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Robust SISO Controller Order Reduction

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***Abstract*—In this paper a method is proposed to reduce the order of a single-input single-output robust controller. The controller ensures system robust performance and reducing its order may result in loss of robust performance. A lower bound on the controller order is provided using balanced truncation technique which guarantees that the robust performance of the closed-loop system is maintained. Simulation results show the effectiveness of the proposed technique.**

I. Introduction

MODELING the dynamic characteristics of the plant is required in the design of a feedback system. Despite the possible differences between the actual plant and the model, the controller designed based on the provided model of the plant must be able to control the actual plant too. The designed controller may not always guarantee robust stability under the new circumstances caused by the plant uncertainties [1]. Also, tracking or regulation errors often have to be controlled in order to ensure the closed-loop performance, meaning that the stability is not the only property of a closed-loop system that must be robust to plant perturbations. These errors are caused by combined effect of exogenous disturbances acting on the system and plant perturbations can cause these errors to increase greatly [2].

In common system order reduction problems, it is desired to approximate a system with a lower order model. Two important factors in doing so are to preserve the stability of the system and also come up with an error bound describing how close the reduced-order model is to the original system. In robust controller order reduction problem, it is also important to guarantee that closed-loop robust performance is preserved. One of the most powerful techniques of model order reduction is balanced truncation [3], [4] which is widely used for stable high-order systems.

In typical robust control problems, the generated controller can be of high order compared to other system components which is not desirable, specially in case of real-time implementation on slow industrial hardware which may limit the achievable sampling rate [5], [6]. The control order reduction has been studied by various researchers, specially in $\mathcal{H}_\infty$-control problems [7-10]. The main objective of this work is to reduce the order of high-order SISO robust controllers. Doing so can significantly reduce the complexity of the device required for realization of the controller while maintaining system robust performance. An upper bound is derived on controller uncertainty which, when combined with the error bound provided by balanced realization, provides a lower limit on the order of the reduced controller to maintain robust performance. As it will be discussed in this article, the SISO structure allows the controller uncertainty upper bound calculation to be performed solely based on the magnitude response of the system components, therefore speeding up the necessary calculations.

The paper is organized as follows. Section II contains the general problem formulation, explaining uncertainty modeling, $M-\Delta$ interconnection, structured singular value $\mu_\Delta(\bullet)$ and main loop theorem through Subsections A-D. The main results are presented in Section III where the upper bound providing sufficient robust performance condition is derived in Subsection III-A. The controller order reduction error is then modeled as an additive uncertainty, leading to an application of the balanced truncation procedure to the stable part of the robust controller in Subsection III-B. Simulation results are given in Section IV which demonstrate the effectiveness of the proposed scheme. Finally, concluding remarks are drawn in Section V.

II. Problem Formulation

A. Uncertainty Modeling

Consider the closed-loop control system in Fig. 1.

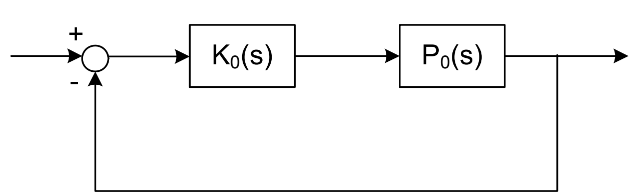

Fig. 1. Typical feedback control system with nominal models for the controller and plant.

The transfer functions $K_0(s)$ and $P_0(s)$ represent the nominal controller and plant models, respectively. Assume

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now that the plant and controller are both subject to perturbation, and consider the additive models $P(s) = P_0(s) + \Delta_P(s)$ and $K(s) = K_0(s) + \Delta_K(s)$ for the uncertainties of the plant and controller, respectively, where [11]:

$$\Delta_P(s) = W_P(s)\tilde{\Delta}_P(s), \quad \tilde{\Delta}_P(s) \in \mathcal{H}_\infty, \quad \left\|\tilde{\Delta}_P(s)\right\|_\infty < 1, \tag{1}$$

$$\Delta_K(s) = W_K(s)\tilde{\Delta}_K(s), \quad \tilde{\Delta}_K(s) \in \mathcal{H}_\infty, \quad \left\|\tilde{\Delta}_K(s)\right\|_\infty < 1. \tag{2}$$

and where $W_P(s)$ and $W_K(s)$ are weighting functions bounding the uncertainties. The control loop of Fig. 1 can now be modified as shown in Fig. 2 to account for uncertainties:

