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Density-based Control of Multiple Robots

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Abstract— **In recent years, the Smoothed Particle Hydrodynamic (SPH) method has been successfully applied to model swarm robotic systems as incompressible/compressible fluids. Essentially, the SPH approach models inter-robot interactions using attraction-repulsion force profiles and in this respect is reminiscent of traditional analytical frameworks used in swarm systems such as Artificial Potential Field based methods. However, in contrast to other virtual force based approaches, the SPH method provides a much more effective way to control the density of the robots; a particularly useful feature in several applications of swarm systems including pattern generation and coverage control. In this paper, we revisit the SPH method from a control point of view with an emphasis on density control, and propose the idea of density-based control for multiple robots. In addition, we modify the original SPH method by fully decentralizing the SPH controller while retaining its density control feature, and introducing an inter-robot collision avoidance mechanism. This enhances the capability of the model in controlling a swarm of real-world robots. Finally, the effectiveness of our density-based control of a large number of robots is demonstrated through implementing two important tasks in multi-robot control: group motion and shape control, and group segregation.**

I. INTRODUCTION

The development of efficient control methodologies for a large number of robots designed to collectively carry out specified tasks (e.g., surveillance) continues to be an active area of research. In this paper, we focus on a control model inspired by physics called the Smoothed Particle Hydrodynamic (SPH) method. The method has been successfully applied in recent years to model multiple robots as a stream of incompressible/compressible fluid [1]–[3] resulting in demonstrable capability in tasks such as pattern generation [3], unknown area coverage and corridor sweeping [1, 2]. For instance, Pac et. al. [1, 2] applied the SPH model to adaptively deploy sensor nodes in an unknown environment. While these and other papers offer a comprehensive understanding of the SPH framework, the nature of the relationship between the SPH model and the more commonly employed Potential Field based methods (or Virtual Force based methods) remains an open question. On the other hand, several mathematical approaches to the problem have been attempted. For instance, Pimenta et. al. [3] have investigated the pattern generation problem in which multiple robots are driven to collectively form a pattern defined by an arbitrary curve. They also prove the stability of the SPH controller in this case. However, to the best of our knowledge, density-based control has not been investigated in the SPH framework. In particular, whilst inter-robot distance is often the control variable of primary interest (e.g., in problems such as rigid formation control and flocking), the density of the robots could emerge as the key variable in certain applications. For instance, if the objective is to drive a large number of robots from one place to another, the robot density is the quantity of key importance while the inter-robot distance plays only a secondary role as a metric useful for collision avoidance. Moreover, certain tasks can be done in an extremely simple manner via controlling density, such as the task of group segregation which will be discussed in detail later. Motivated thus, in this paper we propose the idea of density based control using the framework of the SPH model.

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In order to validate our ideas, we apply the proposed framework to two problems involving the control of multiple robots, the first of which is group motion and shape control. This is motivated by the fact that a human operator can exercise control over a large number of robots with relative ease by sending a small set of commands used to define the shape and motion of the group. The second problem is that of group separation which arises when several sub-groups of robots are required to perform tasks simultaneously, for instance, in tracking multiple intruders. The paper is set as follows. The original SPH model is introduced in Section 2. In Section 3 we explore the model from a control perspective and discuss certain modifications. Section 4 discusses in detail the controllers designed for the two tasks. Simulation results presented in Section 5 demonstrate the effectiveness of the density based controllers.

II. SMOOTHED PARTICLE HYDRODYNAMICS

There are different versions of the SPH model [2, 3], which are considered equivalent according to [4]. In this paper, we choose the model used in [3]. The mathematical definition of robot density is of prime importance in density control. In the SPH model one defines the density of a robot as the weighted sum of distances to its nearby robots within a certain range D. The weight function (or kernel function), considered in this paper, is the cubic spline function as in [3]:

$$W(\mathbf{q},h) = \frac{10}{7\pi h^2}\begin{cases} 1-\frac{3}{2}\kappa^2+\frac{3}{4}\kappa^3 & if\ 0\le\kappa\le 1\\ \frac{1}{4}(2-\kappa)^3 & if\ 1\le\kappa\le 2\\ 0 & otherwise\end{cases} \quad (1)$$

where $\kappa = \|\mathbf{q}\|/h$ and $\mathbf{q}$ is a position vector. The function support is determined by $2h$ where $2h = D$.

