2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Induction Motor Control in Presence of Magnetic Saturation: Speed Regulation and Power Factor Correction

A. El Fadili, F. Giri, A. El Magri, L. Dugard, H. Ouadi.

Abstract—**Controlling induction motors have been given a great deal of interest. Generally, the control issue has been dealt with, neglecting the saturation effect of the magnetic characteristic and ignoring the presence of the AC/DC/AC converter. The originality of the present work is twofold: (i) the magnetic saturation effect is accounted for in the control design model; (ii) the induction motor is considered together with its AC/DC/AC converter. The control objectives for the association 'converter-machine' are: (i) forcing the motor speed to track a varying reference signal and optimizing the rotor flux reference, (ii) regulating the DC Link voltage, (iii) assuring a satisfactory power factor correction (PFC) with respect to the power supply net. A nonlinear multi-loop controller is designed using the backstepping technique and formally analyzed using Lyapunov stability and averaging theory. In addition to closed-loop stability, it is proved that all control objectives (motor speed tracking, DC link voltage regulation and unitary power factor) are asymptotically achieved, up to unavoidable, but small, harmonic errors (ripples).**

I. Introduction

When three-phase induction motors are involved, speed variation is performed through (three-phase) DC/AC inverters due to their high capability of performing flexible voltage and frequency variation. The inverters are generally powered by an AC supply net through (a transformer and) an AC/DC rectifier. The connection line between the rectifier and the inverter is called DC link. The control problem at hand is to design a controller ensuring a wide speed range regulation for the system including the AC/DC rectifier, the DC/AC inverter and the induction motor. The point is that such a system behaves (vis-à-vis to the AC supply grid) as a nonlinear load causing generation of undesirable current harmonics that reduce the rectifier efficiency, induce voltage distortion in the AC supply line and cause electromagnetic compatibility issues. To overcome this drawback, the control objective must consider not only the motor speed regulation but also the current harmonics rejection. The last feature is referred to as the power factor correction (PFC) [8]

Previous works on induction machine speed control simplified the control problem by: (i) ignoring the dynamics of the AC/DC rectifier (focusing thus only on the set 'DC/AC inverter - Motor'); (ii) neglecting the saturation effects in the motor magnetic circuit. The simplified control problem has been dealt with, using several control strategies ranging from simple techniques, e.g. field-oriented control [3], to more sophisticated nonlinear approaches, e.g. direct torque control [6]. Neglecting the AC/DC rectifier was coped with, supposing the DC link voltage to be perfectly regulated and considering the PFC requirement not to be an issue. Of course, this assumption is not necessarily satisfied. On the other hand, neglecting the magnetic saturation effects makes it impossible to consider state-dependent flux references. Specifically, the reference flux (in previous control solutions) is taken constant equal to its nominal value, [3], [6]. Obviously, the machine efficiency is then maximal only when it operates in the neighborhood of its nominal point. If the operation point is below the nominal value (small loads), the useless energy stored in stator inductances reduces the machine efficiency. On the other hand, if the operation point is above the nominal value, the (overloaded) machine operates in the saturation zone of its magnetic characteristic and the control performances are not ensured because the control model based upon is no longer representative of the machine. To overcome the above shortcomings in speed control, the flux reference must be dependent on both the speed reference and torque-load. But this requirement is only achievable if the nonlinear nature of the magnetic characteristic is accounted for in the model.

In the present work, a new control strategy is developed for the whole system including the 'AC/DC rectifier' and the association 'DC/AC inverter-induction motor'. The new strategy involves a nonlinear multi-loop controller obtained using a model that accounts for the saturation effect of the motor magnetic circuit [7]. It is designed using the Lyapunov and backstepping design techniques, bearing in mind three main control objectives, namely motor speed tracking of varying reference trajectories, rotor flux optimization and unitary power factor. Flux optimality consists in minimizing the absorbed stator current that is necessary to produce a given motor torque. It is formally proved that all control objectives are achieved with a good accuracy.

II. Modeling the 'AC/DC/AC Converter-Induction Motor' Association

The controlled system, illustrated by Fig 1, includes an AC/DC boost rectifier and a combination 'inverter-induction motor'. The inverter is a DC/AC converter operating, like the AC/DC rectifier, according to the known Pulse Wide Modulation (PWM) principle.

