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Sliding Mode Optimal Regulator for a Bolza-Meyer Criterion with Non-Quadratic State Energy Terms

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Abstract— **This paper presents the solution to the optimal control problem for a linear system with respect to a Bolza-Meyer criterion with non-quadratic state energy terms. A distinctive feature of the obtained result is that a part of the optimally controlled state trajectory occurs to be in a sliding mode, i.e., represents an enforced motion along a certain manifold. The optimal solution is obtained as a sliding mode control, whereas the conventional linear feedback control fails to provide a causal solution. Performance of the obtained optimal controller is verified in the illustrative example against the conventional LQ regulator that is optimal for the quadratic Bolza-Meyer criterion. The simulation results confirm an advantage in favor of the designed sliding mode control.**

I. Introduction

Since the sliding mode control was invented in the beginning of 1970s (see a historical review in [1]), it has been applied to solve several classes of problems. For instance, the sliding mode control methodology has been used in stabilization [2], [3], tracking [4], observer design [5], identification [6], frequency domain analysis [7], and other control problems. Promising modifications of the original sliding mode concept, such as integral sliding mode [8] and higher order sliding modes [9], [10], have been developed. Application of the sliding mode method is extended even to stochastic systems [11], [13] and stochastic filtering problems [16], [17]. However, although it is possible to design a sliding manifold so that an infinite-horizon quadratic cost functional including the system state only is minimized [1], it seems, to the best of authors' knowledge, that no sliding mode algorithms, solving the optimal control problem for a Bolza-Meyer criterion with the quadratic control term [18], [19], have been designed. Meanwhile, simply the fact that the sliding mode control has a transparent physical sense [1] and is successfully applied to many technical problems [20] leads to a conjecture that the optimal control problems whose solution is given by a sliding mode control should exist. One of those optimal control problems is considered in this paper.

This paper presents the solution to the optimal control problem for a linear system with a Bolza-Meyer criterion, where the control energy term is quadratic and the state energy terms are of the first degree. That type of criteria would be useful in the joint control and parameter identification problems where the objective should be reached for a finite time. It is shown that optimal solution is given by a causal sliding mode control, whereas the conventional linear feedback control does not lead to a causal solution and, therefore, fails. A distinctive feature of the obtained result is that a part of the optimally controlled state trajectory occurs to be in a sliding mode, i.e., represents an enforced motion along a certain, *a priori* determined manifold. The theoretical result is complemented with an illustrative example verifying performance of the designed control algorithm. The optimal sliding mode regulator is compared to the conventional LQR corresponding to the quadratic Bolza-Meyer criterion. The simulation results confirm an advantage in favor of the designed sliding mode control.

The paper is organized as follows. Section 2 states the optimal control problem for a linear system with a non-quadratic Bolza-Meyer criterion. The sliding mode solution to the optimal control problem is given in Section 3. The proof of the obtained results is given in Appendix. Section 4 contains an illustrative example.

II. Optimal Control Problem Statement

Consider a conventional linear time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0, \tag{1}$$

where $x(t) \in R^n$ is the system state and $u(t) \in R^m$ is the control input. The coefficients $A(t)$ and $B(t)$ are considered continuous functions of time. Without loss of generality, the system (1) (pair (A,B)) is assumed to be controllable, i.e, the uncontrollable state components are removed from the consideration.

In the classical linear optimal control problem [18], [19], the criterion to be minimized is defined as a quadratic Bolza-Meyer functional:

$$J_2 = \frac{1}{2}[x(T)]^T \psi [x(T)] + \frac{1}{2}\int_{t_0}^{T} u^T(t)R(t)u(t)dt + \frac{1}{2}\int_{t_0}^{T} x^T(t)L(t)x(t)dt, \tag{2}$$

where $R(t)$ is positive and ψ, $L(t)$ are nonnegative definite continuous symmetric matrix functions, and $T > t_0$ is a certain time moment. The solution to this problem is well-known [18], [19] and considered fundamental for the optimal linear systems theory.