Now we can write down the definition of density ρ_i for robot i as:

$$\rho_i = \sum_j W(\mathbf{q_{ij}}, h) \tag{2}$$

where $\mathbf{q}_{ij} = \mathbf{q}_i - \mathbf{q}_j$ and $\mathbf{q}_i \in \mathbf{R}^2$ is the robot's position in a two dimensional environment. Given the definition of density, the SPH controller can then be defined as:

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right)\nabla_i W_{ij} \tag{3}$$

where $W_{ij} = W(\mathbf{q}_{ij})$, and $\mathbf{v}_i$ is the vector velocity of robot i, $P_i$is the pressure variable defined below. The plot of $-\|\nabla_i W_{ij}\|$ as a function of κ is shown in Fig. 1. Π_{ij} is a dissipative term for handling shocks given by:

$$\Pi_{ij} = \begin{cases} \frac{1}{\bar{\rho}_{ij}}(-\xi_1 \mu_{ij} + \xi_2 \mu_{ij}^2) & if\ \mathbf{v}_{ij} \cdot \mathbf{q}_{ij} < 0 \\ 0 & if\ \mathbf{v}_{ij} \cdot \mathbf{q}_{ij} \geq 0 \end{cases} \tag{4}$$

where

$$\mu_{ij} = \frac{h\mathbf{v}_{ij} \cdot \mathbf{q}_{ij}}{\|\mathbf{q}_{ij} - R_{safe}\|^2} \tag{5}$$

R_{safe} is the minimal distance between a pair of robots that guarantees collision avoidance, $\bar{\rho}_{ij}$ is the average density of robots i and j, and $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$.

The SPH model can incorporate either compressible or incompressible flow. For modeling robots as a compressible fluid, we define the pressure P_i as:

$$P_i = K\rho_i \tag{6}$$

For modeling robots as an incompressible fluid, we define:

$$P_i = K\rho_i \left[\left(\frac{\rho_i}{\rho_0}\right)^\gamma - 1\right] \tag{7}$$

where ρ_0 is the reference density, K and γ are model specific coefficients. For illustrative clarity, we maintain $K = 200$ and $\gamma = 7$ in all the simulations reported in this paper.

Keeping in mind the decentralized control , we define $\mathcal{N}_i$ as the set of all the robots in the neighborhood of robot i:

$$\mathcal{N}_i = \{j \neq i \mid \|\mathbf{q}_j - \mathbf{q}_i\| < D\} \tag{8}$$

Since we define $2h = D$, robot i only needs to know the position and velocity of the robots in $\mathcal{N}_i$.

In this paper, all robots are assumed to obey double-integrator dynamics given by $\ddot{\mathbf{q}}_i = \mathbf{u}_i$. Hence the controller for each robot is:

$$\mathbf{u}_i = \mathbf{f}_i^{SPH} - \xi\mathbf{v}_i \tag{9}$$

where $\mathbf{f}_i^{SPH}$ is the SPH force defined as:

$$\mathbf{f}_i^{SPH} = -\sum_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right)\nabla_i W_{ij} \tag{10}$$

ξ is a positive coefficient which is identically set to unity for all simulations in this paper. The damping term $-\xi\mathbf{v}_i$ helps stabilize the system.

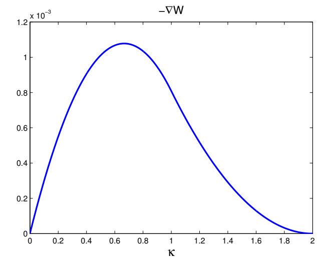


Fig. 1: The plot of $-\|\nabla_i W_{ij}\|$ as a function of κ.

III. Modified SPH Model

A. Modifications

1) Collision Avoidance: Pimenta et al. [3] use the dissipative term Π_{ij} solely as a mechanism to avoid inter-robot and robot-obstacle collisions. However, computation of Π_{ij} requires the knowledge of the velocities of all the robots in the neighborhood. Moreover, Π_{ij} acts exclusively as a damping force the effect of which is the same as the term $-\xi\mathbf{v}_i$ in (9). Hence, instead of using two separate damping forces in this system, we get rid of Π_{ij} and introduce a new mechanism to avoid inter-robot collisions.