Manuscript received September 2, 2010.
F. Giri is with GREYC Lab, UMR CNRS, University of Caen, France.
A. El Fadili, A. El Magri H. Ouadi are with EMI, LAII, Rabat, Marocco.
L. Dugard is with GIPSA Lab, UMR CNRS, Grenoble-INP, France.
Corresponding author: A. El Fadili, elfadili_abderrahim@ yahoo.fr

A. AC/DC rectifier modeling

The power supply net is connected to an H-bridge converter which consists of four IGBT's with anti-parallel diodes for bidirectional power flow mode. This is expected to accomplish two main tasks: (i) providing a constant DC link voltage; (ii) providing an almost unitary power factor. Recall here the average model of (AC / DC) rectifier developed in [9]:

$$\dot{x}_1 = (v_e - u_1 x_2)/L_1\,,\ \dot{x}_2 = (u_1 x_1 - \bar{i}_s)/2C \tag{1a}$$

where:

$$x_1 = \bar{i}_e\,,\ x_2 = \bar{v}_{dc}\,,\ u_1 = \bar{s} \tag{1b}$$

are respectively the average values of i_e, v_{dc} and s, over cutting periods. The switch position s is a function taking values in the discrete set $\{-1, 1\}$. Specifically:

$$s = \begin{cases} 1 & \text{if} \quad S \text{ is } ON \text{ and } S' \text{ is } OFF \\ -1 & \text{if} \quad S \text{ is } OFF \text{ and } S' \text{ is } ON \end{cases} \tag{1c}$$

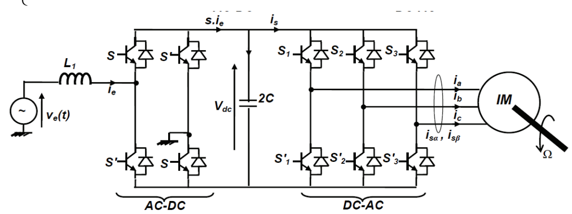

Fig.1. AC/DC/AC drive circuit with a three-level inverter

B. Inverter-Motor modeling

In [7], a new model was developed and experimentally validated for the considered induction motor. Its originality lies in the fact that it takes into account the saturation effect of the machine magnetic characteristics (fig 2). It is defined by the following state space representation:

$$J\dot{\Omega} = -f\Omega + p(\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - T_L \tag{3a}$$

$$\dot{i}_{s\alpha} = -a_2 i_{s\alpha} + \delta\,\phi_{r\alpha} + a_3 p\Omega\phi_{r\beta} + a_3 v_{s\alpha} \tag{3b}$$

$$\dot{i}_{s\beta} = -a_2 i_{s\beta} - a_3 p\Omega\phi_{r\alpha} + \delta\,\phi_{r\beta} + a_3 v_{s\beta} \tag{3c}$$

$$\dot{\phi}_{r\alpha} = a_1 i_{s\alpha} - L_{seq}\delta\,\phi_{r\alpha} - p\Omega\phi_{r\beta} \tag{3d}$$

$$\dot{\phi}_{r\beta} = a_1 i_{s\beta} - L_{seq}\delta\,\phi_{r\beta} + p\Omega\phi_{r\alpha} \tag{3e}$$

δ is a varying parameter that depends on the machine magnetic state (see Fig 2). In [7], this dependence was given a polynomial approximation, i.e.:

$$\delta = \Gamma(\Phi_r) = q_0 + q_1\Phi_r + \ldots + q_m\Phi_r^m \tag{3f}$$

The involved coefficients have been experimentally identified in [7] using Fig 3. Φ_r denotes the amplitude of the (instantaneous) rotor flux, denoted ϕ_r. Consequently:

$$\Phi_r = \sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2} \tag{3g}$$

where $\phi_{r\alpha}, \phi_{r\beta}$ denote the rotor flux $\alpha\beta$-components. $(i_{s\alpha}, i_{s\beta})$ are the αβ-components of the stator current, Ω represents the motor speed, R_s, R_r denote the stator and rotor resistances, T_L represents the load torque, J is the combined rotor and load inertia, p is the number of pole pairs, L_{seq} is the equivalent inductance (of both stator and rotor leakages) as this is seen from the stator,

$$a_1 = R_r\,,\ a_3 = (L_{seq})^{-1}\,,\ a_2 = a_3(R_s + R_r) \tag{4a}$$

The numerical values of the model parameters are those of [7] where the model is experimentally validated using an induction motor of 7.5 KW power.

In (3a-e), $v_{s\alpha}, v_{s\beta}$ denote the stator voltage components in αβ-coordinates (Park's transformation of the three-phase stator voltages). The inverter is featured by the fact that the stator α- and β-voltages can be controlled independently. To this end, these voltages are expressed in function of the corresponding control action

$$v_{s\beta} = v_{dc}\,u_3\,,\ v_{s\alpha} = v_{dc}\,u_2 \tag{4b}$$

where (u_2, u_3) represent the average α- and β-axes of the three-phase duty ratio system (s_1, s_2, s_3). The latter are obtained from (1c) replacing there (S, S') by (S_i, S_i') ($i = 1,2,3$). Now, let us introduce the state variables:

$$x_3 = \bar{\Omega}\,,\quad x_4 = \bar{i}_{s\alpha}\,,\quad x_5 = \bar{i}_{s\beta}\,, x_6 = \bar{\phi}_{r\alpha}\,,\ x_7 = \bar{\phi}_{r\beta} \tag{4c}$$

