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Star Camera Calibration Combined with Independent Spacecraft Attitude Determination

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***Abstract*— A methodology for determining spacecraft attitude and autonomously calibrating star camera, both independent of each other, is presented in this paper. Unlike most of the attitude determination algorithms where attitude of the satellite depend on the camera calibrating parameters (like principal point offset, focal length etc.), the proposed method has the advantage of computing spacecraft attitude independently of camera calibrating parameters except lens distortion. In the proposed method both attitude estimation and star camera calibration is done together independent of each other by directly utilizing the star coordinate in image plane and corresponding star vector in inertial coordinate frame. Satellite attitude, camera principal point offset, focal length (in pixel), lens distortion coefficient are found by a simple two step method. In the first step, all parameters (except lens distortion) are estimated using a closed-form solution based on a distortion free camera model. In the second step lens distortion coefficient is estimated by linear least squares method using the solution of the first step to be used in the camera model that incorporates distortion. These steps are applied in an iterative manner to refine the estimated parameters. The whole procedure is faster enough for onboard implementation.**

I. INTRODUCTION

With the increasing demand for small inexpensive satellite for short period missions, the use of high band width and accurate star tracker are becoming relevant than the use of costly gyroscope. But the accuracy of spacecraft attitude determination depends upon the accuracy of Star camera calibration. In literature all attitude estimation algorithms make use of the star vector pairs in camera coordinate system and inertial coordinate system. The difficulty of this method is that attitude estimation depends upon the accuracy of star camera calibration as star vector in camera coordinate frame is represented by the calibrating parameters. Though star camera is calibrated on the ground with high accuracy before launching but due to many reasons like temperature, vibration, aging electronics etc camera parameters get changed in orbit. This necessitates the fact of on-orbit camera calibration independent of attitude determination. The method proposed in [1] for star camera calibration utilize the fact that interstar angles are an invariant of spacecraft rotation thus there is no need of unknown spacecraft attitude to estimate calibration parameters that offset interstar angles. Ref [2] presents both attitude dependent and attitude independent methods for star camera calibration. The relative merits of two algorithm are also studied for attitude determination and camera calibration. In realisability and robustness issues they have shown that attitude independent algorithm performs better than attitude dependent algorithm. But the method proposed for attitude determination in [2] utilizes the measured star vector in camera coordinate frame which in turn depends on the camera calibration parameters.

In [3], [4] the measured and corresponding inertial vectors are used for computing attitude and attitude rate using EKF. The difficulty in this method is that the estimated attitude is dependent on camera calibration parameters apart from the EKF convergence issue where the solution is far apart from the initial guess.

In the context of star identification, several research have been done for lost-in-space and normal mode of satellite operation. In [5] star identification in lost in space scenario is solved by identifying the star patterns using an indexed star separation database for wide field of view (FOV) star camera. However, for narrow FOV star tracker, the star pair angular matching technique will require large number of catalog stars and associated large star separation database which in turn requires large on board memory. Clouse et al. in [6] has shown this point with a huge onboard memory requirement for a 2×2 FOV star tracker. It is also shown that this method of star identification by star pair angular matching technique may lead to false star identification because the measured star pattern associated with noise has to be identified among many such closely matching patterns. The false star identification method is resolved in [7] by using magnitude of the stars along with the angular separation database. Also, for normal mode operation of satellite (where attide is known to a priori) a fast access to the onboard star catalog using hashing method is presented in [8].

In this paper, calibration of principal point offset and focal length in pixel is done so as to estimate the internal parameters of the star camera. These parameters relate the star vectors in camera coordinate frame to the corresponding star image coordinates in the image plane. Attitude of the satellite is estimated as an estimation of the camera external parameters which relate star vectors in the inertial coordinate frame to the corresponding star vectors in the camera coordinate frame. A simple closed form solution method is used to get all the calibration parameters of the star camera (internal parameters) and attitude of the satellite (external parameters) using distortion free camera model. The method described here solves the original parameters by solving a set of intermediate parameters which relate the star image points to the corresponding star vectors in

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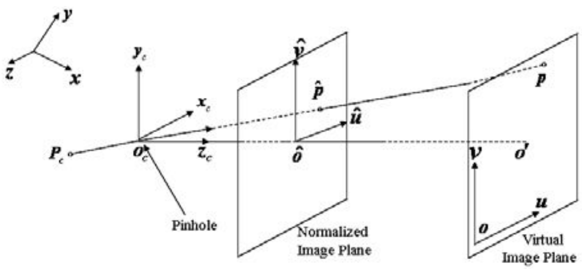

