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Fault diagnosis in nonlinear dynamical systems based on left invertibility condition: a real–time application to three-tank system

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***Abstract*— This work deals with the fault diagnosis problem, some new properties are found using the *left invertibility condition* through the concept of differential output rank. Two schemes of nonlinear observers are used to estimate the fault signals for comparison purposes, one of these is a reduced order observer and the other is a sliding mode observer. The methodology is tested in a real time implementation of a three-tank system.**

I. INTRODUCTION

A fault can be considered as a process degradation or degradation of the equipment performance caused by the change in the physical characteristic of the process, the input process or the external conditions.

The fault detection and isolation problem has been studied for more than three decades, many papers dealing with this problem can be found, see for instance the surveys [1]-[4] and the books [5]-[7]. For the case of nonlinear systems a variety of approaches have been proposed [1]. Some model-based approaches can be found, such as those based upon differential geometric methods [8], [9]. On the other hand, for the fault diagnosis problem, alternative approaches have been proposed based on an algebraic and differential framework [10]-[19]. These approaches consist in the estimation of the fault variables, which are defined as uncertain inputs [10].

Currently, the diagnosis problem is playing an important role in modern industrial processes. This has led control theory into a wide variety of model–based approaches which rely on descriptions via differential and/or difference equations, contrary to other standpoints developed mainly among computer scientist (see [16],[17] and references therein). The primary objectives of fault diagnosis are fault detectability and isolability, i.e., the possible location and determination of the faults present in a system and the time of their occurrences. The tasks of fault detection and isolation are to be accomplished by measuring only the input and the output variables.

This paper focuses on diagnosis of nonlinear systems and the goal is to determine malfunctions in the dynamics. In this communication, the outputs are mainly signals obtained from the sensors. Their number is important to know whether a system is diagnosable or not.

In this article, the diagnosis problem is tackled as a left invertibility problem throughout the concept of differential output rank ρ. Two schemes of observers are proposed in order to estimate the fault signals, one of them is a reduced-order observer based on a free-model approach and another is a sliding-mode observer based on a Generalized Observability Canonical Form (GOCF) [16]. Both schemes are proved to possess asymptotic convergence properties.

The type of faults considered in this work are additive and bounded, however, the algebraic approach can also be used to deal with multiplicative faults (see [11]).

This paper is organized as follows. In section II, some definitions of differential algebra are given. In section III, we discuss the left invertibility condition. In sections IV and V we give a description of the proposed observers. In section VI the three-tank system is analyzed. Finally, in section VII we illustrate this methodology with some experimental results to the three-tank system Amira DTS200 three-tank system [20],[21].

II. SOME DEFINITIONS

Some basic definitions are introduced. Further details can be found in [10]-[13], [22] and references therein.

Definition 1: Let $\mathcal{L}$ and $\mathcal{K}$ be differential fields. A differential field extension $\mathcal{L}/\mathcal{K}$ is given by $\mathcal{K}$ and $\mathcal{L}$ such that: 1) $\mathcal{K}$ is a subfield of $\mathcal{L}$ and; 2) the derivation of $\mathcal{K}$ is the restriction to $\mathcal{K}$ of the derivation of $\mathcal{L}$.

Definition 2: Let $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ be a set of elements of $\mathcal{L}$. If it satisfies an algebraic differential equation $P\left(\xi, \dot{\xi}, \ddot{\xi} \dots\right) = 0$ with coefficients in $\mathcal{K}$ it is called differentially $\mathcal{K}$-algebraically dependent, otherwise, ξ is called differentially $\mathcal{K}$-algebraically independent.

Definition 3: Any set of elements of $\mathcal{L}$ which is differentially $\mathcal{K}$-algebraically independent and maximal with respect to inclusion forms is a differential transcendence basis of $\mathcal{L}/\mathcal{K}$. Two such basis have the same cardinality. This is called the *differential transcendence degree* of $\mathcal{L}/\mathcal{K}$ and denoted by $\mathit{diff}\ tr\ d^{\circ}\ \mathcal{L}/\mathcal{K}$.

