta}\sin\varphi = 0\,. \tag{13}$$

To show the GPME based Algorithm application, consider robot navigation along a trajectory of a specified change of its curvature profile. It results in the constraint

$$\dot{\kappa} = F_0 + \frac{\dot{x}\dddot{y} - \dddot{x}\dot{y}}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}\,, \tag{14}$$

where F_0 does not contain terms with third order time derivatives of variables. For simulations take the curvature profile $\kappa = 2\sin t + 1$. Both constraints (13) and (14) are transformed to the form (1). The reference dynamics (7) is derived using the Algorithm for $p=3$. The control dynamics (9) is derived for $p=1$ and it takes the material constraints (13) into account. Assuming that only control forces act upon the robot, its motion according to (13) and (14) is presented in Fig. 2. The controller is the computed torque.

For the program specified by the third order differential equation (14), the Lagrange-based approach fails [2,9]. Different task based constraints and control laws can be applied to navigate the robot with no changes in the strategy.

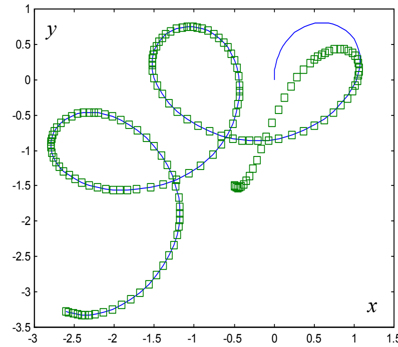

Fig. 2. Programmed motion tracking for the reference motion (14) with σ=3: reference motion (~), controlled motion (□□□).

B. Constraints on a Holonomic System

Consider a two-link planar manipulator model whose two degrees of freedom are described by joint angles Θ_1, Θ_2. Select the constraint (14) for the end-effector motion. In the joint space it has the form

$$\dddot{\Theta}_2 = F_1 - F_2\dddot{\Theta}_1, \tag{15}$$

where F_1 and F_2 do not contain third time derivatives of the angles and include data about the end-effector trajectory curvature κ, which is $\kappa = 0.6 + 0.02t$. The constraint (15) may mimic tasks like writing, scribing or painting. The reference dynamics is generated applying the Algorithm for $p=3$ and the control dynamics is developed as for any holonomic system. Fig. 3 shows the reference motion on the (x,y) plane. It was selected to show that the programmed motion may be reachable for the end-effector for some time only. After reaching the position marked by the arrow, links of a given length cannot follow the program any more. This demonstrates that a program formulated for a system should be inspected via the reference motion outputs.

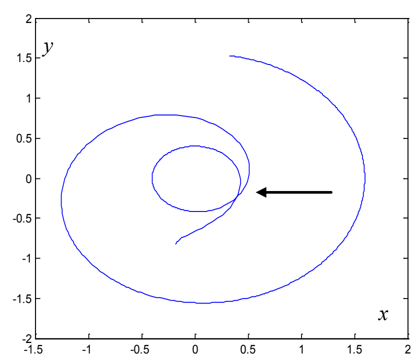

Fig. 3. Reference programmed motion for the end-effector.

C. Constraints from an Underactuation

Consider again the manipulator model from Example *B*. It is now equipped with one actuator in the first joint. A control objective is to move the end-effector according to a programmed motion specified by (15). The reference dynamics for the underactuated manipulator is the same as in Example *B* but its control model is

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{\Theta}_1 \\ \ddot{\Theta}_2 \end{bmatrix}+\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \end{bmatrix}=\begin{bmatrix} \tau_1 \\ 0 \end{bmatrix}, \tag{16}$$

The equation for the unactuated joint is second order nonholonomic, since Θ_2 is present in the inertia matrix [22]. From the first of equations (16) $\ddot{\Theta}_2$ may be obtained as

$$\ddot{\Theta}_2=-\frac{1}{\delta}\left[(\delta+\beta\cos\Theta_2)\ddot{\Theta}_1+\beta\sin\Theta_2\dot{\Theta}_1^2\right]$$

and inserted to the first one yields

$$\left[(\alpha+2\beta\cos\Theta_2)-\frac{1}{\delta}(\delta+\beta\cos\Theta_2)^2\right]\ddot{\Theta}_1- \\ \dot{\Theta}_2\beta\sin\Theta_2(2\dot{\Theta}_1+\dot{\Theta}_2)-\frac{\dot{\Theta}_1^2\beta\sin\Theta_2}{\delta}(\delta+\beta\cos\Theta_2)=\tau_1. \tag{17}$$

