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A Two-Zone Control Oriented SI-HCCI Hybrid Combustion Model for the HIL Engine Simulation

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Abstract: **To make HCCI (Homogeneous Charge Compression Ignition) combustion feasible in a SI (Spark Ignition) engine, it is necessary to have the smooth mode transition between SI and HCCI combustions. The SI-HCCI hybrid combustion studied in this paper describes the combustion mode that starts with SI and ends with HCCI combustions. The main motivation of studying the hybrid combustion mode is that the percentage of the HCCI combustion is a good indicator for combustion mode transition control when the hybrid combustion mode is used during the transition. This paper presents a control oriented model for the SI-HCCI hybrid combustion mode, where the SI combustion phase is modeled under a two-zone assumption and the HCCI combustion phase under a one-zone one. Note that SI and HCCI combustions are special cases in this model. The developed model is capable of simulating the entire engine operating range, and it was validated in a HIL (Hardware-In-the-Loop) simulation environment. The simulation results were compared with those of the corresponding GT-Power model, and good correlations were found for both SI and HCCI combustions.**

I. Introduction

THE continuing pursuit of improving fuel economy and the increasingly stringent emission regulations rekindle research interest in the homogeneous charge compression ignition (HCCI) combustion in recent years. The flameless nature of the HCCI combustion and its high dilution operation capability lead to low combustion temperature. As a result, the formation of NOx (Nitrogen Oxides) can be significantly reduced. Furthermore HCCI engine is capable of un-throttled operation that greatly reduces pumping loss and improves fuel economy [1].

On the other hand, HCCI combustion has its own limitations. The HCCI combustion is limited at high engine load due to the audible knock, and at low load due to misfire caused by lack of sufficient thermal energy to trigger the auto-ignition of the gas-fuel mixture late in the compression stroke [2]. In order to take advantages of the HCCI combustion mode in an internal combustion engine, other combustion mode, such as SI (Spark Ignition) combustion, is needed at high load, at ultra-low load (such as idle), and at certain operational conditions such as cold start and at high engine speed. However, it is fairly challenging to operate the engine in two distinct combustion modes, and it is even more difficult to have smooth combustion mode transition between SI and HCCI combustions [3], since the favorable thermo conditions for one combustion mode are always adverse to the other one. Due to the significant response delay of the hydraulic and electric variable valve timing (VVT) system, cycle-to-cycle residue gas dynamics and response delay in engine air handling system, it is almost impossible to achieve desired thermo conditions for combustion mode switch in one engine cycle [4].

To address this issue, SI-HCCI hybrid combustion mode has been proposed in [1], [5], and [6]. The hybrid combustion starts in SI combustion mode with a relatively low heat release rate; and once the thermo and chemical conditions of the unburned gas satisfy the start of HCCI (SOHCCI) combustion criteria, the combustion continues in HCCI combustion mode, which is illustrated by the dashed curves of mass fraction burned (MFB) shown in Figure 1. During an ideal SI to HCCI combustion transition process, the percentage of SI combustion (ST to SOHCCI, see Figure 1) decreases gradually while the HCCI combustion percentage (SOHCCI to end of combustion or MFB = 1) increases gradually. For the HCCI to SI combustion transition, the process is reversed. This hybrid combustion mode transition process allows using conventional cam phase system.

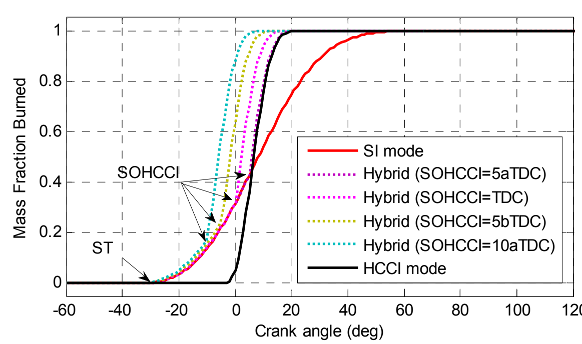

Figure 1: MFB in SI-HCCI hybrid combustion mode transition process

Using the SI-HCCI hybrid combustion mode, smooth mode transition can be realized along with the appropriate control strategy. To accurately control the mode transition between SI and HCCI combustions, a precise combustion model is required, and it needs to be control oriented. Multi-zone, three dimensional CFD models with detailed chemical kinetics are presented in [7], and they are able to precisely describe the thermodynamic, fluid-flow, heat transfer, and pollutant formation phenomena of the HCCI combustion. Similar combustion models have also been

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implemented into commercial codes such as GT-Power and Wave. However, these models with high fidelity cannot be used for control strategy development since they are too complicated to be used for real-time simulation, but they can be used as reference models for developing simplified (or control oriented) combustion models.

