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Algebraic identification of a DC servomechanism using a Least Squares algorithm

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Abstract— **This paper proposes a parameter identification methodology based on a discrete-time Least Squares algorithm and a parametrization obtained using the Operational Calculus. References [4] and [5] proposed previously this parametrization and developed an Algebraic Identification Method (AIM) for parameter estimation of linear systems. The AIM employs the Operational Calculus to obtain analytical expressions for the parameter estimates. These expressions have a singularity at t=0 and certain excitation signals may also produce singularities at other time instants. The proposed approach employs the same parametrization obtained using the Operational Calculus, which is linear in the parameters, and employs a standard on-line discrete-time Least Squares algorithm. In this way, the proposed approach completely eliminates the problem of singularities; moreover, it is experimentally shown that the AIM and the proposed approach have similar performances.**

Key words: Parameter estimation, algebraic parametrization, servomechanism.

I. INTRODUCTION

Parameter estimation plays a key role in today servo drives since the identified model allows tuning a controller applied to a servomechanism. Moreover, the identification procedure is applied under open loop or closed-loop conditions. There exist in the literature several references dealing with parameter identification techniques [1]. Reference [2] shows a procedure for identifying a velocity-controlled servomotor using chaotic excitation signals. The Authors conclude from experiments in a laboratory prototype that the choice of the excitation signals has fundamental role in the identification procedure. Reference [3] presents an output error closed loop identification method. In this case, the servomechanism and a two-parameter model are simultaneously controlled through a Proportional Derivative (PD) controller. The error between the output of the model and the servomechanism feeds a gradient algorithm. Subsequently, the parameter estimates allows computing a Proportional Integral Derivative (PID) controller using a Linear Quadratic Regulator approach. On the other hand, references [4] and [5] present a novel algebraic identification approach based on the Operational calculus. This approach, which in the sequel will be termed as the Algebraic Identification Method (AIM), was applied for the parameter identification of a DC motor [6] and [7]. This approach has several interesting features. Firstly, applying the Operational Calculus to the model of a DC motor allows eliminating constant disturbances and the effect of the initial conditions; moreover, it also filters-out high frequency noise an provides parameter estimates in a very short time period. The approach works with almost all kind of excitation signals; however, there exists the possibility of singularities in the solutions of the parameter estimates. Reference [6] employs the AIM for closed-loop identification of a DC motor model. The method simultaneously estimates the servomotor inertia and viscous friction; then, these estimates allow obtaining an estimate of the Coulomb friction. Reference [7] shows the AIM applied for identifying the second-order model of a velocity-controlled DC servomotor under constant loads.

The aim of this work is to present an on-line identification procedure based on the parametrization obtained using the Operational Calculus combined with a discrete-time Least Squares algorithm. This approach, which will be named Algebraic Recursive Identification Method (ARIM), exploits the advantages of this parametrization and does not exhibit the problem of singularities. The paper has the following structure. Section II presents the servomechanism model. Section III shows the parametrization obtained using the Operational Calculus as well as the AIM and the ARIM. Section IV points up the method employed for validating the identified model. Section V depicts the experimental results obtained using both methods. The paper ends with some concluding remarks.

II. SERVOMECHANISM MODEL

The servomechanism consists of a DC servomotor driving a brass disk, a power amplifier and a position sensor; Fig. 1 depicts its block diagram.

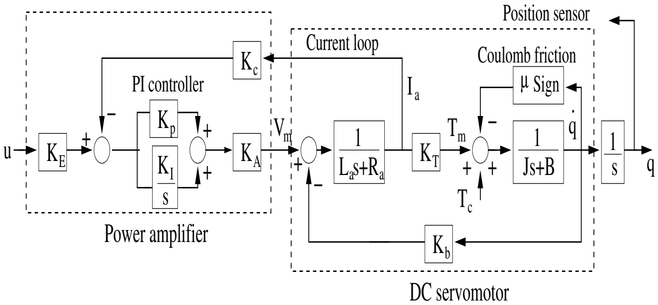


Fig. 1. Servomechanism model.