Using the partial feedback linearizing controller

$$\tau_1=\left[(\alpha+2\beta\cos\Theta_2)-\frac{1}{\delta}(\delta+\beta\cos\Theta_2)^2\right]u- \\ \left[\dot{\Theta}_2\beta\sin\Theta_2(2\dot{\Theta}_1+\dot{\Theta}_2)+\frac{\dot{\Theta}_1^2\beta\sin\Theta_2}{\delta}(\delta+\beta\cos\Theta_2)\right]$$

equations (16) become

$$\begin{aligned} \ddot{\Theta}_1&=u, \\ \ddot{\Theta}_2&=-\frac{1}{\delta}(\delta+\beta\cos\Theta_2)\ddot{\Theta}_1-\frac{1}{\delta}\beta\sin\Theta_2\dot{\Theta}_1^2. \end{aligned} \tag{18}$$

Equations (18) can be expressed in the state space control form. Defining $x_1=\Theta_1, x_2=\Theta_2, x_3=\dot{\Theta}_1, x_4=\dot{\Theta}_2$, we obtain

$$\begin{aligned} \dot{x}_1&=x_3, \\ \dot{x}_2&=x_4, \\ \dot{x}_3&=u, \\ \dot{x}_4&=-\frac{x_3^2\beta\sin x_2}{\delta}-\frac{1}{\delta}(\delta+\beta\cos x_2)u, \end{aligned} \tag{19}$$

where $f(x)=(x_3,x_4,0,-x_3^2\beta\sin x_2/\delta)$, $g(x)=(0,0,e_1,-(\delta+\beta\cos x_2)/\delta)$, with e_1 - the standard basis vector in R^1, are the drift and control vector fields on $\Omega=(-\pi/2,\pi/2)\times(-\pi/2,\pi/2)\times R^2$. The selected controller is PD with gains $k_s=20$, $k_d=10$. Tracking results are presented in Fig. 4 and 5.

D. Conservation Laws

Consider a model of a space manipulator. It is the same as in Example *B* with a base added to it, which is described by a moment of inertia J and ϕ - orientation angle relative to a fixed axis. Let θ_1 be the angle of the first link of mass m_1 and length l_1 relative to the base, and θ_2 - the angle of the second link of mass m_2 and length l_2 relative to the first. Masses are concentrated at link ends. The base is pinned to the ground at its center and it permits the body to rotate freely but prevents translation. Holonomic constraints arising from the linear momentum conservation in a real space manipulator are replaced with holonomic pinned constraints.

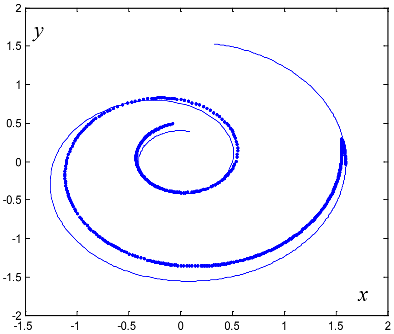

Fig. 4. Programmed motion tracking by the PD controller.

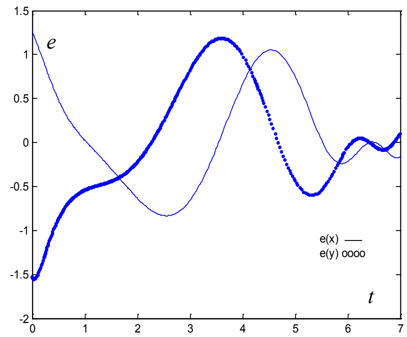

Fig. 5. End-effector position tracking errors e_x , e_y.

