2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Adaptive H_∞ Formation Control for Euler-Lagrange Systems by Utilizing Neural Network Approximators

Yoshihiko Miyasato

***Abstract*— Design methods of adaptive H_∞ formation control of multi-agent systems composed of Euler-Lagrange systems by utilizing neural network approximators are presented in this paper. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters, error terms in potential functions, and approximate and algorithmic errors in neural network estimation schemes are regarded as external disturbances to the process. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable formations are achieved asymptotically via adaptation schemes.**

I. Introduction

Recently, formation control problems of multi-agent systems have attracted much attentions, and several formation control schemes were proposed based on various strategies (for example, leader-follower [1], behavior-based [2], virtual structure [3], and potential function approaches [4], [5], [6]). Among those, the potential function approaches seem to be useful tools from the view points of flexibility of configurations of swarms, automatic avoidance of collisions of agents, and stability of maintaining formations. In those research works, adaptive control or sliding mode control methodologies were applied in order to deal with uncertainties of agents, and stability of control systems was assured via Lyapunov function analysis. Furthermore, robustness properties of the control schemes were also discussed in those works. However, so much attention has not been paid on control performance such as optimal property or transient performance in those approaches.

On the contrary, in recent decades, stable controller designs for nonlinear and adaptive control systems have been investigated from the view point of inverse optimality [7], [8]. In those research works, the resulting control systems are shown to be optimal to certain meaningful cost functionals, and stability of the overall systems is also assured. Those approaches are extended to the design of inverse optimal H_∞ adaptive control systems, and various adaptive control systems are derived from those strategies together with additional control performances such as robustness to uncertain time-varying elements of system parameters [9], [10] and nonlinear parametric models [11], [12].

The purpose of the present paper is to present design methods of adaptive formation control of multi-agent systems composed of Euler-Lagrange systems based on the notion of inverse optimality. The neural network approximators are introduced to estimate nonlinear parametric elements in the agents. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters, artificial error terms in potential functions concerned with formations, and approximate and algorithmic errors in neural network estimation schemes are regarded as external disturbances to the process. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable formations are achieved asymptotically via adaptation schemes.

Y. Miyasato is with Department of Mathematical Analysis and Statistical Inference, The Institute of Statistical Mathematics, 190-8562 Tachikawa, Tokyo, Japan miyasato@ism.ac.jp

II. Problem Statement

We consider a multi-agent system composed of N fully actuated mobile robots which are described as a class of Euler-Lagrange systems [4], [5] written as follows:

$$M_i(y_i)\ddot{y}_i + C_i(y_i, \dot{y}_i)\dot{y}_i + F_i(y_i, \dot{y}_i) = \tau_i, \quad (i = 1, \cdots, N), \tag{1}$$

where $y_i \in \mathbf{R}^n$ is an output (a generalized coordinate), $\tau_i \in \mathbf{R}^n$ is a control input (a force vector), $M_i(y_i) \in \mathbf{R}^{n\times n}$ is an inertia matrix, and $C_i(y_i, \dot{y}_i) \in \mathbf{R}^{n\times n}$ is a matrix of Coriolis and centripetal forces. $F_i(y_i, \dot{y}_i)$ is a nonlinear term whose parametric structure is not specified in advance. Each component has the following properties as a Euler-Lagrange system.

Properties of Euler-Lagrange Systems [13]

1) $M_i(y_i)$ is a bounded, positive definite, and symmetric matrix.
2) $\dot{M}_i(y_i) - 2C_i(y_i, \dot{y}_i)$ is a skew symmetric matrix.
3) A part of the left-hand side of (1) can be written into
$$M_i(y_i)a_i + C_i(y_i, \dot{y}_i)b_i = -Y_i(y, \dot{y}_i, a_i, b_i)\theta_i, \tag{2}$$
where $Y_i(y_i, \dot{y}_i, a_i, b_i)$ is a known function of y_i, $\dot{y}_i$, a_i, b_i (a regressor matrix), and θ_i is an unknown system parameter vector.

Furthermore, it is assumed that $F_i(y_i, \dot{y}_i)$ is approximated by an three-layered neural network (a nonlinear parametric model) as follows:

$$F_i(y_i, \dot{y}_i) = \begin{bmatrix} W_{i1}^{\mathsf{T}} S(V_{i1}^{\mathsf{T}} \bar{z}_i) + \mu_{i11}(z_i) \\ \vdots \\ W_{in}^{\mathsf{T}} S(V_{in}^{\mathsf{T}} \bar{z}_i) + \mu_{i1n}(z_i) \end{bmatrix} \equiv W_i^{\mathsf{T}} S(V_i^{\mathsf{T}} \bar{z}_i) + \mu_{i1}(z_i) \ \in \mathbf{R}^n, \tag{3}$$

$$\bar{z}_i = [z_i^{\mathsf{T}}, 1]^{\mathsf{T}} \in \mathbf{R}^{2n+1}, \quad z_i = [y_i^{\mathsf{T}}, \dot{y}_i^{\mathsf{T}}]^{\mathsf{T}} \in \mathbf{R}^{2n}, \tag{4}$$

$$W_{ij} = [w_{ij1}, \cdots, w_{ijm}]^{\mathsf{T}} \in \mathbf{R}^m, \quad (1 \le j \le n), \tag{5}$$

$$V_{ij} = [v_{ij1}, \cdots, v_{ijm}] \in \mathbf{R}^{(2n+1)\times m},$$

$$v_{ijk} \in \mathbf{R}^{2n+1}, \quad (1 \le j \le n,\ 1 \le k \le m), \tag{6}$$

$$S(V_{ij}^{\mathsf{T}}\bar{z}_i) = [s(v_{ij1}^{\mathsf{T}}\bar{z}_i),\ \cdots,\ s(v_{ijm}^{\mathsf{T}}\bar{z}_i)]^{\mathsf{T}} \ \in \mathbf{R}^m, \tag{7}$$

$$s(v^{\mathsf{T}}\bar{z}) = \frac{1}{1+\exp\{-\gamma(v^{\mathsf{T}}\bar{z})\}}, \qquad (\gamma > 0), \tag{8}$$