Definition 4: Let $\mathcal{G}$, $\mathcal{K}\langle u\rangle$ be differential fields. A nominal dynamic consists in a finitely generated differential algebraic extension $\mathcal{G}/\mathcal{K}\langle u\rangle$, $\quad (\mathcal{G} = \mathcal{K}\langle u, \xi\rangle\,,\ \xi \in \mathcal{G})$. Any element of $\mathcal{G}$ satisfies an algebraic differential equation with coefficients over $\mathcal{K}$ in the components of u and their time derivatives.

Definition 5: Any unknown variable x in a dynamic is said to be *algebraically observable* with respect to $\mathcal{K}\langle u, y\rangle$ if x satisfies a differential algebraic equation with coefficients over $\mathcal{K}$ in the components of u, y and a finite number of their derivatives. Any dynamic with output y is said to be

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algebraically observable if, and only if any state variable has this property.

Definition 6: Let $\mathcal{G}$, $\mathcal{K}\langle u\rangle$ be differential fields. A fault dynamics consists in a finitely generated differential algebraic extension $\mathcal{G}/\mathcal{K}\langle u, f\rangle$, $\mathcal{G} = \mathcal{K}\langle u, f, \xi\rangle$, $\xi \in \mathcal{G}$. Any element of $\mathcal{G}$ satisfies an algebraic differential equation with coefficients over $\mathcal{K}$ in the components of u, f and their time derivatives.

Definition 7: A fault $f \in \mathcal{G}$ is said to be diagnosable if it is algebraically observable over $R\langle u, y\rangle$, i.e. if it is possible to estimate the fault from the available measurements of the system.

Let us consider the class of nonlinear systems with faults described by the following equation

$$\left\{ \begin{array}{l} \dot{x}(t) = A(x, \bar{u}) \\ y(t) = h(x, \bar{u}) \end{array} \right. . \tag{1}$$

Where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ is a state vector, $u = (u_1, \ldots, u_m) \in \mathbb{R}^m$ is a known input vector, $f = (f_1, \ldots, f_\mu) \in \mathbb{R}^\mu$ is an unknown input vector, $\bar{u} = (u, f) \in \mathbb{R}^{m+\mu}$, $y(t) \in \mathbb{R}^p$ is the output vector. A and h are assumed to be analytical vector functions.

Example 1: Let us consider the nonlinear system with one fault (f_1) on the actuator and one fault (f_2) on the sensor of output y_1.

$$\left\{ \begin{array}{ll} \dot{x}_1 = & x_1 x_2 + f_1 + u \\ \dot{x}_2 = & x_1 \\ y_1 = & x_1 + f_2 \\ y_2 = & x_2 \end{array} \right. . \tag{2}$$

Since f_1, f_2 satisfy the differential algebraic equations

$$\begin{array}{rcl} f_1 - \ddot{y}_2 + y_2\dot{y}_2 + u & = & 0 \\ f_2 - y_1 + \dot{y}_2 & = & 0 \end{array} \tag{3}$$

the system (2) is diagnosable and the faults can be reconstructed from the knowledge of u, y and their time derivatives.

Remark 1: The diagnosability condition is independent of the observability of a system.

III. On the left invertibility condition

We have some definitions concerning on the differential output rank of a system.

Definition 8: The differential output rank ρ of a system is equal to the differential transcendence degree of the differential extension $\mathcal{K}\langle y\rangle$ over the differential field $\mathcal{K}$, i.e.,

$$\rho = diff\ tr\ d^o\mathcal{K}\langle y\rangle/\mathcal{K}.$$

Property 1 [23]: Let $\mathcal{K}$, $\mathcal{L}$, $\mathcal{M}$, be differential fields such that $\mathcal{K} \subset \mathcal{L} \subset \mathcal{M}$. Then

$$\begin{array}{rl} diff\ tr\ d^o(\mathcal{M}/\mathcal{K}) = & diff\ tr\ d^o(\mathcal{M}/\mathcal{L}) \\ & +diff\ tr\ d^o(\mathcal{L}/\mathcal{K}) \end{array} \tag{4}$$

■

Property 2: The differential output rank ρ of a system is smaller or equal to $\min(m, p)$, i.e., $\rho = diff\ tr\ d^o\mathcal{K}\langle y\rangle/\mathcal{K} \leq \min(m, p)$, where m and p are the total number of inputs and outputs, respectively. ■

The differential output rank ρ is also the maximum number of outputs that are related by a differential polynomial equation with coefficients over $\mathcal{K}$ (independent of x and u).