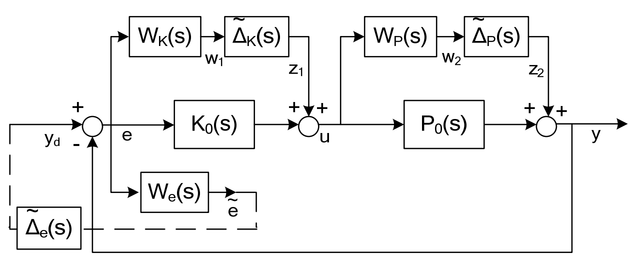


Fig. 2. Expanded control loop with additive uncertainties and a fictitious performance block.

The block $\tilde{\Delta}_e(s)$ is a fictitious uncertainty which appears only in the control design stage to obtain robust performance using the main loop theorem, and is characterized as:

$$\Delta_e(s) = W_e(s)\tilde{\Delta}_e(s), \quad \tilde{\Delta}_e(s) \in \mathcal{H}_\infty, \quad \left\|\tilde{\Delta}_e(s)\right\|_\infty < 1. \tag{3}$$

The uncertainty block $\tilde{\Delta}_e(s)$ and the corresponding weighting function $W_e(s)$ are considered in control design in order to ensure that the error e is maintained within acceptable limits in closed-loop operation.

Throughout the reminder of this paper, omitting the complex variable *s* means that unless specified, the system is sampled at frequency ω, e.g. $P = P(s)|_{s=j\omega}$.

A. M-Δ Interconnection

By partitioning the structure shown in Fig. 2, the perturbed closed-loop system at frequency ω can be represented as an $M-\Delta$ interconnection as shown in Fig. 3 [12] where the complex matrices M and Δ are given by:

$$\Delta := \begin{bmatrix} \overbrace{\begin{bmatrix} \tilde{\Delta}_K & 0 \\ 0 & \tilde{\Delta}_P \end{bmatrix}}^{\hat{\Delta}} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} \tilde{\Delta}_e \end{bmatrix} \end{bmatrix} \tag{4}$$

and:

$$M := (I + K_0 P_0)^{-1} \begin{bmatrix} \overbrace{\begin{bmatrix} -W_K P_0 & -W_K \\ W_P & -W_P K_0 \end{bmatrix}}^{\hat{M}} & \begin{bmatrix} W_K \\ W_P K_0 \end{bmatrix} \\ \begin{bmatrix} -W_e P_0 & -W_e \end{bmatrix} & \begin{bmatrix} W_e \end{bmatrix} \end{bmatrix}. \tag{5}$$

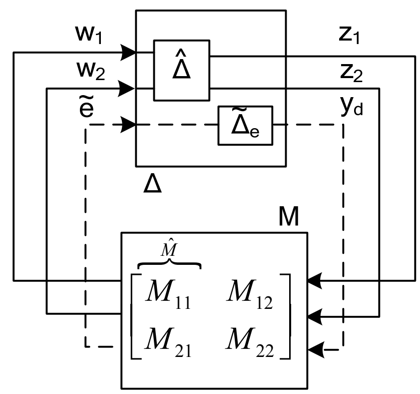


Fig. 3. The $M-\Delta$ interconnection isolating uncertainty blocks.

Note that the top left 2-by-2 sub-matrices in (4) and (5) are $\hat{\Delta}$ and $\hat{M}$, representing the conventional $M-\Delta$ interconnection of the perturbed control loop without taking performance into account.

