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Global robust output regulation for a class of multivariable systems and its application to a motor drive system

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Abstract— **In this paper, we study the global robust output regulation problem for a class of multivariable nonlinear systems. The problem is first converted into a stabilization problem of an augmented system composed of the original plant and an internal model. The augmented system is a two-input system containing both dynamic uncertainty and time-varying static uncertainty. By decomposing the two-input control problem into two single-input control problems, we can solve the problem via a recursive approach utilizing the changing supply function technique. Finally, the theoretical result is applied to the speed tracking control and load torque disturbance rejection problem of a surface-mounted PM synchronous motor.**

I. Introduction

In this paper, we consider the global robust output regulation problem of the following 2-input 2-output nonlinear system

$$
\begin{aligned}
\dot{z} &= f_0(z, x_{1,1}, v, w) + \varphi_{1,0}(z, x_{1,1}, x_{2,1}, v, w)x_{2,1} \\
\dot{x}_{1,1} &= f_{1,1}(z, x_{1,1}, v, w) + b_{1,1}(v, w)x_{1,2} \\
&\quad +\varphi_{1,1}(z, x_{1,1}, x_{2,1}, v, w)x_{2,1} \\
&\ \vdots \\
\dot{x}_{1,r_1} &= f_{1,r_1}(z, x_1, v, w) + b_{1,r_1}(v, w)u_1 \\
&\quad +\varphi_{1,r_1}(z, x_1, x_{2,1}, v, w)x_{2,1} \\
\dot{x}_{2,1} &= f_{2,1}(z, x_1, x_{2,1}, v, w) + b_{2,1}(v, w)x_{2,2} \\
&\ \vdots \\
\dot{x}_{2,r_2} &= f_{2,r_2}(z, x_1, x_2, v, w) + b_{2,r_2}(v, w)u_2 \\
\dot{v} &= A_1 v \\
e &= \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} x_{1,1} - q_{d_1}(v, w) \\ x_{2,1} - q_{d_2}(v, w) \end{bmatrix} \qquad (1)
\end{aligned}
$$

where $z \in R^{n_z}$, for $k = 1, 2$, $x_k = \mathrm{col}(x_{k,1}, \cdots, x_{k,r_k})$ with $x_{k,i} \in R, i = 1, \cdots, r_k$, are the states, and, $u_k \in R$ are the inputs, $e_k \in R$ are the plant outputs representing the tracking errors, $w \in R^{n_w}$ is the plant uncertain parameter vector, $v \in R^{n_v}$ is the exogenous signal representing the disturbance and/or the reference input, and the system $\dot{v} = A_1 v$ is called by exosystem.

It is assumed that all the functions are sufficiently smooth satisfying $f_0(0, \cdots, 0, w) = 0$, and $f_{k,i}(0, \cdots, 0, w) = 0$, $q_{d_k}(0, w) = 0$ for $k = 1, 2$, $i = 1, \cdots, r_k$, $\varphi_{1,i}(0, \cdots, 0, w) = 0$ for $i = 0, 1, \cdots, r_1$ and all the eigenvalues of A_1 are distinct with zero real parts.

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We will study the global robust output regulation problem for system (1) by a dynamic state feedback control law of the following form:

$$u = u_{\scriptscriptstyle K}(\zeta, e, x_1, x_2),\ \dot{\zeta} = g_{\scriptscriptstyle K}(\zeta, x_1, x_2) \qquad (2)$$

where both $u_{\scriptscriptstyle K}$ and $g_{\scriptscriptstyle K}$ are globally defined smooth functions.

Various versions of the output regulation problem have been extensively studied since the 1990s. Here our version follows from [4] and is repeated as follows: given any V and W, which are compact subsets of R^{n_v} and R^{n_w} containing the origins of R^{n_v} and R^{n_w}, respectively, find a control law of the form (2) such that for any $v \in V$ and $w \in W$, the trajectory of the closed-loop system starting from any initial state exists and is bounded for all $t > 0$, and is such that $\lim_{t\to\infty} e(t) = 0$. It is known from the general framework detailed in [4] that the above problem can be tackled in two steps. The first step is to convert the robust output regulation problem for the given plant into a robust stabilization problem for the so-called augmented system composed of the given plant and a dynamic compensator called internal model, and the second step aims to robustly stabilize the augmented system. This framework has been successfully applied to address the robust output regulation problem for various classes of single-input and single-output nonlinear systems [2], [3], [4]. However, few papers have handled the multivariable nonlinear systems [7], [9]. The system considered in [9] is a class of interconnected output feedback systems. The special structure of this class of systems allows the global output regulation problem to be solved by a decentralized error feedback control scheme. The system considered in [7] is an MIMO system in the normal form. However, system (1) may not have well defined relative degree and cannot be converted into normal form. Moreover, [7] only considered semi-global output regulation problem while we consider the global problem here.

What challenge the global robust output regulation problem of system (1) are as follows. First, system (1) is not in the normal form, and there is no clue if the above problem can be solved by output feedback control. Therefore, we have to resort to state feedback control of the form (2) which in turn leads to a much more complicated augmented system. Second, due to the introduction of the internal model, the augmented system is a two-input nonlinear system containing both dynamic uncertainty and time-varying static uncertainty. The stabilization problem of such a system has never been handled. We need to first find conditions under which the augmented system has certain special structure, and then find some recursive approach to convert the two-input stabilization problem into the stabilization problem of two single-input systems. We have indeed found a systematic approach for doing so.

The problem in this paper is motivated by the well-known speed tracking and disturbance rejection problem of the surface-mounted PM synchronous motor [1], [6], [10]. The problem has been handled by feedback linearization method. However, this method needs the exact knowledge of the reference input as well as its derivatives. In contrast, our formulation allows us to handle any reference input generated by some exosystem. Additionally, by utilizing the internal model design, we allow all the motor parameters to be uncertain.

The rest of the paper is organized as follows. In Section II, following the framework of [4], we convert the global robust output regulation problem of the given system into the global stabilization problem of an augmented system composed of the original plant and an internal model. In Section III, we solve the stabilization problem of the augmented system. In Section IV, we apply the result in Section III to the speed tracking control and load torque disturbance rejection of the surface-mounted PM synchronous motor. In Section V, we conclude the paper with a brief remark.

