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Quasi-decentralized Control of Process Systems Using Wireless Sensor Networks with Scheduled Sensor Transmissions

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Abstract— **This paper develops a model-based networked control and scheduling framework for plants with interconnected units and distributed control systems that exchange information using a resource-constrained wireless sensor network (WSN). The framework aims to enforce closed-loop stability while simultaneously minimizing the rate at which each node in the WSN must collect and transmit measurements so as to conserve the limited resources of the wireless devices and extend the lifetime of the network as much as possible. Initially, the exchange of information between the local control systems is reduced by embedding, within each control system, dynamic models that provide forecasts of the evolution of the plant units when measurements are not transmitted through the WSN, and updating the state of each model when communication is re-established at discrete time instances. To further reduce WSN utilization, only a subset of the deployed sensor suites are allowed to transmit their data at any given time to provide updates to their target models. By formulating the networked closed-loop plant as a combined discrete-continuous system, an explicit characterization of the maximum allowable update period is obtained in terms of the sensor transmission schedule, the transmission times of the different sensor suites, the uncertainty in the models as well as the controller design parameters. It is shown that by judicious selection of the transmission schedule and the models, it is possible to enhance the savings in WSN resource utilization over what is possible with concurrent transmission condigurations. Finally, the results are illustrated using a network of chemical reactors with recycle.**

I. Introduction

The convergence of recent advances in sensor manufacturing, wireless communications and digital electronics has produced low-cost wireless sensor networks (WSNs) that can be installed for a fraction of the cost of wired devices (e.g., [1], [2], [3], [4]). WSNs offer unprecedented flexibility ranging from high density sensing capabilities to deployment in areas where wired devices may be difficult or impossible to deploy (such as inside waterways and high-temperature areas in oil refineries). Augmenting existing process control systems with additional WSNs has the potential to expand the capabilities of the existing control technology beyond what is feasible with the wired networked architectures alone. Specifically, deploying additional WSNs throughout the plant and interfacing those devices with the existing control systems, permit collecting and broadly disseminating additional real-time information about the state of the plant units which in turn can be used to enhance the performance and robustness of the plant operations. The extra information, together with the increased levels of sensor redundancy achieved with WSNs, also enable achieving proactive fault-tolerance and real-time plant reconfiguration based on anticipated market demand changes. These are appealing goals that coincide with the recent calls over the last few years for expanding the traditional process control and operations paradigm in the direction of smart plant operations [5], [6].

To harness the full potential of WSNs in process control, there is a need to address the fundamental challenges introduced by this technology from a control point of view. One of the main challenges to be addressed when deploying a low-cost WSN for control is that of handling the inherent constraints on network resources, including the limitations on the computation, processing and communication capabilities. Other constraints such as limited power (battery energy) are also important when the WSN is deployed in harsh or inaccessible environments where a continuous power supply is not feasible and the wireless devices have to rely on battery power instead [4]. A tradeoff exists between the achievable control performance and the extent of network resource utilization. Specifically, maximizing the control performance requires continuous (or at least frequent) collection of data and disseminating it broadly to the target control systems. On the other hand, the limited resources of a WSN, together with the difficulty of frequent battery replacement in a plant environment, suggest that sensing and communication should be reduced in order to aggressively conserve resources and extend the lifetime of the network as much as possible. Realizing the potential of WSNs to improve process control requires proper characterization and management of this tradeoff.

An effort to address this problem was initiated in [7] where a quasi-decentralized networked control architecture was developed for plants with interconnected units that exchange information over a shared communication network and the minimum allowable communication frequency was characterized for the case when all sensor suites are given simultaneous access to the network. In addition to controlling the transmission frequencies of individual sensors in the network, another important way of conserving the WSN resources is to select and activate only a subset of the deployed sensor suites at any given time to communicate with the rest of the plant. Under this restriction, the stability and performance of each unit in the plant becomes dependent

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not only on the controller design but also on the selection of the scheduling strategy that, at any time, determines the order in which the sensor suites of the neighboring units transmit. The scheduling problem is also important in cases where access constraints in the wireless communications medium limit the number of available channels so that at any one time only some of the sensors and actuators can exchange information, while others must wait (e.g., see [8], [9], [10], [11]; see also [12], [13], [14], [15], [16] for additional results and references on networked control).

