2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A pendulum-like system approach to the anticipating synchronization of RCL-shunted Josephson junctions

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***Abstract*— In virtue of the nonlinear pendulum-like system theory, this study addresses the problem of anticipating synchronization of two coupled chaotic RCL-shunted Josephson junctions (RCLSJ) based on time-delayed feedback control. Sufficient conditions are established, under which the existence of anticipating synchronizing slave systems is guaranteed. The design of a desired feedback controller can be achieved by solving a group of linear matrix inequalities by utilizing an available numerical software. In the presence of parameter uncertainties, robust anticipating synchronization is further explored. These results are demonstrated through numerical simulations that under the derived conditions, the slave RCLSJ model could respond in exactly the same way as the master would do in the future, hence it allows us to anticipate the nonlinear chaotic dynamics.**

I. INTRODUCTION

Understanding chaos synchronization in coupled oscillators is currently a focus of research interest [1], [2]. In the case of unidirectional coupling, the evolution of one of the coupled chaotic systems is unaltered by the other system, and accordingly, the two systems are respectively called master and slave systems. Different types of schemes for master-slave synchronization have been proposed and observed in dynamical systems, such as complete synchronization, anticipating synchronization, phase synchronization, generalized synchronization, and intermittent synchronization (see [3], [4] and the references therein). The focus here in this study is the situation when the coupling involves a delay in time, which may lead to anticipating synchronization. Such synchronization regime describes the remarkable phenomenon that it is possible that the slave dynamics act as a predictor of the master dynamics in spite of the inherent unpredictability of chaotic systems [5]. Recently, the phenomenon of anticipating synchronization has been theoretically demonstrated and experimentally vindicated in disparate dynamical systems [6]-[9].

The Josephson effect is the phenomenon of electric current across two weakly coupled superconductors, separated by a very thin insulating barrier. This arrangement, two superconductors linked by a non-conducting barrier, is known as a Josephson junction. In recent years, we witness an increasing interest during recent years in the study of Josephson junctions, which are among important early example systems that show chaotic behavior [10]-[12]. Different models have been introduced to represent Josephson junctions, among which, the interest in shunted nonlinear resistive-capacitive-inductance junction (RCLSJ) models has been significantly increased in the past decades [13]-[16]. Such models generate chaotic oscillations by a pure dc bias only, which are found to be useful in, but not limited to, high-frequency application [13]. More recently, interest on the complex dynamics of Josephson junction has been extended to the synchronization issue [17]. For instance, Dana et al. [17] investigated the synchronization behavior of unidirectionally coupled RCLSJ by means of a negative pulse forcing and observed intermittent synchronization; while the condition for phase synchronization has been examined in a system that consists of two coupled Josephson junctions [18]. Inspired by the close relationship between synchronization and the observer problem in control theory [19], recent synchronization techniques address the problem of chaotic synchronization based on the view point of control theory. By setting up the complete synchronization scheme for two-coupled RCLSJ models, Uçar et al. [20] carried out the study of the dynamics of coupled Josephson junction through suitably designed active controls. In virtue of the backstepping design method, the synchronization issue in parallel has been investigated by Vincent et al. [21]. Robust synchronization has been reported in a more recent study, aiming at the development of a variable structure controller for synchronizing two coupled RCLSJ models subject to uncertainties [22].

Although there is extensive work on synchronization of coupled Josephson junctions, studies on chaotic anticipating synchronization is much less. In this study, we shall focus on the anticipating synchronization of RCLSJ models and present a master-slave configuration for the anticipating synchronization by utilizing the time-delayed feedback control strategy. Illuminated by the fact that chaos in the dc current driven Josephson junction can be studied by measurements on a phase-locked loop [11], this method allows a direct display that we could examine the anticipating synchronization by utilizing the basic knowledge of nonlinear pendulum-like systems, since phase-locked loops are frequently adopted as paradigms of nonlinear pendulum-like systems [23]. In the coupling setups considered here, the feedback controllers are acted upon the master system in order to synchronize the two

This work is supported by the National Science and Technology Infrastructure Program (Grant No. 2008BAA13B07), the China Postdoctoral Science Foundation funded project (Grant No. 20100480242), and the Science Technology Project of State Grid Corporation of China (Research on Safety and Stability of large scale power system based on complex system theory), and the National Science Foundation of China (Grant No. 60874011).

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systems. The determination of anticipating synchronization is further converted into an equivalent stabilizing problem for the error dynamics between master and slave systems, which turns to be a standard form of pendulum-like system with multiple-equilibria [24]. A criterion for anticipating synchronization is presented, which shows that the design of the time-delayed feedback controller can be realized by solving a group of linear matrix inequalities (LMIs) [25], which are readily solvable by available numerical softwares. Moreover, due to the ubiquity of parameter uncertainties, the robust anticipating synchronization between RCLSJ models with parameter uncertainty is further explored.