In this paper, similar to [1] and [8], mean-value method is used to model averaged chemical kinetics and thermodynamic properties, and the model calculation is updated at every crank degree to have the resolution high enough for control strategy development. In [1], the SI combustion process was modeled using a one-zone approach, which introduces large unburned gas temperature estimation error. To address the issue, this paper presents a control oriented model for the SI-HCCI hybrid combustion mode, where the SI combustion phase is modeled under a two-zone assumption and the HCCI combustion phase under a one-zone one. Simulation results show significant modeling error reduction, comparing with that of the one-zone SI combustion model. Also, the developed hybrid combustion model can be executed in real-time due to the low computational requirement.

II. Model Architecture

The purpose of the combustion modeling is to correlate the trapped in-cylinder gas properties, such as air-to-fuel ratio, in-cylinder gas pressure and temperature, to the combustion characteristics such as misfire, knock, burn duration, and IMEP (Indicated Mean Effective Pressure). The developed combustion model will be used for model-based control strategy development and validation of the mode transition between SI and HCCI combustions into the real-time HIL simulation environment.

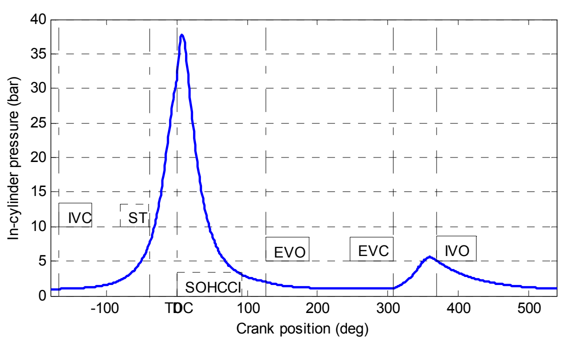

Figure 2: Combustion related events and phases

As mentioned in the last section, the combustion model is updated every crank degree, which is different from other modeling approaches presented in [8] and [9], where the model is updated at a fixed time step. There are many motivations of using the crank-based approach. The first is due to the fact that most combustion characteristics are usually function of the crank angle, such as burn duration and peak pressure location; Secondly, the entire combustion process is divided into several combustion phases related to these events associated with crank position. As shown in Figure 2, these events are intake valve closing (IVC), spark timing (ST), SOHCCI, exhaust valve opening (EVO), exhaust valve closing (EVC), and intake valve opening (IVO). The in-cylinder behaviors (such as pressure, temperature, etc.) are modeled differently during each combustion phase that is defined between two combustion events.

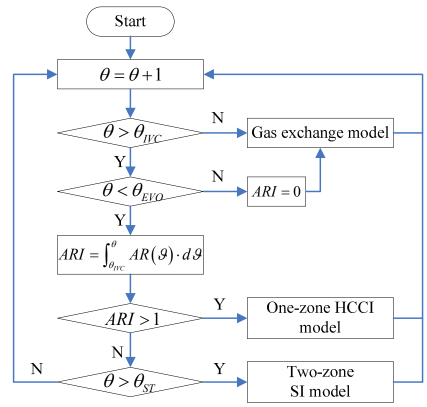

Figure 3: Combustion modeling diagram

The crank-based modeling approach has its own limitation, too. During the real-time simulation the entire model needs to be executed within the time period associated with one crank degree. This leads to short computational time window at high engine speed. For example, at 6000 rpm of engine speed one crank degree corresponds to 28 micro seconds. In order to avoid the overrun in the simulation, the combustion model must be as simple as possible but with required accuracy.

In Figure 2, ST and SOHCCI are marked in the boxes with dashed box to distinguish them from other events, since the occurrences of these two events depend on combustion mode. The rest of events exist for any 4-stroke internal combustion engine. Accordingly, the main difference among each combustion mode lies in the phase between IVC and EVO. When the SOHCCI event does not occur between IVC and EVO events, the engine is operated at the SI combustion mode; and when the SOHCCI event occurs before the SI event, the engine is operated at the HCCI combustion mode; while the SI-HCCI hybrid combustion occurs inter-between the HCCI and SI combustion modes. Note that both SI and HCCI combustion modes are special cases of the SI-HCCI hybrid combustion mode. The combustion modeling architecture is illustrated in the diagram shown in Figure 3. It illustrates the relationship among all sub-system models. The detailed model of each combustion mode and gas exchange will be discussed in the following sections.