Motivated by these considerations, we present in this work a model-based sensor scheduling approach for enhancing power management in WSNs deployed within a large-scale distributed plant with interconnected processing units. The objective is to find an optimal strategy for establishing and terminating communication between the sensors suites (or nodes) of the WSN and the target controllers that minimizes the rate at which each node in the WSN must collect and disseminate data to the rest of the plant without jeopardizing closed-loop stability. The rest of the paper is organized as follows. Following some preliminaries in Section II, the problem of scheduling sensor transmissions in a WSN is formulated. Section III then presents the quasi-decentralized control structure and describes its implementation using the WSN with the aid of appropriate process models and sensor transmission scheduling. The closed-loop system is then formulated and analyzed in Section IV where precise conditions for closed-loop stability are provided. The approach is illustrated in Section V using a chemical plant example, and conclusions are given in Section VI.

II. Preliminaries

A. Plant description

We consider a large-scale distributed plant composed of n interconnected processing units, each of which is modeled by a continuous-time linear system, and represented by the following state-space description:

$$
\begin{aligned}
\dot{x}_1 &= A_1 x_1 + B_1 u_1 + \textstyle\sum_{j=2}^{n} A_{1j} x_j \\
\dot{x}_2 &= A_2 x_2 + B_2 u_2 + \textstyle\sum_{j=1, j\neq 2}^{n} A_{2j} x_j \\
&\;\vdots \\
\dot{x}_n &= A_n x_n + B_n u_n + \textstyle\sum_{j=1}^{n-1} A_{nj} x_j
\end{aligned}
\tag{1}
$$

where $x_i := [x_i^{(1)} \; x_i^{(2)} \; \cdots \; x_i^{(p_i)}]^T \in \mathbb{R}^{p_i}$ denotes the vector of process state variables associated with the i-th processing unit, $u_i := [u_i^{(1)} \; u_i^{(2)} \; \cdots \; u_i^{(q_i)}]^T \in \mathbb{R}^{q_i}$ denotes the vector of manipulated inputs associated with the i-th processing unit, x^T denotes the transpose of a column vector x, A_i, B_i, and A_{ij} are constant matrices. The interconnection term $A_{ij}x_j$, where $i \neq j$, describes how the dynamics of the i-th unit are influenced by the j-th unit in the plant. Note from the summation notation in Eq.1 that each processing unit can in general be connected to all the other units in the plant.

B. Problem formulation and solution methodology

Referring to the schematic plant in Fig.1, we consider a quasi-decentralized control structure in which each unit in the plant has a local control system with its sensors and actuators connected to the local controller through a dedicated wired communication network. An additional suite of wireless sensors is deployed within each unit to transfer measurements of the local state variables to the plant supervisor as well as the other distributed control systems in the plant in order to account for the interactions between the units and minimize disturbance propagation. The various sensor suites form a plant-wide WSN through which the plant units communicate. The overall objective

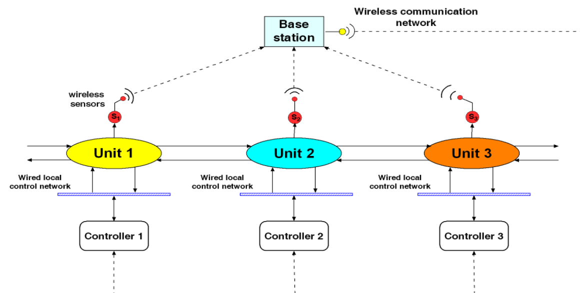

Fig. 1. Quasi-decentralized control structure with cross communication over a plant-wide wireless sensor network.

is to stabilize all the plant units at the origin while keeping the data collection and dissemination by the WSN to a minimum. Minimizing the frequency at which each suite (or node) in the WSN needs to collect and broadcast its data is desired to avoid the unnecessary utilization of the WSN power resources and help prolong the service life of the network. To address the resource-constraint problem, we develop in the next section an integrated model-based quasi-decentralized control and scheduling strategy that reduces the exchange of information between the plant units without loss of stability. This is accomplished by (1) including models within each control system to estimate the interaction terms when measurements are not available, and (2) limiting the number of WSN nodes that, at any given time, transmit their data to update the corresponding target models. The problem is to find an optimal scheduling strategy for establishing and terminating communications between the sensor suites and the target controllers. To illustrate the main ideas, we will consider as an example the configuration where only one wireless sensor suite is allowed to transmit its measurement updates to the appropriate units at any one time, while the others remain dormant for some time before the next suite is allowed to transmit its data. Also, to simplify the presentation of our results, we will focus in this work on the full state feedback problem where the states of all the units are available as measurements. Extensions to the output feedback case are possible and the subject of other research work.

