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Stability and robust regulation of battery driven boost converter with simple feedback

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Abstract— **This paper investigates the problem of regulating the output voltage of a battery driven boost converter where the load is uncertain or changes within a certain range. By using the state-space averaging method, the open-loop system for regulating the output voltage is described as a 6th order differential equation with a bilinear term and input constraints. A simple saturated state feedback is designed by solving some optimization problem with linear matrix inequality constraints. The optimized controller is very close to an integrator feedback. Using the newly developed Lyapunov method, we analyze the stability and regulation of the closed-loop bilinear system. Computation shows that both the optimized state feedback and the integrator feedback can achieve practically global regulation in the presence of uncertain load and uncertain battery voltage. The results are validated by experimental systems.**

Keywords: battery model, bilinear system, boost converter, LMI, robust regulation, stability, saturation

I. INTRODUCTION

There are many factors that affect the terminal voltage of a battery, such as the load current, the state of charge, the temperature, its age and even the history of charge and discharge. When a battery is used to supply power to an electronic device which has strict requirements for the input voltage, a DC-DC converter needs to be constructed to regulate the voltage to a desired value under changing circumstances. Battery driven DC-DC converters have been studied in many works, e.g., [1], [2], [8], [11], [12]. In [8], boost, buck-boost and other types of converters are used to condition the power supplied by photovoltaic batteries. In [1], [12], high-efficiency buck converters were constructed to achieve low-voltage regulation for portable applications. The efficiency of battery powered converters was further addressed in [11]. In [2], a front end boost converter is used to regulate the voltage for the control of switched-reluctance motor drive.

In every power electronic converter, feedback control is invariably used to achieve regulation of voltage, current or power. For a control system, the fundamental requirement is stability. While numerous high efficient converters have been constructed for various applications, there have been continuing efforts devoted to the stability analysis of power electronic converters [4], [7], [10], [13]. It is known that each DC-DC converter is a nonlinear system. The stability is traditionally analyzed via linearization and the feedback control is usually designed via a linearized model, e.g, see [3], [10]. Some recent works have attempted to take into account the nonlinearities in stability analysis [5], [7], [13].

In [5], a nonlinear system approach was developed for analysis and design of power electronic converters, where the DC-DC converter was modeled as a differential equation with a bilinear term and input saturation. The nonlinear model was obtained by using the state-space averaging method developed by Middlebrook in [9]) and utilized in many other works (e.g., see [3]). The problems considered in [5] include controller design for robust stability, and estimation of stability region and tracking domain. These analysis and design problems were converted into numerically efficient optimization algorithms involving linear matrix inequalities (LMI).

In this paper, we will extend the tools developed in [5] to investigate battery driven DC-DC converters.

II. THE BATTERY DRIVEN BOOST CONVERTER - OPEN LOOP SYSTEM DESCRIPTION

We use the circuit in Fig. 1 to model the dynamics of a battery. The parameters of the battery can be identified with the method in [6]. The circuit diagram for the battery powered boost converter is illustrated in Fig. 2. The capacitor C_1 and the resistor R_1 are used to smoothen the spikes of the voltage across the two terminals of the battery. Without these components the voltage will be very noisy due to the fast switching of the MOSFET. The design objective is to keep the voltage across the load R_2 at a desired value for different resistance R_2 and different state of charge of the battery, by automatically adjusting the duty cycle via feedback control.

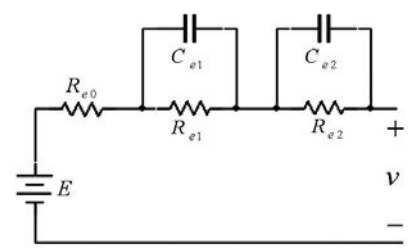

Fig. 1. The battery model.