The amplifier works in current model, i.e. a Proportional Integral (PI) controller closes a loop around the amplifier using the armature current I_a. Variable q is the servomotor position, u is the control voltage, J and B are respectively the inertia and viscous friction. The inertia J comprises the motor and the brass disk inertias. Parameter μ defines the Coulomb friction coefficient and the term T_c corresponds to constant disturbances or parasitic constant voltages produced inside the power amplifier.

The following equation describes the servomechanism model

$$\ddot{q}(t) = -a\dot{q}(t) + bu(t) - c\text{sign}(\dot{q}(t)) + d \qquad (1)$$

Variables are defined as $a = B/J$, $b = K/J$, $K = \frac{K_E K_T}{K_c}$, $c = \mu/J$ y $d = T_c/J$. This model assumes a high value of integral gain K_I; in this way, the servomechanism electric time constant is much smaller than its mechanical time constant.

III. Identification algorithms

Fig. 2 shows how the AIM and the ARIM take the signals from the servomechanism. A PD controller stabilizes the loop without knowledge about the servomechanism parameters.

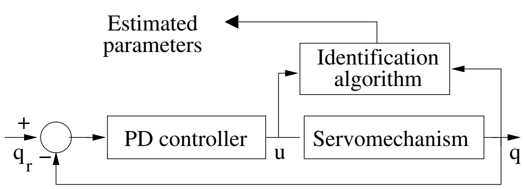


Fig. 2. Closed loop identification.

A. Algebraic Identification Method

In order to apply the AIM, assume that the motor rotates only in one direction. This assumption allows writing equation (1) as follows

$$\ddot{q}(t) = -a\dot{q}(t) + bu(t) + \nu \qquad (2)$$

Note that $\nu = -c + d$ if sign($\dot{q}(t)$)=1 and $\nu = c + d$ if sign($\dot{q}(t)$) $= -1$. Applying the Operational Calculus [4] to this last expression yields the following parametrization [6]

$$z_1(t) = \phi_{11}(t)a + \phi_{12}(t)b \qquad (3)$$

Where[1]

$$\begin{aligned} z_1(t) &= t^3 q - 9\int t^2 q + 18\int^{(2)} tq - 6\int^{(3)} q \\ \phi_{11}(t) &= -\int t^3 q + 6\int^{(2)} t^2 q - 6\int^{(3)} tq \\ \phi_{12}(t) &= \int^{(2)} t^3 u - 3\int^{(3)} t^2 u \end{aligned} \qquad (4)$$

[1] $\int^n \sigma(t)$ represents the iterated integral $\int_0^t \int_0^{\gamma_1} \ldots \int_0^{\gamma_{n-1}} \sigma(\gamma_n) d\gamma_n \ldots d\gamma_2 d\gamma_1$. Moreover, $\left(\int^1 \sigma(t)\right) = \left(\int \sigma(t)\right) = \left(\int_0^t \sigma(\gamma_1) d\gamma_1\right)$.

The following expression gives an alternative writing of (4)

$$\begin{aligned} \dot{z}_1 &= z_{1a} + t^3 q \\ \dot{z}_{1a} &= z_{1b} - 9t^2 q \\ \dot{z}_{1b} &= z_{1c} + 18tq \\ \dot{z}_{1c} &= -6q \end{aligned} \qquad (5)$$

$$\begin{aligned} \dot{\phi}_{11} &= \phi_{11a} & \dot{\phi}_{12} &= \phi_{12a} \\ \dot{\phi}_{11a} &= \phi_{11b} - t^3 q & \dot{\phi}_{12a} &= \phi_{12b} \\ \dot{\phi}_{11b} &= \phi_{11c} + 6t^2 q & \dot{\phi}_{12b} &= \phi_{12c} + t^3 u \\ \dot{\phi}_{11c} &= -6tq & \dot{\phi}_{12c} &= -3t^2 \end{aligned} \qquad (6)$$