B. Structured Singular Value and Robust Stability

The structured singular value of $M \in \mathbb{C}^{n\times n}$ denoted by $\mu_\Delta(M)$, is defined as:

$$\mu_\Delta(M) := \frac{1}{\min\left\{\bar{\sigma}(\Delta_s) : \Delta_s \in \mathbf{\Delta}, \det(I - M\Delta_s) = 0\right\}} \tag{6}$$

unless no $\Delta_s \in \mathbf{\Delta}$ makes $I - M\Delta_s$ singular, in which case $\mu_\Delta(M) = 0$. The set $\mathbf{\Delta}$ in this paper is composed of all structured complex uncertainty matrices as in (4) [11], [12], [13]. The robust stability theorem states that the $M-\Delta$ interconnection given in Fig. 2. is stable for all stable structured perturbations $\Delta(j\omega) \in \mathbf{\Delta}$, $\|\Delta\|_\infty < 1$, if and only if

$$\sup_{\omega} \mu_{\Delta}\left(M\left(j\omega\right)\right) \le 1 . \tag{7}$$

Having $\tilde{\Delta}_e$ and W_e included in (4) and (5), and assuming that (7) holds, implies that the gain from the desired output to the error is bounded, leading to robust performance of the perturbed system. It is desired to find a condition on the maximum size of the controller uncertainty $|W_K|$ such that the augmented $M(s)-\Delta(s)$ system interconnection is robustly stable (i.e., the system with controller uncertainty has robust performance).

C. Main Loop Theorem

The main loop theorem states that for a general $M-\Delta$ interconnection of complex matrices:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad \Delta = \begin{bmatrix} \tilde{\Delta}_1 & 0 \\ 0 & \tilde{\Delta}_2 \end{bmatrix}, \tag{8}$$

the following statement holds [4]:

$$\mu_{\Delta}(M) \le 1 \Leftrightarrow \begin{cases} \mu_1(M_{11}) \le 1, \text{ and} \\ \sup\limits_{\|\tilde{\Delta}_1\|<1} \mu_2\left(F_u\left(M, \tilde{\Delta}_1\right)\right) \le 1. \end{cases} \tag{9}$$

where the $\mu_1(\bullet)$ and $\mu_2(\bullet)$ operators are the structured singular values with respect to $\tilde{\mathbf{\Delta}}_1$ and $\tilde{\mathbf{\Delta}}_2$, and $F_u\left(M, \tilde{\Delta}_1\right)$ is the upper linear fractional transformation (LFT) of M by $\tilde{\Delta}_1$ [12].

II. MAIN RESULT

A. Sufficient Condition for Robust Performance

Following the definition of Δ and M in (4) and (5) with the representation given in (8) and (9), the main loop theorem for the SISO system is written as:

$$\mu_{\Delta}(M) \le 1 \Leftrightarrow \begin{cases} \mu_{\hat{\Delta}}(\hat{M}) \le 1 \text{ and} \\ \sup\limits_{\|\hat{\Delta}\|<1} \mu_e\left(F_u\left(M, \hat{\Delta}\right)\right) \le 1. \end{cases} \tag{10}$$

The top inequality on the right-hand side of (10) neglects the effect of $\tilde{\Delta}_e$. In other words, it guarantees the robust stability of the perturbed system without taking performance into account.

Since $\hat{M}$ is a 2-by-2 matrix, the main loop theorem can be applied again to obtain:

$$\mu_{\hat{\Delta}}(\hat{M}) \le 1 \Leftrightarrow \begin{cases} \mu_K\left(\hat{M}_{11}\right) \le 1 \\ \sup\limits_{|\tilde{\Delta}_K|<1} \mu_P\left(F_u\left(\hat{M}, \tilde{\Delta}_K\right)\right) \le 1 \end{cases} \tag{11}$$