II. Analysis of the RCLSJ model

Throughout this study, we deal with a standard form of RCLSJ model that obeys the following dimensionless dynamics [13]:

$$\begin{cases} \dot{p} = q \\ \dot{q} = \dfrac{1}{\beta_C}[i - g(q)q - sin(p) - r] \\ \dot{r} = \dfrac{1}{\beta_L}(q - r) \end{cases} \tag{1}$$

where p, q, r represents the phase difference, junction voltage and current through shunted inductance of system, respectively. β_C is the capacitive constant, and β_L denotes the inductance constant. In this system, i is an external current consisting of a dc component only. The nonlinear damping term $g(q)$ is approximated by a current-voltage relation between the two junctions and is given by the following step function:

$$g(q) = \begin{cases} 0.366 & \text{if } |q| > 2.9 \\ 0.061 & \text{if } |q| \leq 2.9. \end{cases} \tag{2}$$

Since we are tackling an inhomogeneous differential equation with an explicit time dependence, the extended phase space in which the dynamics is taking place is three-dimensional. On the other hand, the nonlinear property of equation (1) means that its solution allows the possibility of periodic and chaotic orbits. It has been observed that if the parameters are chosen as $\beta_C = 0.707$, $\beta_L = 2.6$, the RCLSJ model (1) exhibits chaotic dynamics [13] for the dc external current in the region $i \in (1, 1.3)$. For the purpose of illustration, Fig. 1 shows that the system with initial conditions $(p(0), q(0), r(0)) = (0, 0, 0)$ behaves chaotically for $i = 1.17$ as observed in the phase portrait of junction voltage q and inductance current r as well as the time-domain plot of p, q and r.

To facilitate the synchronization analysis, the RCLSJ model given in (1) can be mathematically recast into

$$\begin{cases} \dot{x} = Ax + B\varphi(y) \\ \dot{y} = Cx \end{cases} \tag{3}$$

where the matrix parameters are given as

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{\beta_C}g & -1 \\ 0 & \frac{1}{\beta_L} & -\frac{1}{\beta_L} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \frac{1}{\beta_C} \\ 0 \end{bmatrix},\ C = [0\ 1\ 0],$$

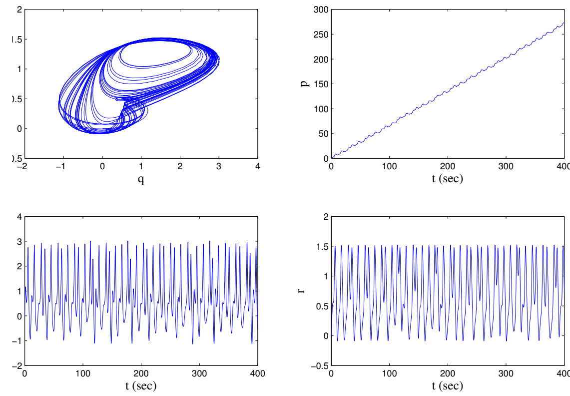


Fig. 1. The phase portrait as well as time-domain plots of the RCLSJ model (1).

with state variables $x = \begin{bmatrix} p & q & r \end{bmatrix}^T$, $y = p$, and nonlinearity $\varphi(y) = i - sin(y)$. Herein, the nonlinearity is 2π periodic with respect to the variable y, which will play an important role in the forth coming sections. The following sections will explore the anticipating phenomenon of two RCLSJ models coupled in a master-slave fashion in virtue of the feedback control scheme.

III. Anticipating Synchronization via Feedback Control

The notion of anticipating synchronization have received particular attention recently ever since the seminal work of Voss [5]. Within the synchronization schemes considered here, controllers are introduced to the master system in order to synchronize the states of two identical RCLSJ models. In this regard, the master and slave systems RCLSJ, both in form of (3), can be mathematically recast into

$$\begin{aligned} &\textbf{(M)} \quad \begin{cases} \dot{x}_1(t) = Ax_1(t) + B\varphi(y_1(t)) + u_1(t), \\ \dot{y}_1(t) = Cx_1(t) + u_2(t), \end{cases} \\ &\textbf{(S)} \quad \begin{cases} \dot{x}_2(t) = Ax_2(t) + B\varphi(y_2(t)), \\ \dot{y}_2(t) = Cx_2(t). \end{cases} \end{aligned} \tag{4}$$