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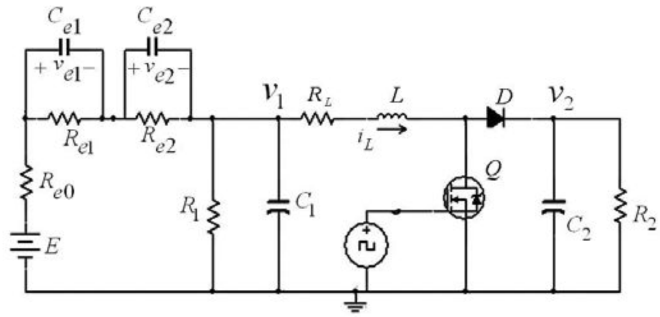

Fig. 2. A battery powered boost converter

We will use a state-space averaged model (see [3], [9]) to describe the dynamics of the circuit. To derive this model, we examine the circuit when the MOSFET is on and the circuit when the MOSFET is off separately.

A. Two operation modes

When the MOSFET is turned on, it behaves like a resistor with a low resistance R_{on} and the voltage at the drain is very low so that there is no current through the diode. The circuit can be equivalently drawn in Figure 3.

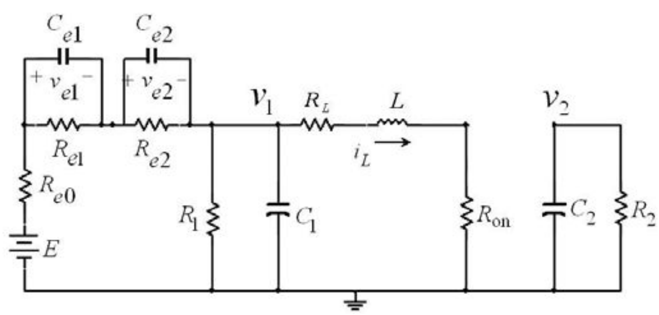

Fig. 3. Equivalent circuit when the MOSFET is on

Define

$$A_1=\begin{bmatrix} -\frac{R_{e0}+R_{e1}}{R_{e0}R_{e1}C_{e1}} & -\frac{1}{R_{e0}C_{e1}} & -\frac{1}{R_{e0}C_{e1}} & 0 & 0 \\ -\frac{1}{R_{e0}C_{e2}} & -\frac{R_{e0}+R_{e2}}{R_{e0}R_{e2}C_{e2}} & -\frac{1}{R_{e0}C_{e2}} & 0 & 0 \\ -\frac{1}{R_{e0}C_1} & -\frac{1}{R_{e0}C_1} & -\frac{R_{e0}+R_1}{R_{e0}R_1C_1} & 0 & -\frac{1}{C_1} \\ 0 & 0 & 0 & -\frac{1}{R_2C_2} & 0 \\ 0 & 0 & \frac{1}{L} & 0 & -\frac{R_L+R_{on}}{L} \end{bmatrix}$$

$$B_1=\begin{bmatrix} \frac{1}{R_{e0}C_{e1}} & 0 \\ \frac{1}{R_{e0}C_{e2}} & 0 \\ \frac{1}{R_{e0}C_1} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Let v_D be the forward voltage drop across the diode and $v=[E \quad v_D]^{\mathrm{T}}$. Denote the state variable as $\xi=[v_{e1} \quad v_{e2} \quad v_1 \quad v_2 \quad i_L]^{\mathrm{T}}$. The differential equation for the state variable ξ can be expressed as

$$\dot{\xi}=A_1\xi+B_1v \tag{1}$$

When the MOSFET is off, it behaves like an open circuit. Since the inductor current is positive and continuous, the diode is in the conducting mode with the forward voltage drop v_D. The circuit can be equivalently drawn in Figure 4:

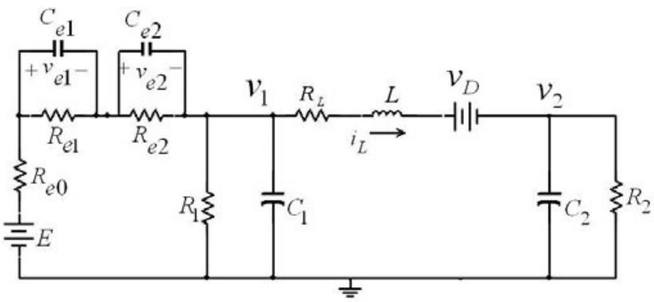

Fig. 4. Eqivalent circuit when the MOSFET is off.