$$W_i = \begin{bmatrix} W_{i1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & W_{in} \end{bmatrix} \in \mathbf{R}^{mn\times n}, \tag{9}$$

$$S(V_i^{\mathsf{T}}\bar{z}_i) = [S(V_{i1}^{\mathsf{T}}\bar{z}_i)^{\mathsf{T}},\ \cdots,\ S(V_{in}^{\mathsf{T}}\bar{z}_i)^{\mathsf{T}}]^{\mathsf{T}} \ \in \mathbf{R}^{mn}, \tag{10}$$

$$\mu_{i1}(z_i) = [\mu_{i11}(z_i),\ \cdots,\ \mu_{i1n}(z_i)]^{\mathsf{T}} \ \in \mathbf{R}^n, \tag{11}$$

where V_{ij} and W_{ij} are layer weights of the j-th neural network for the i-th agent, and m is a number of cells of each neural network. $s(v^{\mathsf{T}}\bar{z})$ is a sigmoid function, and $\mu_{i1}(z_i)$ is a vector of an approximation error for $F_i(y_i, \dot{y}_i)$.

The control objective is to construct an adaptive formation control system for a swarm of mobile robots (1) in which desirable configurations are achieved asymptotically via adaptation schemes.

III. Neural Network Approximator

Based on the fact that any continuous function over a compact set can be approximated by a three-layered neural network with an arbitrary small approximate error [14], the following assumption is introduced.

Assumption 4 *There exist layer weights V_{ij} and W_{ij} satisfying the following relations.*

$$|\mu_{i1j}(z_i)| \le d_{i1j}\psi_{ij}(z_i), \quad (1 \le j \le n), \tag{12}$$

where d_{i1j} are unknown positive constants, and $\psi_{ij}(z_i)$ are known positive functions.

It should be noted that (12) does not mean that the approximate errors $\mu_{i1j}(z_i)$ are small over $z_i \in \mathbf{R}^{2n}$, but says that the magnitudes of those are evaluated from above utilizing known functions $\psi_{ij}(z_i)$. In fact, we can choose $\psi_{ij}(z_i)$ such that $\psi_{ij}(z_i) \to \infty$ as $\|z_i\| \to \infty$.

The estimates of the layer weights V_{ij} and W_{ij} are denoted by $\hat{V}_{ij}$ and $\hat{W}_{ij}$, respectively. Then, the neural network estimation error $\hat{W}_{ij}^{\mathsf{T}} S(\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i) - W_{ij}^T S(V_{ij}^T\bar{z}_i)$ is evaluated in the following lemma.

Lemma 1 [15] *For the three-layered neural network, the estimation error is described as follows:*

$$\begin{aligned} &\hat{W}_{ij}^{\mathsf{T}} S(\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i) - W_{ij}^T S(V_{ij}^T\bar{z}_i) \\ &\quad = \tilde{W}_{ij}^{\mathsf{T}}(\hat{S}_{ij} - \hat{S}_{ij}'\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i) + \hat{W}_{ij}^{\mathsf{T}}\hat{S}_{ij}'\tilde{V}_{ij}^{\mathsf{T}}\bar{z}_i + \mu_{i2j}, \end{aligned} \tag{13}$$

$$\begin{aligned} |\mu_{i2j}| \le\ & \|V_{ij}\| \cdot \|\bar{z}_i\hat{W}_{ij}^{\mathsf{T}}\hat{S}_{ij}'\| \\ & + \|W_{ij}\| \cdot \|\hat{S}_{ij}'\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i\| + |W_{ij}|_1, \end{aligned} \tag{14}$$

$$\tilde{W}_{ij} = \hat{W}_{ij} - W_{ij}, \quad \tilde{V}_{ij} = \hat{V}_{ij} - V_{ij}, \tag{15}$$

$$\hat{S}_{ij} = S(\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i), \tag{16}$$

$$\hat{S}_{ij}' = \mathrm{diag}(\hat{s}_{ij1}',\ \cdots,\ \hat{s}_{ijm}'), \tag{17}$$

$$\hat{s}_{ijk}' = s'(\hat{v}_{ijk}^{\mathsf{T}}\bar{z}_i) = \left[\frac{ds(z)}{dz}\right]_{z=\hat{v}_{ijk}^{\mathsf{T}}\bar{z}_i.} \tag{18}$$

For convenience' sake, $\tilde{W}_{ij}^{\mathsf{T}}(\hat{S}_{ij} - \hat{S}_{ij}'\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i)$ and $\hat{W}_{ij}^{\mathsf{T}}\hat{S}_{ij}'\tilde{V}_{ij}^{\mathsf{T}}\bar{z}_i$ in (13) are rewritten into the following regression forms.

$$\tilde{W}_{ij}^{\mathsf{T}}(\hat{S}_{ij} - \hat{S}_{ij}'\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i) = \tilde{W}_{ij}^{\mathsf{T}}\omega_{ij0}, \tag{19}$$

$$\hat{W}_{ij}^{\mathsf{T}}\hat{S}_{ij}'\tilde{V}_{ij}^{\mathsf{T}}\bar{z}_i = \sum_{k=1}^{m}\hat{w}_{ijk}\hat{s}_{ijk}'\tilde{v}_{ijk}^{\mathsf{T}}\bar{z}_i = \sum_{k=1}^{m}\tilde{v}_{ijk}^{\mathsf{T}}\omega_{ijk}, \tag{20}$$

$$\omega_{ij0} = \hat{S}_{ij} - \hat{S}_{ij}'\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i, \qquad \omega_{ijk} = (\hat{w}_{ijk}\hat{s}_{ijk}')\bar{z}_i, \tag{21}$$

$$\tilde{v}_{ijk} = \hat{v}_{ijk} - v_{ijk}. \tag{22}$$

Then the overall representation for (13) is obtained such as

$$\begin{aligned} &\hat{W}_i^{\mathsf{T}} S(\hat{V}_i^{\mathsf{T}}\bar{z}_i) - W_i^{\mathsf{T}} S(V_i^{\mathsf{T}}\bar{z}_i) \\ &\quad = \tilde{W}_i^{\mathsf{T}}(\hat{S}_i - \hat{S}_i'\hat{V}_i^{\mathsf{T}}\bar{z}_i) + \hat{W}_i^{\mathsf{T}}\hat{S}_i'\tilde{V}_i^{\mathsf{T}}\bar{z}_i + \mu_{i2} \\ &\quad = \tilde{\Phi}_i^{\mathsf{T}}\Omega_i + \mu_{i2}, \end{aligned} \tag{23}$$