Here, the controllers $u_1(t), u_2(t)$ in the following form which are added to the master system:

$$\begin{aligned} u_1(t) &= K_1(x_1(t) - x_2(t-\tau)), \\ u_2(t) &= K_2[\varphi(y_1(t)) - \varphi(y_2(t-\tau))]. \end{aligned} \tag{5}$$

where τ and ω are positive constants, and $K_1 \in \mathbb{R}^{3\times 3}, K_2 \in \mathbb{R}$ are the controller gain matrices. The objective of this section is to develop an LMI based approach to the design of a time-delayed controller such that the anticipating synchronization is achieved. To this end, denote $e(t) = x_1(t) - x_2(t-\tau), \varepsilon(t) = y_1(t) - y_2(t-\tau)$, then we arrive at the following error dynamics:

$$\textbf{(E)} \quad \begin{cases} \dot{e}(t) = (A + K_1)e(t) + B\phi(\varepsilon, y_2), \\ \dot{\varepsilon}(t) = Ce(t) + K_2\phi(\varepsilon, y_2). \end{cases} \tag{6}$$

where $\phi(\varepsilon, y_2) = \varphi(y_1(t)) - \varphi(y_2(t-\tau)) = \varphi(\varepsilon(t) + y_2(t-\tau)) - \varphi(y_2(t-\tau))$ is a periodic function about ε. According to the periodic property of the nonlinearity $\varphi(\cdot)$ mentioned in the previous section, the period of $\phi(\varepsilon, y_2)$ is $T = 2\pi$.

System (6) could be characterized by the transfer function of the linear part from the input $\phi(\varepsilon, y_2)$ to the output $-\dot{\varepsilon}$, namely, $G(s) = C(sI - A - K_1)^{-1}B + K_2$. Furthermore, the following assumption is made on system (6) with periodic variable ε.

Assumption 1: The transfer function matrix $G(0)$ is supposed to be nonsingular.

It is an immediate consequence that the matrix $A + K_1$ is Hurwizian under Assumption 1. Any equilibrium $(e_0(t), \varepsilon_0(t))$ of the error dynamics (6) satisfies

$$(A+K_1)e_0(t) + B\phi(\varepsilon_0, y_2) = 0,$$

$$Ce_0(t) + K_2\phi(\varepsilon_0, y_2) = 0.$$

which arrives at $\phi(\varepsilon_0, y_2) = 0$ and $e_0 = 0$. Since $\phi(\varepsilon_0, y_2)$ is periodic about ε, the system has infinitely many isolated equilibria. Under such circumstances, the error dynamics (6) is similar to that of a pendulum-like system with multiple equilibria [23]. In allusion to such kind of systems, the following statements are borrowed from [24].

Definition 1: Nonlinear feedback system (6) is said to be globally asymptotically stable if every solution $(e(t), \xi(t)) \to (e_0(t), \xi_0(t))$ as as $t \to \infty$.

Remark 1: It can be ensured by Definition 1 that if solution $(e(t), \varepsilon(t))$ for the error dynamics (6) is globally asymptotically stable, then the master and slave RCLSJ models would achieve generalized synchronization.

Remark 2: Alternatively, one could consider a time-delayed feedback controller $u(t) = K(y(t-\tau_1) - x(t-\tau_2))$, with $\tau_1 > 0$ and $\tau_2 > 0$ in (5), similar to the techniques adopted in [26]. Whenever $x(t-\tau_2) - y(t-\tau_1)$ tends to 0 as t tends to ∞, different types of synchronization can be achieved depending on the varying of τ_1 and τ_2. If $\tau_1 = \tau_2$, the synchronization would be complete; the anticipating (lag) synchronization would be achieved if $\tau_1 - \tau_2 > 0$ (< 0).

Before presenting the main criteria, first let us consider the frequency-domain criterion of global asymptotical stability of the error dynamics (6), which is derived based on the results in [24].

Lemma 1: Suppose $G(s)$ is stable and there exist scalars κ, $\delta > 0$ and $\eta > 0$ satisfying the following conditions:

$$\frac{1}{2}\mathrm{He}\kappa G(j\omega) - G^*(j\omega)\delta G(j\omega) - \eta \geq 0 \quad \text{for all } \omega \in \mathbb{R}, \tag{7}$$

$$4\delta\eta > (\kappa\nu)^2, \tag{8}$$

where

$$\nu = \frac{\left|\int_0^{2\pi}\int_0^{2\pi}[\varphi(\varepsilon(t) + y_2(t-\tau)) - \varphi(y_2(t-\tau))]d\varepsilon dy_2\right|}{\int_0^{2\pi}\int_0^{2\pi}|\varphi(\varepsilon(t) + y_2(t-\tau)) - \varphi(y_2(t-\tau))|d\varepsilon dy_2}, \tag{9}$$

then the nonlinear system (6) is globally asymptotically stable.

To derive our main results, we also need the following lemma.