In this paper, the criterion to be minimized includes non-quadratic integral and terminal state energy terms and is

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defined as follows:

$$J_1 = \sum_{i=1}^{n} \psi_{ii} \mid x_i(T) \mid +$$

$$\frac{1}{2}\int_{t_0}^{T} u^T(t)R(t)u(t)dt + \int_{t_0}^{T}\sum_{i=1}^{n} L_{ii}(t) \mid x_i(T) \mid dt, \qquad (3)$$

where $R(t)$ is positive and $L(t)$, ψ are nonnegative definite continuous diagonal matrix functions, and $\mid x \mid = [\mid x_1 \mid, \ldots, \mid x_n \mid]$ is the vector of the absolute values of the component x_j, $j = 1, \ldots, n$ of the vector $x \in R^n$.

The optimal control problem is to find the control $u^*(t)$, $t \in [t_0, T]$, that minimizes the criterion J_1 in (3) along the trajectory $x^*(t)$, $t \in [t_0, T]$, generated upon substituting $u^*(t)$ into the state equation (1).

A solution to the stated optimal control problem is given in the next section and then proved in Appendix. As demonstrated, the obtained solution is a sliding mode control that is optimal with respect to the criterion (3).

III. Optimal Control Problem Solution

The solution to the optimal control problem for the linear system (1) and the criterion (3) is given as follows. The optimal control law takes the sliding mode control form

$$u^*(t) = R^{-1}(t)B^T(t)Q(t)Sign[x(t)], \qquad (4)$$

where the Signum function of a vector $x = [x_1, \ldots, x_n] \in R^n$ is defined as $Sign[x] = [sign(x_1), \ldots, sign(x_n)] \in R^n$, and the signum function of a scalar x is defined as $sign(x) = 1$, if $x > 0$, $sign(x) = 0$, if $x = 0$, and $sign(x) = -1$, if $x < 0$.

The matrix function $Q(t)$ satisfies the matrix equation with time-varying coefficients

$$\dot{Q}(t) = L(t) - A^T(t)Q(t). \qquad (5)$$

The terminal condition for the equation (5) is defined as $Q(T) = -\psi$, if the state $x(t)$ does not reach the sliding manifold $x(t) = 0$ within the time interval $[t_0, T]$, $x(t) \neq 0$, $t \in [t_0, T]$. Otherwise, if the state $x(t)$ reaches the sliding manifold $x(t) = 0$ within the time interval $[t_0, T]$, then the initial condition for $Q(t)$ is set to zero at the time moment t^*, $Q(t^*) = 0$, where t^* is the maximum possible time of reaching the sliding manifold $x(t) = 0$. In other words, there exists no such solution to the system of equations (1), (4), (5) satisfying the conditions $x(t_0) = x_0$ and $Q(t_1) = 0$, $t_1 > t^*$, that $x(t) \neq 0$ for $t < t_1$ and $x(t) = 0$ for some $t \geq t_1$. Note that $x(t) = 0$, $t \geq t^*$, and, consequently, $u(t) = 0$, $t \geq t^*$; therefore, the value of $Q(t)$, $t \geq t^*$, is not needed.

Upon substituting the optimal control (4) into the state equation (1), the optimally controlled state equation is obtained

$$\dot{x}(t) = A(t)x(t) + B(t)R^{-1}(t)B^T(t)Q(t)Sign[x(t)], \quad x(t_0) = x_0. \qquad (6)$$

Consequently, the main result is formulated in the following theorem and proved in Appendix.

Theorem 1. The optimal regulator for the linear system (1) with respect to the criterion (3) is given by the sliding mode control law (4) and the gain matrix differential equation (5). The optimally controlled state of linear system (1) is governed by the equation (6).

Remark 1. Note that Theorem 1 suggests a feasible algorithm for numerical solution of the gain matrix equation (5). Indeed, first, the system of equations (1), (4), (5) is solved with a given initial condition $x(t_0) = x_0$ and the terminal condition $Q(T) = -\psi$ corresponding to the non-integral term in the criterion (3). Any known numerical method, such as "shooting," which consists in varying initial conditions for (5) until a given terminal condition is satisfied, could be used. If the system state $x(t)$ does not reach zero in the interval $[0, T]$ or reaches zero exactly at the final moment $t = T$, then the optimal trajectory and the optimal control are found. If $x(t)$ reaches zero at any point $t_1 < T$, then the initial value for $Q(0)$ should be varied increasing the value of the quadratic form $x_0^T Q(0) x_0$, thus decreasing the energy of the control (4), until the utmost value $-x_0^T \psi_0 x_0$ that the sliding mode still exists. In other words, the solution $x(t)$ of the system of equations (1),(4),(5) with the initial conditions $x(t_0) = x_0$ and $Q(0) = -\psi_0$ reaches zero at a time moment t^*, $x(t^*) = 0$; however, $x(t) \neq 0$, $t \in [0, T]$, for any solution of (1),(4),(5) with initial conditions $x(t_0) = x_0$ and $Q(0)$ such that $-x_0^T Q(0) x_0 < -x_0^T \psi_0 x_0$. The solution $x(t)$ of the equations (1), (4), (5), corresponding to $x(t_0) = x_0$ and $Q(0) = -\psi_0$, yields the optimal trajectory. The formula (4) with the substituted optimal trajectory and the matrix $Q(t)$ corresponding to $Q(0) = -\psi_0$ yields the optimal control as a function of time.