Using the power conservation principle, one gets:

$$\bar{i}_s = (u_2 x_4 + u_3 x_5) \tag{5}$$

Substituting (4a-c) and (5) in (3a-f), the state space equations obtained up to now are put together to get a state space model of the whole system including the AC/DC/AC converters combined with the induction motor. For convenience, the whole averaged model is rewritten here for future reference:

$$\dot{x}_1 = (v_e - u_1 x_2)/L_1 \tag{6a}$$

$$\dot{x}_2 = (u_1 x_1 - \bar{i}_s)/2C \tag{6b}$$

$$J\dot{x}_3 = -f x_3 + p(x_6 x_5 - x_7 x_4) - T_L \tag{6c}$$

$$\dot{x}_4 = -a_2 x_4 + \delta\, x_6 + a_3 p x_3 x_7 + a_3 u_2 x_2 \tag{6d}$$

$$\dot{x}_5 = -a_2 x_5 - a_3 p x_3 x_6 + \bar{\delta}\, x_7 + a_3 u_3 x_2 \tag{6e}$$

$$\dot{x}_6 = a_1 x_4 - L_{seq}\bar{\delta}\, x_6 - p x_3 x_7 \tag{6f}$$

$$\dot{x}_7 = a_1 x_5 - L_{seq}\bar{\delta}\, x_7 + p x_3 x_6 \tag{6g}$$

$$\bar{\Phi}_r = \sqrt{(x_6{}^2 + x_7{}^2)} \tag{6h}$$

$$\bar{\delta} = \Gamma(\bar{\Phi}_r) = q_0 + q_1\bar{\Phi}_r + \ldots + q_m\bar{\Phi}_r^m \tag{6j}$$

III. CONTROLLER DESIGN

A. Control objectives

Two operational control objectives are looked for:

(i) Speed regulation with optimization of the rotor flux norm reference: the machine speed Ω must track a varying reference Ω_{ref} as closely as possible while the rotor flux norm reference must be online adjusted so that the stator current consumption is minimized by acting on.

(ii) PFC requirement: the rectifier input current i_e must be sinusoidal and in phase with the AC supply voltage v_e.

As three control inputs are at hand (namely u_1, u_2 and u_3) two additional control objectives will be pursued:

(iii) Controlling the continuous voltage v_{dc}, making it track a given reference signal v_{dcref}. This reference is generally set to a constant value equal to the nominal voltage entering the inverter.

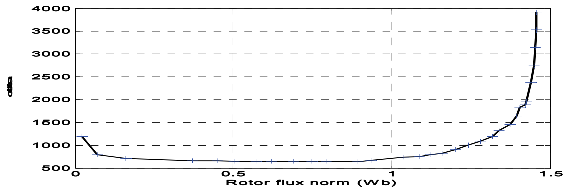

Fig.2. Characteristic (δ, Φ_r): directly computed points (++) and polynomial interpolation (solid). Unities: $\delta(\Omega H^{-2})$, Φ_r (Wb)

B. AC/DC rectifier control design

1) Controlling rectifier input current to meet PFC

The PFC objective means that the average input current rectifier must be sinusoidal and in phase with the AC supply voltage. This amounts to enforce the current x_1 to track a reference signal x_1^* of the form:

$$x_1^* = k\,v_e \tag{7}$$

At this point k is any real parameter that is allowed to be time-varying. Introduce the current tracking error:

$$z_1 = x_1 - x_1^* \tag{8}$$

In view of (6a), the above error undergoes the following equation:

$$\dot{z}_1 = (v_e - u_1\,x_2)/L_1 - \dot{x}_1^* \tag{9}$$

To get a stabilizing control law for this first-order system, consider the quadratic Lyapunov function $V_1 = 0.5z_1^2$. It can be easily checked that the time-derivative $\dot{V}_1$ is a negative definite function of z_1 if the control input is chosen to be:

$$u_1 = L_1\left(c_1\,z_1 + (v_e/L_1) - \dot{x}_1^*\right)/x_2 \tag{10}$$

where $c_1 > 0$ is a design parameter. The properties of such a control law are summarized in the following proposition.

Proposition 1. *Consider the system, next called current (or inner) loop, composed of the current equation (6a) and the control law (10) where $c_1 > 0$ is arbitrarily chosen by the user. If the reference $x_1^* = k\,v_e$ and its first time derivative are available, then one has the following properties:*

a) The current loop satisfies the equation $\dot{z}_1 = -c_1\,z_1$ which is globally exponentially stable i.e. z_1 exponentially vanishes, whatever the initial conditions.

b) If in addition k converges (to a finite value), then the PFC requirement is asymptotically fulfilled i.e. the (average) input current x_1 tends (exponentially fast) to its reference $k\,v_e$ as $t \to \infty$ □

2) DC link voltage regulation

The aim is now to design a tuning law for the ratio k in (7) so that the rectifier output voltage $x_2 = \bar{v}_{dc}$ is steered to a given reference value v_{dcref}. As mentioned above, v_{dcref} is generally (but not mandatory) chosen to be the constant nominal inverter input voltage amplitude (i.e. the nominal stator voltage).

a) Relationship between k and x_2

The first step in designing such a loop is to establish the relation between the ratio k (control input) and the output voltage x_2. This is the subject of the following proposition.