Lemma 2: (KYP Lemma [27]) Given $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, M = M^T \in \mathbb{R}^{(n+m)\times(n+m)}$, with $\det(j\omega I - A) \neq 0$ for $\omega \in \mathbb{R}$ and (A, B) controllable, the following two statements are equivalent:

1° $\begin{bmatrix}(j\omega I - A)^{-1}B \\ I\end{bmatrix}^* M \begin{bmatrix}(j\omega I - A)^{-1}B \\ I\end{bmatrix} \leq 0,\ \forall \omega \in \mathbb{R};$

2° there exists a matrix $P = P^T \in \mathbb{R}^{n\times n}$ such that

$$M + \begin{bmatrix}A^TP + PA & PB \\ B^TP & 0\end{bmatrix} \leq 0$$

The corresponding equivalence for strict inequalities holds even if (A, B) is not controllable.

In the rest of this section, a time-domain equivalent condition of Lemma 1 is proposed in form of linear matrix inequality (LMI) [25].

Theorem 1: Suppose $A + K_1$ is Hurwitzian. Given a constant scalar ρ, if there exist a positive definite matrix $P > 0$, any matrix W, V, as well as diagonal matrices $\eta > 0, \delta > 0$ such that the following LMIs hold:

$$\begin{bmatrix}\mathrm{He}(PA + W) & \frac{1}{2}\rho C^T\delta + PB & C^T\delta \\ * & \eta + \rho V & V^T \\ * & * & -\delta\end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix}2\eta & \rho\delta\nu \\ * & 2\delta\end{bmatrix} > 0, \tag{11}$$

where ν is defined as in (9), then for all $t \geq 0$, the error dynamics (6) is globally asymptotically stable, and the master system **(M)** and the slave **(S)** achieve generalized synchronization, with the controller parameters given as $K_1 = P^{-1}W$ and $K_2 = \delta^{-1}V$.

Proof. The frequency inequality (7) in Lemma 1 can be written into the following condition

$$\begin{bmatrix}[j\omega - (A+K_1)]^{-1}B \\ 1\end{bmatrix}^* \Lambda \begin{bmatrix}[j\omega - (A+K_1)]^{-1}B \\ 1\end{bmatrix} < 0, \tag{12}$$

with

$$\Lambda = \begin{bmatrix}C^T\delta C & \frac{1}{2}C^T\kappa + C^T\delta K_2 \\ * & \eta + K_2^T\delta K_2 + \kappa K_2\end{bmatrix}.$$

By applying Lemma 2, the frequency expression (12) is equivalent to the following matrix inequality

$$\begin{bmatrix}\mathrm{He}(P(A+K_1)) + C^T\delta C & PB + \frac{1}{2}C^T\kappa + C^T\delta K_2 \\ * & \eta + K_2^T\delta K_2 + \kappa K_2\end{bmatrix} < 0. \tag{13}$$

On the other hand, the condition (11) could be directly derived from inequality (8) in virtue of the Schur's Lemma. Accordingly, matrix inequalities (11) along with (13) could guarantee the global asymptotical stability of (6).

However, since the controller parameter to be solved K_1, K_2 existing in the derived condition (13) are coupled with other matrix variables, matrix inequality (13) is nonlinear and hard to solve. To this end, in what follows, the nonlinear matrix inequality is to be converted to a group of conditions that are easier to be dealt with. Based on the Schur's Lemma, matrix inequality (13) can be reformulated as

$$\begin{bmatrix}\mathrm{He}(PA + PK_1) & \frac{1}{2}C^T\kappa + PB & C^T\delta \\ * & \eta + \kappa K_2 & K_2\delta \\ * & * & -\delta\end{bmatrix} < 0. \tag{14}$$

In order to make (14) linear with respect to variables K_1, K_2, let $\kappa = \rho\delta$ with ρ being a prescribed constant. By defining

$$W = PK_1,\ V = \delta K_2,$$

condition (14) is equivalent to an LMI expression as given in (10), thus could be solved efficiently by some available numerical packages. Consequently, if the LMIs (10)-(11) hold, then anticipating synchronization the master and slave systems will be realized, thus the proof is completed. □

IV. Extension to the Robust Anticipating Synchronization

It has been shown that parameter uncertainty may lead to a serious degradation of the system performance if the controller is not well designed. Although there are extensive results on synchronization of two identical RCLSJ, a review of the published literature reveals that the problem of synchronization between RCLSJ models subject to parameter uncertainties has not received sufficient attention yet. In consequence, we shall extend the results derived in the previous section to the master and slave RCLSJ with parameter uncertainties, whose models are given as below:

$$(\mathbf{M}_u)\quad \begin{cases} \dot{x}_1(t) = (A+\Delta A)x_1(t) + (B+\Delta B)\varphi(y_1(t)) + u_1(t) \\ \dot{y}_1(t) = Cx_1(t) + u_2(t) \end{cases}$$

$$(\mathbf{S}_u)\quad \begin{cases} \dot{x}_2(t) = (A+\Delta A)x_2(t) + (B+\Delta B)\varphi(y_2(t)) \\ \dot{y}_2(t) = Cx_2(t) \end{cases} \tag{15}$$