On the other hand, integrating both sides of (3) yields

$$z_2(t) = \phi_{21}(t)a + \phi_{22}(t)b \qquad (7)$$

With

$$z_2(t) = \int_0^t z_1(\tau)d\tau, \qquad \phi_{21}(t) = \int_0^t \phi_{11}(\tau)d\tau,$$

$$\phi_{22}(t) = \int_0^t \phi_{12}(\tau)d\tau$$

Expressions (3) and (7) form the following set of simultaneous equations

$$\begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \qquad (8)$$

Solving the above system gives the estimates of a and b, i.e.

$$\begin{aligned} \hat{a} &= \frac{n_{\hat{a}}(t)}{\det(\phi(t))} = \frac{z_1(t)\phi_{22}(t) - z_2(t)\phi_{12}(t)}{\phi_{11}(t)\phi_{22}(t) - \phi_{12}(t)\phi_{21}(t)} \\ \hat{b} &= \frac{n_{\hat{b}}(t)}{\det(\phi(t))} = \frac{z_2(t)\phi_{11}(t) - z_1(t)\phi_{21}(t)}{\phi_{11}(t)\phi_{22}(t) - \phi_{12}(t)\phi_{21}(t)} \end{aligned} \qquad (9)$$

Note that $\det(\phi(t)) = \phi_{11}(t)\phi_{22}(t) - \phi_{12}(t)\phi_{21}(t) = 0$ for $t = 0$ and the solution has a singularity. Moreover, according to [4], for $t > 0$, almost any signals u and q are persistent, i.e., these signals satisfy a Persistent Condition if they produce $\det(\phi(t)) \neq 0$. Moreover, it is possible to obtain a unique solution $\hat{a}$ and $\hat{b}$ in a finite time interval $[0, \rho]$, $\rho > 0$. After this time, the AIM stops functioning. It is also worth remarking that the signals z_i and the regressors $\phi_{ij}, i, j = 1, 2$ in (3) and (7) remain bounded for a finite time interval. The following filtering procedure [5] attenuates the effects of zero-mean measurement noise.

$$\hat{a} = \frac{g * n_{\hat{a}}(t)}{g * \det(\phi(t))} \qquad \hat{b} = \frac{g * n_{\hat{b}}(t)}{g * \det(\phi(t))} \qquad (10)$$

The term g corresponds to a filter with transfer function $G(s)$ and $*$ is the convolution operator. This work uses $G(s) = 1/s$ and then the expressions for the estimates becomes

$$\hat{a} = \frac{\int n_{\hat{a}}(t)}{\int \det(\phi(t))} \qquad \hat{b} = \frac{\int n_{\hat{b}}(t)}{\int \det(\phi(t))} \qquad (11)$$

B. Algebraic Recursive Identification Method

The ARIM uses the parametrization (3) but the integrals defined in (4) are reset every T seconds; this resetting prevents these signals from becoming unbounded. Note that (3) is also valid for $t = kh$; $k = 0, 1, 2, \ldots$, where h is the sampling period. The above remark allows writing (3) as

$$z_1(kh) = \phi_{11}(kh)a + \phi_{12}(kh)b \tag{12}$$

Omitting the sampling period h and defining $\phi(k) = [\phi_{11}(k)\ \phi_{12}(k)]^T$ and $\theta = [a\ b]^T$ finally leads to

$$z_1(k) = \phi^T(k)\theta \tag{13}$$

This parametrization allows using the following standard Least Squares algorithm [8], [9]:

$$\hat{\theta}(k) = \begin{pmatrix} \hat{a}(k) \\ \hat{b}(k) \end{pmatrix} = \hat{\theta}(k-1) + L(k)\epsilon(k)$$

$$L(k) = \frac{P(k-1)\phi(k)}{1 + \phi^T(k)P(k-1)\phi(k)} \tag{14}$$

$$P(k) = P(k-1) - \frac{P(k-1)\phi(k)\phi^T(k)P(k-1)}{1 + \phi^T(k)P(k-1)\phi(k)}$$

$$\epsilon(k) = z(k) - \phi^T(k)\hat{\theta}(k-1)$$

Vector $\hat{\theta}$ is an estimate of θ, $P(k)$ is the covariance matrix inverse and $\epsilon(k)$ the estimation error. Compared with the AIM, it is clear that the ARIM completely eliminates the singularity problem since it does not stem on the solution (9). Note that the following Persistence of Excitation condition [10], [11] replaces the Persistent Condition given in [4].