$$\Phi_i = \begin{bmatrix} \Phi_{i1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Phi_{in} \end{bmatrix}, \quad \left(\tilde{\Phi}_i = \hat{\Phi}_i - \Phi_i\right), \tag{24}$$

$$\Phi_{ij} = [W_{ij}^{\mathsf{T}},\ v_{ij1}^{\mathsf{T}},\ \cdots,\ v_{ijm}^{\mathsf{T}}]^{\mathsf{T}}, \tag{25}$$

$$\Omega_i = [\Omega_{i1}^{\mathsf{T}},\ \cdots,\ \Omega_{in}^{\mathsf{T}}]^{\mathsf{T}}, \tag{26}$$

$$\Omega_{ij} = [\omega_{ij0}^{\mathsf{T}},\ \omega_{ij1}^{\mathsf{T}},\ \cdots,\ \omega_{ijm}^{\mathsf{T}}]^{\mathsf{T}}, \tag{27}$$

$$\mu_{i2} = [\mu_{i21},\ \cdots,\ \mu_{i2n}]^{\mathsf{T}}, \tag{28}$$

$$\hat{S}_i = S(\hat{V}_i^{\mathsf{T}}\bar{z}_i) = [S(\hat{V}_{i1}^{\mathsf{T}}\bar{z}_i)^{\mathsf{T}},\ \cdots,\ S(\hat{V}_{in}^{\mathsf{T}}\bar{z}_i)^{\mathsf{T}}]^{\mathsf{T}} \tag{29}$$

$$\hat{S}_i' = \begin{bmatrix} \hat{S}_{i1}' & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{S}_{in}' \end{bmatrix}, \tag{30}$$

$$\hat{V}_i = [\hat{V}_{i1}, \cdots, \hat{V}_{in}]. \tag{31}$$

Also, the right-hand sides of (12) and (14) are summarized into the following forms, respectively.

$$\begin{bmatrix} d_{i11}\psi_{i1} \\ \vdots \\ d_{i1n}\psi_{in} \end{bmatrix} = \begin{bmatrix} \psi_{i1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \psi_{in} \end{bmatrix} \begin{bmatrix} d_{i11} \\ \vdots \\ d_{i1n} \end{bmatrix} \equiv \Psi_{i1}D_{i1}, \tag{32}$$

$$\begin{aligned} &\begin{bmatrix} \|V_{i1}\| \cdot \|\bar{z}_i\hat{W}_{i1}^{\mathsf{T}}\hat{S}_{i1}'\| + \|W_{i1}\| \cdot \|\hat{S}_{i1}'\hat{V}_{i1}^{\mathsf{T}}\bar{z}_i\| + |W_{i1}|_1 \\ \vdots \\ \|V_{in}\| \cdot \|\bar{z}_i\hat{W}_{in}^{\mathsf{T}}\hat{S}_{in}'\| + \|W_{in}\| \cdot \|\hat{S}_{in}'\hat{V}_{in}^{\mathsf{T}}\bar{z}_i\| + |W_{in}|_1 \end{bmatrix} \\ &\quad = \begin{bmatrix} \Psi_{i21} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Psi_{i2n} \end{bmatrix} \begin{bmatrix} D_{i21} \\ \vdots \\ D_{i2n} \end{bmatrix} \equiv \Psi_{i2}D_{i2}, \end{aligned} \tag{33}$$

$$\Psi_{i2j} = \left[\|\bar{z}_i\hat{W}_{ij}^{\mathsf{T}}\hat{S}_{ij}'\|,\ \|\hat{S}_{ij}'\hat{V}_{ij}^{\mathsf{T}}\bar{z}_i\|,\ 1\right]_, \tag{34}$$

$$D_{i2j} = [\|V_{ij}\|,\ \|W_{ij}\|,\ |W_{ij}|_1]^{\mathsf{T}}_. \tag{35}$$

Additionally, the next description is introduced to evaluate the term $|\tilde{\Phi}_{ij}^{\mathsf{T}}\Omega_{ij}|$.

$$\tilde{\Phi}_i^{\mathsf{T}}\Omega_i = \begin{bmatrix} \tilde{\Phi}_{i1}^{\mathsf{T}}\Omega_{i1} \\ \vdots \\ \tilde{\Phi}_{in}^{\mathsf{T}}\Omega_{in} \end{bmatrix}, \tag{36}$$

$$|\tilde{\Phi}_{ij}^{\mathsf{T}}\Omega_{ij}| \le \|\tilde{\Phi}_{ij}\|\|\Omega_{ij}\|, \tag{37}$$

$$\begin{bmatrix} \|\tilde{\Phi}_{i1}\|\|\Omega_{i1}\| \\ \vdots \\ \|\tilde{\Phi}_{in}\|\|\Omega_{in}\| \end{bmatrix}$$

$$= \begin{bmatrix} \|\Omega_{i1}\| & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \|\Omega_{in}\| \end{bmatrix} \begin{bmatrix} \|\tilde{\Phi}_{i1}\| \\ \vdots \\ \|\tilde{\Phi}_{in}\| \end{bmatrix}$$
$$\equiv \bar{\Omega}_i D_{i3}. \tag{38}$$

IV. Adaptive H_∞ Formation Control I

First, we consider a formation control problem [4], [6] in which all agents continue to move with a desired velocity $\dot{y}_r$ (39) and with a desired relative configuration defined by (40).

$$\dot{y}_i(t) = \dot{y}_r(t), \tag{39}$$
$$\|y_i(t) - y_j(t)\| = d_{ij}, \quad (d_{ij} = d_{ji},\, i \neq j), \tag{40}$$

where y_r is a reference point (a virtual leader) of the agents.