where the controllers $u_1(t), u_2(t)$ in form of (5) are added to the master system. Parameter matrices ΔA and ΔB are unknown real matrices representing norm-bounded parameter uncertainty, partly due to the changing of environment. The admissible uncertainties are assumed to be of the following form:

$$[\Delta A \quad \Delta B] = DF[E_a \quad E_b] \tag{16}$$

Similarly, the error dynamics can be derived as

$$(\mathbf{E}_u)\quad \begin{cases} \dot{e}(t) = (A+K_1+\Delta A)e(t) + (B+\Delta B)\phi(\varepsilon, y_2), \\ \dot{\varepsilon}(t) = Ce(t) + K_2\phi(\varepsilon, y_2). \end{cases} \tag{17}$$

where $e(t) = x_1(t) - x_2(t-\tau), \varepsilon(t) = y_1(t) - y_2(t-\tau)$, and $\phi(\varepsilon, y_2) = \varphi(\varepsilon(t) + y_2(t-\tau)) - \varphi(y_2(t-\tau))$ is periodic with respect to ε. If the error dynamics is globally asymptotically stable with respect to any parametric variations ΔA, ΔB satisfying (16), then the master and slave systems are said to achieve robust anticipating synchronization. In the rest part of this section, we shall design appropriate controllers such that the slave system robustly anticipates the master. The following lemma will be of significance in the derivation of the main results.

Lemma 3: Let $S_1 = S_1^T$, S_2, S_3 be real matrices with appropriate size, then the following statements are equivalent:
1° $S_1 + \mathrm{He}(S_2\Delta S_3) < 0\ \forall\Delta$: $\Delta^T\Delta \le \lambda^2 I$;
2° There exists a positive number $\xi > 0$ such that $S_1 + \xi\lambda^2 S_2 S_2^T + \xi^{-1}S_3^T S_3 < 0$;
3° There exists a positive number $\xi > 0$ such that

$$\begin{bmatrix} T_1 + \xi\lambda^2 T_3^T T_3 & T_2 \\ T_2^T & -\xi I \end{bmatrix} < 0. \tag{18}$$

Remark 3: The advantage of the representation given in Lemma 3 is that when there is an LMI variable S_2 which can result in a product term with ξ, we can resort to using (18) for convex optimization and vice versa. In fact, the above result is not new and is well known in the robustness analysis literatures. Here, for the sake of calculation, we give it in an alternative form of LMI.

According to the results derived in the previous section, we have the following corollary as an immediate consequence.

Corollary 1: Suppose $A + K_1 + \Delta A$ is Hurwitzian. If there exist a positive definite matrix $P > 0$, any matrix W, V, as well as diagonal matrices $\eta > 0, \delta > 0$ such that the following LMIs hold:

$$\begin{bmatrix} \Pi & \frac{1}{2}\rho C^T\delta + P(B+DFE_b) & C^T\delta \\ * & \eta + \rho V & V^T \\ * & * & -\delta \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} 2\eta & \rho\delta\nu \\ * & 2\delta \end{bmatrix} > 0, \tag{20}$$

where $\Pi = \mathrm{He}(PA + PDFE_a + W)$; ν is defined as in (9) and ρ is a prescribed constant, then the master system $(\mathbf{M}_u)$ and the slave $(\mathbf{S}_u)$ achieve robust anticipating synchronization.

In order to facilitate the design of controllers, matrix inequalities (19)-(20) in Corollary 1 can be further converted into the following theorem, which determines whether the master and slave systems with parameter uncertainties achieve robust anticipating synchronization.

Theorem 2: Given a constant scalar $\xi > 0$. The master system $(\mathbf{M}_u)$ and the slave $(\mathbf{S}_u)$ with parameter uncertainties are said to achieve robust anticipating synchronization, if for a prescribed constant scalar ρ, the matrix $A + K_1$ is Hurwitzian, and there exist a positive definite matrix $P > 0$, matrix W, V, as well as diagonal matrices $\eta > 0, \delta > 0$ such that

$$\begin{bmatrix} \mathrm{He}(PA+W) & \frac{1}{2}\rho C^T\delta + PB & C^T\delta & PD & E_a^T \\ * & \eta + \rho V & V^T & 0 & E_b^T \\ * & * & -\delta & 0 & 0 \\ * & * & * & -\xi^{-1} & 0 \\ * & * & * & * & -\xi \end{bmatrix} < 0, \tag{21}$$

as well as (20) hold. Moreover, the controller parameter can be derived by $K_1 = P^{-1}W$ and $K_2 = \delta^{-1}V$.