Fig. 1. Coordinate Systems

the inertial coordinate frame. This calibration procedure has long been used in photogrammetry and computer vision field but never been used for onboard satellite attitude estimation and star camera calibration. In closed form solution method, parameter values are calculated, through a fast onboard implementable noniterative algorithm (e.g., [9], [10]) by solving linear equations. Utilizing all the parameter values calculated in the first step, camera lens distortion coefficient is calculated in the second step based on a camera model that incorporates distortion, using linear least squares method. Then the whole procedure is repeated for a number of time, using the inversed formulation of distortion model [14], to improve all the parameters. In [12], Weng et al. used an iterative nonlinear optimization approach in the second step, that computes and improves all the parameters. But this iterative nonlinear optimization approach has difficulty for onboard implementation apart from the divergence issue of the solution.

II. Camera Models

Two type of camera models have been considered in this section. The first model is considered as a pinhole model where we ignore the fact that, a point is only in focus when its depth and the distance between the optical center of the camera and its image plane obey the thin lens equation. We assume that the camera (equipped with the lens) is focused at infinity so that the distance between the pinhole and the image plane is equal to the focal length [10]. We also assume the image plane is in front of the pinhole. In the first model we neglect the optical distortion associated with the real lenses. The second model takes into account the radial distortion of the lens. In both the camera model [10] we assume that camera is skew less.

A. Camera Model Without Distortion

Let (x, y, z) represent the coordinate of any scene point P_s in a world coordinate system. In the case of star camera mounted in the satellite the world coordinate system is the Earth fixed inertial coordinate system with respect to which star direction P_s is known. Let (x_c, y_c, z_c) represents the coordinates of the same star vector P_c in a camera centered coordinate system. The origin O_c of the camera centered coordinate system is the optical center of the camera, and z_c is the optical axis. A normalized image plane can be associated with the camera at a unit distance [10] from the pinhole parallel to the physical image plane as shown in Fig. 1. A coordinate system $(\hat{u}, \hat{v})$ is attached to the normalized image plane with the center $\hat{O}$ where the optical axis pierces the plane. The perspective projection equation for the normalized coordinate system is as follows:

$$\hat{u} = \frac{x_c}{z_c} \tag{1}$$

$$\hat{v} = \frac{y_c}{z_c} \tag{2}$$

The above equations can be written in an alternative form as

$$\hat{\boldsymbol{p}} = \frac{1}{z_c}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} P_c \\ 1 \end{pmatrix} \tag{3}$$

where $\hat{\boldsymbol{p}} \triangleq (\hat{u}, \hat{v}, 1)^T$ is the homogeneous coordinates of the point $\hat{p}$. The true image plane, which corresponds to the sensing array, is located at a distance f from the camera pinhole with the image coordinates (u, v) of the image point p (Fig. 1) with the center at O. The coordinates of the image point on the image plane are expressed in number of pixels. O' is the principal point, where the optical axis pierces the image plane (usually not at the center of the image plane), with the coordinates (u_0, v_0). As the pixels are normally not square so we assume another two parameters s_u and s_v representing pixel length along u and v direction. The normalized image coordinates are related with the true image coordinates by the relation given as follows:

$$\boldsymbol{p} = \mathcal{K}\hat{\boldsymbol{p}}, \quad \mathcal{K} = \begin{pmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \tag{4}$$

where $\alpha = \frac{f}{s_u} = f_u$ and $\beta = \frac{f}{s_v} = f_v$ are focal length in pixels along u and v direction respectively. Hence, from Eqs (3) and (4) we obtain

$$\boldsymbol{p} = \frac{1}{z_c}\mathcal{M}\boldsymbol{P}_c, \quad where \quad \mathcal{M}_{3\times4} = \begin{pmatrix} \mathcal{K}_{3\times3} & \boldsymbol{0}_{3\times1} \end{pmatrix} \tag{5}$$

where $\boldsymbol{P}_c = (x_c, y_c, z_c, 1)^T$ represents the homogeneous coordinate vector of P_c in camera coordinate system. The parameters in M, relates the vectors in camera coordinate system to the vectors in image coordinate system in image plane, are called intrinsic parameters of the camera.