A. H_∞ Formation Control I

We introduce a positive potential function $J(y) \in \mathbf{R}$ $(y = [y_1^\mathsf{T}, \cdots, y_N^\mathsf{T}]^\mathsf{T} \in \mathbf{R}^{nN})$ in order to handle the desired configuration (40), where the minimal point of $J(y)$ such as

$$J(y) \to \min, \quad \left(\frac{\partial J(y)}{\partial y_i} = 0, \quad (1 \leq i \leq N)\right), \tag{41}$$

corresponds to the relative configuration (40). It is assumed that $J(y)$ is twice differentiable.

Define a control error s_i by

$$s_i = \Delta \dot{y}_i + g_i(y), \tag{42}$$
$$\Delta y_i = y_i - y_r, \tag{43}$$
$$g_i(y) = \frac{\partial J(y)}{\partial y_i}. \tag{44}$$

Then, we obtain the next relation.

$$\begin{aligned} &M_i \dot{s}_i + C_i s_i + F_i \\ &\quad = M_i(\ddot{y}_i - \ddot{y}_r + \dot{g}_i) + C_i(\dot{y}_i - \dot{y}_r + g_i) + F_i \\ &\quad = \tau_i - Y_i(y_i, \dot{y}_i, a_i, b_i)\theta_i, \end{aligned} \tag{45}$$

where a_i and b_i are defined such as

$$a_i = \dot{g}_i - \ddot{y}_r, \quad b_i = g_i - \dot{y}_r. \tag{46}$$

We determine the control law as follows:

$$\tau_i = Y_i(y_i, \dot{y}_i, a_i, b_i)\hat{\theta}_i + \hat{W}_i^\mathsf{T} S(\hat{V}_i^\mathsf{T} \bar{z}_i) - k_g g_i + v_i, \tag{47}$$

where v_i is a stabilizing signal to be determined later based on an H_∞ criterion, and k_g is a positive constant. We consider the following positive function V_0.

$$V_0 = \frac{1}{2}\sum_{i=1}^{N} s_i^\mathsf{T} M_i s_i + (k_g + \delta) J(y), \tag{48}$$

where $\delta(> 0)$ is an artificial error added to $J(y)$. We take the time derivative of V_0 along the trajectories of s_i and y.

$$\begin{aligned} \dot{V}_0 = \sum_{i=1}^{N} &\left\{ s_i^\mathsf{T}(v_i + Y_i\tilde{\theta}_i + \tilde{\Phi}_i^\mathsf{T}\Omega_i - \mu_{i1} + \mu_{i2} - k_g g_i) \right. \\ &\left. + (k_g + \delta) g_i^\mathsf{T}(s_i - g_i) + (k_g + \delta) g_i^\mathsf{T} \dot{y}_r \right\}, \end{aligned} \tag{49}$$
$$Y_i \equiv Y_i(y_i, \dot{y}_i, a_i, b_i), \tag{50}$$
$$\tilde{\theta}_i \equiv \hat{\theta}_i - \theta_i. \tag{51}$$

Here, we assume that

$$\sum_{i=1}^{N} g_i = 0. \tag{52}$$

It should be noted that the potential function $J(y)$ satisfying (52), is easily realized by choosing $d_{ij} = d_{ji}$ $(i \neq j)$ and by adjusting other parameters. Then, the following relation holds,

$$\begin{aligned} \dot{V}_0(t) = \sum_{i=1}^{N} &\left\{ s_i^\mathsf{T}(v_i + Y_i\tilde{\theta}_i + \tilde{\Phi}_i^\mathsf{T}\Omega_i - \mu_{i1} + \mu_{i2}) \right. \\ &\left. - (k_g + \delta) g_i^\mathsf{T} g_i + \delta g_i^\mathsf{T} s_i) \right\}. \end{aligned} \tag{53}$$

From the evaluation of $\dot{V}_0$, we introduce the following virtual system.

$$\begin{aligned} \dot{s}_i = f_i &+ g_{i1}\tilde{\theta}_i + g_{i2} D_{i3} + g_{i3} D_{i1} \\ &+ g_{i4} D_{i2} + g_{i5}\delta + g_{i6} v_i, \end{aligned} \tag{54}$$
$$\begin{aligned} &f_i = 0,\ g_{i1} = Y_i,\ g_{i2} = \bar{\Omega}_i,\ g_{i3} = \Psi_{i1}, \\ &g_{i4} = \Psi_{i2},\ g_{i5} = g_i,\ g_{i6} = 1. \end{aligned} \tag{55}$$

We are to stabilize the virtual system via a control input v_i by utilizing H_∞ criterion, where $\tilde{\theta}_i$, D_{i1}, D_{i2}, D_{i3} and δ are regarded as external disturbances to the process [9], [10]. For that purpose, we introduce the following Hamilton-Jacobi-Isaacs (HJI) equation and its solution V_{0i}.

$$\begin{aligned} \mathcal{L}_{f_i} V_{0i} + \frac{1}{4}&\left\{ \sum_{j=1}^{5} \frac{\|\mathcal{L}_{g_{ij}} V_{0i}\|^2}{\gamma_{ij}^2} - (\mathcal{L}_{g_{i6}} V_{0i}) R_i^{-1} (\mathcal{L}_{g_{i6}} V_{0i})^\mathsf{T} \right\} \\ &+ q_i = 0, \end{aligned} \tag{56}$$
$$V_{0i} = \frac{1}{2}\|s_i\|^2, \tag{57}$$

where q_i and R_i are a positive function and a positive definite matrix, respectively, and those are derived from HJI equation based on inverse optimality [7], [8], [9], [10] for the given solution V_{0i} and the positive constants $\gamma_{i1} \sim \gamma_{i5}$. The substitution of the solution V_{0i} (57) into HJI equation (56) yields

$$\begin{aligned} \frac{1}{4}&\left\{ \frac{s_i^\mathsf{T} Y_i Y_i^\mathsf{T} s_i}{\gamma_{i1}^2} + \frac{s_i^\mathsf{T} \bar{\Omega}_i \bar{\Omega}_i^\mathsf{T} s_i}{\gamma_{i2}^2} + \frac{s_i^\mathsf{T} \Psi_{i1} \Psi_{i1}^\mathsf{T} s_i}{\gamma_{i3}^2} \right. \\ &\left. + \frac{s_i^\mathsf{T} \Psi_{i2} \Psi_{i2}^\mathsf{T} s_i}{\gamma_{i4}^2} + \frac{s_i^\mathsf{T} g_i g_i^\mathsf{T} s_i}{\gamma_{i5}^2} - s_i^\mathsf{T} R_i^{-1} s_i \right\} + q_i = 0. \end{aligned} \tag{58}$$