II. Preliminaries

Our approach will be based on the framework of [4]. Let us first put the system (1) in the following compact form:

$$\dot{x} = f(x,u,v,w), \; e = h(x,u,v,w) \tag{3}$$

where

$$x = \begin{bmatrix} z \\ x_1 \\ x_2 \end{bmatrix}, h(x,u,v,w) = \begin{bmatrix} x_{1,1} - q_{d_1}(v,w) \\ x_{2,1} - q_{d_2}(v,w) \end{bmatrix},$$

$$\begin{aligned} \Delta_1 &= f_{1,1}(z,x_{1,1},v,w) + b_{1,1}(v,w)x_{1,2} \\ &\quad + \varphi_{1,1}(z,x_{1,1},x_{2,1},v,w)x_{2,1}, \\ &\vdots \\ \Delta_{r_1} &= f_{1,r_1}(z,x_1,v,w) + b_{1,r_1}(v,w)u_1 \\ &\quad + \varphi_{1,r_1}(z,x_1,x_{2,1},v,w)x_{2,1}, \end{aligned}$$

$$f(x,u,v,w) = \begin{bmatrix} f_0(z,x_{1,1},v,w) + \varphi_{1,0}(z,x_{1,1},x_{2,1},v,w)x_{2,1} \\ \Delta_1 \\ \vdots \\ \Delta_{r_1} \\ f_{2,1}(z,x_1,x_{2,1},v,w) + b_{2,1}(v,w)x_{2,2} \\ \vdots \\ f_{2,r_2}(z,x_1,x_2,v,w) + b_{2,r_2}(v,w)u_2 \end{bmatrix}.$$

Associated with (3) are the following partial differential equations:

$$\begin{aligned} \frac{\partial \mathbf{x}(v,w)}{\partial v}A_1 v &= f(\mathbf{x}(v,w),\mathbf{u}(v,w),v,w) \\ 0 &= h(\mathbf{x}(v,w),\mathbf{u}(v,w),v,w) \end{aligned} \tag{4}$$

where $\mathbf{x} : R^{n_v} \times R^{n_w} \mapsto R^{n_z+r_1+r_2}$ and $\mathbf{u} : R^{n_v} \times R^{n_w} \mapsto R^2$ are two smooth functions vanishing at the origin. (4) is known as regulator equations [5].

Two standard assumptions are as follows:

Assumption 2.1: The solution of regulator equations (4) exists and is polynomial in v.

Assumption 2.2: For $i = 1,\cdots,r_1, j = 1,\cdots,r_2$, $b_{1,i}(v,w) > 0, b_{2,j}(v,w) > 0$ for all $v \in R^{n_v}$ and $w \in R^{n_w}$.

Remark 2.1: Under Assumption 2.2, if there exists a globally defined smooth function $\mathbf{z} : R^{n_v} \times R^{n_w} \mapsto R^{n_z}$ with $\mathbf{z}(0,0) = 0$ such that

$$\begin{aligned} \frac{\partial \mathbf{z}(v,w)}{\partial v}A_1 v &= f_0(\mathbf{z}(v,w),q_{d_1}(v,w),v,w) \\ &\quad +\varphi_{1,0}(\mathbf{z}(v,w),q_{d_1}(v,w),q_{d_2}(v,w), \\ &\quad v,w)q_{d_2}(v,w) \end{aligned}$$

for all $(v,w) \in R^{n_v} \times R^{n_w}$, then the other components of the solution of the regulator equations of system (1) can be obtained as follows:

$$\begin{aligned} &\mathbf{x}_{1,1}(v,w) = q_{d_1}(v,w) \\ &\mathbf{x}_{1,2}(v,w) = b_{1,1}^{-1}(v,w)[\frac{\partial q_{d_1}(v,w)}{\partial v}A_1 v - f_{1,1}(\mathbf{z}(v,w), \\ &\quad q_{d_1}(v,w),v,w) - \varphi_{1,1}(\mathbf{z}(v,w),q_{d_1}(v,w), \\ &\quad q_{d_2}(v,w),v,w)q_{d_2}(v,w)] \\ &\mathbf{x}_{1,i+1}(v,w) = b_{1,i}^{-1}(v,w)[\frac{\partial \mathbf{x}_{1,i}(v,w)}{\partial v}A_1 v - f_{1,i}(\mathbf{z}(v,w), \\ &\quad q_{d_1}(v,w),\mathbf{x}_{1,2}(v,w),\cdots,\mathbf{x}_{1,i}(v,w),v,w) \\ &\quad -\varphi_{1,i}(\mathbf{z}(v,w),q_{d_1}(v,w),\mathbf{x}_{1,2}(v,w),\cdots, \\ &\quad \mathbf{x}_{1,i}(v,w),q_{d_2}(v,w),v,w)q_{d_2}(v,w)], 2 \le i \le r_1 \\ &\mathbf{x}_{2,1}(v,w) = q_{d_2}(v,w) \\ &\mathbf{x}_{2,2}(v,w) = b_{2,1}^{-1}(v,w)[\frac{\partial q_{d_2}(v,w)}{\partial v}A_1 v - f_{2,1}(\mathbf{z}(v,w), \\ &\quad \mathbf{x}_1(v,w),q_{d_2}(v,w),v,w)] \\ &\mathbf{x}_{2,j+1}(v,w) = b_{2,j}^{-1}(v,w)[\frac{\partial \mathbf{x}_{2,j}(v,w)}{\partial v}A_1 v - f_{2,j}(\mathbf{z}(v,w), \\ &\quad \mathbf{x}_1(v,w),q_{d_2}(v,w),\mathbf{x}_{2,2}(v,w),\cdots,\mathbf{x}_{2,j}(v,w), \\ &\quad v,w)], 2 \le j \le r_2 \\ &\mathbf{x}_{1,r_1+1} \triangleq \mathbf{u}_1, \mathbf{x}_{2,r_2+1} \triangleq \mathbf{u}_2. \end{aligned} \tag{5}$$