When the angular momentum is conserved, e.g. it is zero, it become a nonholonomic constraint of the form

$$[J+(m_1+m_2)l_1^2+m_2l_2^2]\dot{\phi}+[(m_1+m_2)l_1^2+m_2l_2^2]\dot{\theta}_1 \\ +m_2l_2^2\dot{\theta}_2+m_2l_1l_2\cos\theta_2(2\dot{\phi}+2\dot{\theta}_1+\dot{\theta}_2)=0. \tag{20}$$

The structure of (20) is the same as the material constraint (13). Motion planning for the space manipulator as well as its navigation in space can be done in the same way as in Example *A*; for a case of trajectory tracking see [23].

E. A Multibody Nonholonomic System

A leader-follower system is usually treated as a separate control system comparing to a single robot. Let us show that it may be modeled and controlled in the same way as other constrained systems. Take a leader, which is the robot as in Example *A*. Two followers are robots of the same kinematics. Nonholonomic material constraints for the leader and followers are specified by (13) and the task based by (14) for $\kappa=2sint+1$. Using the Algorithm for *p=3* and the strategy for programmed motion tracking, we obtain robot formation navigation presented in Fig. 6.

F. A Fully Specified Program

Consider a task of navigating a unicycle from Example *A* along a desired trajectory to a rest position. To this end, supplement the constraints (13) by one equation that specifies the trajectory, e.g.

$$\varphi(t)=\cos(0.5\pi-t) \tag{21}$$

and the second for the termination of motion after a specified time, say 20s. Select an initial velocity $v=10$ m/s and $v=f(t)$. The reference kinematics is

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\\\dot{\varphi}\\v\end{bmatrix}=\begin{bmatrix}v\cos\varphi\\v\sin\varphi\\\cos t\\-\sin t\\f(t)\end{bmatrix} \tag{22}$$

where v is an extra state. Fig. 7 and 8 present navigation according to (22) using the Wen-Bayard controller.

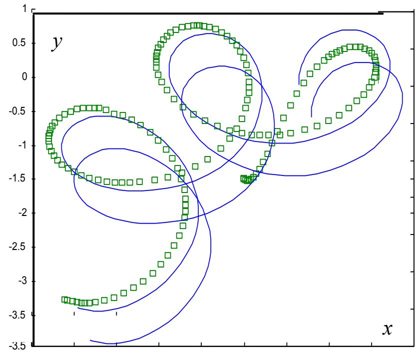

Fig. 6. Navigation of a robot formation for $\kappa = 2\sin t + 1$. Squared line - leader motion, solid line – motion of the followers.

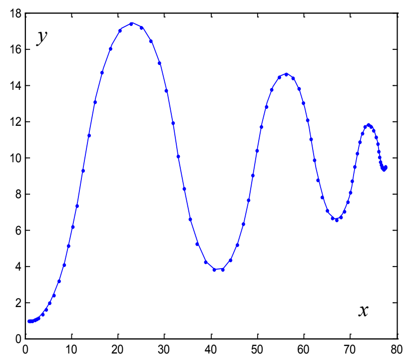

Fig. 7. Programmed motion tracking by the Wen-Bayard controller(•••).

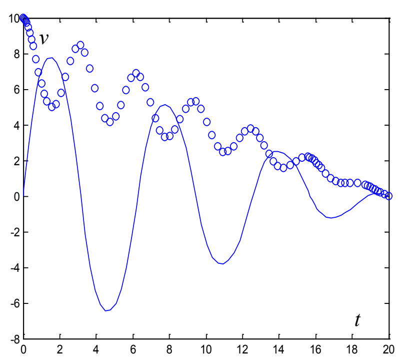

Fig. 8. Unicycle velocity components v_x (––), v_y (ooo) vs. time.

VI. Conclusions

The paper presents the unified nonlinear control strategy for tracking motions specified by the equations of constraints referred to as programmed. The GPME modeling method is applied to design the strategy. The strategy is not sensitive to the kind and order of constraints put upon systems. Control laws dedicated to holonomic and nonholonomic systems may be employed in it. The tracking strategy surpasses other tracking strategies, since tracking any motion specified by equations of constraints of arbitrary order is available.

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