Define

$$A_2=\begin{bmatrix} -\frac{R_{e0}+R_{e1}}{R_{e0}R_{e1}C_{e1}} & -\frac{1}{R_{e0}C_{e1}} & -\frac{1}{R_{e0}C_{e1}} & 0 & 0 \\ -\frac{1}{R_{e0}C_{e2}} & -\frac{R_{e0}+R_{e2}}{R_{e2}R_{e0}C_{e2}} & -\frac{1}{R_{e0}C_{e2}} & 0 & 0 \\ -\frac{1}{R_{e0}C_1} & -\frac{1}{R_{e0}C_1} & -\frac{R_{e0}+R_1}{R_{e0}R_1C_1} & 0 & -\frac{1}{C_1} \\ 0 & 0 & 0 & -\frac{1}{R_2C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{L} & -\frac{1}{L} & -\frac{R_L}{L} \end{bmatrix}$$

Let B_2 be modified from B_1 by replacing the 0 at the lower right corner with $-1/L$. The differential equation for the state variable can be expressed as

$$\dot{\xi}=A_2\xi+B_2v \tag{2}$$

B. The averaged model for continuous conduction mode

We assume continuous conduction mode (e.g., see [3]) for the circuit. Let the switching period be T and the duty cycle be D. During one switching period, the state variable $\xi(t)$ obeys equation (1) for DT and obeys (2) for $(1-D)T$. Let $\bar{\xi}$ be the state variable averaged over one switching period. Then by [9],

$$\dot{\bar{\xi}}=(DA_1+(1-D)A_2)\bar{\xi}+(DB_1+(1-D)B_2)v. \tag{3}$$

Since the design objective is to regulate the voltage across the load, we choose the averaged voltage across C_2 as the system output, denoted as $\bar{y}$. Let $C=[0 \ 0 \ 0 \ 1 \ 0]$. Then

$$\bar{y}=C\bar{\xi}$$

If the boost converter is given a fixed duty cycle D, a steady state will be reached where $\dot{\bar{\xi}}=0$. In this case,

$$\bar{\xi}=-((DA_1+(1-D)A_2))^{-1}(DB_1+(1-D)B_2)v=:\bar{\xi}_{ss}$$

and the steady state output voltage is $\bar{y}_{ss}=C\bar{\xi}_{ss}$.

We consider a boost converter that is used to supply power to an uncertain load or a load that changes among different values. Also, depending on the state of charge, the open circuit voltage of the battery is slowly decreasing. So E and R_2 may take different values.

We would like to design a feedback law to automatically adjust the duty cycle, so that the output voltage stays at a fixed desired value v_{2d} when E and R_2 are subject to possible changes. To design such a feedback law, we first choose a nominal working condition E_0, R_{20}, and a proper duty cycle D_0 which produce the desired output voltage v_{2d}. Let the matrices A_1, A_2 corresponding to R_{20} be A_{10}, A_{20} and let the v corresponding to E_0 be v_0. Let the steady state be $\bar{\xi}_0$. Then

$$0=(D_0A_{10}+(1-D_0)A_{20})\bar{\xi}_0+(D_0B_1+(1-D_0)B_2)v_0 \quad (4)$$

$$v_{2d} = C\bar{\xi}_0 \quad (5)$$

For a circuit with general E, R_2 and D, we define $\bar{x} = \bar{\xi} - \bar{\xi}_0$, $u = D - D_0$ and $y = \bar{y} - v_{2d}$. After tedious computation (by using (4) and (3), and adding/subtracting terms), we obtain