Let $g(x,u) = \text{col}(x_{1,2},\cdots, x_{1,r_1},u_1,x_{2,2},\cdots,x_{2,r_2},u_2)$ with its i-th component being denoted by $g_i(x,u)$. Under Assumptions 2.1 and 2.2, for $i = 1,\cdots,r_1+r_2$, there exist integers σ_i and real numbers $a_{i,1},\cdots,a_{i,\sigma_i}$ such that $g_i(\mathbf{x}(v,w),\mathbf{u}(v,w))$ satisfies, for all trajectories $v(t)$ of the exosystem and all $w \in R^{n_w}$,

$$\begin{aligned} &\frac{d^{\sigma_i} g_i(\mathbf{x}(v,w),\mathbf{u}(v,w))}{dt^{\sigma_i}} \\ &\quad = a_{i,1}g_i(\mathbf{x}(v,w),\mathbf{u}(v,w)) + a_{i,2}\frac{dg_i(\mathbf{x}(v,w),\mathbf{u}(v,w))}{dt} \\ &\quad\quad + \cdots + a_{i,\sigma_i}\frac{d^{(\sigma_i-1)}g_i(\mathbf{x}(v,w),\mathbf{u}(v,w))}{dt^{(\sigma_i-1)}}. \end{aligned} \tag{6}$$

Let

$$\begin{aligned} \tau_i(v,w) = &\Big[\; g_i(\mathbf{x}(v,w),\mathbf{u}(v,w)) \quad \tfrac{dg_i(\mathbf{x}(v,w),\mathbf{u}(v,w))}{dt} \\ &\cdots \quad \tfrac{d^{(\sigma_i-1)}g_i(\mathbf{x}(v,w),\mathbf{u}(v,w))}{dt^{(\sigma_i-1)}} \;\Big]^T \end{aligned} \tag{7}$$

and (Ψ_i, Φ_i) be an observable pair as follows:

$$\Phi_i = \left[\begin{array}{c|c} 0_{(\sigma_i-1)\times 1} & I_{(\sigma_i-1)\times(\sigma_i-1)} \\ \hline a_{i,1} & a_{i,2} \ \cdots \ a_{i,\sigma_i} \end{array}\right]$$
$$\Psi_i = \left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array}\right]_{1\times\sigma_i}. \quad (8)$$

Let $\theta_i(v,w) = T_i\tau_i(v,w)$ where $T_i \in R^{\sigma_i\times\sigma_i}$ is a nonsingular matrix to be specified later. Then for all v, w

$$\begin{aligned} \frac{d\theta_i(v,w)}{dt} &= T_i\Phi_i T_i^{-1}\theta_i(v,w) \\ g_i(\mathbf{x}(v,w), \mathbf{u}(v,w)) &= \beta_i(\theta_i(v,w)) \end{aligned} \quad (9)$$

where $\beta_i(\theta_i) = \Psi_i T_i^{-1}\theta_i$. System (9) is called steady-state generator with output $g_i(x,u), i = 1,\cdots,r_1+r_2$ [4].

Let $M_i \in R^{\sigma_i\times\sigma_i}$ and $N_i \in R^{\sigma_i\times 1}$ be a pair of controllable matrices with M_i Hurwitz. Then we call the following dynamic compensator

$$\dot{\eta}_i = M_i\eta_i + N_i g_i(x,u), i = 1,\cdots,r_1+r_2 \quad (10)$$

as the internal model of (1) with output $g_i(x,u)$. The composition of the plant (1) and the internal model (10) is called the augmented system.

Remark 2.2: Since (M_i, N_i) is controllable and (Φ_i, Ψ_i) is observable, there exists a unique nonsingular matrix T_i satisfying the Sylvester equation [8]

$$T_i\Phi_i - M_iT_i = N_i\Psi_i, i = 1,\cdots,r_1+r_2. \quad (11)$$

With T_i defined this way, $(\eta_i - \theta_i)$ will satisfy

$$\begin{aligned} (\dot{\eta}_i - \dot{\theta}_i) &= M_i(\eta_i-\theta_i) + N_i(g_i(x,u) - g_i(\mathbf{x}(v,w), \\ &\quad \mathbf{u}(v,w))). \end{aligned} \quad (12)$$

Thus $\lim_{t\to\infty}(\eta_i - \theta_i) = 0$ if $\lim_{t\to\infty}(g_i(x,u) - g_i(\mathbf{x}(v,w), \mathbf{u}(v,w))) = 0$.

Performing on the augmented system composed of (1) and (10) the following coordinate and input transformation

$$\begin{aligned} z_0 &= z - \mathbf{z}(v,w) \\ z_{1,i} &= \eta_i - \theta_i(v,w) - b_{1,i}^{-1}(v,w)N_i\bar{x}_{1,i}, \\ &\quad i = 1,\cdots,r_1 \\ \bar{x}_{1,1} &= x_{1,1} - q_{d_1}(v,w) \\ \bar{x}_{1,i+1} &= x_{1,i+1} - \beta_i(\eta_i), i = 1,\cdots,r_1-1 \\ \bar{u}_1 &= u_1 - \beta_{r_1}(\eta_{r_1}) \\ z_{2,j} &= \eta_{r_1+j} - \theta_{r_1+j}(v,w) - b_{2,j}^{-1}(v,w)N_{r_1+j}\bar{x}_{2,j}, \\ &\quad j = 1,\cdots,r_2 \\ \bar{x}_{2,1} &= x_{2,1} - q_{d_2}(v,w) \\ \bar{x}_{2,j+1} &= x_{2,j+1} - \beta_{r_1+j}(\eta_{r_1+j}), j = 1,\cdots,r_2-1 \\ \bar{u}_2 &= u_2 - \beta_{r_1+r_2}(\eta_{r_1+r_2}) \end{aligned} \quad (13)$$

yields the following system

$$\begin{aligned} \dot{z}_0 &= \bar{f}_0(z_0, \bar{x}_{1,1}, \bar{x}_{2,1}, \mu) \\ \dot{z}_{1,i} &= M_iz_{1,i} + Q_{1,i}(z_0, z_{1,1},\cdots,z_{1,i-1}, \bar{x}_{1,1},\cdots, \\ &\quad \bar{x}_{1,i}, \bar{x}_{2,1}, \mu) \\ \dot{\bar{x}}_{1,i} &= \bar{f}_{1,i}(z_0, z_{1,1},\cdots,z_{1,i}, \bar{x}_{1,1},\cdots,\bar{x}_{1,i}, \bar{x}_{2,1}, \mu) \\ &\quad + b_{1,i}(\mu)\bar{x}_{1,i+1} \\ \dot{z}_{2,j} &= M_{r_1+j}z_{2,j} + Q_{2,j}(\bar{X}_{r_1}, z_{2,1},\cdots,z_{2,j-1}, \bar{x}_{2,1}, \\ &\quad \cdots, \bar{x}_{2,j}, \mu) \\ \dot{\bar{x}}_{2,j} &= \bar{f}_{2,j}(\bar{X}_{r_1}, z_{2,1},\cdots,z_{2,j}, \bar{x}_{2,1},\cdots,\bar{x}_{2,j}, \mu) \\ &\quad + b_{2,j}(\mu)\bar{x}_{2,j+1} \end{aligned} \quad (14)$$