Definition 1: A vector $\phi(k) \in R^n$ satisfies a Persistence of Excitation (PE) condition if for all j there exist some α such that

$$\alpha_1 \geq \sum_{\kappa=j}^{j+\alpha} [\chi^T \phi(\kappa)]^2 \geq \alpha_2 \tag{15}$$

For positive constants $\alpha_1, \alpha_2 > 0$ and for $\chi \in R^n$ with $\|\chi\|$=1.

The estimates $\hat{\theta}$ obtained using the AIM and the ARIM allow computing estimates $\hat{c}$ and $\hat{d}$ of the remaining parameters c and d. Assume that after a time t_0 the AIM and the Least Squares algorithm provide estimates $\hat{a}$ and $\hat{b}$ and define the triangle reference $q_r(t)$ shown in Fig. 3

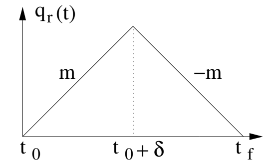


Fig. 3. Reference signal $q_r(t)$.

$$q_r(t) = \begin{cases} m(t - t_0) + q_r(t_0), & \text{if } t \in [t_0,\ t_0 + \delta] \\ -m[t - (t_0 + 2\delta)] + q_r(t_0), & \text{if } t \in [t_0 + \delta,\ t_f] \end{cases} \tag{16}$$

The terms m and $-m$ correspond to the slopes of $q_r(t)$ and $\delta = \dfrac{t_f - t_0}{2}$. If the gain of the PD controller stabilizing the servomechanism is high enough, then $q(t) \approx q_r(t)$. As a consequence, in the time interval $[t_0,\ t_0 + \delta]$ the equalities $\dot{q}(t) = m$, sign($\dot{q}$)=1, and $\ddot{q} = 0$ hold. Therefore, the estimated model corresponds to

$$\hat{c} - \hat{d} = -\hat{a}m + \hat{b}u(t), \qquad t \in [t_0,\ t_0 + \delta] \tag{17}$$

Equivalently, during the time interval $[t_0 + \delta,\ t_f]$ the inequalities $\dot{q}(t) = -m$, sign($\dot{q}$) $= -1$, and $\ddot{q} = 0$ hold. The corresponding estimated model is

$$-\hat{c} - \hat{d} = \hat{a}m + \hat{b}u(t), \qquad t \in [t_0 + \delta,\ t_f] \tag{18}$$

Equation (17) is equivalent to

$$\hat{c} - \hat{d} = -\hat{a}m + \hat{b}u(t - \delta), \qquad t \in [t_0 + \delta,\ t_f] \tag{19}$$

Since the estimates $\hat{a}$, $\hat{b}$, $\hat{c}$, and $\hat{d}$ are constants so do the control signals $u(t)$ and $u(t-\delta)$. Hence, define $u_m = u(t-\delta)$ and $u_{-m} = u(t)$, $t \in [t_0 + \delta,\ t_f]$. Using this definition and adding (18) and (19) yields

$$-2\hat{d} = \hat{b}[u_m + u_{-m}] \qquad t \in [t_0 + \delta,\ t_f] \tag{20}$$

Therefore, the following expressions give parameter estimates $\hat{c}$ and $\hat{d}$

$$\hat{c} = -[\hat{a}m + \hat{b}u_m + \hat{d}]$$

$$\hat{d} = -\frac{\hat{b}[u_m + u_{-m}]}{2} \tag{21}$$

IV. MODEL VALIDATION

In order to validate the identified models using the AIM and the ARIM, they are employed for computing a model reference tracking control law (see Fig. 4). The tracking error $e = q_m - q$ is sampled p times and every sampled value e_i is used for computing the mean square error, which is a measure of the tracking quality