Remark 2. The initial condition for $Q(t)$, $Q(0) = -\psi_0$, established in Remark 1, leads to the same condition $Q(t^*) = 0$ as defined in Theorem 1. Indeed, for all values $Q(0)$, such that $-x_0^T Q(0) x_0 < -x_0^T \psi_0 x_0$, the solution $x(t)$ of the equations (1), (4), (5), corresponding to $x(t_0) = x_0$ and $Q(0)$ does not reach the sliding manifold $x(t) = 0$. Therefore, the resulting values of the criterion J_1 would be even higher than that for the solution corresponding to the terminal value $Q(T) = -\psi$, where ψ is the matrix from the integral term in (3). On the other hand, for all values $Q(0)$, such that $-x_0^T Q(0) x_0 \geq -x_0^T \psi_0 x_0$, the resulting values of the criterion J_1 are equal to $x_0^T Q(0) x_0$; therefore, the value $x_0^T \psi_0 x_0$ would be the least among them.

Remark 3. As follows from Theorem 1, application of the sliding mode control (4) leads to a causal terminal condition for the gain matrix equation (5), which makes the optimal control problem numerically solvable. In contrast, application of the linear feedback control $u^*(t) = K(t)x(t)$ leads to the terminal condition $Q(T) * \mid x(T) \mid = -\psi$, which explicitly depends on the unknown value $x(T)$, and, therefore, is non-causal. As well-known, non-causal problems are not numerically solvable and unusable in practice. Thus, in case of a criterion (3), the sliding mode control allows one to obtain a feasible solution to the optimal control problem, whereas the classical linear feedback control fails.

IV. EXAMPLE

This section presents an illustrative example of designing the optimal regulator for a system (1) with a criterion (3), using the scheme (4)–(6).

Consider a scalar linear system

$$\dot{x}(t) = x(t) + u(t), \quad x(0) = 1. \tag{7}$$

The control problem is to find the control $u(t)$, $t \in [0,T]$, $T = 5$, that minimizes the criterion

$$J_1 = \frac{1}{2}\int_0^T u^2(s)ds + \int_0^T \mid x(t) \mid dt, \tag{8}$$

where $\mid x \mid$ denotes the absolute value of a scalar variable x.

Applying the optimal regulator (4)–(6), the control law (4) is given by

$$u^*(t) = Q^*(t)sign[x(t)], \tag{9}$$

where $Q^*(t)$ satisfies the equation

$$\dot{Q}^*(t) = 1 - Q^*(t), \tag{10}$$

with the initial condition $Q^*(t^*) = 0$, where t^* is the maximum possible time of reaching the sliding manifold $x(t) = 0$ by the system state $x(t)$.

Upon substituting the control (9) and the obtained expression for $Q^*(t)$ into (7), the optimally controlled system takes the form

$$\dot{x}(t) = x(t) + Q^*(t)sign[x(t)], \quad x(0) = 1. \tag{11}$$

The obtained system (10),(11) can be solved using simple numerical methods, such as ”shooting.” This method consists in varying initial conditions of (10) until the given terminal condition is satisfied (see Remark 1).

The results of applying the regulator (9)–(11) to the system (7) are shown in Figs. 1 and 2, which present the graphs of the control (9) $u^*(t)$, the state (11) $x(t)$, and the corresponding values of the criterion (8) $J_1(t)$ in the interval $[0,5]$ as solid lines. Figure 1 shows the graphs in the entire simulation interval $[0,5]$ and Figure 2 demonstrates the graph details in the interval $[4.995,5]$ to provide better comparison. The value of the criterion (8) at the final moment $T = 5$ is $J_1(5) = 2.7323$.