In this paper, we use the additional repulsive force $\mathbf{f}_i^{repel}$ that is defined as:

$$\mathbf{f}_i^{repel} = K_{repel} \sum_{j \in \mathcal{N}_i^r} \frac{1}{\|\mathbf{q}_{ij}\|} \cdot \frac{\mathbf{q}_{ij}}{\|\mathbf{q}_{ij}\|} \tag{11}$$

where K_{repel} is a coefficient used to adjust the magnitude of the repulsive force, and $\mathcal{N}_i^r$ is a set of robots inside the R_{safe} range:

$$\mathcal{N}_i^r = \{j \neq i \mid \|\mathbf{q}_j - \mathbf{q}_i\| < R_{safe}\} \tag{12}$$

2) SPH model: Since our emphasis has now turned to density control from the simulation of the fluid motion, it is no longer imperative to keep the SPH model intact if altering the model can achieve more efficient density control. The original SPH model requires the knowledge of densities of all nearby robots (see (10)) which increases the communication requirement. In fact, we can safely get rid of the P_j/ρ_j^2 term in (10) without jeopardizing the feature of density control. Thus the SPH force can be rewritten as:

$$\mathbf{f}_i^{SPH} = -\frac{P_i}{\rho_i^2} \sum_j \nabla_i W_{ij} \tag{13}$$

and the control law becomes:

$$\mathbf{u}_i = \mathbf{f}_i^{SPH} + \mathbf{f}_i^{repel} - \xi\mathbf{v}_i \tag{14}$$

In the new controller, each robot only needs to know its own density and velocity in order to compute the control force, thereby obviating the need for further communication between robots.

The equations of collective motion of the system of N robots are given by:

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= -\nabla V(\mathbf{q}) - \xi\mathbf{v} \end{aligned} \tag{15}$$

where, $\mathbf{q} \in R^{2N}$ is the stacked position vector of all robots, and $\mathbf{v} \in R^{2N}$ is the stacked velocity vector of all robots, and

$$V(\mathbf{q}) = \sum_i \left[V_i^{SPH} + V_i^{repel}\right] \tag{16}$$

where

$$V_i^{SPH} = K\left[\frac{1}{\gamma}\left(\frac{\rho_i}{\rho_0}\right)^{\gamma} - \ln \rho_i\right] \tag{17}$$

$$V_i^{repel} = K_{repel}\sum_j \ln\left(\|\mathbf{q}_{ij}\|\right) \tag{18}$$

for the incompressible fluid version. For the compressible fluid version:

$$V_i^{SPH} = K \ln \rho_i \tag{19}$$

The SPH controller can be written as:

$$\mathbf{u}_i = -\nabla V_i^{SPH} - \nabla V_i^{repel} - \xi v_i \tag{20}$$

In order to carry out the stability analysis of the collective motion of all robots, we consider the following positive definite function as the Lyapunov function:

$$\phi(\mathbf{q}, \mathbf{v}) = V(\mathbf{q}) + \frac{1}{2}\mathbf{v}^T\mathbf{v} \tag{21}$$

Lemma 1: Consider a system of N mobile robots, the dynamics of which is dictated by (15) and the control law is given by (20). For all initial conditions belonging to the level set of $\phi(\mathbf{q},\mathbf{v})$ given by $\Omega_C = \{(\mathbf{q},\mathbf{v}) : \phi(\mathbf{q},\mathbf{v}) \le C, C > 0\}$, the system asymptotically converges to the largest invariant set in $\Omega_I \subset \Omega_C$. In Ω_I, the velocities of all the robots vanish and the total potential of all robots given by (16) approaches a local minimum.

Proof: Differentiating $\phi(\mathbf{q},\mathbf{v})$ with respect to time and using (15), one obtains:

$$\begin{aligned}\dot{\phi}(\mathbf{q},\mathbf{v}) &= \mathbf{v}^T\nabla V(\mathbf{q}) + \mathbf{v}^T\dot{\mathbf{v}} \\ &= \mathbf{v}^T\nabla V(\mathbf{q}) + \mathbf{v}^T\left(-\nabla V(\mathbf{q}) - \xi\mathbf{v}\right) \\ &\le 0.\end{aligned} \tag{22}$$