Proposition 2. *Consider the power rectifier described by (6a-b) together with the control law (10). Under the same assumptions as in Proposition 1, one has the following properties:*

1) The output voltage x_2 varies, in response to the tuning ratio k, according to the equation:

$$\dot{x}_2 = (k\,v_e^2 + z_1 v_e)/2C\,x_2 - (u_2 x_4 + u_3 x_5)/2C \tag{11}$$

2) The squared voltage ($y = x_2^2$) varies, in response to the tuning ratio k, according to the equation:

$$\dot{y} = k\,v_e^2/C + z_1 v_e/C + \chi(x,t) \tag{12}$$

with

$$\chi(x,t) = -x_2(u_2 x_4 + u_3 x_5)/C \tag{13}$$ ■

b) Squared DC-link voltage regulation.

The ratio k appears as a control signal in the system defined by (12). As said before, the reference signal $y_{ref} \overset{def}{=} v_{dcref}^2$ (of the squared DC-link voltage $x_2 = \bar{v}_{dc}$) is chosen to be constant and is given the nominal inverter input voltage value. Then, it follows from (12) that the tracking error $z_2 = y - y_{ref}$ undergoes the following equation:

$$\dot{z}_2 = E^2 k/C + E^2 k\cos(2\omega_e t)/C + \sqrt{2}\,E z_1\cos(\omega_e t)/C + \chi(x,t) - \dot{y}_{ref} \tag{15}$$

where we have used the fact that $v_e = \sqrt{2}.E.\cos(\omega_e t)$ and $v_e^2 = E^2(1+\cos(2\omega_e t))$. To get a stabilizing control law for the system (15), consider the following Lyapunov function:

$$V_2 = 0.5z_2^2 \tag{16}$$

It is easily checked that the time-derivative $\dot{V}_2$ can be made negative definite in the state z_2 by letting:

$$k\,E^2 + k\,E^2\cos(2\omega_e t) + \sqrt{2}\,E\,z_1\cos(\omega_e t) = C\left(-c_2 z_2 - \chi(x,t)\right) + C\,\dot{y}_{ref} \tag{17}$$

where $c_2 > 0$ is a design parameter. Bearing in mind the fact that the first derivative of the control ratio k must be available (Proposition 1), one suggests, as a tuning law for such a ratio, the following filtered version of the above solution:

$$\dot{k} + d\,k = d\,C\left(-c_2 z_2 - \chi(x,t)\right)/E^2 + dC\dot{y}_{ref}/E^2 \tag{18}$$

At this point, the regulator parameters (d, c_2) are any positive real constants. The way these parameters should be chosen will be made clear later (see Theorem 1). For now, let us describe the two control loops we have designed.

Proposition 3. *Consider the control system consisting of the AC/DC rectifier described by (6a-b) together with the control laws (10) and (18). Using Proposition 1 (Part 1), it follows that the resulting closed-loop undergoes, in the* (z_1, z_2, k)*-coordinates, the following equation where* $z_2 = y - y_{ref}$, $b_0 = E^2/C$*; and* $b_1 = dC/E$ *:*

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{k} \end{pmatrix} = \begin{pmatrix} -c_1 & 0 & 0 \\ 0 & 0 & b_0 \\ 0 & -b_1c_2 & -d \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ -b_1 \end{pmatrix} \chi(x,t)$$
$$+ \begin{pmatrix} 0 \\ E^2 k \cos(2\omega_e t)/C + \sqrt{2}E\, z_1 \cos(\omega_e t)/C \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ -b_1 \end{pmatrix} \dot{y}_{ref} \qquad (19)$$

C. Motor speed and rotor flux norm regulation

a) Optimal flux reference generation

A key feature of the model (6a-j) is that it accounts for the the saturation feature of the motor magnetic characteristic. This feature makes possible the generation of optimal flux references. Presently, optimality is intended in the sense of minimizing the stator current necessary to produce a given load torque. In [1], it was shown that the optimal flux reference Φ_{ref} is a function of the stator current. This function was also given a polynomial representation of the form:

$$\Phi_{ref} = \xi(\bar{I}_s) = h_0 + h_1\bar{I}_s + h_2\bar{I}_s^2 + \ldots + h_n\bar{I}_s^n \qquad (20)$$

with

$$\bar{I}_s{}^2 = x_4{}^2 + x_5{}^2 \qquad (21)$$

b) Speed and flux controller design and analysis

A regulator will now be designed to make the rotor speed and flux norm follow their references Ω_{ref} and $\Phi_{ref} = \xi(\bar{I}_s)$. The signal Ω_{ref} is any bounded and derivable function of time and its two first derivatives are available and bounded. These properties can always be achieved by filtering the reference through a second-order linear filter. The regulator design is performed in two steps using the backstepping technique [5]. First, introduce the tracking errors:

$$z_3 = \Omega_{ref} - x_3 \qquad (22)$$

$$z_4 = \Phi_{\text{ref}}^2 - (x_6{}^2 + x_7{}^2) \qquad (23)$$

Step 1. It follows from (6c) and (6f-g) that the errors z_3 and z_4 undergo the following differential equations:

$$\dot{z}_3 = \dot{\Omega}_{ref} - p(x_6x_5 - x_7x_4)/J + T_{L/J} + fx_3/J \qquad (24)$$

$$\dot{z}_4 = 2\Phi_{\text{ref}}\dot{\Phi}_{\text{ref}} - 2a_1(x_6x_4 + x_7x_5) + 2L_{seq}\,\bar{\delta}\,(\Phi_{\text{ref}}^2 - z_4) \qquad (25)$$

In (24)-(25), the quantities $p(x_6x_5 - x_7x_4)/J$ and $2a_1(x_6x_4 + x_7x_5)$ stand up as virtual control signals. If these were the actual control signals, the error system (24)-(25) could be globally asymptotically stabilized letting $p(x_6x_5 - x_7x_4)/J = \mu_1$ and $2a_1(x_6x_4 + x_7x_5) = \nu_1$ with:

$$\mu_1 \overset{def}{=} c_3z_3 + \dot{\Omega}_{ref} + T_L/J + f(\Omega_{ref} - z_3)/J \qquad (26)$$

$$\nu_1 \overset{def}{=} c_4z_4 + 2\Phi_{\text{ref}}\dot{\Phi}_{\text{ref}} + 2L_{seq}\,\bar{\delta}\,(\Phi_{\text{ref}}^2 - z_4) \qquad (27)$$

where c_3 and c_4 are any positive design parameters. As the quantities $p(x_6x_5 - x_7x_4)/J$ and $2a_1(x_6x_4 + x_7x_5)$ are not the actual control signals, they cannot be let equal to μ_1 and ν_1, respectively. Nevertheless, the expressions of μ_1 and ν_1 are retained as first stabilizing functions. Introduce the errors:

$$z_5 = \mu_1 - p(x_6x_5 - x_7x_4)/J \qquad (30)$$

$$z_6 = \nu_1 - 2a_1(x_6x_4 + x_7x_5) \qquad (31)$$

Then, using the notations (26) to (31), the dynamics of z_3 and z_4, can be rewritten as follows:

$$\dot{z}_3 = -c_3z_3 + z_5 \qquad (32)$$

$$\dot{z}_4 = -c_4z_4 + z_6 \qquad (33)$$

Similarly, the time-derivative of V_3 can be expressed, in function of the new errors, as follows:

$$\dot{V}_3 = -c_3z_3^2 - c_4z_4^2 + z_3z_5 + z_4z_6 \qquad (34)$$

Step 2. The second design step consists in choosing the actual control signals, u_2 and u_3, so that the system with states (z_3, z_4, z_5, z_6) be asymptotically stable. It follows from (30):

$$\dot{z}_5 = \dot{\mu}_1 - p(\dot{x}_6x_5 + x_6\dot{x}_5 - \dot{x}_7x_4 - x_7\dot{x}_4)/J \qquad (35)$$

Using (6c-g) and (26), (35) gives:

$$\dot{z}_5 = \mu_2 + pa_3x_2(x_7u_2 - x_6u_3)/J \qquad (36)$$

with

$$\begin{aligned} \mu_2 &= c_3(-c_3z_3 + z_5) + \ddot{\Omega}_{ref} + \dot{T}_L/J \\ &\quad + p(f/J + L_{seq}\bar{\delta} + a_2)(x_6x_5 - x_7x_4)/J \\ &\quad + p^2a_3x_3(x_6{}^2 + x_7{}^2)/J - (fT_L)/J^2 - f^2x_3/J^2 \\ &\quad + p^2x_3(x_7x_5 + x_6x_4)/J \end{aligned} \qquad (37)$$

Similarly, it follows from (31) that z_6 undergoes the following differential equation:

$$\dot{z}_6 = \dot{\nu}_1 - 2a_1(\dot{x}_6x_4 + x_6\dot{x}_4 + \dot{x}_7x_5 + x_7\dot{x}_5) \qquad (38)$$