A practical way to determinate the differential output rank is by taking into account all possible differential polynomials of the form

$$h_r(y_1, \ldots, y_p) = 0 \tag{5}$$

and if is possible to find r independent relations of the form (5), then the differential output rank is given by $\rho = p - r$, that is to say, there exists only $p - r$ independent outputs.

Proposition 1 [24]: Let consider a class of systems given by (1). A system is said to be left invertible if and only if

$$\rho = diff\ tr\ d^o\mathcal{K}\langle y\rangle/\mathcal{K} = diff\ tr\ d^o\mathcal{K}\langle u, f\rangle/\mathcal{K}. \qquad ■$$

Property 1 is the main tool used to prove the following theorem that looks quite natural. The theorem shows the relationship between the diagnosability and the left invertibility condition.

Theorem 1: If system (1) is left invertible, then the fault vector f can be obtained by means of the output vector.

Proof: let us consider the following field towers:

$$\mathcal{K} \subset \mathcal{K}\langle u\rangle \subset \mathcal{K}\langle u, f\rangle \subset \mathcal{K}\langle u, y, f\rangle, \tag{6}$$

$$\mathcal{K} \subset \mathcal{K}\langle y\rangle \subset \mathcal{K}\langle u, y\rangle \subset \mathcal{K}\langle u, y, f\rangle, \tag{7}$$

From (6) and property 1, we have:

$$\begin{aligned} diff\ tr\ d^o\mathcal{K}\langle u, y, f\rangle/\mathcal{K} = & \ diff\ tr\ d^o\mathcal{K}\langle u, y, f\rangle/\mathcal{K}\langle u, f\rangle \\ & +diff\ tr\ d^o\mathcal{K}\langle u, f\rangle/\mathcal{K}\langle u\rangle \\ & +diff\ tr\ d^o\mathcal{K}\langle u\rangle/\mathcal{K} \\ = & \ 0 + m + \mu \end{aligned} \tag{8}$$

From proposition 1, $diff\ tr\ d^o\mathcal{K}\langle y\rangle/\mathcal{K} = m + \mu$. From (7) we obtain

$$diff\ tr\ d^o\mathcal{K}\langle u, y, f\rangle/\mathcal{K}\langle u, y\rangle = -diff\ tr\ d^o\mathcal{K}\langle u, y\rangle/\mathcal{K}\langle y\rangle \tag{9}$$

Since the transcendence degree is always positive, we have the following:

$$diff\ tr\ d^o\mathcal{K}\langle u, y, f\rangle/\mathcal{K}\langle u, y\rangle = 0 \tag{10}$$

This means that f is differentially algebraic over $\mathcal{K}\langle u, y\rangle$. Thus, the diagnosability condition is satisfied and the theorem is proven. ■

IV. Reduced-order observer

Let consider system (1). The fault vector f is unknown and it can be assimilated as a state with uncertain dynamics. Then, in order to estimate it, the state vector is extended to deal with the unknown fault vector. The new extended system is given by

$$\begin{array}{c} \dot{x}(t) = A(x, \bar{u}) \\ \dot{f} = \Omega(x, \bar{u}) \\ y(t) = h(x, u) \end{array} \tag{11}$$

where $\Omega(x, \bar{u}) = [\Omega_1(x, \bar{u}), ..., \Omega_\mu(x, \bar{u})]^T : \mathbb{R}^{n+m+\mu} \rightarrow \mathbb{R}^\mu$ is an uncertain function. Note that a classic Luenberger observer can not be constructed because the term $\Omega(x,\bar{u})$ is unknown. This problem is overcome by using a reduced order uncertainty observer in order to estimate the failure variable f. Next Lemma describes the construction of a proportional reduced order observer for (11).