III. Two-zone SI Combustion Model

During the combustion the spark ignited flame front divides the in-cylinder gas mixture into two zones, the burned and unburned zones as shown in Figure 4. The temperature of the unburned zone is quite different from that of burned zone as seen in Figure 6 and Figure 7. However, this difference is ignored in one-zone SI combustion model discussed in [1], and an averaged value of temperature is used for both zones. This may not lead to large modeling error for SI combustion, in which the temperature difference does not impact much on the combustion model parameters such as in-cylinder pressure. However, for the SI-HCCI hybrid combustion, the unburned zone temperature is a key parameter for predicting the start of HCCI combustion, and it is the motivation for developing this two-zone combustion model.

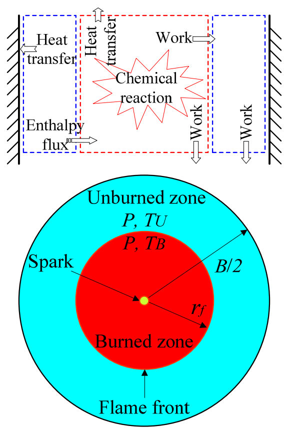

Figure 4: Two-zone combustion model

For simplicity, in the two-zone model, gas mixture pressure is assumed to be evenly distributed throughout both burned and unburned zones. Inertial gas is assumed to remain in the unburned zone after spark ignition. The fuel, air, and residue gas charge are uniformly premixed. The fuel burn rate is approximated by the Wiebe function [10]:

$$x(\theta_i)=1-e^{-a\left(\frac{\theta-\theta_{ST}}{\Delta\theta}\right)^{m+1}} \tag{1}$$

where x is the mass fraction burned (MFB); θ is the current crank position; the predicted burn duration $\Delta\theta$ and Weibe exponent m are calibration parameters of engine speed, load and coolant temperature. Coefficient a depends on how the burn duration $\Delta\theta$ is defined. If $\Delta\theta$ is specified as 10% to 90% MFB, a is calculated by

$$a=\left\{\left[-\ln(1-0.9)\right]^{\frac{1}{m+1}}-\left[-\ln(1-0.1)\right]^{\frac{1}{m+1}}\right\}^{m+1} \tag{2}$$

During the combustion phase between IVC and ST, in-cylinder gas is under isentropic compression, and the governing equations can be found in [1] and [11]. After SI combustion starts, the mass of burned zone is calculated based upon the mass of burned fuel that can be calculated from (1), assuming that the burned zone air-to-fuel ratio is known. According to the first law of thermodynamics [17], the energy balance equation of burned zone is represented by

$$\frac{d(M_B\cdot e_B)}{d\theta}+P\frac{dV_B}{d\theta}+Q_B=M_f\frac{dx}{d\theta}h_{LHV}\cdot\eta_{SI}+\frac{dM_B}{d\theta}h_U \tag{3}$$

where M_B, V_B, and e_B are the mass, volume, and internal energy of burned zone; Q_B is the heat transfer from the burned zone; M_f is the mass of fuel injected into each cylinder for every engine cycle; P is the gas pressure of both zones; h_{LHV} is the lower heat value of fuel; h_U is the specific enthalpy of unburned zone; and η_{SI} is the combustion efficiency due to incomplete combustion.

The energy balance equation of unburned zone is

$$\frac{d(M_U\cdot e_U)}{d\theta}+P\frac{dV_U}{d\theta}+Q_U=\frac{dM_U}{d\theta}h_U \tag{4}$$

where M_U, V_U, and e_U are the mass, volume and internal energy of burned zone, respectively; and Q_U is the heat transfer from the burned zone.

Moreover, the gas of both burned and unburned zones can be considered as ideal gas [12], therefore ideal gas law holds for both zones. For burned zone, we have

$$\frac{P\cdot V_B}{R\cdot T_B}=M_B=x\cdot M_f\cdot(1+\sigma_0) \tag{5}$$

where σ_0 is the stoichiometric air-to-fuel ratio of the fuel; T_B is the burned zone gas temperature; and R is gas constant.