Using (6c-g) and (27), it follows from (38):

$$\dot{z}_6 = \nu_2 - 2a_1a_3x_2(x_6u_2 + x_7u_3) \qquad (39)$$

with

$$\begin{aligned} \nu_2 &= (c_4 - 2L_{seq}\,\bar{\delta})(-c_4z_4 + z_6) + 2\Phi_{ref}\ddot{\Phi}_{ref} - 2(a_1)^2\bar{I}_s{}^2 \\ &\quad + 2\dot{\Phi}_{ref}^2 + 4L_{seq}\,\bar{\delta}\,\Phi_{ref}\dot{\Phi}_{ref} + 2(L_{seq}\,\dot{\bar{\delta}} - a_1\bar{\delta})\overline{\Phi}_r^2 \end{aligned}$$

$$+2a_1a_3p(x_7x_4-x_5x_6)+2a_1(L_{seq}\bar{\delta}+a_2)(x_4x_6+x_5x_7) \quad (40)$$

where the derivative of $\bar{\delta}$ is obtained from (6i):

$$\dot{\bar{\delta}}=\frac{d\bar{\delta}}{d\bar{\Phi}_r}\frac{d\bar{\Phi}_r}{dt}=\frac{d\bar{\delta}}{d\bar{\Phi}_r}\left(\frac{x_6}{\bar{\Phi}_r}\dot{x}_6+\frac{x_7}{\bar{\Phi}_r}\dot{x}_7\right) \quad (41)$$

The derivatives $\dot{\Phi}_{ref}$ and $\ddot{\Phi}_{ref}$ are obtained using (20) and (21). To analyze the error system, composed of equations (32-33), (36) and (39), let us consider the following augmented Lyapunov function candidate:

$$V_4=V_3+0.5(z_5^2+z_6^2) \quad (42)$$

Using (34), (36) and (39), its time-derivative along the trajectory of the state vector (z_3, z_4, z_5, z_6) is:

$$\dot{V}_4=-c_3z_3^2-c_4z_4^2+z_5\left(z_3+\mu_2+pa_3x_2(x_7u_2-x_6u_3)/J\right)$$
$$+z_6\left(z_4+\nu_2-2a_1x_2a_3(x_7u_3+x_6u_2)\right) \quad (43)$$

Adding $c_5z_5^2-c_5z_5^2+c_6z_6^2-c_6z_6^2$ to the right side of (43) and rearranging terms, yield:

$$\dot{V}_4=-c_3z_3^2-c_4z_4^2-c_5z_5^2-c_6z_6^2$$
$$+z_5(z_3+\mu_2+c_5z_5+pa_3x_2(x_7u_2-x_6u_3)/J)$$
$$+z_6\left(z_4+\nu_2+c_6z_6-2a_1x_2a_3(x_7u_3+x_6u_2)\right) \quad (44)$$

where c_5 and c_6 are new arbitrary positive real design parameters. Equation (44) suggests that the control signals u_2, u_3 must set to zero the two quantities between curly brackets (on the right side of (44)). Letting these quantities equal to zero and solving the resulting second-order linear equation system with respect to (u_2, u_3), give the following control law:

$$\begin{bmatrix}u_2\\u_3\end{bmatrix}=\begin{bmatrix}\lambda_0 & \lambda_1\\ \lambda_2 & \lambda_3\end{bmatrix}^{-1}\begin{bmatrix}-z_3-c_5z_5-\mu_2\\-z_4-c_6z_6-\nu_2\end{bmatrix} \quad (45)$$

with:

$$\lambda_0=pa_3x_7x_2/J,\ \lambda_3=-2a_1x_7x_2,\ \lambda_2=-2a_1x_6x_2,$$
$$\lambda_1=-pa_3x_6x_2/J, \quad (46)$$

Note that the determinant D of the matrix on the right side of (45) never vanishes in practice due to the machine remnant flux. $D=\lambda_0\lambda_3-\lambda_1\lambda_2=-2pa_1a_3x_2{}^2(x_6{}^2+x_7{}^2)/J$.

Substituting the control law (45) to (u_2, u_3) on the right side of (44) yields: $\dot{V}_4=-c_3z_3^2-c_4z_4^2-c_5z_5^2-c_6z_6^2$ (47)

As this is a negative definite function of the state vector (z_3, z_4, z_5, z_6), the closed-loop system is globally asymptotically stable [4].

The result thus established is more precisely formulated in the Theorem 1, where the following notations are used:

$$Z_1=[z_1\quad z_2\quad k]^T;\ Z_2=[z_3\quad z_4\quad z_5\quad z_6]^T\ Z=[Z_1\quad Z_2]^T \quad (48a)$$

$$b_2=\sqrt{2}EC^{-1};\ b_3=dE^{-2};\ \varepsilon=\omega_e{}^{-1} \quad (48b)$$

$$A=\begin{pmatrix}A_1 & O_{(3,4)}\\ O_{(4,3)} & A_2\end{pmatrix}\in IR^{7\times 7} \quad (48c)$$

with

$$A_1=\begin{pmatrix}-c_1 & 0 & 0\\ 0 & 0 & b_0\\ 0 & -b_1 & -d\end{pmatrix}\ A_2=\begin{pmatrix}-c_3 & 0 & 1 & 0\\ 0 & -c_4 & 0 & 1\\ -1 & 0 & -c_5 & 0\\ 0 & -1 & 0 & -c_6\end{pmatrix} \quad (48d)$$

$$f(Z,t)=\left[0\quad \left(b_0k\cos(2\omega_et)+b_2z_1\cos(\omega_et)\right)\quad 0\quad 0_4{}^T\right]\in IR^7 \quad (48e)$$