Lemma 1 [22]: If the following hypotheses are satisfied:

H1: $\Omega(x,\bar{u})$ is bounded, i.e., $|\Omega_i(x,\bar{u})| \leq N \in \mathbb{R}^+ \ \forall\ 1 \leq i \leq \mu$.

H2: $f(t)$ is algebraically observable over $\mathbb{R}\langle u,y\rangle$.

Then the system

$$\dot{\hat{f}}_i = k_i\left(f_i - \hat{f}_i\right),\ 1 \leq i \leq \mu \tag{12}$$

is a reduced order observer for system (11), where $\hat{f}_i$ denotes the estimate of fault f_i and $k_i \in \mathbb{R}^+ \ \forall i = 1,..,\mu$ are positive real coefficients that determine the desired convergence rate of the observer. ■

Lemma 2: If a fault signal f_i, $i \in \{1,...,\mu\}$ of system (1) is algebraically observable and can be written in the following form

$$f_i = a_i\dot{y} + b_i(u,y) \tag{13}$$

where $a_i = [a_{i1},...,a_{im}] \in \mathbb{R}^m$ is a constant vector and $b_i(u,y)$ is a bounded function, then there exists a function $\gamma_i \in C^1$, such that the reduced order observer (12) can be written as the following asymptotically stable system

$$\begin{aligned} \dot{\gamma}_i &= -k_i\gamma_i + k_ib_i(u,y) - k_i^2a_iy \\ \hat{f}_i &= \gamma_i + k_ia_iy, \end{aligned} \tag{14}$$

with $\gamma_i(0) = \gamma_{i0} \in \mathbb{R}$ ■

V. Sliding-mode observer

Consider the nonlinear system with faults given by (1), assuming that the fault vector f is algebraically observable over $\mathbb{R}\langle u,y\rangle$ and therefore it satisfies a differential algebraic polynomial

$$\bar{\psi}(f,y,\dot{y},\ddot{y},...,\overset{(r)}{y},u,\dot{u},...) = 0 \tag{15}$$

Where r is the maximum order of the output time derivatives.

Introducing the following change of coordinates

$$\eta_1 = y,\ \eta_2 = \dot{y},\ ...,\eta_r = \overset{(r-1)}{y} \tag{16}$$

we obtain the following representation of (15) which is the so-called Generalized Observability Canonical Form [16].

$$\begin{gathered} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = \eta_3 \\ \cdots \\ \dot{\eta}_r = \Phi(f,\eta_1,\eta_2,...\eta_r,u,\dot{u},..\overset{(r-1)}{u}) \\ y = \eta_1 \end{gathered} \tag{17}$$

Where $\Phi(\cdot)$ is considered as an unmodeled dynamics.

The observer structure. The following system is a sliding-mode observer for the system (17).

$$\begin{gathered} \dot{\hat{\eta}}_1 = \hat{\eta}_2 + m_1 sign(y-\hat{y}) \\ \cdots \\ \dot{\hat{\eta}}_{r-1} = \hat{\eta}_r + m_{r-1} sign(y-\hat{y}) \\ \dot{\hat{\eta}}_r = m_r sign(y-\hat{y}) \\ \text{with } \hat{y} = \hat{\eta}_1 \end{gathered} \tag{18}$$

where $m_j > 0$, $\forall\ 1 \leq j \leq r$, and

$$sign(y-\hat{y}) = \begin{cases} 1 & if\ (y-\hat{y}) > 0 \\ -1 & if\ (y-\hat{y}) < 0 \\ undefined & if\ (y-\hat{y}) = 0 \end{cases}.$$

Then returning to the original coordinates and taking into account (15), the fault can be estimated from the following relationship

$$\bar{\psi}(\hat{f},\hat{\eta},\dot{\hat{\eta}},\ddot{\hat{\eta}},...,\overset{(r)}{\hat{\eta}},u,\dot{u},...) = 0 \tag{19}$$

Observer Convergence Analysis.