where

$$\begin{aligned} &\bar{f}_0(z_0, \bar{x}_{1,1}, \bar{x}_{2,1}, \mu) \\ &= f_0(z, x_{1,1}, v, w) + \varphi_{1,0}(z, x_{1,1}, x_{2,1}, v, w)x_{2,1} \\ &\quad - f_0(\mathbf{z}(v,w), q_{d_1}(v,w), v, w) - \varphi_{1,0}(\mathbf{z}(v,w), \\ &\quad q_{d_1}(v,w), q_{d_2}(v,w), v, w)q_{d_2}(v,w) \end{aligned}$$

and $\mu = \text{col}(v,w)$, $l = n_v + n_w$, $\bar{x}_{1,r_1+1} = \bar{u}_1$, $\bar{x}_{2,r_2+1} = \bar{u}_2$, $\bar{X}_i = \text{col}(z_0, z_{1,1}, \bar{x}_{1,1},\cdots,z_{1,i}, \bar{x}_{1,i})$, $i = 1,\cdots,r_1$, $j = 1,\cdots,r_2$. For $i = 1,\cdots,r_1$, $j = 1,\cdots,r_2$, $Q_{1,i}, \bar{f}_{1,i}, Q_{2,j}, \bar{f}_{2,j}$ are globally defined smooth functions satisfying $Q_{1,i}(0,\cdots,0,\mu) = 0$, $\bar{f}_{1,i}(0,\cdots,0,\mu) = 0$, and $Q_{2,j}(0,\cdots,0,\mu) = 0$, $\bar{f}_{2,j}(0,\cdots,0,\mu) = 0$, for all $\mu \in R^l$. The expressions of these functions are omitted due to the space limit.

Remark 2.3: Now the global robust output regulation problem of system (1) has been converted to the global robust stabilization problem of system (14). Thus, if a state feedback control law of the form

$$\bar{u}_1 = \alpha_1(\bar{x}_1), \ \bar{u}_2 = \alpha_2(\bar{x}_2) \quad (15)$$

where $\bar{x}_1 = \text{col}(\bar{x}_{1,1},\cdots,\bar{x}_{1,r_1}), \bar{x}_2 = \text{col}(\bar{x}_{2,1},\cdots,\bar{x}_{2,r_2})$, α_1 and α_2 are globally defined smooth functions vanishing at the origin globally stabilizes the augmented system (14), then the following control law

$$\begin{aligned} u_1 &= \alpha_1(\bar{x}_1) + \beta_{r_1}(\eta_{r_1}) \\ u_2 &= \alpha_2(\bar{x}_2) + \beta_{r_1+r_2}(\eta_{r_1+r_2}) \\ \dot{\eta}_i &= M_i\eta_i + N_ig_i(x,u), i = 1,\cdots,r_1+r_2 \end{aligned} \quad (16)$$

solves the global robust output regulation problem of the original plant (1).

III. Main Results

Having derived the augmented system (14), all we need to do is to stabilize (14). Our idea is to decompose the stabilization problem of the augmented system into the stabilization problem of two single-input systems. While the first single-input system has $\bar{X}_{r_1}$ as the state and $\bar{u}_1$ as the control input, the second single-input system has $\text{col}(\bar{X}_{r_1}, z_{2,1}, \bar{x}_{2,1},\cdots,z_{2,r_2}, \bar{x}_{2,r_2})$ as the state and $\bar{u}_2$ as the control input. Both systems contain some dynamic uncertainties. To handle these uncertainties, we need one more assumption.

Assumption 3.1: Consider the z_0-subsystem of (14) where $\mu : [0,\infty) \mapsto R^l$ is a bounded piecewise continuous function. For any compact subset $\Sigma \subset R^l$, there exists a C^1 function V_{z_0} satisfying $\underline{\alpha}_0(\|z_0\|) \le V_{z_0}(z_0) \le \overline{\alpha}_0(\|z_0\|)$ for some class K_∞ function $\underline{\alpha}_0(\cdot), \overline{\alpha}_0(\cdot)$ such that for all $\mu \in \Sigma$,

$$\dot{V}_{z_0} \le -\alpha_0(\|z_0\|) + \gamma_0(\bar{x}_{1,1}, \bar{x}_{2,1})$$

where $\alpha_0(\cdot)$ is some class K_∞ function satisfying $\lim_{s\to 0^+} \sup(\frac{\alpha_0^{-1}(s^2)}{s}) < \infty$ and $\gamma_0(\cdot)$ is a known smooth positive definite function.

Lemma 3.1: Consider the following system

$$\begin{aligned} \dot{\zeta}_1 &= \varphi_1(\zeta_1, x, y, \mu(t)) \\ \dot{\zeta}_2 &= A\zeta_2 + \varphi_2(\zeta_1, x, y, \mu(t)) \\ \dot{x} &= \phi(\zeta_1, \zeta_2, x, y, \mu(t)) + b(\mu(t))u \end{aligned} \quad (17)$$

where $\zeta_1 \in R^{n_1}, \zeta_2 \in R^{n_2}, x \in R, y \in R, \mu : [0,\infty) \mapsto R^l$ is a bounded piecewise continuous function, $A \in R^{n_2 \times n_2}$ is a Hurwitz matrix, $\varphi_1(\zeta_1, x, y, \mu)$, $\varphi_2(\zeta_1, x, y, \mu)$ and $\phi(\zeta_1, \zeta_2, x, y, \mu)$ are sufficiently smooth with $\varphi_1(0,0,0,\mu) = 0$, $\varphi_2(0,0,0,\mu) = 0$ and $\phi(0,0,0,0,\mu) = 0$ for all $\mu \in R^l$. Assume, given any compact subset $\Sigma \subset R^l$, there exists a C^1 function $\bar{V}_1(\zeta_1)$ satisfying $\underline{\gamma}_1(\|\zeta_1\|) \le \bar{V}_1(\zeta_1) \le \overline{\gamma}_1(\|\zeta_1\|)$ for some class K_∞ functions $\underline{\gamma}_1(\cdot)$ and $\overline{\gamma}_1(\cdot)$ such that, for all $\mu(t) \in \Sigma$, along the trajectory of system $\dot{\zeta}_1 = \varphi_1(\zeta_1, x, y, \mu)$,