The optimal regulator (9)–(11) is compared to the best linear regulator for the criterion (2) with the quadratic integral terms

$$J_2 = \frac{1}{2}\int_0^T (u^2(t)dt + x^2(t))dt. \tag{12}$$

As follows from the optimal LQR theory [18], [19], the linear control law is given by

$$u(t) = Q(t)x(t), \tag{13}$$

where $Q(t)$ satisfies the Riccati equation

$$\dot{Q}(t) = -A^T(t)Q(t) - Q(t)A(t) + L(t) -$$
$$Q(t)B(t)R^{-1}(t)B^T(t)Q(t),$$

with the terminal condition $Q(T) = -\psi$. Substituting numerical values from (7), (12) for the parameters $A(t)$, $B(t)$, $L(t)$, and $R(t)$, the last equation turns to

$$\dot{Q}(t) = 1 - 2Q(t) - Q^2(t), \quad Q(5) = 0. \tag{14}$$

Upon substituting the control (13) into (7), the controlled system takes the form

$$\dot{x}(t) = x(t) + Q(t)x(t), \quad x(0) = 1. \tag{15}$$

Note that the comparison of the sliding mode optimal regulator (9)–(11) to the best linear regulator (13)–(15) with respect to the criterion (8) is conducted for illustration purposes, since the sliding mode optimal regulator (9)–(11) should theoretically yield a better result, as follows from Theorem 1.

The results of applying the regulator (13)–(15) to the system (7) are shown in Figs. 1 and 2, which presents the graphs of the control (13) $u(t)$, the state (15) $x(t)$, and the corresponding values of the criterion (8) $J_1(t)$ in the interval $[0,5]$ as dashed lines. Figure 1 shows the graphs in the entire simulation interval $[0,5]$ and Figure 2 demonstrates the graph details in the interval $[4.995,5]$ to provide better comparison. The value of the criterion (8) at the final moment $T = 5$ is $J_1(5) = 2.77$.

It can be observed that the optimal sliding mode control (9) yields a certainly better value of the criterion (8) in comparison to the linear feedback control (13).

V. APPENDIX

Proof of Theorem 1. Necessity. Define the Hamiltonian function [18] for the optimal control problem (1), (3) as

$$H(x,u,q,t) = \frac{1}{2}u^T R(t)u + L(t) \mid x(t) \mid + q^T \dot{x}(t) =$$
$$= \frac{1}{2}u^T R(t)u + L(t) \mid x(t) \mid + q^T [A(t)x + B(t)u]. \tag{19}$$

Applying the maximum principle condition $\partial H/\partial u = 0$ to this specific Hamiltonian function (19) yields

$$\partial H/\partial u = 0 \Rightarrow R(t)u(t) + B^T(t)q(t) = 0.$$

Accordingly, the optimal control law is obtained as

$$u^*(t) = -R^{-1}(t)B^T(t)q(t).$$

Let us seek $q(t)$ as the Signum function of $x(t)$ multiplied by a gain matrix

$$q(t) = -Q(t)Sign[x(t)], \tag{20}$$

where $Q(t)$ is a square symmetric matrix of dimension $n \times n$. This yields the complete form of the optimal control

$$u^*(t) = R^{-1}(t)B^T(t)Q(t)Sign[x(t)]. \tag{21}$$

Using the co-state equation $dq(t)/dt = -\partial H/\partial x$, which gives

$$-dq(t)/dt = L(t)Sign[x(t)] + A^T(t)q(t), \tag{22}$$

and substituting (20) into (22), we obtain

$$\dot{Q}(t)Sign[x(t)] + Q(t)d(Sign[x(t)])/dt =$$

$$= L(t)Sign[x(t)] - A^T(t)Q(t)Sign[x(t)]. \quad (23)$$

Taking into account that $d(Sign[x(t)])/dx = 0$ almost everywhere outside the sliding manifold $x(t) = 0$, the following equation is obtained

$$\dot{Q}(t)Sign[x(t)] = L(t)Sign[x(t)] - A^T(t)Q(t)Sign[x(t)]. \quad (24)$$

Note that if $x(t) = 0$, then $u(t) = 0$; therefore, the value of $Q(t)$ is no longer needed. The equation (24) is satisfied, if $Q(t)$ is assigned as a solution of the equation (5).