We will analyze the convergence properties of the proposed observer considering the presence of a noise signal δ contaminating the output measurements, such that

$$y = \eta_1 + \delta. \tag{20}$$

Let us define the state estimation errors as

$$e_1 = \eta_1 - \hat{\eta}_1,\quad e_i = (\eta_i - \hat{\eta}_i)/m,\quad i = 2,...,r\ , \tag{21}$$

where $m > 0$, it follows that the estimation error vector $e = \begin{bmatrix} e_1 & ... & e_r \end{bmatrix}^T$ verifies the relationship

$$\dot{e} = A_{\bar{\mu}}e - Ksign(Ce+\delta) + \Delta s \tag{22}$$

where $\bar{\mu} > 0$ is a regularizing parameter, $A_{\bar{\mu}} = \begin{bmatrix} -\bar{\mu} & m & 0 & ... & 0 \\ 0 & -\bar{\mu} & m & & 0 \\ 0 & 0 & -\bar{\mu} & & \vdots \\ & & & \ddots & m \\ 0 & 0 & 0 & ... & -\bar{\mu} \end{bmatrix}$, $K = \begin{bmatrix} m_1 \\ m_2 \\ ... \\ m_r \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & ... & 0 \end{bmatrix}$ and $\Delta s = \begin{bmatrix} \bar{\mu}e_1 \\ ... \\ \bar{\mu}e_{r-1} \\ \Phi + \bar{\mu}e_r \end{bmatrix}$ is an uncertainty term.

Assumption A1. There exist nonnegative constants L_{0s}, L_{1s}, such that the following generalized quasi-Lipschitz condition holds

$$\|\Delta s\| \leq L_{0s} + (L_{1s} + \|A_{\bar{\mu}}\|)\|e\|. \tag{23}$$

Assumption A2. The additive output noise δ, is bounded, namely

$$|\delta| \leq \delta^+ < \infty, \tag{24}$$

Assumption A3. There exists a positive definite matrix $Q_0 = Q_0^T > 0$, such that the following matrix Riccati equation

$$PA_{\bar{\mu}} + A_{\bar{\mu}}^TP + PRP + Q = 0 \tag{25}$$

with $R := \Lambda_s^{-1} + 2\left\|\Lambda_s\right\| L_{1s} I,\ \Lambda_s = \Lambda_s^T > 0,$

$$Q = Q_0 + 2(L_{1s} + \|A_{\bar{\mu}}\|^2) I$$

has a positive definite solution $P = P^T > 0$.

Theorem 2: If assumptions from A1 to A3 are satisfied, then

$$[V - V^*]_+ \to 0 \tag{26}$$

where

$$V = V(e) = \|e\|_P^2 := e^T P e,$$

$$V^* := \frac{2\left\|\Lambda_s\right\| L_{0s}^2 + 4k\delta^+}{\lambda_{\text{mín}}(P^{-1/2} Q^T Q P^{-1/2})},$$

and the function $[\cdot]_+$ is defined as follows

$$[x]_+ = \begin{cases} x & if\ x \geq 0 \\ 0 & if\ x < 0 \end{cases}. \tag{27}$$

■

Remark 2: Theorem 2 states that the weighted estimation error norm $V(e)$ asymptotically converges to the zone bounded by V^*. In other words, it is ultimately bounded.

VI. Application to the three-tank system

VI-A. Description of the three-tank system

The Amira DTS200 is described in figure 1. The corresponding model with faults is given by the following equations [21]

$$\begin{aligned} \dot{x}_1 &= \frac{1}{A}(u_1 - q_{13} + f_1) \\ \dot{x}_2 &= \frac{1}{A}(u_2 + q_{32} - q_{20} + f_2) \\ \dot{x}_3 &= \frac{1}{A}(q_{13} - q_{32}) \end{aligned} \tag{28}$$

where $u_1 = q_1$ and $u_2 = q_2$ are the manipulable input flows, $x_i = h_i =$ level in the tank i. A is the transversal constant section of any of the identical tanks, and q_{ij} represents the water flow from tank i to tank j, $(1 \leq i, j \leq 3)$ which according to the generalized Torricelli's rule, valid for laminar flow

$$q_{ij} = a_i S\ sign(h_i - h_j)\sqrt{2g\left|h_i - h_j\right|} \tag{29}$$

with $q_{20} = a_2 S\ \sqrt{2gh_2}$. Where S is the transversal area of the pipe that interconnects the tanks (see figure 1) and a_i are the output flow coefficients, which are not exactly known, so they are considered as uncertain parameters. We assume the existence of actuator faults denoted by f_1 and f_2 $(\mu = 2)$, each one of these faults represents a variation in the respective pump driver gain, which can be originated by an electronic component malfunction, or even by a leakage or an obstruction in the pump pipes.