We now relate the star vector in inertial coordinate frame to the vector in camera coordinate frame through the extrinsic parameters R (rotation matrix defining the orientation of the camera) and t (translation vector defining the position of the camera). The relation is giving by

$$\boldsymbol{P}_c = \begin{pmatrix} R & \boldsymbol{t} \\ \boldsymbol{0}^T & 1 \end{pmatrix}\boldsymbol{P}_s \tag{6}$$

where $\boldsymbol{P}_s$ is the homogeneous coordinate vector of the world scene point P_s. The rotation matrix

$$R = \begin{pmatrix} \mathbf{r}_1{}^T & \mathbf{r}_2{}^T & \mathbf{r}_3{}^T \end{pmatrix}_{3\times3}^T$$

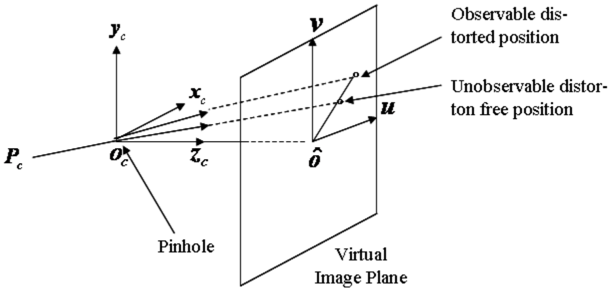

Fig. 2. Distortion Model

and translation vector

$$\boldsymbol{t} = \begin{pmatrix} t_1 & t_2 & t_3 \end{pmatrix}^T_{3\times 1}$$

On substituting (6) into (5) we get

$$\boldsymbol{p} = \frac{1}{z_c}\mathcal{N}\boldsymbol{P}_s \quad where \quad \mathcal{N}_{3\times 4} = \mathcal{K}_{3\times 3}\begin{pmatrix} R_{3\times 3} & \boldsymbol{t}_{3\times 1} \end{pmatrix} \quad (7)$$

The projection matrix $\mathcal{N}$ can be written in terms of four intrinsic parameters $(u_0, v_0, \alpha, \beta)$ and six extrinsic parameters (three angles defining three degree of freedom of R and three components of t) as

$$\mathcal{N} = \begin{pmatrix} \alpha\mathbf{r}_1^T + u_0\mathbf{r}_3^T & \alpha t_1 + u_0 t_3 \\ \beta\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \beta t_2 + v_0 t_3 \\ \mathbf{r}_3^T & t_3 \end{pmatrix} \quad (8)$$

Defining the three rows of $\mathcal{N}$ as $\boldsymbol{n}_1{}^T$, $\boldsymbol{n}_2{}^T$, $\boldsymbol{n}_3{}^T$, (7) can be rewritten as follows:

$$u = \frac{\boldsymbol{n}_1 \cdot \boldsymbol{P}_s}{\boldsymbol{n}_3 \cdot \boldsymbol{P}_s} \quad (9)$$

$$v = \frac{\boldsymbol{n}_2 \cdot \boldsymbol{P}_s}{\boldsymbol{n}_3 \cdot \boldsymbol{P}_s} \quad (10)$$

Star camera calibration and satellite attitude determination would be possible by estimating four intrinsic and six extrinsic parameters where inertial or world coordinates of the stars and their corresponding pixel location in the sensor or image plane are known. But in real scenario the star locations in the image plane can not be known exactly as there are some noise associated in centroid calculation.

B. Camera Model With Distortion

The position of the image points in the image plane is influenced by several types of distortion due to imperfection of camera lenses. In this paper we only consider radial distortion (Fig. 2) among many others because distortion function is totally dominated by radial distortion and especially by first term [11]. Other distortions can also be taken into account with more elaborate model as given in [12], [14]. It has been shown that taking more type of distortions into the model sometimes cause numerical instability [11]. In the present context we have only considered the first term of distortion coefficient. The projection matrix, for the case where only first order radial distortion of lens is present, is given as follows [10]

$$\boldsymbol{p} = \frac{1}{z_c}\begin{pmatrix} 1/\lambda & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}\mathcal{N}\boldsymbol{P}_s, \quad \lambda = k_1 d^2 \quad (11)$$

p be the distorted image point coordinate in pixel, k_1 be the distortion coefficient and $d^2 = \hat{u}^2 + \hat{v}^2$ be the squared distance of the image point from the image center in the normalized image plane. Rewriting d^2 in terms of u and v using (4) we get