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{A}_b\bar{x}\,u + \bar{B}u + \bar{g}, \quad y = C\bar{x}, \quad (6)$$

where $\dot{\bar{x}}$ denotes the time derivative of $\bar{x}$ and

$$\begin{aligned} \bar{A} &= D_0A_1 + (1-D_0)A_2, \ \ \bar{A}_b = A_1 - A_2, \\ \bar{B} &= (A_1 - A_2)\bar{\xi}_0 + (B_1 - B_2)v \\ \bar{g} &= (D_0(A_1 - A_{10}) + (1-D_0)(A_2 - A_{20}))\bar{\xi}_0 \\ &\quad +(D_0B_1 + (1-D_0)B_2)(v - v_0) \end{aligned}$$

Corresponding to the desired output value of $\bar{y} = v_{2d}$, the desired value of y is 0. For the purpose of bringing y to 0 and keeping it there, we integrate it to obtain a new state

$$x_a = \int y dt = \int C\bar{x}dt.$$

Define the augmented state as $x := \begin{bmatrix} \bar{x} \\ x_a \end{bmatrix}$. Let

$$A = \begin{bmatrix} \bar{A} & 0 \\ C & 0 \end{bmatrix}, \ A_b = \begin{bmatrix} \bar{A}_b & 0 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix}, \ g = \begin{bmatrix} \bar{g} \\ 0 \end{bmatrix}$$

where the 0's have compatible dimensions. Then

$$\dot{x} = Ax + A_bx\,u + Bu + g. \quad (7)$$

where $x \in \mathbb{R}^6$.

If a control law $u = f(x)$ yields a stable equilibrium point for (7), then every state, in particular, $x_a = \int y dt$, will go to a constant. This means that $y(t)$ must go to zero and the voltage output is regulated to the desired value v_{2d}.

The control input $u = D - D_0$ is a perturbation of the duty cycle D. Assume that $D \in [D_1, D_2] \subset [0,1]$. We need to impose the constraint: $-D_0 + D_1 \le u \le D_2 - D_0$. Denote

$$u_m = D_0 - D_1, \quad u_p = D_2 - D_0.$$

Then $u_m, u_p > 0$ and the input constraint can be written as

$$-u_m \le u \le u_p. \quad (8)$$

III. Design of saturated state-feedback law for stability

If a state feedback is to be implemented, we must have all the state variables available for measurement. This is impossible for x_1, x_2, the voltages of the capacitors inside the battery. We will first present a general method for state feedback design. Then we propose some methods to deal with these state variables that can not be measured. The resulting feedback law will be simple but turns out to be effective on our circuits.

A. Design state feedback gain via LMI optimization

We first design a stabilizing state feedback law for the system under nominal working condition. Due to the constraint on the input, $u \in [-u_m, u_p]$, we use a saturated feedback law $u = \text{sat}(Kx)$, where $\text{sat}(\cdot)$ is the saturation function (usually with asymmetric saturation levels $-u_m$ and u_p):

$$\text{sat}(s) = \begin{cases} u_p, & s > u_p \\ s, & s \in [-u_m, u_p] \\ -u_m, & s < -u_m \end{cases}$$

Under the state feedback and nominal working condition, $g = 0$ and the closed loop system is

$$\dot{x} = Ax + A_bx\,\text{sat}(Kx) + B\text{sat}(Kx). \quad (9)$$

This system has similar description as the system (5) we studied in [5]. The only difference is that in [5], the saturation function is assumed to be symmetric for simplicity, which is a special case when $u_m = u_p$. The design method in [5] can be easily modified to the general case where u_m and u_p may not be the same. For convenience, we outline the main ideas in [5] (with slight modification for the general case) for how to design the feedback gain K.