The system (28) has four state regions in which the corresponding model is differentiable [15], any of these regions can be chosen to do the analysis, just avoiding loss of differentiability by crossing from one to another. In this work $x_1 > x_3 > x_2 > 0$ is the only considered region of operation, which experimentally is easy to operate.

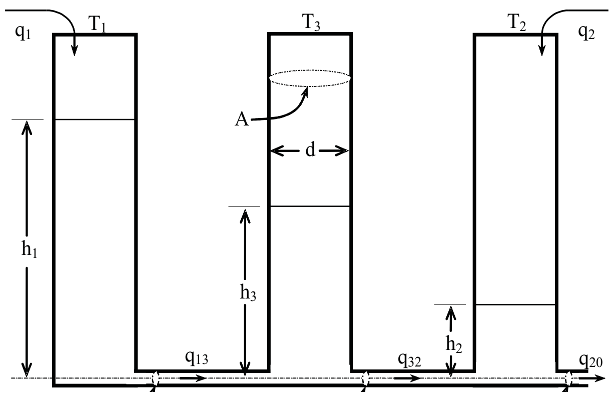

Fig. 1. Schematic diagram of the three-tank system.

VI-B. Diagnosability analysis

According to theorem 1 we need two or more measured outputs, this can only happen in the following cases:

- Case 0. $p = 3$ (h_1, h_2, and h_3 measurable)
- Case 1. $p = 2$ (h_1 not measurable, h_2, and h_3 measurable)
- Case 2. $p = 2$ (h_2 not measurable, h_1, and h_3 measurable)
- Case 3. $p = 2$ (h_3 not measurable, h_1, and h_2 measurable)

VI-B.1. Case 0: The simplest case (and the only one reported in previous works [15], with numerical results) takes place when we can measure the full state vector, that is to say, we have three outputs: $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3$; in this case, from (28) we have

$$f_1 = A\dot{y}_1 + a_1 S\sqrt{2g\left(y_1 - y_3\right)} - u_1 \tag{30}$$

$$f_2 = A\ \dot{y}_2 - a_3 S\sqrt{2g\left(y_3 - y_2\right)} + a_2 S\sqrt{2gy_2} - u_2 \tag{31}$$

System (28) is left invertible because the differential output rank is equal to 2. This means that faults f_1 and f_2 are diagnosable.

VI-B.2. Case 1: We consider only the outputs: $y_2 = x_2$ and $y_3 = x_3$. By taking into account (28) we have

$$A\dot{y}_3 = a_1 S\sqrt{2g\left(x_1 - y_3\right)} - a_3 S\sqrt{2g\left(y_3 - y_2\right)}, \tag{32}$$

we get

$$x_1 = y_3 + \frac{1}{2ga_1^2 S^2}\left(A\dot{y}_3 + a_3 S\sqrt{2g\left(y_3 - y_2\right)}\right)^2 \tag{33}$$

Then, by replacing x_1 in (30) we obtain a set of two differential equations with coefficients in $\mathbb{R}\left\langle u, y\right\rangle$ with two unknowns f_1 and f_2, this means system (28) is left invertible (i.e., faults f_1 and f_2 are diagnosable) with the two considered outputs.

VI-B.3. Case 2: We consider only the outputs: $y_1 = x_1$ and $y_3 = x_3$. By taking into account (32) we obtain

$$x_2 = y_3 - \frac{1}{2ga_3^2 S^2}\left(-A\ \dot{y}_3 + a_1 S\sqrt{2g\left(y_1 - y_3\right)}\right)^2. \tag{34}$$

From (31) in a similar way we can obtain system (28) is left invertible (i.e., faults f_1 and f_2 are diagnosable) with the two considered outputs.