We have:

$$\begin{aligned} &\mu_K\left(\hat{M}_{11}\right) \le 1 \\ &\Leftrightarrow \sup_{\|\tilde{\Delta}_K\|<1} \rho\left(\hat{M}_{11}\tilde{\Delta}_K\right) \le 1 \\ &\Leftrightarrow \sup_{|\tilde{\Delta}_K|<1} \left(\left|\hat{M}_{11}\right|\left|\tilde{\Delta}_K\right|\right) \le 1 \\ &\Leftrightarrow \left|\hat{M}_{11}\right| \le 1 \Leftrightarrow \left|W_K\right| \le \left|\frac{1+K_0P_0}{P_0}\right|. \end{aligned} \tag{12}$$

Using the definition of the LFT:

$$\begin{aligned} F_u\left(\hat{M}, \tilde{\Delta}_K\right) &= \hat{M}_{22} + \hat{M}_{21}\tilde{\Delta}_K\left(I - \hat{M}_{11}\tilde{\Delta}_K\right)^{-1}\hat{M}_{12} \\ &= -\frac{W_P\left(K_0 + W_K\tilde{\Delta}_K\right)}{1+P_0\left(K_0 + W_K\tilde{\Delta}_K\right)}. \end{aligned} \tag{13}$$

Hence,

$$\begin{aligned} &\sup_{|\tilde{\Delta}_K|<1} \mu_P\left(F_u\left(\hat{M}, \tilde{\Delta}_K\right)\right) \le 1 \\ &\Leftrightarrow \sup_{\substack{|\tilde{\Delta}_P|<1 \\ |\tilde{\Delta}_K|<1}} \rho\left(-\frac{W_P\left(K_0 + W_K\tilde{\Delta}_K\right)}{1+P_0\left(K_0 + W_K\tilde{\Delta}_K\right)}\tilde{\Delta}_P\right) \le 1 \\ &\Leftrightarrow \sup_{\substack{|\tilde{\Delta}_P|<1 \\ |\tilde{\Delta}_K|<1}} \left(\left|\frac{W_P\left(K_0 + W_K\tilde{\Delta}_K\right)}{1+P_0\left(K_0 + W_K\tilde{\Delta}_K\right)}\right|\left|\tilde{\Delta}_P\right|\right) \le 1 \\ &\Leftrightarrow \sup_{|\tilde{\Delta}_K|<1} \left(\left|\frac{W_P\left(K_0 + W_K\tilde{\Delta}_K\right)}{1+P_0\left(K_0 + W_K\tilde{\Delta}_K\right)}\right|\right) \le 1, \end{aligned} \tag{14}$$

yielding to the following proposition.

Proposition 1: The $\hat{M}(s)-\hat{\Delta}(s)$ system interconnection of Fig. 3 is robustly stable if the following inequality holds for every ω:

$$\left|W_K\right| \le \frac{\left|1+P_0K_0\right| - \left|W_PK_0\right|}{\left|P_0\right| + \left|W_P\right|}. \tag{15}$$

Proof: In the case that the controller has no uncertainty, the last inequality in (14) reduces to $\left|W_PK_0\right| \le \left|1+P_0K_0\right|$. The same inequality can be expressed as:

$$\left|W_PK_0 + W_PW_K\tilde{\Delta}_K\right| \le \left|1+P_0K_0 + P_0W_K\tilde{\Delta}_K\right|, \ \forall\left|\tilde{\Delta}_K\right| < 1. \tag{16}$$

A sufficient condition for (16) to hold is given by

$$\left|W_P K_0\right| + \left|W_P W_K\right| \le \left|1 + P_0 K_0\right| - \left|P_0 W_K\right| \tag{17}$$

which after rearrangement leads to (15), completing the proof.