Then, q_i and R_i are given as follows:

$$q_i = \frac{1}{4} s_i^\mathsf{T} K_i s_i, \tag{59}$$
$$\begin{aligned} R_i = &\left(\frac{Y_i Y_i^\mathsf{T}}{\gamma_{i1}^2} + \frac{\bar{\Omega}_i \bar{\Omega}_i^\mathsf{T}}{\gamma_{i2}^2} + \frac{\Psi_{i1}\Psi_{i1}^\mathsf{T}}{\gamma_{i3}^2} \right. \\ &\left. + \frac{\Psi_{i2}\Psi_{i2}^\mathsf{T}}{\gamma_{i4}^2} + \frac{g_i g_i^\mathsf{T}}{\gamma_{i5}^2} + K_i \right)^{-1}, \end{aligned} \tag{60}$$
$$K_i = K_i^\mathsf{T} > 0, \quad (K_i \in \mathbf{R}^{n \times n}), \tag{61}$$

where K_i is a free parameter. By utilizing R_i, v_i is deduced as a solution for the corresponding H_∞ control problem.

$$
\begin{aligned}
v_i &= -\frac{1}{2}R_i^{-1}\mathcal{L}_{g_{i6}}V_{0i} = -\frac{1}{2}R_i^{-1}s_i \\
&= -\frac{1}{2}\left(\frac{Y_iY_i^{\mathsf{T}}}{\gamma_{i1}^2} + \frac{\bar{\Omega}_i\bar{\Omega}_i^{\mathsf{T}}}{\gamma_{i2}^2} + \frac{\Psi_{i1}\Psi_{i1}^{\mathsf{T}}}{\gamma_{i3}^2}\right. \\
&\qquad \left. + \frac{\Psi_{i2}\Psi_{i2}^{\mathsf{T}}}{\gamma_{i4}^2} + \frac{g_ig_i^{\mathsf{T}}}{\gamma_{i5}^2} + K_i\right)^{-1} s_i. \qquad (62)
\end{aligned}
$$

Then, we obtain the following theorem for the multi-agent system (1).

Theorem 1 *It is assumed that $J(y)$ satisfies the condition (52). Then, the H_∞ formation control system composed of (1), (47) and (62) is uniformly bounded for arbitrary bounded design parameters $\hat{\theta}_i$ and $\hat{\Phi}_i$. Furthermore, v_i is a sub-optimal control solution which minimizes the upper bound of the following cost functional J_{cost}.*

$$
\begin{aligned}
J_{cost} = &\sup_{\tilde{\theta}_i,\tilde{\Phi}_i,D_{i1},D_{i2},\delta\in\mathcal{L}^2}\left[\sum_{i=1}^{N}\int_0^t (q_i + v_i^{\mathsf{T}}R_iv_i)d\tau + V_0(t)\right. \\
&-\sum_{i=1}^{N}\left\{\gamma_{i1}^2\int_0^t\|\tilde{\theta}_i\|^2d\tau + \gamma_{i2}^2\int_0^t\|\tilde{\Phi}_i\|^2d\tau\right. \\
&+\gamma_{i3}^2\int_0^t\|D_{i1}\|^2d\tau + \gamma_{i4}^2\int_0^t\|D_{i2}\|^2d\tau \\
&\left.\left.+\gamma_{i5}^2\int_0^t\delta^2d\tau\right\}\right]. \qquad (63)
\end{aligned}
$$

Additionally, the next inequality holds for any finite $t\,(>0)$.

$$
\begin{aligned}
&\sum_{i=1}^{N}\int_0^t(q_i + v_i^{\mathsf{T}}R_iv_i)d\tau + V_0(t) \\
&\quad \le \sum_{i=1}^{N}\left\{\gamma_{i1}^2\int_0^t\|\tilde{\theta}_i\|^2d\tau + \gamma_{i2}^2\int_0^t\|\tilde{\Phi}_i\|^2d\tau\right. \\
&\qquad +\gamma_{i3}^2\int_0^t\|D_{i1}\|^2d\tau + \gamma_{i4}^2\int_0^t\|D_{i2}\|^2d\tau \\
&\qquad \left.+\gamma_{i5}^2\int_0^t\delta^2d\tau\right\} + V_0(0). \qquad (64)
\end{aligned}
$$

Proof: By considering HJI equation, we take the time derivative of $V_0(t)$ (48) along the trajectories of the multi-agent system (1) and the H_∞ formation control scheme.

$$
\begin{aligned}
\dot{V}_0 \le &\sum_{i=1}^{N}\left(v_i + \frac{1}{2}R_i^{-1}s_i\right)^{\mathsf{T}}R_i\left(v_i + \frac{1}{2}R_i^{-1}s_i\right) \\
&-\sum_{i=1}^{N}v_i^{\mathsf{T}}R_iv_i - \sum_{i=1}^{N}q_i - \sum_{i=1}^{N}(k_g+\delta)g_i^{\mathsf{T}}g_i \\
&-\sum_{i=1}^{N}\gamma_{i1}^2\left\|\tilde{\theta}_i - \frac{Y_i^{\mathsf{T}}s_i}{2\gamma_{i1}^2}\right\|^2 + \sum_{i=1}^{N}\gamma_{i1}^2\|\tilde{\theta}_i\|^2 \\
&-\sum_{i=1}^{N}\gamma_{i2}^2\sum_{j=1}^{n}\left(\|\tilde{\Phi}_{ij}\| - \frac{\|\Omega_{ij}\||s_{ij}|}{2\gamma_{i2}^2}\right)^2 + \sum_{i=1}^{N}\gamma_{i2}^2\sum_{j=1}^{n}\|\tilde{\Phi}_{ij}\|^2 \\
&-\sum_{i=1}^{N}\gamma_{i3}^2\sum_{j=1}^{n}\left(d_{i1j} - \frac{\psi_{ij}|s_{ij}|}{2\gamma_{i3}^2}\right)^2 + \sum_{i=1}^{N}\gamma_{i3}^2\sum_{j=1}^{n}d_{i1j}^2 \\
&-\sum_{i=1}^{N}\gamma_{i4}^2\sum_{j=1}^{n}\left(D_{i2j} - \frac{\Psi_{i2j}|s_{ij}|}{2\gamma_{i4}^2}\right)^2 + \sum_{i=1}^{N}\gamma_{i4}^2\sum_{j=1}^{n}D_{i2j}^2 \\
&-\sum_{i=1}^{N}\gamma_{i2}^2\left|\delta - \frac{g_i^{\mathsf{T}}s_i}{2\gamma_{i5}^2}\right|^2 + \sum_{i=1}^{N}\gamma_{i2}^2\delta^2, \qquad (65)
\end{aligned}
$$