Since $\text{sat}(Kx) \in [-u_m, u_p]$, the system (9) can be simply described (with some conservatism) as a differential inclusion with saturation:

$$\dot{x} \in \text{co}\{A_px + B\text{sat}(Kx), A_mx + B\text{sat}(Kx)\}, \quad (10)$$

where $A_p = A + A_bu_p, A_m = A - A_bu_m$, and $\text{co}\{X\}$ denotes the convex hull of the set X. The objective is to construct the maximal contractively invariant ellipsoid inside the linear region

$$\mathcal{L}(K) := \{x \in \mathbb{R}^n : |Kx| \le \min(u_m, u_p)\}.$$

This objective can be converted into the following optimization problem:

$$\begin{aligned} &\inf_{Q>0,H,\gamma} \ \gamma \quad (11) \\ s.t. \ &1)\ A_pQ + QA_p^{\mathrm{T}} + (BH + H^{\mathrm{T}}B^{\mathrm{T}}) < -2\varepsilon Q \\ &2)\ A_mQ + QA_m^{\mathrm{T}} + (BH + H^{\mathrm{T}}B^{\mathrm{T}}) < -2\varepsilon Q \\ &3)\ \begin{bmatrix} \min(u_m^2, u_p^2) & H \\ H^{\mathrm{T}} & Q \end{bmatrix} > 0 \\ &4)\ I < \gamma Q \end{aligned}$$

where $\varepsilon > 0$ is a given number used to ensure a certain convergence rate. By solving the above problem and letting $P = Q^{-1}, K = HP$, we obtain a contractively invariant ellipsoid $\mathcal{E}(P) = \{x \in \mathbb{R}^n : x^{\mathrm{T}} P x \leq 1\}$ for the system (10), which is inside the stability region.

B. Dealing with capacitor voltages inside the battery

Since the capacitor voltages inside the battery are not available and very hard to estimate, we first need to come up with a method to design a feedback law that does not involve these two state variables. A simple trick that worked is to replace C_{e1} and C_{e2} with very small values, e.g., $10^{-6}F$. With these parameter changes, the feedback gain K resulting from (11) will have nearly 0 values for the first two elements, indicating that we don't need the voltages of the battery capacitors in the feedback law.

The main issues to be addressed are,

- Does this feedback law stabilize the real circuit with very large capacitance in the battery?
- Can robust stability and regulation for the real circuit be confirmed using theoretical and numerical method?
- How this feedback law perform in the real circuit?

We will use numerical computational results and experiment to show that the simple feedback law designed for a modified model (with very small capacitance C_{e1} and C_{e2}) can work very well on the real circuit with very large C_{e1} and C_{e2}. Moreover, the simple feedback law can achieve practically global regulation under different load conditions and battery voltages.

IV. Stability analysis and region of regulation

Assume that a state feedback law has been designed. Under general working condition, the closed-loop system is described as,

$$\dot{x} = Ax + A_b x\, \mathrm{sat}(Kx) + B\mathrm{sat}(Kx) + g. \tag{12}$$

For simplicity, we assume in this section that the battery voltage E and the load R_2 are fixed. This is reasonable since E and R_2 change much slower than the rest of the system.

We would like to analyze each system corresponding to a fixed working condition separately instead of examining the robust stability/regulation of all possible situations in one shot. The performance at the few extreme cases may give us some informative pictures.

Suppose that the system has an equilibrium point at x_e such that

$$0 = Ax_e + A_b x_e(Kx_e) + BKx_e + g, \quad Kx_e \in [-u_m, u_p]. \tag{13}$$

At this equilibrium point, every state variable, in particular, the output of the integrator $\int y dt$, stays at a constant value. This implies that $y(\infty) = 0$ and the output of the boost converter is regulated to the desired value.