The upper bound given in (15) guarantees robust stability as the first step towards maintaining robust performance as the ultimate goal of this paper. Back to (10), the second inequality in the right side should be satisfied. By applying a method similar to the one in (14) to the LFT

$$\begin{aligned} F_u\left(M,\hat{\Delta}\right) &= M_{22} + M_{21}\hat{\Delta}\left(I - M_{11}\hat{\Delta}\right)^{-1} M_{12} \\ &= \frac{W_e}{1 + \left(P_0 + W_P\tilde{\Delta}_P\right)\left(K_0 + W_K\tilde{\Delta}_K\right)}, \end{aligned} \tag{18}$$

one can conclude that:

$$\begin{aligned} &\sup_{\|\hat{\Delta}\|<1} \mu_e\left(F_u\left(M,\hat{\Delta}\right)\right) \le 1 \\ \Leftrightarrow &\sup_{\substack{|\tilde{\Delta}_K|<1 \\ |\tilde{\Delta}_P|<1}} \left(\left|\frac{W_e}{1 + \left(P_0 + W_P\tilde{\Delta}_P\right)\left(K_0 + W_K\tilde{\Delta}_K\right)}\right|\right) \le 1 \end{aligned} \tag{19}$$

which leads to the following proposition.

Proposition 2: The $\hat{M}(s) - \hat{\Delta}(s)$ system interconnection has robust performance if the following inequality holds for every ω:

$$\left|W_K\right| \le \frac{\left|1 + P_0 K_0\right| - \left|W_P K_0\right| - \left|W_e\right|}{\left|P_0\right| + \left|W_P\right|}. \tag{20}$$

Proof: Since the normalized uncertainties in (19) appear as products with their respective weighting functions, one can absorb the phase of the weighting functions into the normalized uncertainties. Rewriting (19) leads to the following inequality which holds for all $\left|\tilde{\Delta}_K\right| < 1$, $\left|\tilde{\Delta}_P\right| < 1$:

$$\left|\left(1 + P_0 K_0 + K_0\left|W_P\right|\tilde{\Delta}_P + P_0\left|W_K\right|\tilde{\Delta}_K + \left|W_P\right|\left|W_K\right|\tilde{\Delta}_P\tilde{\Delta}_K\right)\right| \ge \left|W_e\right|. \tag{21}$$

In the nominal case when the system has no uncertainty, (21) reduces to $\left|1 + P_0 K_0\right| \ge \left|W_e\right|$. Thus, (21) is satisfied if:

$$\left|\left(K_0\left|W_P\right|\tilde{\Delta}_P + P_0\left|W_K\right|\tilde{\Delta}_K + \left|W_P\right|\left|W_K\right|\tilde{\Delta}_P\tilde{\Delta}_K\right)\right| < \left|1 + P_0 K_0\right| - \left|W_e\right|. \tag{22}$$

The left-hand side of (22) reaches its maximum when all three of its components are aligned in the same direction in the complex plane. This can happen if $\measuredangle\tilde{\Delta}_P = \measuredangle P_0$ and $\measuredangle\tilde{\Delta}_K = \measuredangle K_0$. Applying these phase equalities to (22) leads to (20) and completes the proof.

Note that the right-hand side of (20) is always smaller than the right-hand side of (15). Also, it can be seen that (20) is a tighter upper bound than (12). This confirms that (20) is the sufficient robust performance condition to be satisfied.

B. Controller Order Reduction

So far, an upper bound for $\left|W_K\right|$ as a sufficient condition for robust performance has been derived. The reduced-order controller can be modeled as:

$$K_r(s) = K_0(s) + \Delta_r(s) \tag{23}$$

where $\Delta_r(s)$ is the order reduction error associated with $K_0(s)$, and $K_r(s)$ is the reduced-order controller. Recalling the controller perturbation definition in (2), $\Delta_r(s)$ can be considered as $\Delta_K(s)$ with the exception of having a known phase as it can be derived as a rational transfer function rather than the product of a weighting function and an uncertain bounded element. This discussion leads to the following proposition.

Proposition 3: Suppose that $L(\omega)$ is a real function of ω equal to the upper bound derived in (20). The reduced-order controller $K_r(s)$ maintains robust performance of the $\hat{M}(s) - \hat{\Delta}(s)$ system interconnection if for every frequency ω:

$$\left|\Delta_r(j\omega)\right| \le L(\omega). \tag{24}$$

Proof: Since $\Delta_r(j\omega)$ follows the definition of (2), its magnitude is bounded by the magnitude of $W_K(j\omega)$, which in turn is bounded by $L(\omega)$ as in (20), a sufficient condition for robust performance. This completes the proof.