where

$$
s_i(t) = \left[s_{i1}(t),\,\cdots,\,s_{in}(t)\right]^{\mathsf{T}}. \qquad (66)
$$

Then, Theorem 1 is derived from the evaluations of $\dot{V}_0(t)$ (65). ■

B. Adaptive H_∞ Formation Control I

Next, we determine the adaptation scheme of $\hat{\theta}_i$ and $\hat{\Phi}_{ij}$. We assume that the upper bounds of $\|\theta_i\|$ and $\|\Phi_{ij}\|$ are known a priori. Then, $\hat{\theta}_i$ and $\hat{\Phi}_{ij}$ are tuned by the following adaptive laws.

$$
\begin{aligned}
&\dot{\hat{\theta}}_i(t) = \Pr\{-\Gamma_{i1}Y_i^{\mathsf{T}}(t)s_i(t)\}, \qquad (67) \\
&\dot{\hat{\Phi}}_{ij}(t) = \Pr\{-\Gamma_{i2j}\Omega_{ij}(t)s_{ij}(t)\}, \qquad (68) \\
&(\Gamma_{i1} = \Gamma_{i1}^{\mathsf{T}} > 0,\ \Gamma_{i2j} = \Gamma_{i2j}^{\mathsf{T}} > 0), \\
&(i = 1,\,2,\,\cdots,\,N, \quad j = 1,\,2,\,\cdots,\,n),
\end{aligned}
$$

where $\Pr(\cdot)$ are projection operations in which tuning parameters are constrained to bounded regions deduced from upper bounds of those parameters [16]. Then, the tuning parameters $\hat{\theta}_i$ and $\hat{\Phi}_{ij}$ are made uniformly bounded by the projection-type adaptive laws, and we obtain the following theorem for the multi-agent system (1).

Theorem 2 *It is assumed that $J(y)$ satisfies the condition (52). Then the adaptive H_∞ formation control system composed of (1), (47), (62) and the adaptation laws (67), (68) is uniformly bounded, and the following relation holds*

$$
\lim_{T\to\infty}\frac{1}{T}\int_0^T\sum_{i=1}^{N}\|\Delta\dot{y}_i(t)\|^2dt \le \text{const.}\sum_{i=1}^{N}(\gamma_{i3}^2 + \gamma_{i4}^2), \quad (69)
$$

$$
\lim_{T\to\infty}\frac{1}{T}\int_0^T\sum_{i=1}^{N}\|g_i(y(t))\|^2dt \le \text{const.}\sum_{i=1}^{N}(\gamma_{i3}^2 + \gamma_{i4}^2), (70)
$$

and the desirable relative configuration (39), (40) is achieved approximately by the accuracy proportional to $\sum_{i=1}^{N}\sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$, that is, $\sum_{i=1}^{N}(\|\Delta\dot{y}_i\| + \|g_i\|) \sim \sum_{i=1}^{N}\sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$.

Proof: From Theorem 1 and the property of the projection-type adaptive laws, it is shown that the adaptive control system is uniformly bounded. For further stability analysis, a positive function V is defined by

$$
\begin{aligned}
V = &\frac{1}{2}\sum_{i=1}^{N}s_i^{\mathsf{T}}M_is_i + k_gJ(y) \\
&+\frac{1}{2}\sum_{i=1}^{N}\left(\tilde{\theta}_i^{\mathsf{T}}\Gamma_{i1}^{-1}\tilde{\theta}_i + \sum_{j=1}^{n}\tilde{\Phi}_{ij}^{\mathsf{T}}\Gamma_{i2j}^{-1}\tilde{\Phi}_{ij}\right). \qquad (71)
\end{aligned}
$$

We take the time derivative of V along its trajectories, and obtain

$$\dot{V} \leq -\frac{1}{2}\sum_{i=1}^{N} s_i^{\mathsf{T}} K_i s_i - k_g \sum_{i=1}^{N} g_i^{\mathsf{T}} g_i + \sum_{i=1}^{N} \gamma_{i3}^2 \|D_{i1}\|^2 + \sum_{i=1}^{N} \gamma_{i4}^2 \|D_{i2}\|^2. \quad (72)$$

Then, we derive the following inequality

$$\frac{1}{T}\sum_{i=1}^{N}\left(\frac{1}{2}\int_0^T s_i^{\mathsf{T}} K_i s_i dt + k_g \int_0^T g_i^{\mathsf{T}} g_i dt\right) + \frac{V(T)}{T} \leq \sum_{i=1}^{N} \gamma_{i3}^2 \|D_{i1}\|^2 + \sum_{i=1}^{N} \gamma_{i3}^4 \|D_{i2}\|^2 + \frac{V(0)}{T}. \quad (73)$$

Since $V(0)$ and $V(T)$ are uniformly bounded for $\forall T > 0$, we deduce the relations (69), (70) in Theorem 2. ■

Remark 1 In the proposed adaptive control system, it is also shown that $J(y)$ is uniformly bounded. Therefore, the collision of agents ($y_i = y_j$ $(i \neq j)$) is avoided automatically, if we choose $J(y)$ with the property such that $J(y) \to \infty$ as $y_i \to y_j$ $(i \neq j)$ [4], [5], [6].