$$d^2 = \frac{(u-u_0)^2}{\alpha^2} + \frac{(v-v_0)^2}{\beta^2} \quad (12)$$

Let (u, v) be the ideal (undistorted) pixel coordinate of an image point and corresponding distorted image point coordinate in pixel is (u_d, v_d). Then from (11) and (12) we can write

$$u_d = u + (u-u_0)k_1[\frac{(u-u_0)^2}{\alpha^2} + \frac{(v-v_0)^2}{\beta^2}] \quad (13)$$

$$v_d = v + (v-v_0)k_1[\frac{(u-u_0)^2}{\alpha^2} + \frac{(v-v_0)^2}{\beta^2}] \quad (14)$$

The above equation is based on undistorted image coordinates and used in camera calibration. It transforms an undistorted image point (u, v) into a distorted image point (u_d, v_d).

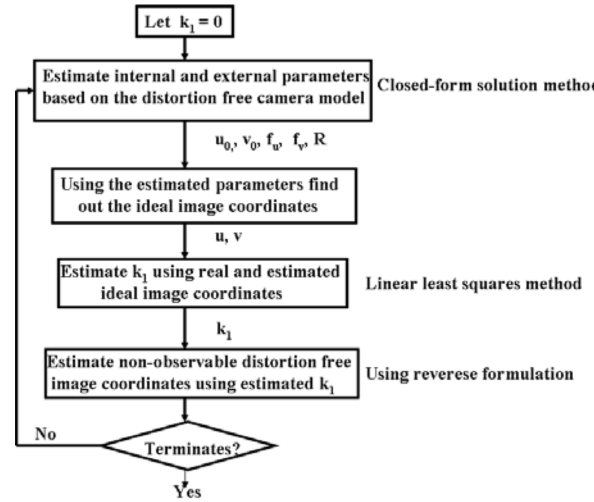

Fig. 3. Flowchart of the Estimation Method

III. Procedure for Estimating Parameters

We adopt two step approach for the calibration of all the camera parameters. As the radial distortion coefficient k_1 is normally assumed to be small so the other four intrinsic and six extrinsic parameters can be estimated [13] using distortion free model given in section II-A taking $k_1 = 0$. The first step consists of a closed form solution of all internal and external parameters based on a distortion free camera model. The second step is a linear least squares based estimation of k_1 after having estimated the other parameters in the first step. These steps are repeated for a number of times for the best estimation of the parameters using closed form linear approach. Algorithm is given below:

1) Let $k_1 = 0$.
2) Compute other parameters (using closed form solution) with k_1 fixed.
3) Compute k_1 with the other parameters fixed at their current estimate.
4) Repeat from step 2 up to a number of iteration by fixing k_1 at its current estimated value. With the estimated value of k_1, unobservable distortion free image points are estimated to be used in the distortion free model at step 2.

In the first iteration we assume that image points are distortion free and we use distortion free camera model to find the other intrinsic and extrinsic parameters. So image points chosen in the first iteration near to the center of the image plane where radial distortion is minimum. But there should be a trade off of selecting the image points as excessive concentration around the center of the image plane hamper the external parameter estimation. Here a radius of one quarter of image side length is considered for selecting image points as in Ref. [12]. A detailed description of parameters estimation procedure is given in the flowchart in Fig. 3.

A. Estimation of Distortion Free Parameters: Closed Form Solution

From (7) the projection matrix $\mathcal{N}$, for distortion free camera model, can be computed if the pixel coordinates of stars and their corresponding inertial coordinates are known. Form this known projection matrix four intrinsic and six extrinsic parameters value can be extracted. The whole procedure is a noniterative linear approach based on closed form solution so very fast enabling it for onboard implementation.

1) Computation of Projection Matrix: From the property of perspective projection, projection matrix in (8) is a nonsingular matrix. Now from perspective projection equation (9), (10), we can write, for ith image point

$$(\boldsymbol{n}_1 - u_i\boldsymbol{n}_3)\cdot\boldsymbol{P}_{s_i} = 0 \tag{15}$$

$$(\boldsymbol{n}_2 - v_i\boldsymbol{n}_3)\cdot\boldsymbol{P}_{s_i} = 0 \tag{16}$$