The balanced realization technique [3] provides an upper bound on the infinity norm of $\Delta_r(s)$. Therefore, after balancing the controller $K_0(s)$ or order N_0 and computing the associated Hankel singular values (HSV) [3], the smallest order of $K_r(s)$ guaranteeing robust performance using sufficient condition of (24), r_0, is:

$$r_0 \triangleq \min\left\{ r \middle| 2\sum_{m=r+1}^{N_0} \sigma_m \le \min_{\omega} L \right\}. \tag{25}$$

In other words, it is safe to remove $N_0 - r_0$ last states of the balanced $K_0(s)$ while maintaining robust performance. In the case of a robust controller with unstable poles, the controller is decomposed into the summation of stable and unstable components, $K_{0_S}(s)$ and $K_{0_U}(s)$. This decomposition can be done by applying a partial fraction expansion to $K_0(s)$ and then collecting stable and unstable parts separately. The model order reduction can then be applied to $K_{0_S}(s)$ using the same upper bound in (20). Fig. 4. shows how this decomposition does not require a new upper bound calculation.

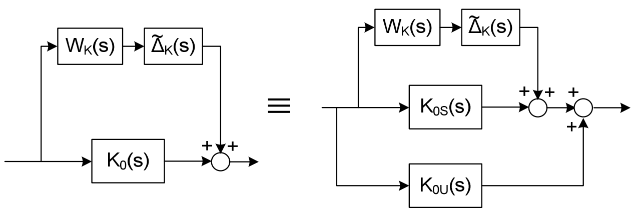


Fig. 4. Separation of stable and unstable controller components

The upper bound in (20) acts as a sufficient robust performance condition. At the same time, balanced realization technique provides an upper bound and not an exact measure of the order reduction error, e.g. its frequency domain behavior. Therefore, there might still be room for controller order reduction comparing to what (25) suggests. As a result, a future improvement would consist of a necessary and sufficient condition for robust performance which eventually will provide more accurate information about the maximum allowed difference between the original and reduced-order controller.

III. Simulation Results

Consider the nominal plant $P_0(s)$, the additive uncertainty weighting function $W_P(s)$ and performance weighting function $W_e(s)$ given by:

$$P_0(s) = \frac{10}{s+2},$$

$$W_P(s) = \frac{4}{s+4},$$

$$W_e(s) = \frac{50}{s+3}.$$

Using DK iteration [2], a robust controller $K_0(s)$ of 7th order is computed as:

$$K_0(s) = c \cdot \frac{\prod_{i=1}^{6} s + b_i}{\prod_{l=1}^{7} s + a_l}$$

where $c = -32.9235 \times 10^7$, $b_1 = 3256$, $b_2 = 4.739$, $b_3 = 4.464$, $b_4 = 3.035$, $b_5 = 2.253$, $b_6 = 2$, $a_1 = 1.905 \times 10^5$, $a_2 = 3242$, $a_3 = 57.79$, $a_4 = 15.28$, $a_5 = 4.966$, $a_6 = 3$ and $a_7 = 2.399$.

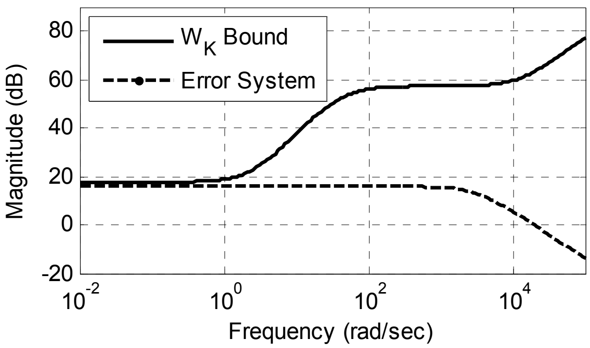
Note that although the plant and weighting functions are all first-order transfer functions, the synthesized robust controller has a higher order due to the constraints imposed by the combination of all system components. This is typical in robust control design.