III. Quasi-decentralized Networked Control with Scheduled Sensor Transmissions

A. Controller synthesis

To realize the quasi-decentralized control structure, the first step is to synthesize for each unit a stabilizing feedback controller of the general form:

$$
u_i(x) = K_i x_i + \textstyle\sum_{j=1, j\neq i}^{n} K_{ij} x_j \tag{2}
$$

where K_i is the local feedback component responsible for stabilizing the i-th subsystem in the absence of interconnections, and K_{ij} is a "feedforward" component that compensates for the effect of the j-th neighboring subsystem on the dynamics of the i-th unit. Note that the implementation of the control law of Eq.2 requires the availability of state measurements from both the local subsystem being controlled and the connected units. Without loss of generality, we consider the case where measurements of the local state are available to the local controller more frequently than measurements from the other connected plant units. This is a reasonable scenario given that the local information is transmitted over a dedicated wired network, while transmission of the other measurements involves using the shared plant-wide WSN, which is potentially unreliable. However, it is possible to generalize the control structure to account for possible limitations in the local networks.

B. Handling network resource constraints

1) A model-based networked control structure: The first step in conserving the plant-wide WSN resources is to reduce the transfer of information between the wireless sensor suites and the target control systems as much as possible without sacrificing closed-loop stability. To this end, and following the idea presented in [7], we include in each control system a set of dynamic models that provide estimates of the evolution of the states of the neighboring units when their state measurements are not available from the WSN. The use of a model to recreate the interactions of a given unit with one of its neighbors allows the wireless sensor suite of each neighboring unit to send its data in a discrete fashion since the model can provide an approximation of that unit's dynamics. "Feedforward" from one unit to another is then performed by updating the states of the model using the actual state measurements provided by the wireless sensors of the corresponding unit at discrete time instances. With this setup, the local control law for each unit is implemented as follows:

$$
\begin{aligned}
u_i(t) &= K_i x_i(t) + \sum_{j=1, j\neq i}^{n} K_{ij}\widehat{x}_j^i(t),\ t \neq t_k^j,\ i = 1, 2, \cdots, n \\
\dot{\widehat{x}}_j^i(t) &= \widehat{A}_j \widehat{x}_j^i(t) + \widehat{B}_j \widehat{u}_j^i(t) + \widehat{A}_{ji} x_i(t) + \sum_{l=1, l\neq i, l\neq j}^{n} \widehat{A}_{jl}\widehat{x}_l^i(t), t \neq t_k^j \\
\widehat{u}_j^i(t) &= K_j \widehat{x}_j^i(t) + K_{ji} x_i(t) + \sum_{l=1, l\neq i, l\neq j}^{n} K_{jl}\widehat{x}_l^i(t),\quad t \neq t_k^j \\
\widehat{x}_j^i(t_k^j) &= x_j(t_k^j),\quad j = 1, \cdots, n, j \neq i,\quad k = 0, 1, 2, \cdots
\end{aligned} \tag{3}
$$

where $\widehat{x}_j^i$ is an estimate of x_j, used by the local control system of the i-th unit, $\widehat{A}_j$, $\widehat{B}_j$, $\widehat{A}_{jl}$ are constant matrices that do not necessarily match the actual dynamics of the j-th process, i.e., in general $\widehat{A}_j \neq A_j$, $\widehat{B}_j \neq B_j$, $\widehat{A}_{jl} \neq A_{jl}$. Notice that since x_i is available continuously, it is used directly by all the models embedded in the i-th control system. The notation t_k^j is used to indicate the k-th transmission time for the j-th sensor suite in the WSN.

Remark 1: By limiting the rate at which measurements from a given unit need to be communicated to the rest of the plant, the quasi-decentralized control structure offers a possible compromise (in terms of implementation) between the complexity of traditional centralized controllers and the performance limitations of fully decentralized control schemes (e.g., see [17], [18], [19]). The problem of controlling large-scale multi-unit plants has also been studied within other frameworks, such as passivity-based control [20], distributed model predictive control (e.g., [21], [22]), agent-based systems [23] and singular perturbation formulations [24]. In these works, however, the problem of integrating WSNs into the plant-wide control structure and the subsequent communication issues that this raises have not been studied.