For unburned zone, the ideal gas law can be represented by

$$\frac{P\cdot V_U}{R\cdot T_U}=M_U=M_{IVC}-x\cdot M_f\cdot(1+\sigma_0) \tag{6}$$

where T_U is the unburned zone gas temperature; M_{IVC} is the gas mass at the crank position of IVC. Note that it is the total gas mass during the phase of IVC-EVO.

Additionally, the cylinder geometry yields

$$V_B+V_U=V \tag{7}$$

where V is the cylinder volume.

For a SI gasoline engine the heat transfer due to radiation is relatively small in comparison with convective heat transfer [10]. Thereby only convective heat transfer is counted in the energy balance equations, and Woschni correlation model [14] is used to calculate Q_B and Q_U in (3) and (4).

$$Q=3.26\cdot B^{m-1}\cdot P^m\cdot w^m\cdot T^{0.75-1.62m}\cdot A\cdot(T-T_W) \tag{8}$$

where B is cylinder bore; w is gas flow speed that is a function of engine speed; A is the contact area between gas and cylinder wall; T_W is average temperature of cylinder wall; and exponent m is used for correlation, where m=0.8 gives best correlation for the combustion mode.

In the five equations from (3) to (7), there are five unknowns: T_B, T_U, V_B, V_U, and P. Due to the nonlinearity in equations (5) and (6), the solutions are not unique and the rational solution is adopted in the model simulation.

IV. One-zone Combustion Model for HCCI Mode

The most common practice in control oriented modeling of HCCI combustion is to assume the combustion process, that converts the fuel and oxidizer into combustion products, is governed by single rate equation of an Arrhenius form [10]

$$AR=A\cdot x_f^a\cdot x_{ox}^b\cdot e^{\left(-\frac{E_a}{R\cdot T}\right)} \tag{9}$$

where AR is the rate of disappearance of unburned fuel; x_f and x_{ox} are unburned fuel and oxidizer mass fractions; exponents a and b are usually selected to be unity; multiplier A and Arrhenius activation energy E_a can be obtained by matching the experimentally determined rates of burning.

Based on [9] and [13], the integral of the Arrhenius function can be used to estimate the crank position, θ_{SOHCCI}, of SOHCCI, The SOHCCI crank position is defined as the crank angle for 1% fuel burned under HCCI combustion, and it can be represented by

$$ARI = \int_{\theta_{IVC}}^{\theta_i} A \cdot x_f^a \cdot x_{ox}^b \cdot e^{\left(-\frac{E_a}{R \cdot T}\right)} d\vartheta \qquad (10)$$

The Arrhenius integration in (10) starts at θ_{IVC}, and is reset at θ_{EVO}. During this period, if spark ignition never happens, the variable T in (10) equals to in-cylinder gas temperature; during the SI combustion phase of the SI-HCCI hybrid combustion mode, only unburned zone temperature is used for T after spark ignition. Once the HCCI combustion is triggered, the fuel burn rate is also modeled using Wiebe function in (1), but exponent m and predicted burn duration $\Delta\theta$ will switch to the calibrations of HCCI mode.

The HCCI combustion is modeled under the one-zone assumption due to its flameless nature. Fuel and air are assumed to be premixed homogeneously throughout the whole cylinder; thermodynamic characteristics such as pressure and temperature are uniformly distributed in the cylinder, and they can be calculated using the simplified dynamics shown in equations (11) and (12) below.

$$T(\theta_i) = T(\theta_{i-1}) \cdot \left(\frac{V(\theta_{i-1})}{V(\theta_i)}\right)^{(\kappa-1)} + \eta_{HCCI} \cdot \frac{M_f \cdot h_{LHV} \cdot \left[x(\theta_i) - x(\theta_{i-1})\right]}{M_{IVC} \cdot C_v} \qquad (11)$$

and

$$P(\theta_i) = P(\theta_{i-1}) \cdot \frac{V(\theta_{i-1})}{V(\theta_i)} \cdot \frac{T(\theta_i)}{T(\theta_{i-1})} \qquad (12)$$

where η_{HCCI} is a scaling factor due to incomplete combustion; κ is the average heat capacity ratio of the cylinder charge.