Proof. Note that the LMI (19) could be rewritten into the expression as follows:

$$\Xi + \begin{bmatrix} PD \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} E_a & E_b & 0 \end{bmatrix} + \begin{bmatrix} E_a^T \\ E_b^T \\ 0 \end{bmatrix} F^T \begin{bmatrix} D^T P & 0 & 0 \end{bmatrix} < 0, \tag{22}$$

where

$$\Xi = \begin{bmatrix} \mathrm{He}(PA+W) & \frac{1}{2}\rho C^T\delta + PB & C^T\delta \\ * & \eta + \rho V & V^T \\ * & * & -\delta \end{bmatrix}.$$

Based on Lemma 3, condition (22) is equivalent to

$$\Xi+\xi\begin{bmatrix}PD\\0\\0\end{bmatrix}\begin{bmatrix}D^TP & 0 & 0\end{bmatrix}+\xi^{-1}\begin{bmatrix}E_a^T\\E_b^T\\0\end{bmatrix}\begin{bmatrix}E_a & E_b & 0\end{bmatrix}<0,$$

and accordingly

$$\begin{bmatrix}\Theta & \frac{1}{2}\rho C^T\delta+PB+\xi^{-1}E_a^TE_b & C^T\delta\\ * & \eta+\rho V+\xi^{-1}E_b^TE_b & V^T\\ * & * & -\delta\end{bmatrix}<0 \quad (23)$$

with $\Theta=\mathrm{He}(PA+W)+\xi PDD^TP+\xi^{-1}E_a^TE_a$. Furthermore, applying Lemma 3 to (23) leads to the LMI (21) directly, which completes the proof. □

V. Numerical Simulations

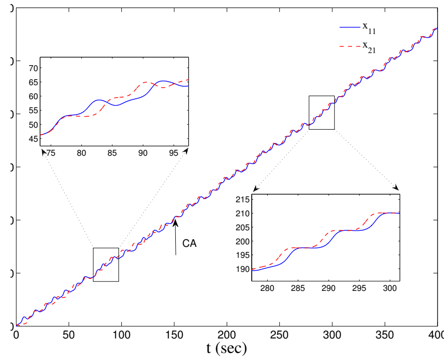
In this part, the effectiveness of the proposed methods in the previous sections will be demonstrated. Throughout the next two numerical examples, the parameters are employed as $\beta_C=0.707$, $\beta_L=2.6$, $i=1.17$, and g is described as in (2). In this manner, the RCLSJ model behaves chaotically as illustrated in Fig. 1.

Example 1 In this example, the anticipating synchronization between the master and slave RCLSJ models in the form of **(M)** and **(S)** as described in (4) will be examined, with feedback controllers added to the master system. By solving the LMIs in (10)-(11), we arrive at

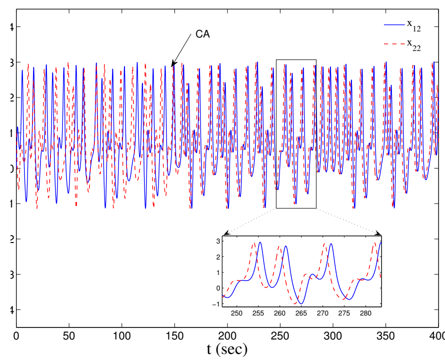
$$K_1=\begin{bmatrix}-0.5234 & -0.4988 & -0.0008\\ -1.3507 & -1.1560 & 0.1764\\ -0.0014 & 0.0633 & -0.1297\end{bmatrix}, \quad K_2=2.5031.$$

It is thus guaranteed by Theorem 1 that, with the derived controller parameters, the master and slave RCLSJ models could achieve anticipating synchronization.

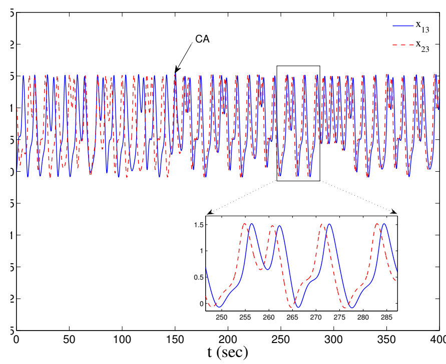
For the purpose of illustration, we shall numerically simulate the coupled RCLSJ systems (4) for $0s\le t\le 400s$ with the control signals (5) activated for $t\ge 150s$. Herein, the initial conditions for the master system are picked as $x_1(0)=[x_{11}(0)\ x_{12}(0)\ x_{13}(0)]^T=[0 0 0]^T$, and those of the slave system are $x_2(0)=[x_{21}(0)\ x_{22}(0)\ x_{23}(0)]^T=[1 1 1]^T$, with constant time delay $\tau=1.5$. Simulation results of the dynamic behaviors shown in Fig. 2(a)-(c) are depicted under the circumstance that the control is switched on for $t\ge 150s$ (marked as CA in the figures). Within these figures, the solid and dashed lines represent the dynamic trajectories of the master and slave RCLSJ systems, respectively. The top-left inset in Fig. 2(a) shows that if no controller is employed, the trajectories of the master and slave RCLSJ models become irrelevant to each other; the bottom-right inset is the zoom illustrating the anticipating behavior after adding the feedback controller. In a similar pattern, the magnified parts of the trajectory in Fig. 2(b) and Fig. 2(c) are depicted in the corresponding zooms, which reveals the anticipating effect between the master and slave RCLSJ models. To make the demonstration even more clear, we plot the waveforms of the corresponding error dynamics $e_i(t)$ $(i=1,2,3)$ in Fig. 3, which shows that the error states converge to zero soon after the feedback controller is switched on at $t=150s$ (marked as CA within these figures).