In what follows, we examine the stability around the equilibrium point x_e. Furthermore, we would like to estimate a *region of regulation* around the equilibrium point. The *region of regulation* is simply defined as the set of initial conditions which will lead to the equilibrium point x_e. Denote the response of (12) to an initial condition x_0 as $\phi(t, x_0)$. Then the region of regulation for x_e can be defined as

$$\mathcal{R}_r(x_e) := \{x_0 : \lim_{t\to\infty} \phi(t, x_0) = x_e\}. \tag{14}$$

The size of region of regulation $\mathcal{R}_r(x_e)$ is very important if the load switches within a wide range. Assume that before the load is switched at $t = 0$, an equilibrium point $x_{e,old}$ has been reached, i.e., $x(0) = x_{e,old}$. When the load is switched to a new value, a new equilibrium point $x_{e,new}$ is generated and the old equilibrium point $x_{e,old}$ becomes the initial condition. If $x_{e,old} \in \mathcal{R}_r(x_{e,new})$, then the new equilibrium point will also be reached. If the region of regulation is global (or practically global), then any (possible) initial condition will lead to the equilibrium point where the output stays at the desired value. If the region of regulation is practically global for every possible working condition, then as the load is switched between any possible values, a new equilibrium point will always be reached, implying that the output voltage will always come back to the desired value.

The region of regulation around an equilibrium point x_e can be converted as the stability region with a change of state variable. Define

$$z := x - x_e$$

and two matrices,

$$A_r := A + A_b K x_e, \quad B_r := B + A_b x_e$$

It can be verified (after tedious computation, by subtracting (13) from (12)), that

$$\dot{z} = A_r z + (A_b z + B_r)(\mathrm{sat}(K(z + x_e)) - Kx_e) \tag{15}$$

Equivalently,

$$\dot{z} = \begin{cases} A_r z + (u_p - Kx_e)(A_b z + B_r), & Kz > u_p - Kx_e \\ (A_r + B_r K)z + A_b z(Kz), & K(z + x_e) \in [-u_m, u_p] \\ A_r z - (u_m + Kx_e)(A_b z + B_r), & Kz < -u_m - Kx_e \end{cases} \tag{16}$$

For the system (16), $z = 0$ is an equilibrium point. Denote the response of (16) to an initial condition z_0 as $\psi(t, z_0)$. The stability region around $z = 0$ can be defined as

$$\mathcal{S} := \{z_0 : \lim_{t\to\infty} \psi(t, z_0) = 0\}. \tag{17}$$

From the definitions, it is straightforward to see that

$$\mathcal{R}_r(x_e) = x_e + \mathcal{S},$$

i.e., the region of regulation can be easily obtained by adding the equilibrium point x_e to every point in the stability region.

The system description in (16) is exactly the same as the general description (11) in [5]. Thus we can use the optimization method in [5] to estimate the region of stability/regulation.

V. Numerical Example for Stability and Robust Regulation

A. Circuit description and feedback law

We used a lead-acid battery rated 6V, 7.2Ah for the design and computational analysis. The parameters change with the load and state of charge but we will see later that the circuit with feedback can tolerate these changes very well. The nominal parameters for the battery are given as follows: $R_{e0} = 0.0219\Omega$, $R_{e1} = 0.033\Omega$, $R_{e2} = 0.1038\Omega$, $C_{e1} = 16.5755F, C_{e2} = 115.4946F$. The open-loop circuit voltage E of the battery varies within $[5.85, 6.45]V$. We choose the nominal value of E as $E_0 = 6.15V$. Parameters for other circuit elements are given as follows: $C_1 = 0.33mF, C_2 = 0.394mF$, $R_1 = 20k\Omega$, $L = 0.33mH$, $R_L = 0.15\Omega$, $R_{on} = 0.036\Omega$ and $v_D = 0.4V$. The load R_2 varies within $[20.3, 194.5]\Omega$. We choose the nominal value of R_2 as $R_{20} = 40\Omega$.

The desired output voltage is $19V$. Under the nominal working condition, this is reached at $D = D_0 = 0.7114$.