From the LaSalle's Invariance Principle, all solutions of the systems starting in Ω_C will converge to the largest invariant set in $\Omega_I = \left\{(\mathbf{q},\mathbf{v}) \in \Omega_C : \dot{\phi}(\mathbf{q},\mathbf{v}) = 0\right\}$. In particular, this occurs when the velocities of all the robots become zero. Furthermore, in the steady state, the velocities of the robots do not change, i.e.$\dot{\mathbf{v}} = 0$. Hence, from (15) we have:

$$\nabla V(\mathbf{q}) = 0 \tag{23}$$

Hence, the total potential of all robots is locally minimized. ■

Proposition 1: A pair of robots i and j with dynamics given by $\ddot{\mathbf{q}}_i = \mathbf{u}_i(\mathbf{q}, t)$ and a control law determined by (14) will never collide with each other.

Proof: Collision between robots i and j is characterized by $\mathbf{q}_{ij} = 0 = \mathbf{q}_{ji}$. Consider the SPH force on robot i given by (13). From the expression for the cubic spline function (1), $\lim_{\mathbf{q}\to 0} W(\mathbf{q}, h) = 10/7\pi h^2$, a finite constant. However, from (11), the repulsive force $\mathbf{f}_i^{repel} \to \infty$ as $\mathbf{q} \to 0$. Hence, the repulsive forces on robots i and j grows unboundedly large as the robots approach each other thereby eliminating the possibility of a collision. ■

B. Density-based Controller

In density-based control, robot density plays an important role, and the controller either controls the density directly or exercises control based on it. The incompressible fluid version of SPH model turns out to be an ideal density controller because the definition of pressure in (7) introduces a typical feedback mechanism to control the density. In particular, since $-\nabla_i W_{ij}$ is always a repulsive force between robot i and robot j (see Fig. 1), when $\rho_i > \rho_0$, the SPH force between robot i and j generates a repulsive force that reduces the density. When $\rho_i < \rho_0$, it generates an attractive force resulting in an increase in the robot's density. In this sense, the SPH model is very similar to Virtual Force or Potential Field based methods because there are only attractive and repulsive forces. The difference is that $\mathbf{f}_i^{SPH}$ can change its type according to the density in real time, which also is the source of the strength of the density-based controller.

The compressible fluid version of SPH implies a varying density. Thus we use (6) to define the pressure. Clearly, we have a typical positive feedback controller. In this case the SPH force will always be repulsive as P_i is always positive and $-\nabla_i W_{ij}$ is always a repulsive force. Furthermore, a closer look at the SPH equation:

$$\frac{P_i}{\rho_i^2} = \frac{K}{\rho_i} \tag{24}$$

reveals that a decrease in ρ_i (when the group is expanding) is accompanied by an increase in the SPH force. This feature is in fact quite useful since when the robot density is large, moving slowly helps avoid potential collisions.

In the next section, we demonstrate the utility of the density control feature of the SPH model by implementing multi-robot controllers in two tasks: group motion and shape control, and group segregation.

IV. Controller Design

A. Group Motion and Shape Control

A simple group-level motion and shape control mechanism is very important for a single human operator to exercise control on a large number of robots. Significant previous contributions to this topic includes the works of Pimenta et al. [3], Cheah et al. [5], and Belta and Kumar [6]. In [3], the authors apply SPH model to drive a swarm of robots to form a pattern defined by a curve, while the motion of the robot resembles that of a fluid. Belta and Kumar [6] specify group level properties by initially providing the first and second moments of the robot distribution and then designing a controller to obtain that distribution. This kind of abstraction method helps in developing high level control command in lower dimensions rather than specifying positions of each robot. They reported that, in the steady state, the robots form an ellipse centered at the point specified by the first moment and with principal axes specified by the second moments. However, the robots are distributed without any fixed boundary (the boundary provided in [6] is excessively conservative when the number of robots is large), and the distribution is only statistical in nature. In contrast, the region-based method proposed in [5, 7] provides a rigid boundary within which robot positions are constrained. However, they do not provide a mechanism to distribute the robots once they are inside the boundary. In this paper, we propose a new controller that fuses the region-based controller with compressible fluid version of the SPH controller. The new controller has the following properties:

- Every robot will be driven into a bounded region specified by several functions.
- Inside the bounded region, the robots will exploit the compressible nature to spread out to occupy the whole space.