V. ADAPTIVE H_∞ FORMATION CONTROL II

Next, we generalize the previous formation control I, and consider a formation control problem of the leader-follower type [5], where all agents continue to move with a desired velocity $\dot{y}_r$

$$\dot{y}_i(t) = \dot{y}_r(t), \quad (74)$$

and also satisfy the formation constraints on the maximum distance from the reference point y_r and on the minimum relative distance from other agents written as below:

$$\|y_i - y_r\| \leq r_i, \quad (r_i > 0,\ 1 \leq i \leq N), \quad (75)$$

$$\|y_i - y_j\| \geq d_{ij}, \quad (d_{ij} = d_{ji} > 0,\ 1 \leq i \neq j \leq N). \quad (76)$$

Instead of (76), the relative configuration (40) can be also adopted as a specified case of the constraint on relative distances from other agents.

A. H_∞ Formation Control II

We introduce a positive potential function $J_G(\Delta y) \in \mathbf{R}$ ($\Delta y = [\Delta y_1^{\mathsf{T}}, \cdots, \Delta y_N^{\mathsf{T}}]^{\mathsf{T}}$) in order to handle the formation constraint on the maximum distance from the reference point y_r (75), and introduce another positive potential function $J_L(y)$ to handle the formation constraint on the minimum relative distance from other agents (76). It is assumed that $J_G(\Delta y)$ and $J_L(y)$ are twice differentiable, and that the desired total configurations (75), (76) correspond to the minimal points of $J_G(\Delta y)$ and $J_L(y)$ such as

$$J_G(\Delta y) \to \min, \quad \left(\frac{\partial J_G(\Delta y)}{\partial \Delta y_i} = 0 \ \ (1 \leq i \leq N)\right), \quad (77)$$

$$J_L(y) \to \min, \quad \left(\frac{\partial J_L(y)}{\partial y_i} = 0 \ \ (1 \leq i \leq N)\right). \quad (78)$$

Or equivalently, (77), (78) hold uniformly in the appropriate region defined by (75), (76).

Define a control error s_i by (42), (43) and g_i is newly defined by

$$g_i(y) = \xi_i + \rho_i, \quad (79)$$

$$\xi_i(y) = \frac{\partial J_G(\Delta y)}{\partial \Delta y_i}, \quad (80)$$

$$\rho_i(y) = \frac{\partial J_L(y)}{\partial y_i}. \quad (81)$$

Then, we obtain the same relation as (45), where a_i and b_i are defined by (46), but the definition of g_i (79) is different from the previous case (44). Furthermore, the control law is the same form as the previous one (47) with the new definition of g_i (79). We consider the following positive function V_0.

$$V_0 = \frac{1}{2}\sum_{i=1}^{N} s_i^{\mathsf{T}} M_i s_i + (k_g + \delta) J_G(\Delta y) + (k_g + \delta) J_L(y), \quad (82)$$

where $\delta\,(> 0)$ is an artificial error added to $J_G(\Delta y)$ and $J_L(y)$. We take the time derivative of V_0 along the trajectories of s_i, Δy_i and y.

$$\dot{V}_0 = \sum_{i=1}^{N}\left\{ s_i^{\mathsf{T}}(v_i + Y_i\tilde{\theta}_i + \tilde{\Phi}_i^{\mathsf{T}}\Omega_i - \mu_{i1} + \mu_{i2} - k_g g_i) + (k_g + \delta) g_i^{\mathsf{T}}(s_i - g_i) + (k_g + \delta)\rho_i^{\mathsf{T}}\dot{y}_r)\right\}. \quad (83)$$

Here, we assume that

$$\sum_{i=1}^{N} \rho_i = 0. \quad (84)$$

Similarly to the previous case, the potential function $J_L(y)$ satisfying (84), is easily realized by choosing $d_{ij} = d_{ji}$ and by adjusting other parameters. Then, the following relation holds,

$$\dot{V}_0 = \sum_{i=1}^{N}\left\{ s_i^{\mathsf{T}}(v_i + Y_i\tilde{\theta}_i + \tilde{\Phi}_i^{\mathsf{T}}\Omega_i - \mu_{i1} + \mu_{i2} - k_g g_i) - (k_g + \delta) g_i^{\mathsf{T}} g_i + \delta g_i^{\mathsf{T}} s_i)\right\}. \quad (85)$$

From the evaluation of $\dot{V}_0$, we introduce the virtual system (54), (55) in which g_i is defined by (79), and are to stabilize the virtual system via a control input v_i by utilizing H_∞ criterion, where the same terms $\tilde{\theta}_i$, D_{i1}, D_{i2}, D_{i3}, δ are regarded as external disturbances to the process [9], [10]. Then, by repeating the same discussion with the new definition of g_i (79), for q_i and R_i defined by (59), (60), (61) and for v_i determined such as (62) we obtain the following theorem.

Theorem 3 *It is assumed that $J_L(y)$ satisfies the condition (84). Then, the H_∞ formation control system composed of (1), (47), (62) with the new definition of g_i (79), is uniformly bounded for arbitrary bounded design parameters $\hat{\theta}_i$ and $\hat{\Phi}_i$. Furthermore, v_i is a sub-optimal control solution which minimizes the upper bound of the cost functional J_{cost} (63). Additionally, the same inequality as (64) holds for any finite $t\,(> 0)$.*

Proof: The proof is carried out by the similar procedure to Theorem 1, where V_0 is defined by (82), and g_i is defined by (79). ■

B. Adaptive H_∞ Formation Control II

Next, we determine the adaptation scheme of $\hat{\theta}_i$ and $\hat{\Phi}_{ij}$. $\hat{\theta}_i$ and $\hat{\Phi}_{ij}$ are tuned by the same adaptive laws as (67), (68) with the new defition of g_i (79). Then, we obtain the following theorem for the multi-agent system (1).