2) WSN power management through scheduled sensor transmissions and model updates: A key parameter in the analysis of the control and update laws in Eq.3 is the update period for each sensor suite, $h^j := t_{k+1}^j - t_k^j$, which determines the frequency at which the j-th wireless sensor suite collects and sends measurements to the other units through the network to update the corresponding model estimates. To simplify the analysis, we consider in what follows only the case when the update period is constant and the same for all units, so that $t_{k+1}^j - t_k^j := h,\ j = 1, 2, \cdots, n$. The update period is also an important measure of the extent of network resource utilization, with a larger h indicating a larger reduction in resource utilization. To further reduce network utilization, we perform sensor scheduling whereby only one wireless sensor suite is allowed to transmit its measurements to the appropriate units at any one time, while the other suites remain dormant for some time before the next suite is allowed to transmit its data (the analysis can be generalized to cases where multiple suites transmit at the same time). The transmission schedule is defined by (1) the sequence (or order) of transmitting suites:

$$\{s_j,\ j = 1, 2, \cdots, n\},\quad s_j \in \mathcal{N} := \{1, 2, \cdots, n\},$$

where s_j is a discrete variable that denotes the j-th transmitting entity in the sequence, and (2) the time at which each node in the sequence collects and transmits measurements. To characterize the transmission times, we introduce the variable:

$$\Delta t_j := t_k^{s_{j+1}} - t_k^{s_j},\quad j = 1, 2, \cdots, n - 1,$$

which is the time interval between the transmissions of two consecutive nodes in the sequence. Fig.2 is a schematic

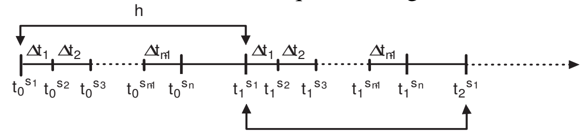

Fig. 2. A schematic showing the time-line for the transmission of each sensor suite in an h-periodic schedule.

representation of how sensor scheduling is performed. Note that the schedule is h-periodic in that the same sequence of transmitting nodes is executed repeatedly every h seconds (equivalently, each node transmits its data every h seconds). Note also from the definitions of both h and Δt_j that we always have the constraint $\sum_{j=1}^{n-1}\Delta t_j < h$. Since the update periods for all units are the same, the time intervals between the transmission times of two specific units are constant, and within any single execution of the schedule (which lasts less than h seconds), each sensor suite can only transmit its measurements through the WSN and update its models in the local control systems of its neighbors once. This can be represented mathematically by the condition: $s_i \neq s_j$ when $i \neq j$. By manipulating the time intervals Δt_j (i.e., the transmission times) and the order in which the nodes transmit, one can search for the optimal sensor transmission schedule that leads to the largest update period (or the smallest communication rate between each sensor suite and its target units).

IV. Networked closed-loop stability analysis

A. A combined discrete-continuous system formulation

The successful implementation of the networked control and scheduling strategies described in Section III requires characterizing the minimum allowable frequency at which each wireless sensor suite must collect and transmit its data to the appropriate target control systems (or maximum allowable update period) for a given sensor transmission schedule. To this end, we define the model estimation errors by $e_j^i = x_j - \widehat{x}_j^i$, for $j \neq i$, and $e_j^i = 0$, for $j = i$, where e_j^i represents the difference between the state of the j-th unit and the state of its model embedded in the local control system of the i-th unit. Note that since measurements of x_i are assumed to be available to the local control system of the i-th unit at all times, we always have $e_i^i = 0$. Introducing the augmented vectors: $\mathbf{e}_j := [(e_j^1)^T\ (e_j^2)^T\ \cdots\ (e_j^n)^T]^T$, $\mathbf{e} := [\mathbf{e}_1^T\ \mathbf{e}_2^T\ \cdots\ \mathbf{e}_n^T]^T$, $\mathbf{x} := [x_1^T\ x_2^T\ \cdots\ x_n^T]^T$, it can be shown that the overall closed-loop plant of Eq.1 and Eq.3 can be formulated as a combined discrete-continuous system of the form:

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \Lambda_{11}\mathbf{x}(t) + \Lambda_{12}\mathbf{e}(t) \\
\dot{\mathbf{e}}(t) &= \Lambda_{21}\mathbf{x}(t) + \Lambda_{22}\mathbf{e}(t), \quad t \neq t_k^j \\
\mathbf{e}_j(t_k^j) &= \mathbf{0}, \quad j = 1, 2, \cdots, n, \quad k = 0, 1, 2, \cdots,
\end{aligned}
\tag{4}
$$

where the plant states evolve continuously in time and the estimation errors are reset to zero at each transmission instance. Note, however, that unlike the case of simultaneous sensor transmission (where no scheduling takes place) which was investigated in [7], not all models within a given unit are updated (and hence not all estimation errors are re-set to zero) at each transmission time. Instead, only the model of the transmitting unit is updated using the measurements provided by the wireless sensor suite of that particular unit.