For possible controller uncertainty, the upper bound for $|W_K|$ or $L(\omega)$ is calculated and shown in Fig. 5. Applying balanced realization to $K_0(s)$ results in the following HSVs:

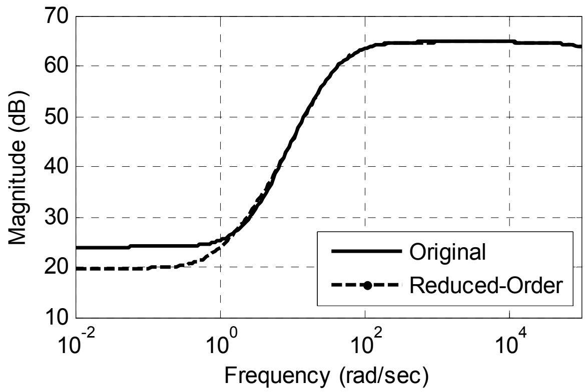
Σ = 934.89, 864.1, 75.65, 3.1, 9.3×10^{-3}, 6.3×10^{-3}, 7.72×10^{-6}.

Using (25), r_0 is found to be equal to 3. Thus, the last 4 states of the balanced controller are removed and the reduced-order controller $K_r(s)$ will consist of 3 states. The magnitude plot of original and reduced-order controllers as well as the error system $K_0(s) - K_r(s)$ versus the derived upper bound is depicted in Fig. 5. Note that the error magnitude is close to the allowed bound. Also it should be mentioned that the upper bound is calculated over the same set of frequency points used to evaluate the necessary frequency responses.

The μ-plot of the system using both original and reduced-order controllers is depicted in Fig. 6. As it can be seen in the graph, μ is bounded by one in both cases meaning that the robust performance is met. It reaches 0.56 in case of using reduced-order controller while the maximum is 0.42 in case of using original robust controller, complying with the error system plot in Fig. 5(a) and different magnitude responses depicted in Fig 5(b).



(a)



(b)

Fig. 5. (a) Magnitude response of the error system versus the derived upper bound; (b) the magnitude response of the original and reduced-order controllers.

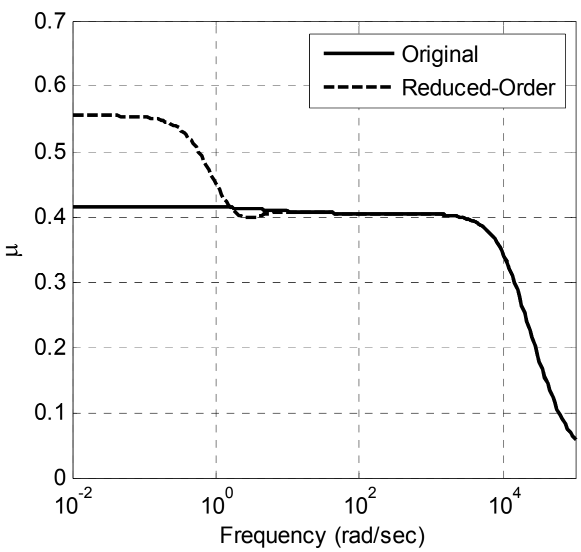


Fig. 6. The μ -plot using original and reduced-order controllers.

IV. Conclusions

A method is proposed to reduce the order of a single-input single-output robust controller. An upper bound on the magnitude of the controller uncertainty is derived based on the frequency response magnitudes of system components which provides a sufficient condition for maintaining system robust performance without using any μ -calculation routines. This is only possible in the SISO case. The bound is then used to provide a lower bound on the controller order which guarantees the robust performance of the closed-loop system. Balanced truncation technique is used for controller order reduction as a powerful and well-developed order reduction method. Simulation results show the efficiency and simplicity of the proposed method. The robust controller order reduction for general MIMO systems is being considered as a parallel research project. It follows a relatively similar but more sophisticated routine, including the use of μ -calculation as an intermediate tight upper bound calculation.

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