Note that if the state $x(t)$ does not reach the sliding manifold $x(t) = 0$ at an interior point of the interval $[0,T]$, the transversality condition [18] for $q(T)$ implies that

$$q(T) = -Q(T)Sign[x(T)] = \partial J/\partial x(T) = \psi Sign[x(T)],$$

which is satisfied if

$$Q(T) = -\psi. \quad (25)$$

However, if $x(t)$ reaches the sliding manifold $x(t) = 0$ before the final moment $t = T$, then the problem becomes a two fixed point problem, where the terminal point is fixed at an *a priori* unknown time moment t_1 when $x(t)$ reaches the sliding manifold $x(t) = 0$. Therefore, the transversality condition results in the condition for $Q(t)$ at the point $t = t_1$, $Q(t_1) = 0$. Among all the moments t_1 when $x(t)$ reaches the sliding manifold $x(t) = 0$, the minimum value of criterion J_1 would be obtained for the maximum possible time t^*, $t^* \geq t_1$, of reaching the sliding manifold $x(t) = 0$. Indeed, the criterion value would be equal to $-x_0^T Q(0)x_0$, where $Q(0)$ is the corresponding initial value of $Q(t)$, and the maximum time value of entering the sliding mode corresponds to the minimum control energy, i.e., the least value of $-x_0^T Q(0)x_0$. Thus, the terminal conditions for the equation (5) are correctly defined by Theorem 1. The necessity part is proved.

Sufficiency. The optimality of the optimal control law $u^*(t)$ given in Theorem 1 and by the formula (21) is proved in a standard way (see details, for example, in [21]): composing the Hamilton-Jacobi-Bellman (HJB) equation, corresponding to the Hamiltonian (19), and demonstrating that it is satisfied with the Bellman function $V(x,t) = -x^T Q(t)Sign[x] = -\sum_{i,j=1}^{n} Q_{ij}(t)sign[x_j]x_i$, where $Q_{ij}(t)$ are the entries of the matrix $Q(t)$ solving the equation (5). The proof mostly repeats the formulas (22)–(25) in the necessity part. Finally, minimizing the right-hand side of the HJB equation over u yields the optimal control $u^*(t)$ in the form (21). The theorem is proved. ■

VI. Conclusions

This paper presents an optimal control problem, whose solution is given by a sliding mode control. The optimal control problem is considered for a linear system with a Bolza-Meyer criterion, where the control energy term is quadratic and the state energy terms are of the first degree. That type of criteria would be useful in the joint control and parameter identification problems where the objective should be reached for a finite time. A distinctive feature of the obtained result is that a part of the optimally controlled state trajectory occurs to be in a sliding mode. It is shown that optimal solution is given by a causal sliding mode control, whereas the conventional linear feedback control fails to provide a feasible solution. The proposed approach based on a sliding mode control is expected to be applicable to other optimal control problems with non-quadratic criteria, where the conventional linear feedback control would not work.

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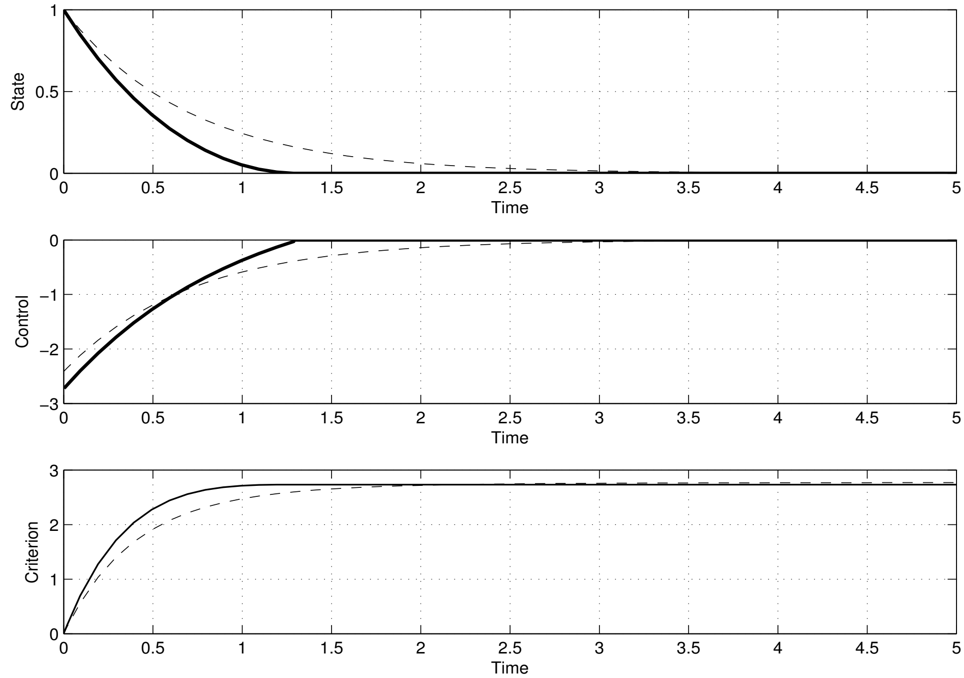