$$E = \sum_{i=1}^{p} e_i^2$$

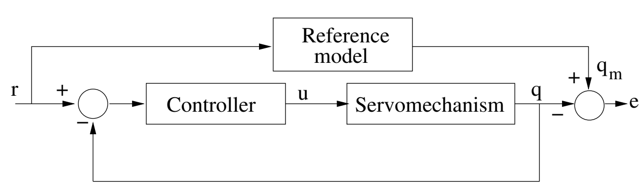


Fig. 4. Model reference control system.

A. Control law

Assume the following reference model

$$\ddot{q}_m(t) = -a_1\dot{q}_m(t) - a_2 q_m(t) + r(t) \tag{22}$$

Parameters a_1 and a_2 are positive constants and $r(t)$ is a reference. The next expression defines the control law applied to the servomechanism

$$u(t) = \frac{1}{\hat{b}}[\lambda_1 \dot{e}(t) + \lambda_2 e(t) + \ddot{q}_m(t) + \hat{a}\dot{q}(t) + \hat{c}\text{sign}(\dot{q}(t)) - \hat{d}] \tag{23}$$

The term $\ddot{q}_m(t)$ is given by (22) and λ_1, λ_2 are positive constants. Adding and subtracting $\hat{b}u$ to $\ddot{e}(t) = \ddot{q}_m(t) - \ddot{q}(t)$ and using (1) leads to

$$\begin{aligned} \ddot{e}(t) &= \ddot{q}_m(t) - \ddot{q}(t) + \hat{b}u(t) - \hat{b}u(t) \\ &= \ddot{q}_m(t) + a\dot{q}(t) - bu(t) + c\text{sign}(\dot{q}(t)) - d + \hat{b}u(t) - \hat{b}u(t) \end{aligned} \tag{24}$$

Substituting the control law (23) in this last equation produces the error dynamics

$$\ddot{e}(t) + \lambda_1 \dot{e}(t) + \lambda_2 e(t) = \tilde{\theta}^T \psi(t) \tag{25}$$

with

$$\tilde{\theta} = \begin{pmatrix} \tilde{a} \\ \tilde{b} \\ \tilde{c} \\ \tilde{d} \end{pmatrix} = \begin{pmatrix} \hat{a} - a \\ \hat{b} - b \\ \hat{c} - c \\ \hat{d} - d \end{pmatrix} \qquad \psi(t) = \begin{pmatrix} -\dot{q}(t) \\ u(t) \\ -\text{sign}(\dot{q}(t)) \\ 1 \end{pmatrix} \tag{26}$$

V. Experimental results

The laboratory prototype consists in a servomotor from Moog, model C34-L80-W40 (Fig. 5 (a)) driven by a Copley Controls power amplifier, model 423, configured in current mode. A BEI optical encoder, model L15 with 2500 pulses per revolution, allows measuring the servomotor position. The algorithms are coded using the MatLab/Simulink software platform under the program Wincon from Quanser Consulting, and a Q8 board also from Quanser Consulting performs data acquisition. The software runs on a Personal Computer using an Intel Core 2 quad processor, and the Q8 board is allocated in a PCI slot inside this computer. The PD controller gains are set to $k_p = 10$ and k_d =0.34. The following linear band-pass filter

$$G(s) = \left(\frac{220s}{s+220}\right)\left(\frac{500}{s+500}\right)$$

estimates the servomotor velocity from position measurements. The sampling period is 50 μs and the ODE5 method allows evaluating the integrals appearing in the signals z_i and the regressors ϕ_{ij}, $i,j = 1,2$ of (3) and (7). The sampling period for the Least Squares algorithm is fixed to h =0.5ms and the time used for resetting the integrals used in the proposed method is set to $T = 2.5s$. The initial value for the covariance matrix in the Least Squares algorithm is $P(0) = $ diag(10,000, 10,000).