Here we show the equations for region-based control and refer to [5] for the details. In region-based control, the boundary function is defined by $f_{G_i}(\Delta \mathbf{q}_i)$, where

$$\Delta \mathbf{q}_i = \mathbf{q}_i - \mathbf{q}_c \tag{25}$$

and $\mathbf{q}_i = [q_{i1}, q_{i2}]^T$ is the two-dimensional coordinate of robot i and $\mathbf{q}_c = [q_{c1}, q_{c2}]^T$ is a reference point of the shape. If $\mathbf{q}_i$ lies on the shape, then $f_{G_i}(\Delta \mathbf{q}_i) = 0$. We define a potential energy function for robot i:

$$P_{Gi}(\Delta \mathbf{q}_i) = \frac{k}{2}[max(0, f_G(\Delta \mathbf{q}_i))]^2 \tag{26}$$

where k is a positive coefficient. Now we can define a force $\mathbf{f}_i^{Shape}$ that is used to drive all robots into the shape:

$$\begin{aligned} \mathbf{f}_i^{Shape} &= \nabla_i P_{Gi} \\ &= \frac{\partial P_{Gi}(\Delta \mathbf{q}_i)}{\partial \mathbf{q}_i} \\ &= \begin{cases} k f_G(\Delta \mathbf{q}_i) \left(\frac{\partial f_G(\Delta \mathbf{q}_i)}{\partial \mathbf{q}_i} \right)^T & f_G(\Delta \mathbf{q}_i) > 0 \\ 0 & f_G(\Delta \mathbf{q}_i) \leq 0 \end{cases} \\ &= k \cdot max(0, f_G(\Delta \mathbf{q}_i)) \left(\frac{\partial f_G(\Delta \mathbf{q}_i)}{\partial \mathbf{q}_i} \right)^T \end{aligned} \tag{27}$$

Hence the region-based SPH controller is defined as:

$$\mathbf{u}_i = \mathbf{f}_i^{Shape} + \mathbf{f}_i^{SPH} + \mathbf{f}_i^{repel} - \xi \mathbf{v}_i \tag{28}$$

To facilitate the mathematical analysis, in this task, the SPH force is defined as in (13).

Proposition 2: Given a system of N mobile robots with dynamics given by $\ddot{\mathbf{q}}_i = \mathbf{u}_i(\mathbf{q}, t)$ and a control law determined by (28), where $\mathbf{f}_i^{Shape} = \nabla_i P_{Gi}$ and P_{Gi} is a shape potential function, the equilibrium points of the system are at an extremum of $\phi_S(\mathbf{q}) = \sum_i P_{Gi}$.

Proof: Since the system is in equilibrium we have $\ddot{\mathbf{q}} = \dot{\mathbf{q}} = 0$. Hence, we have $\mathbf{u}_i = 0$. Therefore $\sum_i \mathbf{u}_i = 0$. Since in (11), we have $\mathbf{q}_{ij} = -\mathbf{q}_{ji}$ and in (13), we have $\nabla_i W_{ij} = -\nabla_j W_{ji}$, we have $\sum_i \mathbf{u}_i = \sum_i \nabla_i P_{Gi} = 0$. However, $\sum_i \nabla P_{Gi} = 0$ is the necessary condition for $\phi_S(\mathbf{q})$ to be an extremum. ■

B. Group Segregation

In multi-robot cooperative control, a very interesting problem is that of segregation, i.e., splitting of a group of robots into spatially identifiable subgroups. The robots can be heterogeneous or homogeneous. Heterogeneous robots can be segregated by using concepts based on differential potential as proposed in [8]. For homogeneous robots, segregation is much more challenging because the robots do not have an identifiable property that distinguishes them to form separate groups. Therefore, the robots need to reach an agreement on which robot belongs to which group. A common way to reach such an agreement in a distributed manner is to use market-based methods in which robots can communicate with each other and reallocate themselves based on expected utility. In contrast to these methods, we propose a controller to separate robots in a self-organizing manner. In this approach, the global behavior, group segregation, emerges from the local simple interactions between robots. The group segregation controller is given as:

$$\mathbf{u}_i = \mathbf{f}_i^{SPH} + \mathbf{f}_i^{repel} - \xi \mathbf{v}_i \tag{29}$$

In this controller, we choose $R_{safe} = D = 2h$, and apply the incompressible fluid version of SPH force where the pressure P_i is defined by (7). From extensive numerical simulations, it was observed that the proposed controller has the following properties:

- Robots can be separated into several subgroups in a distributed manner.
- Each subgroup forms a perfect circular pattern.