$$\dot{\bar{V}}_1 \le -\gamma_1(\|\zeta_1\|) + \bar{\pi}_1(x) + \bar{\pi}_2(y) \quad (18)$$

where $\gamma_1(\cdot)$ is some known class K_∞ function satisfying $\lim_{s\to 0^+} \sup(\frac{\gamma_1^{-1}(s^2)}{s}) < \infty$ and $\bar{\pi}_1(x), \bar{\pi}_2(y)$ are some known smooth positive definite functions. Then there exist a smooth function $\rho : R \mapsto [0,\infty)$, a controller of the form

$$u = -\rho(x)x + \nu \quad (19)$$

with $\nu \in R$, and a C^1 function $U_1(\zeta_1, \zeta_2, x)$ satisfying $\underline{\alpha}_1(\|\zeta_1, \zeta_2, x\|) \le U_1(\zeta_1, \zeta_2, x) \le \overline{\alpha}_1(\|\zeta_1, \zeta_2, x\|)$ for some class K_∞ functions $\underline{\alpha}_1(\cdot)$ and $\overline{\alpha}_1(\cdot)$, such that, along the trajectory of the closed-loop system composed of (17) and (19),

$$\dot{U}_1 \le -\|\zeta_1\|^2 - \|\zeta_2\|^2 - x^2 + \nu^2 + \pi(y) \quad (20)$$

for some known smooth positive definite function $\pi(y)$.

The proof is skipped due to the space limit.

Before establishing the next Lemma, we introduce the following notations:

$$\begin{aligned} \tilde{x}_{1,1} &= \bar{x}_{1,1} \\ \tilde{x}_{1,i+1} &= \bar{x}_{1,i+1} + \rho_{1,i}(\tilde{x}_{1,i})\tilde{x}_{1,i}, i = 1, \cdots, r_1 - 1 \\ \bar{u}_1 &= -\rho_{1,r_1}(\tilde{x}_{1,r_1})\tilde{x}_{1,r_1} \end{aligned} \quad (21)$$

$$\begin{aligned} \tilde{x}_{2,1} &= \bar{x}_{2,1} \\ \tilde{x}_{2,j+1} &= \bar{x}_{2,j+1} + \rho_{2,j}(\tilde{x}_{2,j})\tilde{x}_{2,j}, j = 1, \cdots, r_2 - 1 \\ \bar{u}_2 &= -\rho_{2,r_2}(\tilde{x}_{2,r_2})\tilde{x}_{2,r_2} \end{aligned} \quad (22)$$

where, for $k = 1, 2$, $j = 1, \cdots, r_k$, $\rho_{k,j}(\tilde{x}_{k,j})$ are some nonnegative smooth functions to be specified in the proof of Lemma 3.2.

Also, let $\tilde{X}_i = \text{col}(z_0, z_{1,1}, \tilde{x}_{1,1}, \cdots, z_{1,i}, \tilde{x}_{1,i})$, $i = 1, \cdots, r_1$.

Lemma 3.2: Under Assumption 3.1, given any compact subset $\Sigma \subset R^l$, there exist nonnegative smooth functions $\rho_{1,i}(\tilde{x}_{1,i})$, $i = 1, \cdots, r_1$, that define the control law (21) and a C^1 function $\tilde{U}_{r_1}(\tilde{X}_{r_1})$ satisfying $\underline{\alpha}_{r_1}(\|\tilde{X}_{r_1}\|) \le \tilde{U}_{r_1}(\tilde{X}_{r_1}) \le \overline{\alpha}_{r_1}(\|\tilde{X}_{r_1}\|)$ for some class K_∞ functions $\underline{\alpha}_{r_1}(\cdot)$ and $\overline{\alpha}_{r_1}(\cdot)$ such that, for all $\mu \in \Sigma$,

$$\dot{\tilde{U}}_{r_1} \le -\|\tilde{X}_{r_1}\|^2 + \varpi_{r_1}(\bar{x}_{2,1}) \quad (23)$$

where $\varpi_{r_1}(\cdot)$ is a known smooth positive definite function.

The proof is also skipped due to the space limit.

Remark 3.1: Under control law (21), system (14) can be put in the following form:

$$\begin{aligned} \dot{\tilde{X}}_{r_1} &= F_{r_1}(\tilde{X}_{r_1}, \bar{x}_{2,1}, \mu) \\ \dot{z}_{2,j} &= M_{r_1+j}z_{2,j} + \tilde{Q}_{2,j}(\tilde{X}_{r_1}, z_{2,1}, \cdots, z_{2,j-1}, \bar{x}_{2,1}, \\ &\quad \cdots, \bar{x}_{2,j}, \mu) \\ \dot{\bar{x}}_{2,j} &= \tilde{f}_{2,j}(\tilde{X}_{r_1}, z_{2,1}, \cdots, z_{2,j}, \bar{x}_{2,1}, \cdots, \bar{x}_{2,j}, \mu) \\ &\quad + b_{2,j}(\mu)\bar{x}_{2,j+1} \\ j &= 1, \cdots, r_2 \end{aligned} \quad (24)$$

where

$$\tilde{f}_0(z_0, \tilde{x}_{1,1}, \bar{x}_{2,1}, \mu) = \bar{f}_0(z_0, \bar{x}_{1,1}, \bar{x}_{2,1}, \mu)$$