Theorem 4 *It is assumed that $J_L(y)$ satisfies the condition (84). Then the adaptive H_∞ formation control system composed of (1), (47), (62) and the adaptation law (67), (68) with the new defibition of g_i (79) is uniformly bounded, and the same relation as (69), (70) holds, and the desired velocity tracking (74) is achieved approximately by the accuracy proportional to $\sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$ ($\sum_{i=1}^N \|\Delta\dot{y}_i\| \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$) . Furthermore, by choosing appropriate formation constraints, such as an appropriate desirable region related to $J_G(\Delta y)$ and appropriate relative distances related to $J_L(y)$, the desired formation of the leader-follower type is achieved approximately by the accuracy proportional to $\sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$, that is, $\sum_{i=1}^N (\|\xi_i\|+\|\rho_i\|) \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$.*

Proof: Similarly to Theorem 1, the adaptive control system is shown to be uniformly bounded. For further stability analysis, a positive function V is defined by

$$V = \frac{1}{2}\sum_{i=1}^N s_i^\top M_i s_i + k_g\{J_G(\Delta y)+J_L(y)\} + \frac{1}{2}\sum_{i=1}^N \left(\tilde{\theta}_i^\top \Gamma_{i1}^{-1}\tilde{\theta}_i + \sum_{j=1}^n \tilde{\Phi}_{ij}^\top \Gamma_{i2j}^{-1}\tilde{\Phi}_{ij} \right). \quad (86)$$

We take the time derivative of V along its trajectories, and obtain the same inequality as (72), where (84) is considered. Then, we deduce the same relations as (69), (70).

On the contrary, since $g_i = \xi_i + \rho_i$, it follows that

$$\left\| \sum_{i=1}^N g_i \right\| = \left\| \sum_{i=1}^N \xi_i + \sum_{i=1}^N \rho_i \right\| \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}, \quad (87)$$

and the next relation is derived from the assumption (84).

$$\left\| \sum_{i=1}^N \xi_i \right\| \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}. \quad (88)$$

Here we consider the case where all agents do not satisfy the formation constraint related to $J_G(\Delta)$, and several agents are outside the desired region defined by (75). It should be noted that $\xi_i = 0$ for the agents inside the desired region. If those agents outside the desired region are on the one side of the region, then the corresponding ξ_i have the same sign along one axis, and this shows that (88) means the relation $\sum_{i=1}^N \|\xi_i\| \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$. Next, we consider the case where several agents are on the opposite sides outside the desired region. If we choose a sufficient large region related to $J_G(\Delta)$, then it follows that $\rho_i \to 0$ for the agents outside the region. Hence, $\sum_{i=1}^N \|\xi_i\| \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$ holds for the corresponding ξ_i. In the end, by choosing appropriate formation constraints, such as an appropriate desired region related to $J_G(\Delta y)$ and appropriate relative distances related to $J_L(y)$, the next equation holds for all agents

$$\sum_{i=1}^N (\|\xi_i\|+\|\rho_i\|) \sim \sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}, \quad (89)$$

and the desired formation of the leader-follower type is achieved approximately by the accuracy proportional to $\sum_{i=1}^N \sqrt{\gamma_{i3}^2+\gamma_{i4}^2}$ [5]. ■

VI. CONCLUDING REMARKS

Design methodologies of adaptive H_∞ formation control of multi-agent systems composed of Euler-Lagrange systems by utilizing neural network approximators have been proposed in the present paper. The resulting control strategies are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters, error terms in potential functions, and approximate and algorithmic errors in neural network estimation schemes are regarded as external disturbances to the process. It is shown that the resulting control systems are robust to uncertain system parameters and uncertain nonlinear properties, and that the desirable formations are achieved asymptotically via adaptation schemes.

REFERENCES

[1] J. Shao, G. Xie and L. Wang, "Leader Following Formation Control of Multiple Mobile Vehicles", *IET Control Theory Appl.,* vol.1, pp.545-552, 2007.

[2] T. Balch and R.C. Arkin, "Behavior-Based Formation Control for Multirobot Teams", *IEEE Transactions on Robotics and Automation,* vol.14, pp.926-939, 1998.

[3] M.A. Tan and K. Lewis, "Virtual Structures for High-Precision Cooperative Mobile Robot Control", *Proceedings of the 1996 IEEE/RSJ INternational Conference Intelligent Robots and Systems,* vol.1, pp.132-139, 1996.

[4] A.R. Pereira and L. Hsu, "Adaptive Formation Control Using Artificial Potentials for Euler-Lagrange Agents", *Proceedings of the 17th IFAC World Congress,* pp.10788-10793, 2008.

[5] C.C. Cheah, S.P. Hou and J.J.E. Slotine, "Region-Based Shape Control for a Swarm of Robots", *Automatica*, vol.45, pp.2406-2411, 2009.

[6] J. Yao, R. Ordóñez and V. Gazi, "Swarm Tracking Using Artificial Potentials and Sliding Mode Control", *Proceedings of the 45th IEEE Conference on Decision and Control*, pp.4670-4675, 2006.

[7] M. Krstić and H. Deng, *Stabilization of Nonlinear Uncertain Systems*, Springer, 1998.

[8] Y. Miyasato, "Redesign of Adaptive Control Systems Based on the Notion of Optimality", *Proceedings of the 38th IEEE Conference on Decision and Control*, pp.3315-3320, 1999.

[9] Y. Miyasato, "Adaptive nonlinear H_∞ control for processes with bounded variations of parameters – general relative degree case – ", *Proceedings of the 39th IEEE Conference on Decision and Control*, pp.1453-1458, 2000.

[10] Y. Miyasato, "Adaptive nonlinear H_∞ control for processes with bounded variations of parameters – general forms and general relative degree case –", *Proceedings of ALCOSP 2001*, pp.419-424, 2002.

[11] Y. Miyasato, "Design method of adaptive nonlinear H_∞ control systems via neural network approximators", *Proceedings of the 40th IEEE Conference on Decision and Control*, pp.3158-3163, 2001.

[12] Y. Miyasato, "Adaptive nonlinear H_∞ control systems via neural network approximators", *Proceedings of the 2006 IEEE CCA/CACSD/ISIC,* pp.2349-2354, 2006.

[13] M.W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, 1989.

[14] K. I. Funahashi: On the approximate realization of continuous mappings by neural-networks, *Neural Networks*, Vol.2, 183-192, 1989.

[15] T. Zhang, S.S. Ge and C.C. Hang : Design and performance analysis of a direct adaptive controller for nonlinear systems , *Automatica*, Vol.35, No.11,1809-1817, 1999.

[16] P.A. Ioannou and J. Sun, *Robust Adaptive Control,* PTR Prentice-Hall, 1996.