(a)



(b)



(c)

Fig. 2. The state dynamic behaviors of anticipating synchronization between the master and slave RCLSC models.

Example 2 This example focuses on the robust anticipating synchronization of RCLSJ models with parameter uncertainties, as expressed in (15). Here, the uncertainty parameters are adopted as $\Delta A=DFE_a, \Delta B=DFE_b$, where $D=[0.4\ 0.5\ 0.1]^T$, $E_a=[0.8\ 0.6\ 0]$, $E_b=0.5$, and F is a random scalar that varies from 0 to 1. The controller parameter matrices are obtained through solving the LMIs (20) and (21):

$$K_1=\begin{bmatrix}-0.7775 & -0.5390 & -0.0545\\ -0.7649 & -0.9642 & 0.3836\\ -0.0932 & 0.1839 & -0.1599\end{bmatrix}, \quad K_2=2.0175,$$

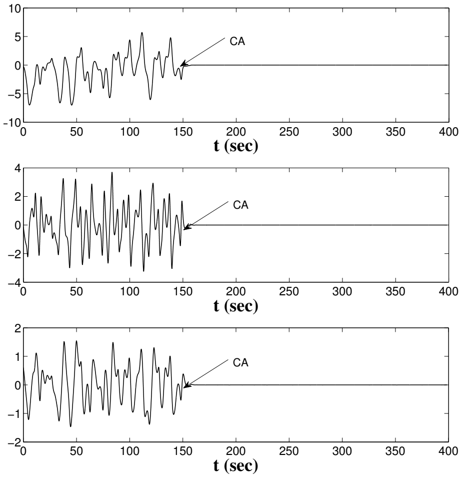


Fig. 3. The error dynamics $e_i(t)$ $(i = 1,2,3)$.

which in turn guarantees by Theorem 2 the realization of robust anticipating synchronization. To further demonstrate the effectiveness of the proposed method, norms of the error dynamics $\|e_i(t)\|$ for $i = 1,2,3$ are plotted in Fig. 4 (a)-(c), respectively, which can be observed to coincide with the theoretical results.

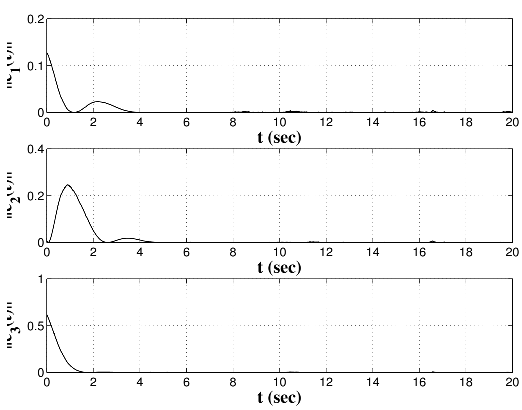


Fig. 4. Robust anticipating synchronization with parameter uncertainties.

VI. Conclusion

In summary, we have carried out analysis of the anticipating synchronization between two master-slave coupled RCLSJ models. Through examining the derived error dynamics, which turns to be of the pendulum-like type, criteria have been established in terms of LMIs which are applicable to guarantee the anticipating synchronization of such systems. Under the circumstance of parameter uncertainties, we further explored the robust anticipation scenario. These results allow one to predict the nonlinear chaotic behaviors by using a copy of the same system that performs as a slave. As for the future work, synchronization of RCLSJ models with more generalized type of uncertainties, such as the polytopic uncertainties, could be taken into account, which would be a natural extension of this study.

References

[1] L.M. Pecora and T.L. Carroll, Synchronization in chaotic Systems, *Phys. Rev. Lett.*, vol. 64, 1990, pp 821-824.

[2] G. Chen and X. Dong, *From Chaos to Order: Methodologies, Perspectives and Applications*, World Scientific, Singapore: 1998

[3] R. Brown and L. Kocarev, A unifying definition of synchronization for dynamical systems, *Chaos*, vol. 10, 2000, pp 344-349.

[4] S. Boccaletti, L.M. Pecora and A. Pelaez, Unifying framework for synchronization of coupled dynamical systems, *Phys. Rev. E*, vol. 63, 2001, pp 066219.