We need to properly choose the range of the duty cycle. If the range exceeds the duty cycle corresponding to the maximal output, the closed-loop system may become unstable. Of all the possible working conditions, the minimal duty cycle to reach the peak is $D = 0.8709$, corresponding to the case where $E = 5.85V$ and $R_2 = 20.3\Omega$. Thus we restrict D in the range $[0.13, 0.8709]$. Since $D_0 = 0.7114$, we have $u = D - D_0 \in [-0.5814, 0.1595] = [-u_m, u_p]$.

To design a state feedback law without involving the voltage of the capacitors in the battery, i.e., $x(1)$ and $x(2)$, we set $C_{e1} = C_{e2} = 0.1\mu F$. By choosing $\varepsilon = 17.5$ in (11), we obtain

$$K = [0.0 \;\; 0.0 \;\; -0.0009 \;\; 0.0002 \;\; -0.006 \;\; -1.4507]; \quad (18)$$

The first two elements of K are nearly 0. Thus we don't need to use the capacitor voltages in the battery for feedback.

B. Practically global stability and regulation

The following computational results are obtained by using $C_{e1} = 16.5755F$, $C_{e2} = 115.4946F$. For robust stability and regulation, the same feedback law $u = \text{sat}(Kx)$ is used for all the working conditions, where sat is the saturation function that restricts $u \in [-0.5814, 0.1595]$.

We first perform stability analysis on the closed-loop system under feedback $u = \text{sat}(Kx)$. Using the algorithm in [5], we determined a stability region which includes $225.36 \times \{x : \|x\|_2 \le 1\}$ for the system under nominal working condition. This stability region is practically global as compared to the possible range of each state variable.

For the extreme working condition $E = 5.85V, R_2 = 20.3\Omega$, the region of regulation includes $x_e + 244 \times \{z : \|z\|_2 \le 1\}$, which also indicates practically global regulation. For another extreme working condition $E = 6.45V, R_2 = 194.5\Omega$, the region of regulation includes $x_e + 189 \times \{z : \|z\| \le 1\}$.

It is interesting to note that all the first 5 elements of K are nearly zero. Can we just set all these 5 elements to 0 and still get good stability and regulation performance? The resulting feedback law will only consist of the integration of the error between the actual output and the desired output, which would be very easy to implement. The following are numerical results for the simple feedback gain $K = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -1.4507]$.

The stability region includes $215 \times \{x : \|x\|_2 \le 1\}$ for the system under nominal working condition. For the working condition $E = 5.85V, R_2 = 20.3\Omega$, the region of regulation includes $x_e + 223 \times \{z : \|z\|_2 \le 1\}$. For the working condition $E = 6.45V, R_2 = 494.5\Omega$, the region of regulation includes $x_e + 187 \times \{z : \|z\|_2 \le 1\}$. Still, practically global stability/regulation has been achieved by using a very simple integrator control.

VI. Experimental Results

We used three 6V lead-acid batteries rated 4.5Ah, 7.2Ah and 13Ah, under different state of charge, to conduct the experiment. The load switches among three values: 20.3Ω, 40.5Ω, and 194.5Ω. All the circuit parameters for the boost converter are the same as those for computation in Section V. The pulse-width-modulation is implemented with two 555 timers with switching frequency 27kHz.

A. Transient responses to load switches

Fig. 5 shows the experimental response with a 7.2Ah battery. The open circuit voltage is 6.07V. The top curve

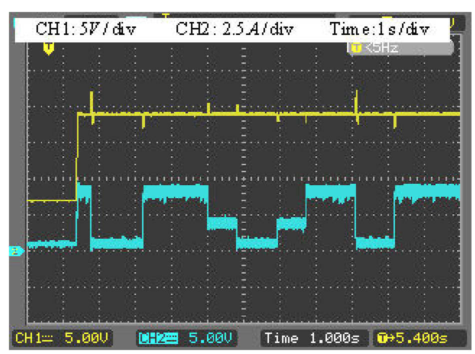

Fig. 5. Experiment under changing load - with a 7.2Ah battery at 6.07V

is the output voltage, the bottom curve (wide band) is the inductor current (not filtered). The width of the current band indicates the size of the ripples during the switching cycles. The three levels of the current band (from high to low) correspond to the load of 20.3Ω, 40.5Ω, 194.5Ω, respectively.