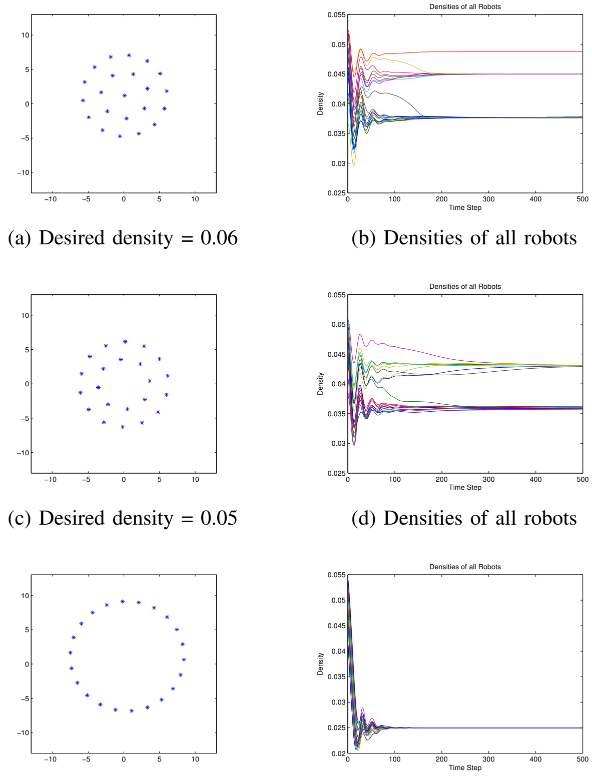
From the behavior-based control point of view, the SPH force $\mathbf{f}_i^{SPH}$ is a behavior to hold the group together, while the additional repulsive force $\mathbf{f}_i^{repel}$ is a behavior to break the group. Group segregation may also be viewed as a process of these two forces reaching a balance. Essentially, this system shows resemblance to a nonlinear time-variant consensus control system [9], and the following proposition shows that the SPH force $\mathbf{f}_i^{SPH}$ almost always points towards a weighted geometric center during the whole dynamic process.

Proposition 3: If $\rho_i < \rho_0$, the SPH force $\mathbf{f}_i^{SPH}$ always points to the point $\mathbf{q}_{wc}^i = \frac{\sum_j \delta_{ij} \mathbf{q}_j}{\sum_j \delta_{ij}}$.

Proof: The SPH force is given by (13).

$$\begin{aligned} \mathbf{f}_i^{SPH} &= -\frac{P_i}{\rho_i^2} \sum_{j \in \mathcal{N}_i} \nabla_i W_{ij} && (30) \\ &= \frac{P_i}{\rho_i^2} \sum_j \delta_{ij}(\mathbf{q}_j - \mathbf{q}_i) && (31) \\ &= \frac{P_i}{\rho_i^2} \left(\sum_j \delta_{ij} \mathbf{q}_j - (\sum_j \delta_{ij}) \cdot \mathbf{q}_i \right) && (32) \\ &= \frac{(\sum_j \delta_{ij}) \cdot P_i}{\rho_i^2} \left(\frac{\sum_j \delta_{ij} \mathbf{q}_j}{\sum_j \delta_{ij}} - \mathbf{q}_i \right) && (33) \\ &= \frac{(\sum_j \delta_{ij}) \cdot P_i}{\rho_i^2} (\mathbf{q}_{wc}^i - \mathbf{q}_i) && (34) \end{aligned}$$

where $\delta_{ij} = g(\|\mathbf{q}_{ij}\|) = \frac{-\nabla_i W(\|\mathbf{q}_{ij}\|)}{\|\mathbf{q}_{ij}\|} \geq 0$ and $\mathbf{q}_{wc}^i = \frac{\sum_j \delta_{ij} \mathbf{q}_j}{\sum_j \delta_{ij}}$. Since $P_i > 0$ because of $\rho_i < \rho_o$, the SPH force



(a) Desired density = 0.06 (b) Densities of all robots

(c) Desired density = 0.05 (d) Densities of all robots

(e) Desired density = 0.028 (f) Densities of all robots

Fig. 2: Robots form various circular patterns in different desired densities. Densities are the same for robots on the same circle.