Fig. 5 (b) shows the excitation signal $q_r(t)$ employed during the experiments. The excitation signal for the time interval [0, 5] is $q_r = 11t + 4\sin(0.8\pi t)$ and corresponds to the identification of parameters a and b. Equation (16) defines the excitation signal for the time interval [5, 15]. Fig. 5 (c) and 5 (d) depict respectively the servomotor speed and the control signal obtained by applying the excitation signal $q_r(t)$.

Fig. 6 shows the time evolution of the parameter estimates produced by the AIM. Estimates $\hat{a}$ and $\hat{b}$ were set to 1 in the time interval [0, 0.5] due to the singularity problem exhibited by this method. Fig. 7 portraits the time evolution of the parameter estimates obtained using the ARIM. It is worth noting that the estimates $\hat{a}$ and $\hat{b}$ obtained using the AIM converge faster than the ones obtained using the ARIM to 0.15 and 137 respectively . However, in both cases the estimates converges to constant values after $2s$. The behavior of the estimates $\hat{c}$ and $\hat{d}$ for the two methods is the same.

Fig. 8 (a) shows the time evolution of $\sum[\chi^T\phi]^2$ with $\chi = [1/\sqrt{2}\ 1/\sqrt{2}]^T$. Therefore, the excitation signal employed for identifying $\hat{a}$ and $\hat{b}$ is fulfills a PE condition. The models identified using the AIM and the ARIM are used for computing control law (23) with a_1 =0.3, $a_2 = 10$, $\lambda_1 = 10$ and $\lambda_2 = 10$. Fig. 8 (b) depicts the output $q_m(t)$ of the reference model, and Fig. 8 (c) and 8 (d) show the tracking error when the control law is computed using the parameter estimates obtained though the AIM and the ARIM respectively.

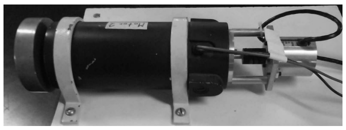
Table I shows the parameter estimates and the mean square error E, which is computed for p=300,000. This table also shows the nominal parameters of the servomechanism obtained using the servomotor and power amplifier technical data. The experimental results indicates a good agreement between the estimated parameters produced by the two methods and the nominal parameters $\hat{a}$ and $\hat{b}$. The parameters associated to the Coulomb friction and the constant disturbance are not available from the technical data. Moreover, the mean square error E is similar for the two identification methods.

VI. Conclusions

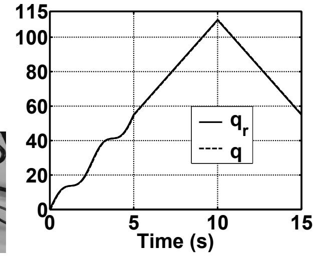
The Algebraic Identification Method (AIM) and the proposed approach (ARIM) successfully identified a four-parameter DC servomotor model. The former provides parameter estimates in a slightly shorter time period than the proposed approach; however, the later is completely free of singularities.

Table I
PARAMETER ESTIMATES.

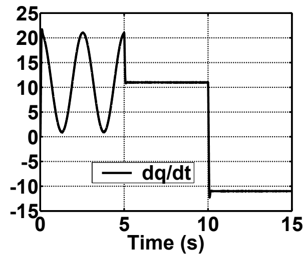
	$\hat{a}$	$\hat{b}$	$\hat{c}$	$\hat{d}$	Mean square error E
Nominal parameters	0.193	137.78	—	—	
AIM	0.149	137.5	4.5	1.05	28.7375
ARIM	0.155	137.3	4.4	0.97	28.8329



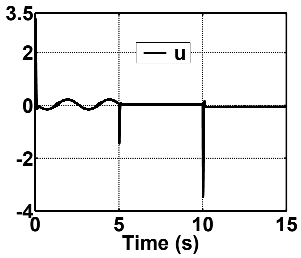
(a) Servomotor.