[5] H.U. Voss, Anticipating chaotic synchronization, *Phys. Rev. E*, vol. 61(5), 2000, pp 5115-5119.

[6] C. Masoller, Anticipation in the Synchronization of Chaotic Semiconductor Lasers with Optical Feedback, *Phys. Rev. Lett.*, vol. 86, 2001, pp 2782-2785.

[7] M. Ciszak, O. Calvo, C. Masoller, C. Mirasso and R. Toral, Anticipating the Response of Excitable Systems Driven by Random Forcing, *Phys. Rev. Lett.*, vol. 90, 2003, pp 204102.

[8] H. Huijberts, H. Nijmeijer and T. Oguchi, Anticipating synchronization of chaotic Lur'e systems, *Chaos*, vol. 17, 2007, pp 013117

[9] S.Y. Xu, Y. Yang and L. Song, Control-oriented approaches to anticipating synchronization of chaotic deterministic ratchets, *Phys. Lett. A*, vol. 373, 2009, pp 2226-2236.

[10] B.A. Hubermann, J.A. Crutchfield and N.H. Packard, Noise phenomena in Josephson junctions, *App. Phys. Lett.*, vol. 37, 1980, pp 750-752.

[11] M. Cirillo, N.F. Pedersen, On bifurcation and transition to chaos in a Josephson junction, *Phys. Lett. A*, vol. 90(3), 1982, pp 150-152.

[12] K.K. Likharev, *Dynamics of Josephson Junctions and Circuits*, Gorden and Breach, New York: 1986.

[13] S. Dana, D. Sengupta and K. Edoh, Chaotic Dynamics in Josephson Junction *IEEE Trans. Circuits Systems I*, vol. 48(8), 2001, 990-996.

[14] C.B. Whan and C.L. Lobb, Complex dynamical behavior in RCL-Shunted Josephson tunnel junctions, *Phys. Rev. E*, vol. 53, 1996, pp 405-413.

[15] A.B. Cawthorne, C.B. Whan and C.L. Lobb, Complex dynamics of resistively and inductively shunted Josephson junction, *J. Appl. Phys.*, vol. 84, 1998, pp 1126-1132.

[16] Ahmad M. Harb and Bassam A. Harb, Controlling chaos in Josephson-junction using nonlinear backstepping controller, *IEEE Trans. Appl. Supercon.* 16(4), 2006, 1988-1998.

[17] S.K. Dana, P.K. Roy, G.C. Sethia, A. Sen and D.C. Sengupta, Taming of chaos and synchronisation in rclshunted josephson junctions by external forcing, *IEEE Proc. Circuits Devices Syst.*, vol. 153(5), 2006, pp 453-460.

[18] R.N. Chitra and V.C. Kuriakose, Dynamics of coupled Josephson junctions under the influence of applied fields, *Phys. Lett. A*, vol. 365, 2007, pp 284.

[19] H. Nimeijer, M.Y. Mareels Ivan, An observer looks at synchronization, *IEEE Trans. Circuits Systems I*, vol. 44, 1997, pp 882-890.

[20] A. Uçar, K.E. Lonngren and E.W. Bai, Chaos synchronization in RCL-shunted Josephson junction via active control, *Chaos, Solitons and Fractals*, vol. 31, 2007, pp 105-111.

[21] U.E. Vincent, A. Uçar, J.A. Laoye and S.O. Kareem, Control and synchronization of chaos in RCL-shunted Josephson junction using backstepping design, *Physica C*, vol. 468, 2008, pp 374-382.

[22] J.J. Yan, C.F. Huang and J.S. Lin, Robust synchronization of chaotic behavior in unidirectional coupled RCLSJ models subject to uncertainties, *Nonlinear Analysis*, vol. 10, 2009, pp 3091-3097.

[23] G.A. Leonov, V. Reitmann and V.B. Smirnova, *Non-local methods for pendulum-like feedback systems*, Teubner-Texte zur Mathematik Bd. 132, B.G., Teubner Stuttgart-Leipzig: 1992.

[24] G.A. Leonov, D.V. Ponomarenko and V.B. Smirnova, *Frequency-Domain Methods for Nonlinear Analysis*, World Scientific, Singapore: 1996.

[25] S. Boyd, L. ELGhaoui, E. Feron and V. Balakrishnam, *Linear Matrix Inequalities in Systems and Control*, SIAM, Philadelphia: 1994.

[26] X. Huang and J.D. Cao, Generalized synchronization for delayed chaotic neural networks: a novel coupling scheme, *Nonlinearity*, vol.19, 2006, pp 2797-2811.

[27] A. Rantzer, On the Kalman-Yakubovich-Popov lemma, *Syst. Contr Lett.*, vol. 28, 1996, pp 7-10.