Initially the reference for the output voltage is 0 so the actual output (about 7V) corresponds to the minimal duty cycle (D=0.13). The reference is turned on at about t=1.3

second when the load is 20.3Ω. The output quickly follows the reference to 19V. Later the load is switched 8 times. Every time as the load is switched, the voltage quickly adjusts to the desired value after a spike. Fig. 6 shows the

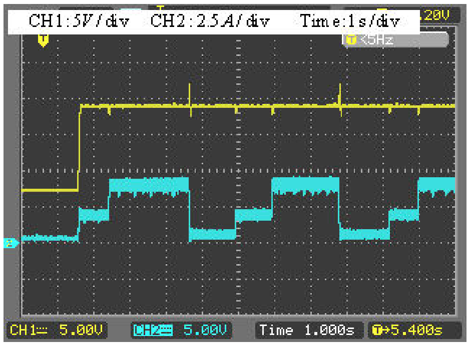

Fig. 6. Experiment under changing load - with a 13Ah battery at 6.33V

experiment with a 13Ah battery. The open circuit voltage is 6.33V. The reference is turned on at about t=1.6 second to an initial load of 40.5Ω, followed by 7 load switches. The response is similar to the case with the 7.2Ah battery.

We then used a 4.5Ah battery with open circuit voltage at about 5.82V. The residual capacity is very low and the terminal voltage drops quickly when the current is above 4A. The response is shown in Fig. 7, where the current

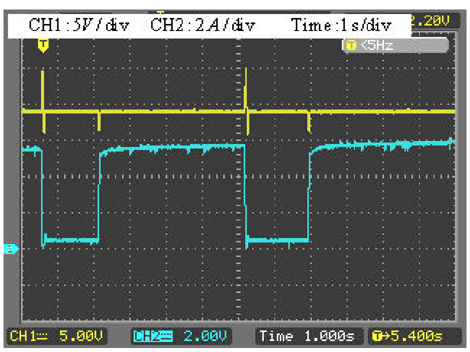

Fig. 7. Experiment under changing load - with a 4.5Ah battery at 5.82V

is filtered for better view of the dynamics. The current corresponding to the 20.3Ω load is more than 5A, visibly larger than those in Figs. 5 and 6, which are less than 4A. This larger current is because the load is absorbing the same power but the battery voltage is lower.

B. Transient response to battery switch

Two batteries are used in this test. One is the 13Ah battery fully charged, with terminal voltage E about 6.55V. The other one is the 4.5Ah battery with very low residue capacity, with terminal voltage about 5.835V. A simple SPDT toggle is used to connect to one of the batteries. Fig. 8 shows the response of v_2 and i_L (filtered) to the switching between the two batteries. The load is fixed at $R_2 = 20.3\Omega$. The switching between batteries yields some spikes but the output voltage quickly returns to 19V.

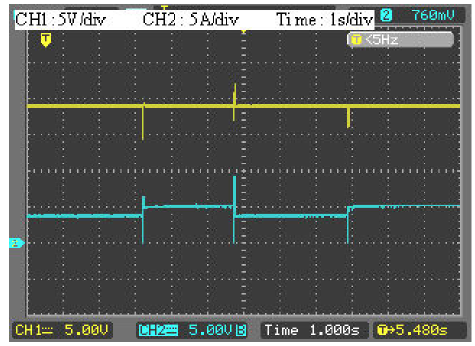

Fig. 8. Response to battery switching

VII. Conclusions

This paper presents a state-space method to study the stability and robust regulation of battery driven DC-DC boost converter with uncertain or changing load. Computational results show that an integrator can achieve practically global stabilization and regulation of output voltage. These results are verified by an experimental system with different batteries under different state of charge and load conditions.

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