Referring to Eq.4, Λ_{11}, Λ_{12}, Λ_{21}, and Λ_{22} are $m\times m$, $m\times mn$, $mn\times m$, and $mn\times mn$ constant matrices, respectively, where $m = \sum_{i=1}^{n} p_i$ and p_i is the dimension of the i-th state vector. These matrices are linear combinations of A_i, B_i, A_{ij}, $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{A}_{ij}$, K_i, K_{ij}, which are the matrices used to describe the dynamics, the models, and the control laws of the different units. The explicit forms of these matrices are omitted for brevity but can be obtained by substituting Eq.3 into Eq.1. Defining the augmented state vector $\xi(t) := [\mathbf{x}^T(t)\ \ \mathbf{e}^T(t)]^T$, the dynamics of the overall closed-loop system can be written as:

$$
\begin{aligned}
\dot{\xi}(t) &= \Lambda\xi(t), \quad t \neq t_k^j \\
\xi(t_k^j) &= \left[\mathbf{x}^T(t_k^j)\ \mathbf{e}_1^T(t_k^j)\ \cdots\ \mathbf{e}_{j-1}^T(t_k^j)\ \mathbf{0}\ \mathbf{e}_{j+1}^T(t_k^j)\ \cdots\ \mathbf{e}_n^T(t_k^j)\right]^T
\end{aligned}
\tag{5}
$$

where $k = 0, 1, 2, \cdots$, and $\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}$.

B. Characterizing the scheduled closed-loop response

In order to derive conditions for closed-loop stability, we need first to express the plant response as a function of the update period and the sensor transmission schedule. The following proposition provides the needed characterization.

Proposition 1: *Consider the closed-loop system described by Eq.5 with a transmission schedule* $\{s_1, s_2, \cdots, s_n\}$ *and the initial condition* $\xi(t_0^{s_1}) = [\mathbf{x}^T(t_0^{s_1})\ \ \mathbf{e}_1^T(t_0^{s_1}),\ \cdots,\ \mathbf{e}_n^T(t_0^{s_1})]^T = \xi_0$, *with* $\mathbf{e}_{s_1}(t_0^{s_1}) = \mathbf{0}$. *Then:*

(a) For $t \in [t_k^{s_j},\ t_k^{s_{j+1}})$, $j = 1, 2, \cdots, n-1$, $k = 0, 1, 2, \cdots$, *the closed-loop system response is given by:*

$$
\xi(t) = e^{\Lambda(t - t_k^{s_j})} I_s^{s_j} \Gamma_j(\Delta t_j, I_s^{s_j}) M^k \xi_0 \tag{6}
$$

where

$$
\Gamma_j = \prod_{j-1-\mu=0}^{j-2} e^{\Lambda\Delta t_\mu} I_s^{s_\mu}, \text{for } j \geq 2, \text{and } \Gamma_j = I, \text{ for } j = 1 \tag{7}
$$

and M as given in Eq.9.

(b) For $t \in [t_k^{s_n},\ t_{k+1}^{s_1})$, $k = 0, 1, 2, \cdots$, *the closed-loop system response is given by:*

$$
\xi(t) = e^{\Lambda(t - t_k^{s_n})} I_s^{s_n} \left[\prod_{n-1-\mu=0}^{n-2} e^{\Lambda\Delta t_\mu} I_s^{s_\mu}\right] M^k \xi_0 \tag{8}
$$

where

$$
M = e^{\Lambda(h - \sum_{j=1}^{n-1}\Delta t_j)} I_s^{s_n} \left[\prod_{n-1-\mu=0}^{n-2} e^{\Lambda\Delta t_\mu} I_s^{s_\mu}\right] \tag{9}
$$

$$
I_s^{s_j} = \begin{bmatrix} I & O & \cdots & O \\ O & H_1 & \cdots & O \\ \vdots & \vdots & & \vdots \\ O & O & \cdots & H_n \end{bmatrix}, \ H_i = \begin{cases} I, & i \neq s_j \\ O, & i = s_j \end{cases} \tag{10}
$$

for $j = 1, 2, \cdots, n$, $t_{k+1}^{s_j} - t_k^{s_j} = h$ *and* $\Delta t_j = t_k^{s_{j+1}} - t_k^{s_j}$, $j = 1, 2, \cdots, n-1$.