In the above equation u_i, v_i and homogeneous world coordinate $\boldsymbol{P}_{s_i}$ are known. Taking out the unknown twelve coefficients of matrix $\mathcal{N}$, we can write system of $2m$ linear homogeneous equations for m image points as [10]

$$\begin{pmatrix} \boldsymbol{P}_{s_1}^T & \mathbf{0}^T & -u_1\boldsymbol{P}_{s_1}^T \\ \mathbf{0}^T & \boldsymbol{P}_{s_1}^T & -v_1\boldsymbol{P}_{s_1}^T \\ \cdots & \cdots & \cdots \\ \boldsymbol{P}_{s_m}^T & \mathbf{0}^T & -u_m\boldsymbol{P}_{s_m}^T \\ \mathbf{0}^T & \boldsymbol{P}_{s_m}^T & -v_m\boldsymbol{P}_{s_m}^T \end{pmatrix}\begin{pmatrix} \boldsymbol{n}_1 \\ \boldsymbol{n}_2 \\ \boldsymbol{n}_3 \end{pmatrix} = \mathbf{0} \tag{17}$$

For $m \geq 6$ linear least squares method can be used to find the solution of projection matrix $\mathcal{N}$ with one constraint of unity norm of vector $\boldsymbol{n} = \begin{pmatrix} \boldsymbol{n}_1 & \boldsymbol{n}_2 & \boldsymbol{n}_3 \end{pmatrix}^T$.

2) Estimation of Internal and External Parameters From Projection Matrix: Using the relation between the projection matrix $\mathcal{N}$ and internal, external parameters as in (8), these parameters can be estimated from the known projection matrix given in [10]. For the shake of completeness the procedure is briefly described below. Writing the projection matrix $\mathcal{N} = \begin{pmatrix} \mathcal{A} & \boldsymbol{b} \end{pmatrix}$ we obtain

$$\mu\begin{pmatrix} \mathcal{A} & \boldsymbol{b} \end{pmatrix} = \mathcal{K}\begin{pmatrix} R & \boldsymbol{t} \end{pmatrix} \tag{18}$$

From Eqs. (8) and (18) we get

$$\mu\begin{pmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \\ \boldsymbol{a}_3^T \end{pmatrix} = \begin{pmatrix} \alpha\mathbf{r}_1^T + u_0\mathbf{r}_3^T \\ \beta\mathbf{r}_2^T + v_0\mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix} \tag{19}$$

where μ is the scale factor added to take care of the fact that the computed projection matrix has unit norm i.e., $|\mathcal{N}| = |\boldsymbol{n}| = 1$ and $\boldsymbol{a}_1^T, \boldsymbol{a}_2^T, \boldsymbol{a}_3^T$ are the rows of $\mathcal{A}$. As the rows of a rotation matrix have unit length and are orthogonal to each other we obtain

$$\mu = \varepsilon/|\boldsymbol{a}_3|, \quad where \quad \varepsilon = \mp 1 \tag{20}$$

$$\boldsymbol{r}_3 = \mu\boldsymbol{a}_3 \tag{21}$$

$$u_0 = \mu^2(\boldsymbol{a}_1\cdot\boldsymbol{a}_3) \tag{22}$$

$$v_0 = \mu^2(\boldsymbol{a}_2\cdot\boldsymbol{a}_3) \tag{23}$$

$$\alpha = \mu^2|\boldsymbol{a}_1\times\boldsymbol{a}_3| \tag{24}$$

$$\beta = \mu^2|\boldsymbol{a}_2\times\boldsymbol{a}_3| \tag{25}$$

Sign of internal parameters α, β is known in advance and can be taken positive. Now computing the remaining external parameters as

$$\boldsymbol{r}_1 = \frac{(\boldsymbol{a}_2\times\boldsymbol{a}_3)}{|\boldsymbol{a}_2\times\boldsymbol{a}_3|} \tag{26}$$

$$\boldsymbol{r}_2 = \boldsymbol{r}_3\times\boldsymbol{r}_1 \tag{27}$$

There are two possible choices for the rotation matrix R depending on the value of ε. The translation parameters can be recovered by using the relation (from (18))

$$\mathcal{K}\boldsymbol{t} = \mu\boldsymbol{b}$$

which gives

$$\boldsymbol{t} = \mu\mathcal{K}^{-1}\boldsymbol{b} \tag{28}$$

the sign of t_3 depends on the fact that whether the origin of the world coordinate system is in front or behind the camera which in turn decides the sign of ε.