$$F_{r_1}(\tilde{X}_{r_1}, \bar{x}_{2,1}, \mu) = \begin{bmatrix} \tilde{f}_0(z_0, \tilde{x}_{1,1}, \bar{x}_{2,1}, \mu) \\ M_1 z_{1,1} + \tilde{Q}_{1,1}(z_0, \tilde{x}_{1,1}, \bar{x}_{2,1}, \mu) \\ \tilde{f}_{1,1}(\tilde{X}_1, \bar{x}_{2,1}, \mu) + b_{1,1}(\mu)\tilde{x}_{1,2} \\ \vdots \\ M_{r_1} z_{1,r_1} + \tilde{Q}_{1,r_1}(\tilde{X}_{r_1-1}, \tilde{x}_{1,r_1}, \bar{x}_{2,1}, \mu) \\ \tilde{f}_{1,r_1}(\tilde{X}_{r_1}, \bar{x}_{2,1}, \mu) + b_{1,r_1}(\mu)\bar{u}_1 \end{bmatrix},$$

and for $i = 1, \cdots, r_1$, $j = 1, \cdots, r_2$, $\tilde{Q}_{1,i}, \tilde{f}_{1,i}, \tilde{Q}_{2,j}, \tilde{f}_{2,j}$ are globally defined smooth functions satisfying $\tilde{Q}_{1,i}(0, \cdots, 0, \mu) = 0$, $\tilde{f}_{1,i}(0, \cdots, 0, \mu) = 0$, and $\tilde{Q}_{2,j}(0, \cdots, 0, \mu) = 0$, $\tilde{f}_{2,j}(0, \cdots, 0, \mu) = 0$, for all $\mu \in R^l$. The expressions of these functions are omitted due to the space limit.

The system (24) is an SISO system and is in lower triangular form and the stabilization problem of system (24) has been extensively studied in the literature. In particular, as $\tilde{X}_{r_1}$-subsystem satisfies inequality (23), we can directly invoke Theorem 4.1 of [3] to conclude that there exist a control law of the form (22) and a C^1 function $\tilde{U}_{r_1+r_2}(\tilde{X}_{r_1+r_2})$ satisfying $\underline{\beta}_{r_1+r_2}(\|\tilde{X}_{r_1+r_2}\|) \le \tilde{U}_{r_1+r_2}(\tilde{X}_{r_1+r_2}) \le \bar{\beta}_{r_1+r_2}(\|\tilde{X}_{r_1+r_2}\|)$ for some class K_∞ functions $\underline{\beta}_{r_1+r_2}(\cdot), \bar{\beta}_{r_1+r_2}(\cdot)$ such that

$$\dot{\tilde{U}}_{r_1+r_2} \le -\|\tilde{X}_{r_1+r_2}\|^2 \quad (25)$$

where $\tilde{X}_{r_1+r_2} = \text{col}(\tilde{X}_{r_1}, z_{2,1}, \tilde{x}_{2,1}, \cdots, z_{2,r_2}, \tilde{x}_{2,r_2})$. Thus, the stabilization problem of system (14) is solved by control law (21) and (22). The control law (21) and (22) can be put in the form (15). By Remark 2.3, we can obtain the following result.

Theorem 3.1: Under Assumptions 2.1, 2.2 and 3.1, the state feedback control law of the following form

$$\begin{aligned} u_1 &= -\rho_{1,r_1}(\tilde{x}_{1,r_1})\tilde{x}_{1,r_1} + \beta_{r_1}(\eta_{r_1}) \\ u_2 &= -\rho_{2,r_2}(\tilde{x}_{2,r_2})\tilde{x}_{2,r_2} + \beta_{r_1+r_2}(\eta_{r_1+r_2}) \\ \dot{\eta}_i &= M_i\eta_i + N_i g_i(x,u), i = 1, \cdots, r_1 + r_2 \end{aligned} \quad (26)$$

solves the global robust output regulation problem of system (1).

IV. Application to Speed Control of Surface-mounted PM Synchronous Motor

Consider the following system [10], [11]:

$$\begin{aligned} \frac{d\theta_r}{dt} &= \omega_r \\ \frac{d\omega_r}{dt} &= \frac{3p\Phi_v}{2J}i_q - \frac{B}{J}\omega_r - \frac{1}{J}T_L \\ \frac{di_d}{dt} &= -\frac{R_s}{L}i_d + pi_q\omega_r + \frac{1}{L}u_d \\ \frac{di_q}{dt} &= -\frac{R_s}{L}i_q - pi_d\omega_r - \frac{p\Phi_v}{L}\omega_r + \frac{1}{L}u_q \end{aligned} \quad (27)$$

where θ_r is rotor position, ω_r is speed, i_d and i_q are dq frame stator currents, T_L is an unknown constant load torque with a known bound, u_d and u_q are dq frame stator voltages, L is dq axes inductance, Φ_v is rotor flux, R_s is stator resistance, J is inertia, B is viscous friction coefficient and p is the number of pole pairs.

System (27) is called surface-mounted PM synchronous motor because dq axes inductances are equal. We now design a dynamic state feedback control law such that the solution of the closed-loop system is globally bounded in the presence of the unknown constant load torque T_L, and

1) the speed ω_r tracks the trajectory $y_d(t) = A\sin(\omega t + \phi)$ with arbitrary unknown amplitude A and initial phase ϕ;
2) the d axis current i_d is asymptotically regulated to zero.

As we mentioned in the introduction, similar control problems have been considered in several papers via feedback linearization approach [10], [11]. This approach needs to know the reference input $y_d(t)$ as well as its first and second derivatives. Here we will solve the above problem using the output regulation theory. For this purpose, define the following exosystem

$$\dot{v} = A_1 v = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v \quad (28)$$

where $v = [v_1, v_2, v_3]^T$. The system (28) can generate any combination of a sine function with arbitrary amplitudes and initial phase and an arbitrary constant. In particular, with initial value given by $v(0) = \left[A\sin\phi, A\cos\phi, \frac{1}{J}T_L\right]^T$, the solution of (28) is such that

$$v_1(t) = y_d(t), v_3(t) = \frac{1}{J}T_L. \quad (29)$$

Let $x_{1,1} = \omega_r, x_{1,2} = i_q, x_{2,1} = i_d, u_1 = u_q, u_2 = u_d, a_{11} = \frac{B}{J}, b_{11} = \frac{3p\Phi_v}{2J}, a_{12} = \frac{R_s}{L}, a_{13} = \frac{p\Phi_v}{L}, b_{12} = \frac{1}{L}, a_{14} = p, a_{21} = \frac{R_s}{L}, a_{22} = p, b_{21} = \frac{1}{L}$. Then the system (27) can be put in the form (1) as follows:

$$\begin{aligned} \dot{x}_{1,1} &= -a_{11}x_{1,1} - v_3 + b_{11}x_{1,2} \\ \dot{x}_{1,2} &= -a_{12}x_{1,2} - a_{13}x_{1,1} + b_{12}u_1 - a_{14}x_{1,1}x_{2,1} \\ \dot{x}_{2,1} &= -a_{21}x_{2,1} + a_{22}x_{1,1}x_{1,2} + b_{21}u_2 \\ e &= \begin{bmatrix} x_{1,1} - v_1 \\ x_{2,1} \end{bmatrix}. \end{aligned} \quad (30)$$