VI-B.4. Case 3: We consider only the outputs: $y_1 = x_1$ and $y_2 = x_2$. By taking into account (30) we get

$$x_3 = y_1 - \frac{1}{2ga_1^2 S^2}\left(-A\ \dot{y}_1 + f_1 + u_1\right)^2. \qquad (35)$$

From (31) we only can obtain one differential equation involving the two faults, therefore, system (28) is not left invertible, i.e., faults f_1 and f_2 are not diagnosable with the two considered outputs.

VII. Experimental Results

We verified the real time performance of the proposed estimators in a laboratory setting of the Amira DTS200 system. The known parameter values for the utilized system are: $A = 0.0149\ m^2$, $S = 5 \times 10^{-5}\ m^2$ and the unknown parameters: a_1, a_2, and a_3. The sample time in all the experiments was $0.001\ s$, this was chosen so small in order to get the best performance from the sliding-mode observer. The experimental results are described as follows

VII-A. Identification results

With no presence of faults, the unknown parameters a_1, a_2, and a_3 were estimated meanwhile the values for the input flows were: $q_1 = 0.000025\ m^3/s$ and $q_2 = 0.000020 m^3/s$, along $1000\ s$ in these conditions the evolution of the estimated values for the unknown coefficients is shown in figure 2a.

At the end of the identification process the estimated values for the flow parameters were obtained:

$$a_1 = 0.418, a_2 = 0.789, a_3 = 0.435. \qquad (36)$$

In figure 2b the simulated and the measured actual levels are shown in order to give a visual comparison between the actual and the estimated model, the actual level measurements are drawn in a gray color, while the levels obtained by simulating the model using the estimated values given by (36) for the flow coefficients are shown in black color.

VII-B. Fault estimation results

In all the experiments described in this subsection the input flows were maintained constant as $q_1 = 0.00002\ m^3/s$ and $q_2 = 0.000015\ m^3/s$, also two faults were artificially generated through the following expressions: $f_1 = 0.00005\ \left[1 + \sin\left(0.2te^{-0.01t}\right)\right]\mathcal{U}(t-220)$, $f_2 = 0.00005\ \left[1 + \sin\left(0.05te^{-0.001t}\right)\right]\mathcal{U}(t-300)$, where $\mathcal{U}(t)$ is the unit step function.

As we do not know the dynamics Φ, we can take as a reference the Lipschitz constants of the fault signals, which are 10.6×10^{-7} and 11.25×10^{-7} respectively, then we choose L_{1s} bigger enough, for example $L_{1s} = 0.001$, in a similar way, we choose $m = 0.1$, $\bar{\mu} = 1$, $\Lambda_s = 20$, $Q_0 = I$, then $R = 0.09I$, $Q = 3.2122I$, with these parameters we obtain

$$P = \begin{bmatrix} 20.4009 & -1.2107 \\ -1.2107 & 20.5446 \end{bmatrix} > 0$$

The two proposed schemes for fault estimation were evaluated in case 1 (x_1 not measurable), the results are described as follows.

Only the two outputs $y_2 = x_2$ and $y_3 = x_3$ were taken into account, an estimation for the unknown state x_1 was necessary to be obtained. In figure 3 we show the resulting estimations achieved with the reduced-order observer. A low-pass filter was necessary in order to reduce the effect of the measurement noise, we chose a second-order Butterwort filter whose transfer function is given by $G_f(s) = 1/\left(32s^2 + 8s + 1\right)$. The gain values chosen for both fault observers were $k = 2$, and for the state observer x_1, $k_{x_1} = 0.3$. As we can observe the estimation results with this scheme are good (figure 3).