B. Estimation of Radial Distortion Coefficient

From Eqs. (13) and (14) we have two equations for ith image point as follows:

$$\begin{bmatrix} (u_i-u_0)[\frac{(u_i-u_0)^2}{\alpha^2}+\frac{(v_i-v_0)^2}{\beta^2}] \\ (v_i-v_0)[\frac{(u_i-u_0)^2}{\alpha^2}+\frac{(v_i-v_0)^2}{\beta^2}] \end{bmatrix} k_1 = \begin{bmatrix} u_{d_i}-u_i \\ v_{d_i}-v_i \end{bmatrix} \tag{29}$$

For m image points we would get $2m$ linear equations which is in matrix form as

$$Dk_1 = d \tag{30}$$

Linear least squares solution for the distortion coefficient is given by [13]

$$k_1 = (D^T D)^{-1} D^T d \tag{31}$$

After estimating the distortion coefficient k_1, unobservable distortion free image points are estimated by the inversed formula that transforms a distorted image point (u_d, v_d) into an undistorted image point (u, v) [14]. The reversed formula can be expressed as

$$u = u_d + (u_d - u_0)k_1[\frac{(u_d - u_0)^2}{\alpha^2} + \frac{(v_d - v_0)^2}{\beta^2}] \tag{32}$$

$$v = v_d + (v_d - v_0)k_1[\frac{(u_d - u_0)^2}{\alpha^2} + \frac{(v_d - v_0)^2}{\beta^2}] \tag{33}$$

These estimated distortion free points are used in the $2nd$ step of the algorithm for estimating internal and external parameters using distortion free camera model.

Using the procedure described above satellite attitude can be estimated by pre multiplying the estimated rotation matrix R with the transpose of sensor attitude matrix with respect to body (satellite).

IV. Simulation Result

We have simulated star observations at 10 hz update rate for a $20° \times 20°$ FOV star camera with 1024×1024 pixel array. The pixel coordinate of principal point offset is taken as $u_0 = 512.75$, $v_0 = 512.25$. The focal length and pixel length of star camera is assumed to be $f = 49.5$mm, $s_u = 0.016$mm, and $s_v = 0.014$mm respectively. The star observations are corrupted by adding centroiding noise $\sigma_{centroid} = 17\mu$rad [2]. Simulation results have been obtained for the first order radial distortion coefficient $k_1 = 5e-05$ and $k_1 = -5e-04$.

A star catalog of 1614 stars is formed by taking stars up to visual magnitude 5.0 from the basic SKYMAP 2000 catalog. In the inertial star catalog only the unit direction of the stars with respect to inertial frame is given. So, in the simulation to generate synthetic star image, world coordinate (unity norm) of the stars are projected to the image plane using perspective projection equations (9) and (10) with $\boldsymbol{t} = \mathbf{0}$. Though translation vector $\boldsymbol{t}$ is used in the formulation, for estimating satellite attitude and camera calibration parameters, we don't require to estimate this external parameter. To show the improvement in the attitude estimation error by our proposed methodology, attitude estimation error by ESOQ2 method [15] for the uncalibrated camera parameters is shown in Fig. 4. Simulation results for satellite attitude estimation and camera calibration parameters obtained by our proposed methodology, are given by the figures 5, 6, 7, 8 for radial distortion coefficient $k_1 = 5e - 04$.