With (28) and (30) ready, the speed control problem of surface-mounted PM synchronous motor can be solved if we can solve the robust output regulation problem of the system composed of (28) and (30) by a state feedback control law of the form (26). An advantage of the nonlinear output regulation approach is that it can render the motor speed to track a sinusoidal signal with arbitrary unknown amplitude and initial phase without actually using $y_d(t)$ and its first and second derivatives. This approach allows all the motor parameters L, Φ_v, R_s, J, B to be unknown with known bounds.

We now verify system (30) satisfies all assumptions of Theorem 3.1. In fact, since system (30) only contains polynomial nonlinearity and the dimension of the z dynamics of system (30) is zero, Assumption 2.1 holds trivially. Also, since $b_{11}, b_{12}, b_{21} > 0$, Assumption 2.2 is satisfied. By (5), the solution of the regulator equations associated with (28) and (30) is given as follows:

$$\begin{aligned} \mathbf{x}_{1,1}(v,w) &= v_1 \\ \mathbf{x}_{1,2}(v,w) &= b_{11}^{-1}a_{11}v_1 + b_{11}^{-1}\omega v_2 + b_{11}^{-1}v_3 \\ \mathbf{x}_{2,1}(v,w) &= 0 \\ \mathbf{u}_1(v,w) &= b_{12}^{-1}(-b_{11}^{-1}\omega^2 + b_{11}^{-1}a_{11}a_{12} + a_{13})v_1 \\ &\quad + b_{12}^{-1}(b_{11}^{-1}a_{11}\omega + b_{11}^{-1}a_{12}\omega)v_2 \\ &\quad + b_{12}^{-1}b_{11}^{-1}a_{12}v_3 \\ \mathbf{u}_2(v,w) &= -b_{11}^{-1}a_{22}b_{21}^{-1}v_1(a_{11}v_1 + \omega v_2 + v_3). \end{aligned}$$

Let $g(x,u) = \text{col}(x_{1,2}, u_1, u_2)$. Then we can obtain a steady-state generator of the form (9) with T_i the solution of the following equations:

$$T_i\Phi_i - M_iT_i = N_i\Psi_i, \quad i = 1,2,3 \quad (31)$$

where (M_i, N_i), $i = 1,2,3$, are any controllable pairs with M_i Hurwitz. By (10), we can construct the internal model with output $x_{1,2}, u_1, u_2$ as follows:

$$\begin{aligned} \dot{\eta}_1 &= M_1\eta_1 + N_1x_{1,2} \\ \dot{\eta}_2 &= M_2\eta_2 + N_2u_1 \\ \dot{\eta}_3 &= M_3\eta_3 + N_3u_2. \end{aligned} \quad (32)$$

Performing the coordinate and input transformation (13) on the system composed of (30) and (32) gives the augmented system (14) which takes the following specific form:

$$
\begin{aligned}
\dot{z}_{1,1} &= M_1 z_{1,1} + d_1 \bar{x}_{1,1} \\
\dot{\bar{x}}_{1,1} &= b_{11}\bar{x}_{1,2} + d_2 z_{1,1} + d_3 \bar{x}_{1,1} \\
\dot{z}_{1,2} &= M_2 z_{1,2} + d_4 z_{1,1} + d_5 \bar{x}_{1,1} + c_1(v)\bar{x}_{2,1} + d_6 \bar{x}_{1,2} \\
&\quad + d_7 \bar{x}_{1,1}\bar{x}_{2,1} \\
\dot{\bar{x}}_{1,2} &= d_8 z_{1,2} + d_9 z_{1,1} + d_{10}\bar{x}_{1,1} + c_2(v)\bar{x}_{2,1} \\
&\quad + d_{11}\bar{x}_{1,2} + d_{12}\bar{x}_{1,1}\bar{x}_{2,1} + b_{12}\bar{u}_1 \\
\dot{z}_{2,1} &= M_3 z_{2,1} + c_3(v) z_{1,1} + c_4(v)\bar{x}_{1,1} + d_{13}\bar{x}_{2,1} \\
&\quad + c_5(v)\bar{x}_{1,2} + d_{14}\bar{x}_{1,1}\bar{x}_{1,2} + d_{15}\bar{x}_{1,1} z_{1,1} \\
&\quad + d_{16}\bar{x}_{1,1}^2 \\
\dot{\bar{x}}_{2,1} &= d_{17} z_{2,1} + c_6(v) z_{1,1} + c_7(v)\bar{x}_{1,1} + d_{18}\bar{x}_{2,1} \\
&\quad + c_8(v)\bar{x}_{1,2} + d_{19}\bar{x}_{1,1}\bar{x}_{1,2} + d_{20}\bar{x}_{1,1} z_{1,1} \\
&\quad + d_{21}\bar{x}_{1,1}^2 + b_{21}\bar{u}_2 \qquad (33)
\end{aligned}
$$

where the expressions of $c_1(v), \cdots, c_8(v), d_1, \cdots, d_{21}$ are omitted due to space limit.

Since the dimension of the z_0-subsystem in (33) is zero, Assumption 3.1 holds trivially. By Theorem 3.1, the stabilization problem of system (33), and hence, the output regulation problem of system (30) are solvable.