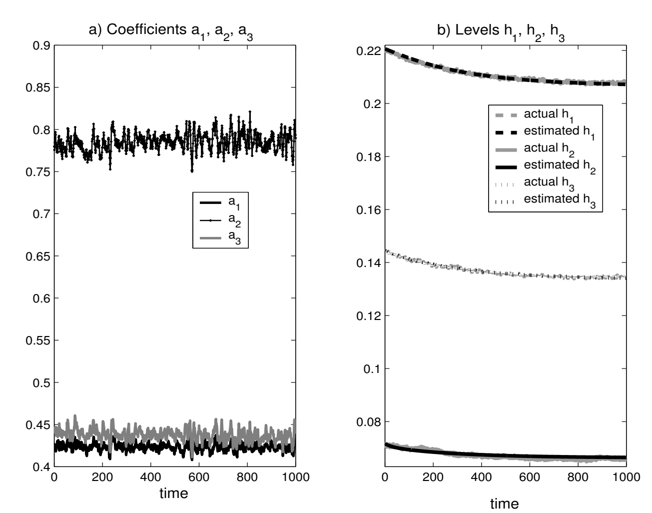

Fig. 2. a) Parameter identification. b) Validation of the estimated model.

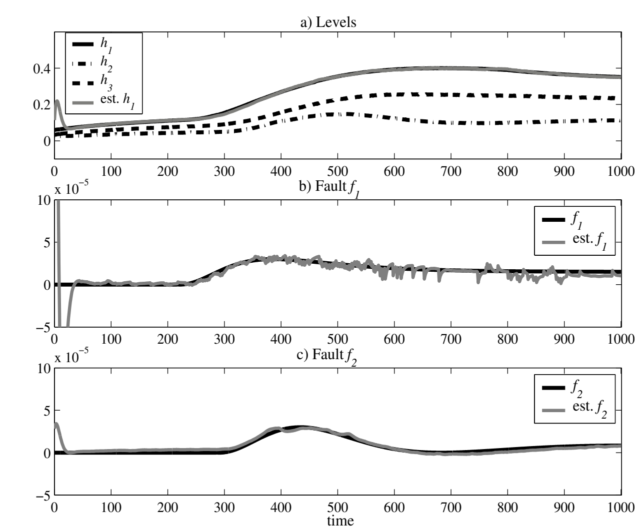

Fig. 3. Fault diagnosis for unknown h_1 using the reduced order observer.

A sliding-mode observer was also tested in this case. In figure 4 the corresponding results achieved with the sliding-mode observer are shown. It is worth to mention that with this observer it was not necessary to include the reducing noise filter providing the inherent robustness of this observer. The gain values chosen for the fault and state observers were $m_1 = 0.1$, $m_2 = 0.01$. As we can observe from figure 4, this scheme also provides good estimation results.

VIII. Concluding Remarks

We have tackled the fault diagnosis problem in nonlinear systems using the condition of left invertibility through the concept of differential output rank. The usefulness of theorem 1, theorem 2, and lemma 2 was shown, this allowed the estimation of two simultaneous faults with less measurements. The theoretical and simulation results were tested in a real-time implementation (three-tank system). The experimental results for the two observers showed similar performance, however the proposed sliding-mode observer is more robust against measurement noise, as it was expected.

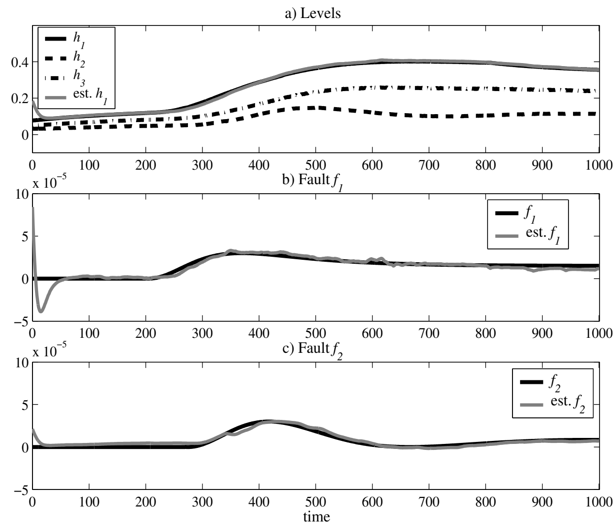

Fig. 4. Fault diagnosis for unknown h_1 using the sliding mode observer.

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