There are two terms in the right hand of equation (11). The first term represents an isentropic compressing or expanding process. While the second term calculates the temperature rise due to the heat released during the combustion. Therefore, the complicated thermodynamic process of the combustion is simplified into an isentropic volume change process without heat exchange in one crank degree period with heat absorption from combusted fuel without volume change in an infinitely small time period.

V. Models for Gas Exchange Phases

The gas exchange process (from EVO to IVC in Figure 2) was modeled by combined correlation and physical modeling approach. It's simple but with fairly high fidelity.

A. Gas Exhaust Process Mode

Assume that the gas exit from cylinder to the exhaust manifold during the EVO-EVC phase for the negative valve overlap (NVO) or during the EVO-IVO for the positive valve overlap (PVO). In this phase, the in-cylinder gas mixture is assumed to isentropically expand in the cylinder, exhaust runner and manifold. The in-cylinder pressure drops quickly but not instantaneously down to the level of the exhaust manifold pressure. It normally takes a few crank degrees for the in-cylinder pressure to approach the exhaust manifold pressure. It is difficult to model this dynamics using simple dynamic equations for real-time simulations. For simplicity, a first order transfer function is used to approximate this dynamic process as:

$$P(z) = \frac{1 - \tau_{EVO}}{1 - \tau_{EVO} \cdot z^{-1}} \cdot P_{EM}(z) \qquad (13)$$

where z is the unit delay operator; τ_{EVO} is the transition time constant for exhaust valve opening; and P_{EM} is the exhaust manifold absolute pressure.

As an isentropic process, the in-cylinder gas temperature can be calculated as a function of the pressure drop. That is:

$$T(\theta_i) = T(\theta_{i-1}) \cdot \left[\frac{P(\theta_i)}{P(\theta_{i-1})}\right]^{\frac{\kappa-1}{\kappa}} \qquad (14)$$

B. Recompression Model for Negative Valve Overlap

Negative valve overlap (NVO) is often used to regulate the HCCI combustions. There are two main advantages. One is to decompose the pilot fuel injected in this phase (EVC-IVO in Figure 2) into short carbon chain molecules [15]; and the other is to adjust the residue gas temperature. As a result the in-cylinder gas temperature at IVC can be optimized for desired SOC of the HCCI combustion. The first effect can hardly be modeled using governing equations. In [7] and [15], the Arrhenius integration is correlated to the experimental data. This paper ignores this effect until experimental data is available. For the second one, it can be approximated to an isentropic volume change process of ideal gas in a closed system. Temperature and pressure are calculated by

$$T(\theta_i) = T(\theta_{i-1}) \cdot \left(\frac{V(\theta_{i-1})}{V(\theta_i)}\right)^{(\kappa-1)} \qquad (15)$$

and

$$P(\theta_i) = P(\theta_{i-1}) \cdot \left(\frac{V(\theta_{i-1})}{V(\theta_i)}\right)^{\kappa} \qquad (16)$$

Another important parameter calculated in this phase is the residue gas mass. It is calculated based upon ideal gas law, and updated once per engine cycle at EVC for NVO.

$$M_r = M(\theta_{EVC}) = \frac{P(\theta_{EVC}) \cdot V(\theta_{EVC})}{R \cdot T(\theta_{EVC})} \cdot \eta_r \qquad (17)$$

where η_r is the discharge coefficient.

C. Gas Exchange for Positive Valve Overlap

The gas exchange behavior during the phase from IVO to EVC for the PVO is also very complicated. Since both intake and exhaust valves are partially opened in this phase, residue gas and fresh charge can flow in many ways depending on the pressure ratio across each valve. For simplification, assume gas exchange across exhaust valve is terminated in this phase; part of residue gas can flow to intake port, but finally it flows back into cylinder during the rest of the intake process. Based upon this assumption, the mass of residue gas for PVO can be calculated by equation (18) at IVO instead of EVC for NVO, and gas exchange of this phase is actually a part of the air intake process.

$$M_r = M(\theta_{IVO}) = \frac{P(\theta_{IVO}) \cdot V(\theta_{IVO})}{R \cdot T(\theta_{IVO})} \cdot \eta_r \tag{18}$$

This modeling approach may result in slight error for M_r. Fortunately, the PVO only occurs in case of SI combustion process, in which the influence of residue gas to the whole engine performance is much less than that during the HCCI combustion process which usually involves the NVO strategy. On the other hand, the multiplier η_r can also be calibrated to match the test data or the high resolution simulation data. For this paper the GT-Power simulation data is used to calibrate the entire combustion model.