$\mathbf{f}_i^{SPH}$ has the same direction with the vector $(\mathbf{q}_{wc}^i - \mathbf{q}_i)$. In other words, $\mathbf{f}_i^{SPH}$ is pointing towards $\mathbf{q}_{wc}^i$ at all times. In fact, $\mathbf{q}_{wc}^i$ is the weighted center of mass and the weight is determined by the kernel function W (see (1)). ■

Extensive simulations demonstrate that in the steady state, each group will form a perfect circular pattern (either a single ring or multiple rings). Fig. 2 shows that the robots form circular patterns that change from multiple ring-like pattern to a single ring as the desired density is decreased. A mathematical explanation of this phenomenon remains an interesting, open problem. An intuitive understanding of this phenomenon can be arrived at by considering the fact that decreasing the desired density results in an equilibrium condition where robots need to be more spatially distributed and this can happen when robots in multiple-ring like pattern move out to form a single ring. If we further decrease the desired density, any pattern with a given number of robots become unstable and separates to exhibit segregation behavior. It is note worthy that equilibrium condition always results in a highly symmetrical circular (with either single or multiple rings) pattern.

V. Simulations

A. Simulation #1 (Group Motion and Shape Control)

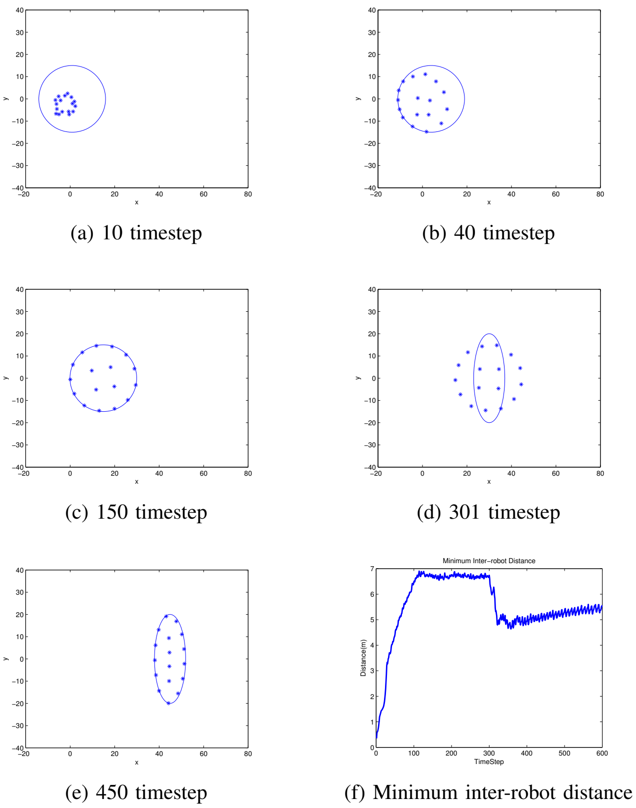
The region based SPH controller is very much applicable to dynamic situations when the shape function changes with time. In order to demonstrate the effectiveness of the proposed method to redistribute when the shape of the region changes, we carried out extensive simulations. As shown in Fig. 3, initially 17 robots were randomly placed in a 5 by 5 square region. The parameters D and K_{repel} were chosen to be 12 and 7 respectively. The initial desired shape function is a circle with radius 15. Further, the circle moves along the x axis towards right at the speed of 1m/s. At the $timestep = 300$ ($1\, timestep = 0.1 sec$ for all simulations in this paper), the shape function is changed into an ellipse with principal axes 7 units and 20 units. As we can see from the figure, the robots converge to the boundary and redistribute themselves inside the ellipse, which is due to the self-spreading property of the compressible fluid. From the plot of minimum inter-robot distance versus time, we can see that the robots never collide with each other in the process.