(b) Reference $q_r(t) \approx q(t)$.



(c) Signal $\dot{q}(t)$.



(d) Control signal $u(t)$.

Fig. 5. Servomotor used in the laboratory test and signals $q(t)$, $\dot{q}(t)$ and $u(t)$.

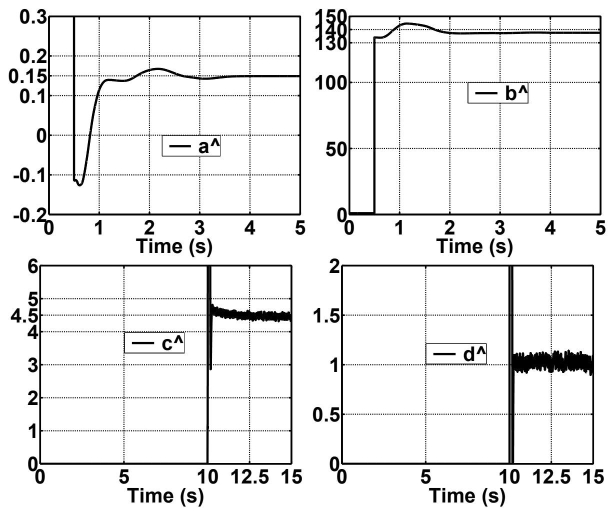


Fig. 6. Parameter estimates produced by the AIM.

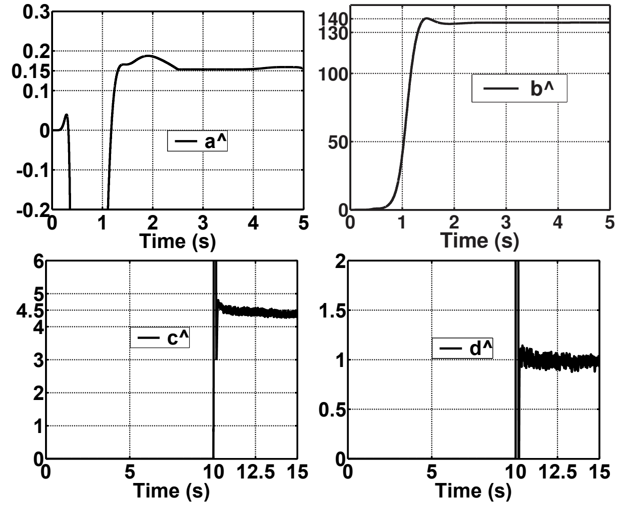
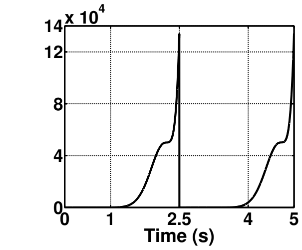
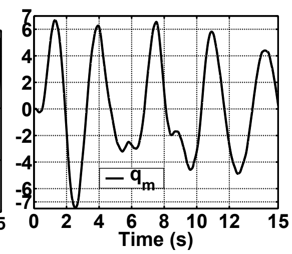


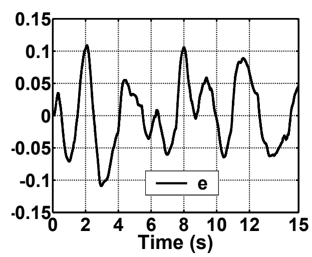
Fig. 7. Parameter estimates obtained by the ARIM.



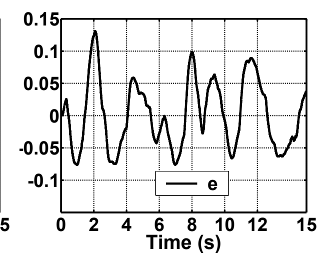
(a) $\sum(\chi^T\phi)^2$ in (15).



(b) Signal q_m.



(c) Error e corresponding to the estimates produced by the AIM.



(d) Error e corresponding to the estimates produced by the ARIM.

Fig. 8. Output q_m of the reference model and tracking errors e.

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