Fig. 1. Sliding mode regulator optimal with respect to criterion J_1 vs. linear feedback regulator in the entire simulation interval $[0,5]$. Graphs of the control (9) $u^*(t)$, the state (11) $x(t)$, and the corresponding values of the criterion (8) $J_1(t)$ are shown as solid lines. Graphs of the control (13) $u(t)$, the state (15) $x(t)$, and the corresponding values of the criterion (8) $J_1(t)$ are shown as dashed lines.

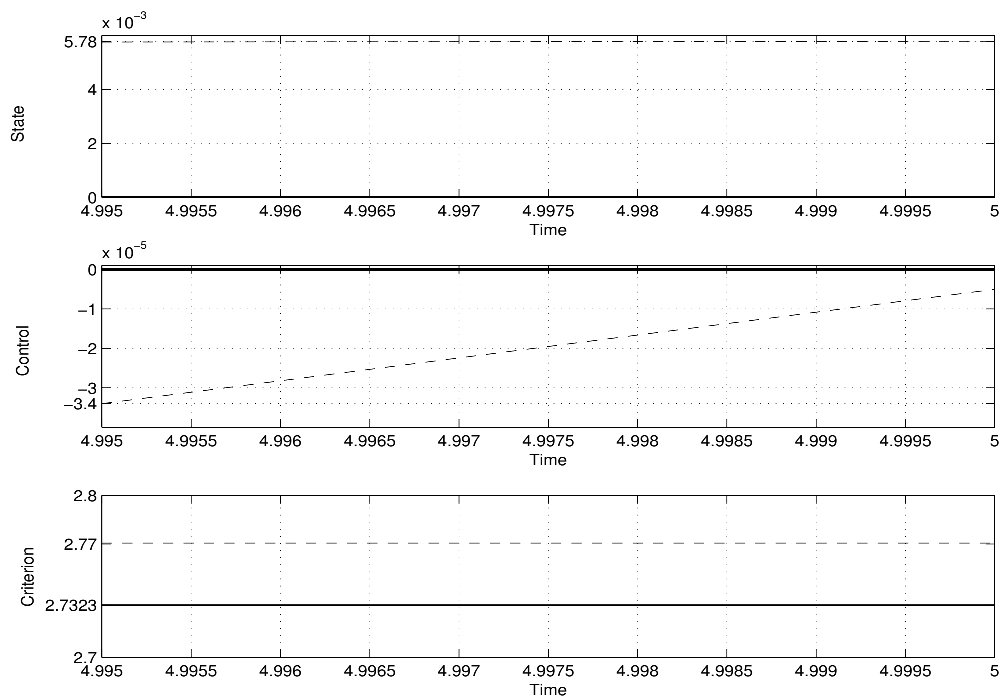

Fig. 2. Sliding mode regulator optimal with respect to criterion J_1 vs. linear feedback regulator in the interval $[0,4.995]$. Graphs of the control (9) $u^*(t)$, the state (11) $x(t)$, and the corresponding values of the criterion (8) $J_1(t)$ are shown as solid lines. Graphs of the control (13) $u(t)$, the state (15) $x(t)$, and the corresponding values of the criterion (8) $J_1(t)$ are shown as dashed lines.