B. Simulation #2 (Group Segregation)

By tuning the parameters D, K_{repel} and ρ_0, we can split the robots into several subgroups. In Fig. 4, initially 46 robots are randomly placed in a 10 by 10 square region. Desired density ρ_0 is 0.06 at the beginning. At 550 timestep we change it to 0.02 and at 1000 timestep we change it back to 0.06 again. The parameters $D = 25$, $K_{repel} = 7$. As we can see, in the first phase, robots are separated into two groups and each group forms a solid circular pattern (with multiple rings). After the desired density is decreased, they are further separated into four smaller groups and each group still forms a circular pattern (with single ring). As the desired density increased again, each ring pattern collapses into a solid circular pattern while the number of groups remains. From the plot of the densities of all robots (Fig. 5), we can clearly see the three different phases with different patterns formed. Also we notice that the densities are the same for robots on the same layer of the ring . From Fig. 5 we can also see that the transition time between two phases is very short (less than 200 timestep or 20 sec), which demonstrates the effectiveness of the density-based controller in group segregation task.

VI. Conclusions and further work

In this paper, we propose the idea of density-based control for controlling multiple robots in the framework of SPH. In fact, the density of robots plays an important role in controlling a large number of robots. Two tasks have been chosen to demonstrate the effectiveness of the density-based controller: group motion and shape control, and group segregation. Mathematical analysis and extensive simulations show the effectiveness of the controller for group motion and shape control. Furthermore, extensive simulations demonstrate the ability of the density based controller to achieve segregation. Future research will focus on the mathematical analysis



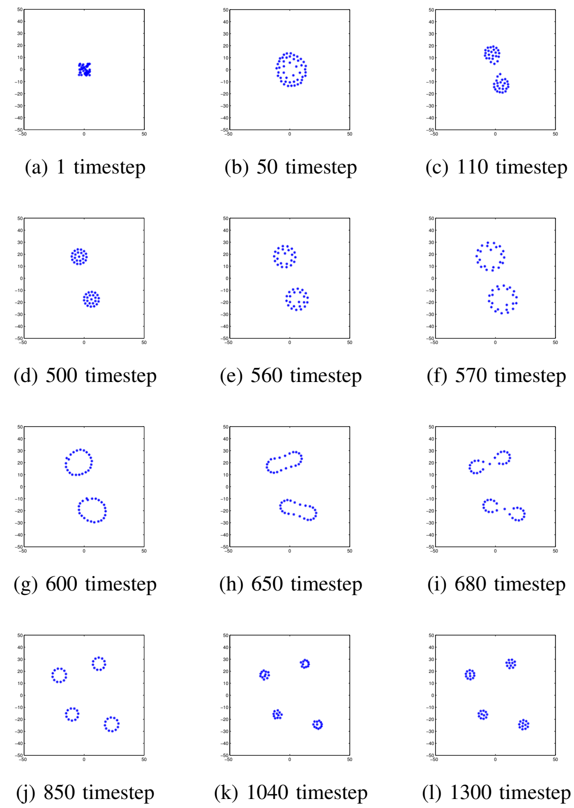
(a) 10 timestep (b) 40 timestep

(c) 150 timestep (d) 301 timestep

(e) 450 timestep (f) Minimum inter-robot distance

Fig. 3: The demonstration of the effectiveness of SPH controller to redistribute robots inside a new shape.

of the controller and development of a theoretical basis to understand the role played by different parameters in achieving different types of self organizing behaviors.



(a) 1 timestep (b) 50 timestep (c) 110 timestep

(d) 500 timestep (e) 560 timestep (f) 570 timestep

(g) 600 timestep (h) 650 timestep (i) 680 timestep

(j) 850 timestep (k) 1040 timestep (l) 1300 timestep

Fig. 4: The demonstration of self-organized group separation behavior

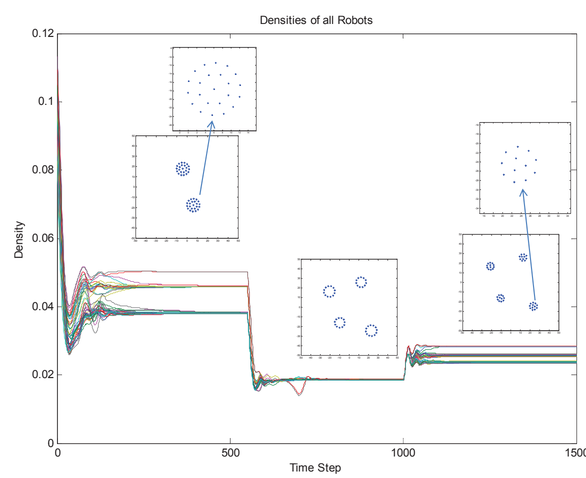


Fig. 5: Densities of all robots

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