Following the design procedure given in Section III, we can obtain a control law in the form of (26). To be more specific, $p = 3$ and the nominal values of motor parameters are taken from [11]: $\bar{R}_s = 1.2\Omega, \bar{B} = 0.0001$N.m.sec/rad, $\bar{\Phi}_v = 0.18$V.sec/rad, $\bar{L} = 0.011$H, $\bar{J} = 0.006$Kg.m^2. The nominal value of load torque is $\bar{T}_L = 0.3$N.m. Assume frequency $\omega = 3$, unknown amplitude $A \leq 2$rad/sec and the actual values of $R_s, B, \Phi_v, L, J, T_L$ satsify $R_s \in [0.5\bar{R}_s, 2\bar{R}_s], B \in [0.5\bar{B}, 2\bar{B}], \Phi_v \in [0.5\bar{\Phi}_v, 2\bar{\Phi}_v], L \in [0.5\bar{L}, 2\bar{L}], J \in [0.5\bar{J}, 2\bar{J}], T_L \in [0.5\bar{T}_L, 2\bar{T}_L]$. Then we can obtain a specific control law as follows:

$$
\begin{aligned}
u_1 &= -\bar{k}_2(1 + \tilde{x}_{1,2}^2)\tilde{x}_{1,2} + \Psi_2 T_2^{-1}\eta_2 \\
u_2 &= -\bar{k}_3(1 + e_2^2)e_2 + \Psi_3 T_3^{-1}\eta_3 \\
\tilde{x}_{1,2} &= x_{1,2} - \Psi_1 T_1^{-1}\eta_1 + \bar{k}_1 e_1 \\
\dot{\eta}_1 &= M_1\eta_1 + N_1 x_{1,2} \\
\dot{\eta}_2 &= M_2\eta_2 + N_2 u_1 \\
\dot{\eta}_3 &= M_3\eta_3 + N_3 u_2 \qquad (34)
\end{aligned}
$$

where $\bar{k}_1 = 2, \bar{k}_2 = 0.6, \bar{k}_3 = 60$, and the specific values of various matrices are omitted due to the space limit.

The performance of the control law is evaluated through computer simulation with the following motor parameters: $R_s = 2\bar{R}_s, B = 0.5\bar{B}, \Phi_v = 0.9\bar{\Phi}_v, L = 2\bar{L}, J = 0.7\bar{J}$. The desired speed is $y_d(t) = 2\sin(3t + \frac{\pi}{2})$rad/sec, and the load $T_L = 0$ for $0 \leq t < 4$s and $T_L = 0.7\bar{T}_L$ for $t \geq 4$s. The initial values of various variables are $\omega_r(0) = 0.1$rad/sec, $i_d(0) = 0.2$A, $i_q(0) = 0.3$A, $v_1(0) = 2, v_2(0) = 0, v_3(0) = 0$, and $\eta_1(0) = \eta_2(0) = \eta_3(0) = 0$. Figures 1 and 2 show the motor speed tracking performance and the current i_d, respectively.

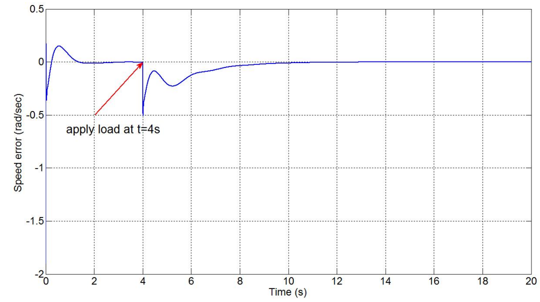

Fig. 1. speed error with load at $t = 4$s

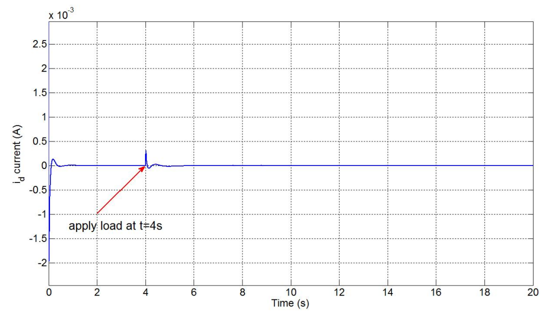

Fig. 2. i_d current with load at $t = 4$s

V. Conclusion

Due to the recursive nature, our approach can also be generalized to an m-input m-output system with a structure similar to (1).

References

[1] I. C. Baik, K. H. Kim, and M. J. Youn. Robust nonlinear speed control of PM synchronous motor using boundary layer intergral sliding mode control technique. *IEEE Trans. Contr. Syst. Tech.*, vol. 8, no. 1, pp. 47-54, 2000.

[2] T. Chen and J. Huang. Global robust output regulation by state feedback for strict feedforward systems, *IEEE Trans. Automat. Contr.*, vol. 54, no. 9, pp. 2157-2163, 2009.

[3] Z. Chen and J. Huang. A general formulation and solvability of the global robust output regulation problem. *IEEE Trans. Automat. Contr.*, vol. 50, no. 4, pp. 448-462, 2005.

[4] J. Huang and Z. Chen. A general framework for tackling the output regulation problem. *IEEE Trans. Automat. Contr.*, vol. 49, no. 12, pp. 2203-2218, 2004.

[5] A. Isidori and C. I. Byrnes. Output regulation of nonlinear systems. *IEEE Trans. Automat. Contr.*, vol. 35. no. 2, pp. 131-140, 1990.

[6] K. H. Kim and M. J. Youn. A nonlinear speed control for a PM synchronous motor using a simple disturbance estimation technique. *IEEE Trans. Ind. Electron.*, vol. 49, no. 3, pp. 524-535, 2002.

[7] N. K. McGregor, C. I. Byrnes, and A. Isidori. Results on nonlinear output regulation of MIMO systems. in *Proc. of 2006 American Control Conference*, Minneapolis, Minnesota, USA, 2006, pp. 5795-5800.

[8] V. O. Nikiforov. Adaptive non-linear tracking with complete compensation of unknown disturbances. *European Journal of Control*, vol. 4, no. 2, pp. 132-139, 1998.

[9] X. Ye and J. Huang. Decentralized adaptive output regulation for a class of large-scale nonlinear systems. *IEEE Trans. Automat. Contr.*, vol. 48, no. 2, pp. 276-281, 2003.

[10] J. Zhou and Y. Wang. Real-time nonlinear adaptive backstepping speed control for a PM synchronous motor. *Control Eng. Pract.*, vol. 13, no. 10, pp. 1259-1269, 2005.

[11] G. Zhu, L. A. Desaint, O. Akhrif, and A. Kaddouri. Speed tracking control of a permanent synchrounous motor with state and load torque observer. *IEEE Trans. Ind. Electron.*, vol. 47, no. 2, pp. 346-355, 2000.