D. Air-intake Process Model

The air-intake process from IVO to IVC is also a process of in-cylinder gases mixing. During this phase the fresh charged air, injected fuel vapor, and residue gas are assumed to be mixed homogeneously, which is an assumed condition for combustion. At the same time, the in-cylinder pressure drops to the pressure level of intake manifold [16]. A first order transfer function is also used in this model.

$$P(z) = \frac{1-\tau_{IVO}}{1-\tau_{IVO} \cdot z^{-1}} \cdot P_{IM}(z) \tag{19}$$

and

$$T(\theta_i) = \frac{M_f \cdot T_f + M_a \cdot T_{IM} + M_r \cdot T_r}{M_f + M_a + M_r} \tag{20}$$

where τ_{EVO} is the transition time constant for intake valve opening; P_{EM} is the intake manifold absolute pressure; M_a is the fresh charge mass (note that it could be the mixture of fresh air and external EGR gas); T_r is the temperature of residue gas. Assuming that the residue gas expanses isentropically throughout the cylinder and intake port it can be calculated using equation (14).

At the end of air-intake process, which is at IVC, the total in-cylinder gas mass M_t needs to be calculated. That is,

$$M_t = M(\theta_{IVC}) = \frac{P(\theta_{IVC}) \cdot V(\theta_{IVC})}{R \cdot T(\theta_{IVC})} \cdot \eta_t \tag{21}$$

VI. Model Calibration and Validation

Equations (1) to (21) were programmed into an *S*-function in Simulink using C language. The developed Simulink model was calibrated by the simulation data of the GT-Power combustion model. The setup of the GT-Power model is the same in the Simulink model. For both models the combustion related engine parameters are the same and listed in Table 1.

Calibrations were generated by sweeping the engine speed and load within the engine operating range. For each test condition, GT-Power simulation results were used as the baseline. Firstly, η_r and η_t were calibrated to make the residue gas and total charge quantities match the GT-Power simulation results; secondly, coefficients such as m and k were calibrated to make MFB profile match; at last, η_{SI} or η_{HCCI} were adjusted to make IMEP match.

Table 1: Combustion related engine specification

Parameter	Model value
bore/stroke/con-rod length	86mm/86mm/143.6mm
compression ratio	9.8:1
intake valve/exhaust valve diameters	31.4mm/22.5mm
Intake / exhaust valve lifts of high stage	8.8mm/8.75mm
Intake / exhaust valve lifts of low stage	4.8mm/4.75mm

The one-zone combustion model for SI and HCCI combustion has been validated during previous work in [1] and [11]. In this paper, the two-zone SI combustion model is mainly investigated. Key combustion related parameters obtained from both GT-Power and the two-zone models were plotted for comparison and shown in Figure 5 to Figure 7.

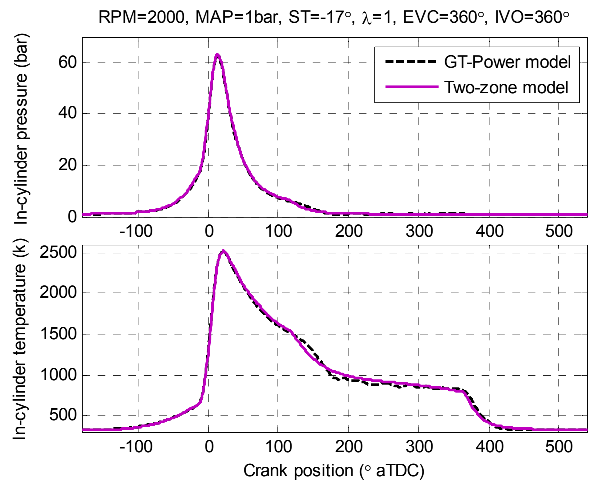

Figure 5: In-cylinder pressure and temperature traces of SI combustion

Good correlations have been demonstrated in Figure 5 for the in-cylinder pressure and temperature. Note that the temperature in Figure 5 is the average temperature of both burned and unburned zone temperatures. In Figure 6 and Figure 7, the burned and unburned zone temperatures are plotted separately for both GT-Power and two-zone models and the correlations are also fairly good.