$$g=\left[0\quad -C^{-1}\quad b_3\quad 0_4^T\right]^T\in IR^7 \quad (48f)$$

$$h=\left[0\quad 1\quad -d\,CE^{-2}\quad 0_4{}^T\right]^T\in IR^7 \quad (48g)$$

where 0_4 denotes the null vector of IR^4

$$\rho(Z_2,t)=\frac{j}{p^2a_3\bar{\Phi}_r{}^2}J^2(z_5-\mu_1)-\frac{l}{(2a_1)^2\bar{\Phi}_r{}^2}(\nu_1-z_6) \quad (48h)$$

$$j=-(z_3+c_5z_5+c_3(-c_3z_3+z_5)+\ddot{\Omega}_{ref}+\dot{T}_L/J-fT_L/J^2$$
$$+(f/J+L_{seq}\bar{\delta}+a_2)(\mu_1-z_5)-f^2(\Omega_{ref}-z_3)/J^2$$
$$+p^2a_3(\Omega_{ref}-z_3)(\Phi_{ref}{}^2-z_4)/J)+p^2(\Omega_{ref}-z_3)(\nu_1-z_6)/2a_1J \quad (48i)$$

$$l=-(z_4+c_6z_6+(c_4-2L_{seq}\bar{\delta})(-c_4z_4+z_6)+4L_{seq}\bar{\delta}\,\Phi_{ref}\dot{\Phi}_{ref}+2\dot{\Phi}_{ref}^2$$
$$+(L_{seq}\bar{\delta}+a_2)(\nu_1-z_6)+2(L_{seq}\dot{\bar{\delta}}-a_1\bar{\delta})(\Phi_{ref}^2-z_4)+2\Phi_{ref}\ddot{\Phi}_{ref}$$
$$-2(a_1)^2(J^2(z_5-\mu_1)^2/p^2+(\nu_1-z_6)^2/(2a_1)^2)/\bar{\Phi}_r^2+2a_1a_3J(z_5-\mu_1) \quad (48j)$$

Theorem 1 *(main result)**.** Consider the system including the AC/DC/AC power converters and the induction motor connected in tandem, as shown in Fig.1. For control design purpose, the system has been represented by its average model (6a-j). Consider the controller defined by the control laws (10), (18) and (45) where all design parameters, namely $c_1, c_2, c_3, c_4, c_5, c_6$ and d are positive. Then, one has the following results:*

1) The resulting closed-loop system satisfies the state-space equation:

$$\dot{Z}=AZ+f(Z,t)+g\,\rho(Z_2,t)+h\,\dot{y}_{ref} \quad (49)$$

2) Let v_{dcref} and Ω_{ref} be either constant or periodic signals, with period $N\pi/\omega_e$ (for some positive integer N), and suppose that they are time derivable (up to second order for Ω_{ref}) with bounded derivatives. Then, there exists a positive real ε^* such that, if $0<\varepsilon<\varepsilon^*$ then:

a) The tracking error $z_2=y-y_{ref}$ and the tuning parameter k are harmonic signals whose amplitudes are continuously depending on ε.

b) Furthermore, one has:

(i) $\lim_{\varepsilon\to 0} z_2(t,\varepsilon)=0$; *(ii)* $\lim_{\varepsilon\to 0} k(t,\varepsilon)=\bar{\rho}(0_4)/b_0C$ (50)

where $\bar{\rho}(0_4)$ denotes the mean value of the periodic time function $\rho(0_4,t)$ □

IV. SIMULATION

The simulated experimental set-up has the following characteristics: Supply network: 220V/50Hz, AC/DC/AC converters: $L_1=15mH$; $C=1.5mF$; Induction motor characteristics: nominal power . Controller design parameters: $c_1 = 1000$, $c_2 = 30$, $c_3 = 100$, $c_4 = 400$, $c_5 = 500$, $c_6 = 1000$, $d = 100$.

The load torque T_L is a step-like function set first to 0 Nm, then, it steps to 10 Nm at t = 6s, to 20 Nm at t = 8s , to 30 Nm at t = 10s, to 40 Nm at t = 12s, to 50 Nm at t = 14s and to -20 Nm at t = 16s.

The controller performances are illustrated by Figs 5 to 8. Fig. 5 shows that the DC-link voltage $x_2 = \bar{v}_{dc}$ is well regulated and quickly settles down after each change in the speed reference or load torque. The resulting input current i_e is illustrated by Fig 8. It is seen that the current amplitude changes whenever the speed reference or the load torque vary. But the current frequency is sensitive to these changes. Specifically, the current remains (almost) all time in phase with the supply net voltage complying with the PFC requirement. Figs 6 and 7 show that the motor speed and the rotor flux norm do perfectly converge to their respective references. The tracking quality is quite satisfactory for both controlled variables (Ω, Φ_r).