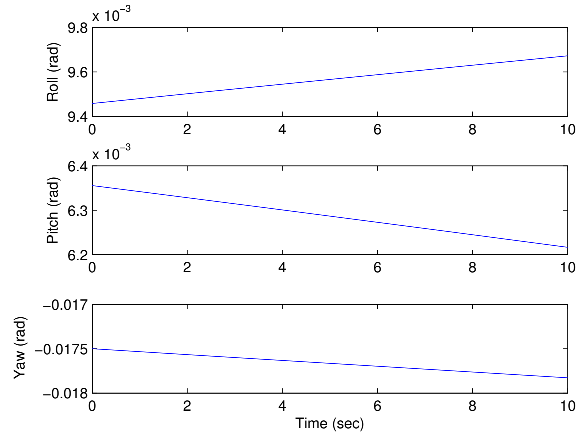

Fig. 4. Attitude Estimation Errors by ESOQ2 for uncalibrated camera

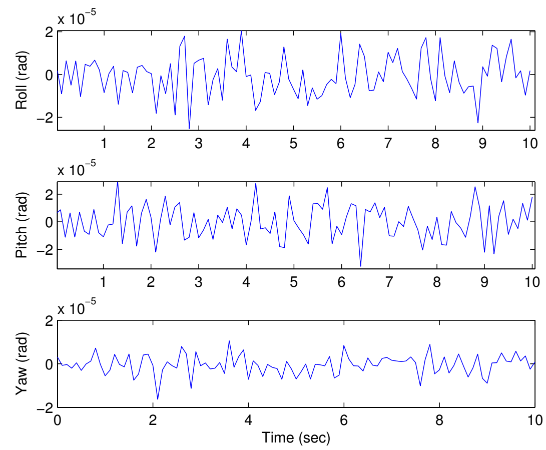

Fig. 5. Attitude Estimation Errors with $k_1 = -5e - 04$

V. Conclusions

This method of estimating satellite attitude and camera parameters uses the star image points on the sensor plane and corresponding star direction with respect to inertial frame. Thus there is a need for catalog star identification process. However, star identification processes use the fact that interstar angles remain unchanged under rotational transformation. But for that star image points need to convert into the star vector in camera coordinate frame utilizing focal length and principal point offset. Hence star identification process must be robust with respect to the uncalibrated camera parameters and lens distortion.

Unlike other attitude estimation method, which uses measured star vector in camera coordinate frame, this method uses measured image points directly. Thus there is no coupling between calibration parameters and attitude matrix.

The actual constraint in the intermediate parameters have not been considered in the closed-form solution of projection matrix. Hence, for higher order lens distortion and centroiding noise the accuracy of the estimation will be poor because intermediate parameters will not satisfy the constraints.

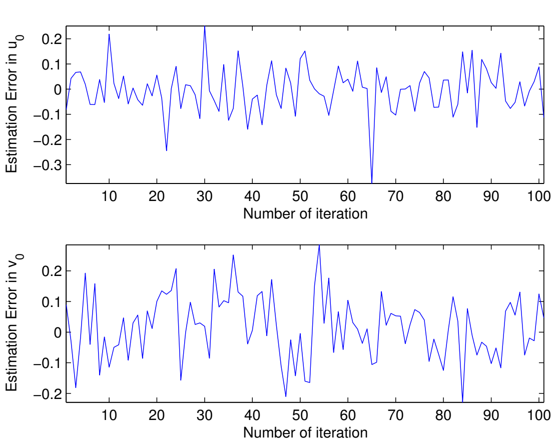

Fig. 6. Estimation Error in Principal Point offset with $k_1 = -5e - 04$

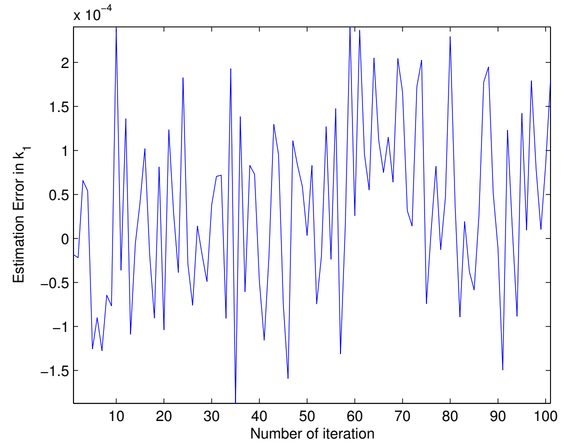

Fig. 8. Estimation Error in k_1

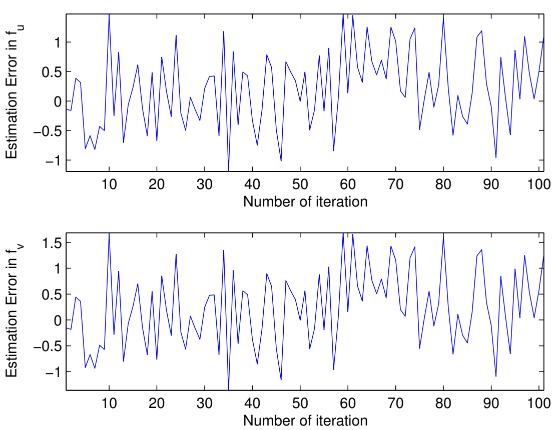

Fig. 7. Estimation Error in Focal Length with $k_1 = -5e - 04$

To get the solution by this method the condition of minimum six stars in the FOV must be satisfied.

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