In Figure 7, after spark ignition (-17^{o} aTDC in this case) the unburned zone temperatures of both GT-Power model and the two-zone model are higher than the in-cylinder gas temperature without combustion, which is calculated by setting T_{IVC} to be same as that in the simulation with combustion and injecting no fuel. This indicates that the burned zone is always pushing the unburned zone during the SI combustion phase and applying work to it, resulting fast increment of the unburned zone temperature. Based on equation (10), the temperature increment advances SOHCCI and triggers HCCI combustion in it. This is the main objective of the SI-HCCI hybrid combustion mode.

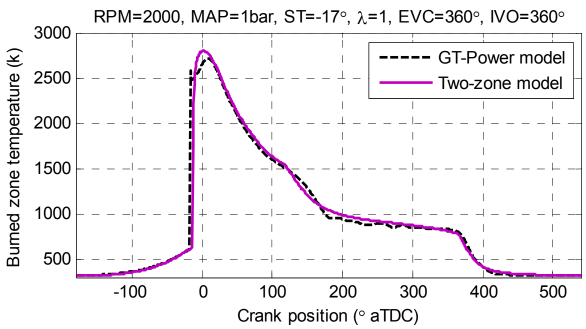

Figure 6: Burned zone temperature traces

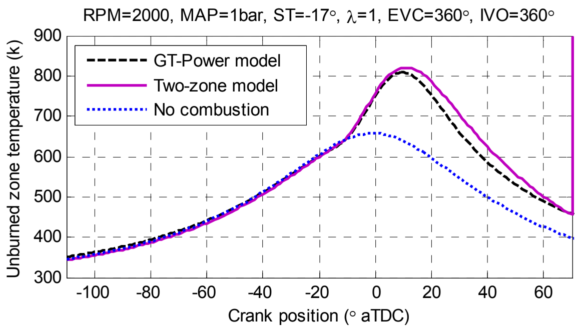

Figure 7: Unburned zone temperature trace

The comparison data shown in Figure 5 to Figure 7 are for one engine operating condition. For the entire engine operating range, several key combustion parameters were selected for comparison in Figure 8. Two load conditions are investigated at MAP = 0.4 bar for low load condition and MAP = 1 bar for full load condition. For each condition, engine speed sweeps from 1000 rpm to 5000 rpm with 1000 rpm increment interval. Fairly good correlations are shown in Figure 8, which indicates that the two-zone model is accurate for modeling SI combustion process.

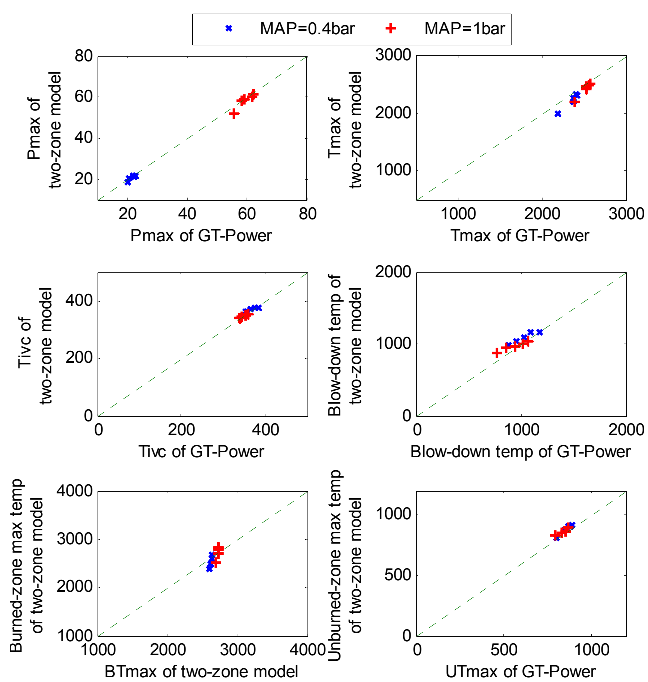

Figure 8: Correlation plots of GT-Power and the two-zone SI model

VII. Conclusions

This paper presents a control oriented combustion model for a HCCI (homogeneously charge compression ignition) capable SI (spark ignited) engine. The SI combustion was modeled using the two-zone method and HCCI combustion is under one-zone assumption. The developed control oriented model can be used to simulate not only the SI and HCCI combustions but also the spark assisted HCCI combustion or the hybrid combustion. For SI and HCCI combustion modes, the model has also been calibrated and validated using the corresponding GT-Power simulation results.

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