V. CONCLUSION

This paper has addressed the problem of controlling associations including an AC/DC rectifier, a DC/AC inverter and an induction motor. Unlike most previous works, the motor magnetic characteristic is let to be what it is i.e. a saturating curve. The system dynamics have been described by the averaged 7th order nonlinear state-space model (6a-j). It was formally established that the proposed controller achieves the objectives for which it was designed: (i) almost unitary power factor; (ii) tight DC-link voltage regulation; (iii) satisfactory rotor speed reference tracking and rotor flux norm regulation, over a wide range of load torque variation. In all operation conditions, the proposed SDOF controller leads to a smaller absorbed stator current, compared to that produced by a constant flux controller (see Fig 9). These results are confirmed by many simulations.

REFERENCES

[1] Elfadili A., Giri F., Ouadi H., Dugard L., and El Magri A. "Induction Machine Control in Presence of Magnetic Saturation. Speed Regulation with Optimized Flux Reference". European Control Conference, August 23-26, 2009, Budapest, Hungary.

[2] Sira.H and R. Silva. 'Control design techniques in power electronics devices'. Springer. (2006).

[3] Heinemann G. and W. Leonhard 'Self-tuning field orientated control of an induction motor drive', Proceedings of international power electroniques conference April 2-6, 1990 Shinjuku Tokyo, Japan.

[4] Khalil H. 'Nonlinear systems'. Prentice Hall. (2003).

[5] Krstic M., I. Kanellakopoulos, and P. Kokotovic. 'Nonlinear and adaptive control design'. John Wilay & Sons, Inc, 1995.

[6] Nuno M. Silva1, P. António Martins, S. Adriano. Carvalho, 'Torque and speed modes simulation of a DTC controlled induction motor' Proceedings of the 10th Mediterranean Conference on Control and Automation - MED2002 Lisbon, Portugal, July 9-12, (2002)Y. Yorozu,

[7] Ouadi. H, F. Giri, and L. Dugard. "Modeling saturated induction motors". IEEE Conference on Control Applications (CCA'04), Taipei, Taiwan. 2004 Vol.1, pp. 75 – 80.

[8] Singh B., G. Bhuvaneswari, and V. Garg. 'Improved Power Quality AC-DC Converter for Electric Multiple Units in Electric Traction'. Power India Conference, pp. 6, 2006.

[9] Elfadili A., Giri F. El Magri A, Ouadi H., Dugard L., and A. Aboulouafa. "“Induction Motor Control through AC/DC/AC Converters Formal analysis of speed regulation performances and power factor correction” IFAC American Control Conference -- ACC2010 Baltimore, Maryland, USA

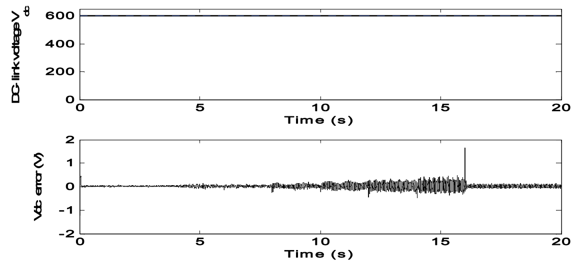

Fig.5. DC-link voltage v_{dc} response Upper: reference and measured DC-link voltage; lower: error control

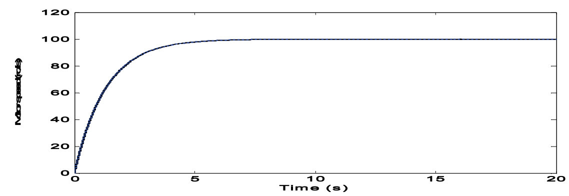

Fig.6. Dotted: Speed reference. Solid: Identical speed responses obtained by the state-dependent optimized flux (SDOF) controller and the constant flux controller.

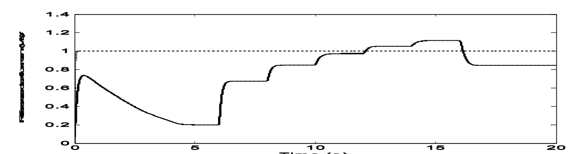

Fig.7. Rotor flux norm reference (Wb) (solid: state-dependent optimized flux, dotted: constant flux)

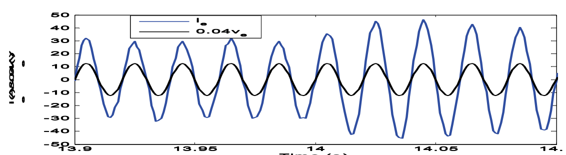

Fig.8. Unitary power factor checking in presence of a varying speed reference and load torque

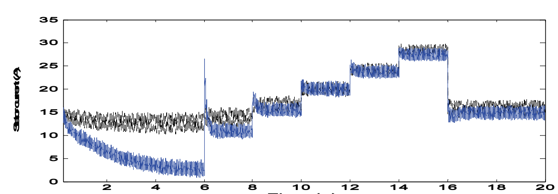

Fig 9: Absorbed stator current (A) (solid: SDOF controller, dotted: constant flux controller).