Proof: First, we have from Eq.5 that at times t_k^j only the error $\mathbf{e}_j(t)$ is reset to zero. This can be represented by writing $\xi(t_k^j) = I_s^j \xi(t_k^{j-})$, where I_s^j is given in Eq.10. Then on the interval $t \in [t_k^{s_j},\ t_k^{s_{j+1}})$, $j = 1, 2, \cdots, n-1$, the system response is:

$$
\xi(t) = e^{\Lambda(t - t_k^{s_j})} \xi(t_k^{s_j}) = e^{\Lambda(t - t_k^{s_j})} I_s^{s_j} \xi(t_k^{s_j -}) \tag{11}
$$

In view of Eq.11, we obtain by induction:

$$
\begin{aligned}
\xi(t) &= e^{\Lambda(t - t_k^{s_j})} I_s^{s_j} e^{\Lambda(t_k^{s_j} - t_k^{s_{j-1}})} \xi(t_k^{s_{j-1}}) \\
&= e^{\Lambda(t - t_k^{s_j})} I_s^{s_j} e^{\Lambda\Delta t_{j-1}} I_s^{s_{j-1}} \xi(t_k^{s_{j-1}^-}) \\
&\ \ \vdots \\
&= e^{\Lambda(t - t_k^{s_j})} I_s^{s_j} \Gamma_j \xi(t_k^{s_1 -})
\end{aligned}
\tag{12}
$$

On the interval $t \in [t_k^{s_n},\ t_{k+1}^{s_1})$, the system response is:

$$\xi(t) = e^{\Lambda(t-t_k^{s_n})}\xi(t_k^{s_n}) = e^{\Lambda(t-t_k^{s_n})}I_s^{s_n}\xi(t_k^{s_n-}) \qquad (13)$$

Using Eq.12 to calculate $\xi(t_k^{s_n-})$ we get:

$$\xi(t) = e^{\Lambda(t-t_k^{s_n})}I_s^{s_n}\prod_{n-1-\mu=0}^{n-2} e^{\Lambda\Delta t_\mu}I_s^{s_\mu}\xi(t_k^{s_1-}) \qquad (14)$$

which can be used to write:

$$\begin{aligned}\xi(t) &= e^{\Lambda(t-t_k^{s_n})}I_s^{s_n}\Gamma_n e^{\Lambda(t_k^{s_1}-t_{k-1}^{s_n})}I_s^{s_n}\Gamma_n\xi(t_{k-1}^{s_1-})\\ &\vdots\\ &= e^{\Lambda(t-t_k^{s_n})}I_s^{s_n}\Gamma_n\left[e^{\Lambda(h-\sum_{j=1}^{n-1}\Delta t_j)}I_s^{s_n}\Gamma_n\right]^k\xi(t_0^{s_1})\\ &= e^{\Lambda(t-t_k^{s_n})}I_s^{s_n}\Gamma_n M^k\xi(t_0^{s_1}), t\in[t_k^{s_n},\ t_{k+1}^{s_1})\\ &,\quad k=0,1,2,\cdots\end{aligned}$$

where Γ_n can be obtained from Eq.7. In the same way, we obtain from Eq.12 that:

$$\begin{aligned}&= e^{\Lambda(t-t_k^{s_j})}I_s^{s_j}\Gamma_j e^{\Lambda(t_k^{s_1}-t_{k-1}^{s_n})}I_s^{s_n}\Gamma_n\xi(t_{k-1}^{s_1-})\\ &\vdots\\ &= e^{\Lambda(t-t_k^{s_j})}I_s^{s_j}\Gamma_j[e^{\Lambda(h-\sum_{j=1}^{n-1}\Delta t_j)}I_s^{s_n}\Gamma_n]^k\xi(t_0^{s_1})\\ &= e^{\Lambda(t-t_k^{s_j})}I_s^{s_j}\Gamma_j M^k\xi(t_0^{s_1}), t\in[t_k^{s_j},\ t_k^{s_{j+1}})\end{aligned}$$

This completes the proof of the proposition.

Remark 2: The expression in Eqs.6-7 captures the closed-loop response during the time periods between the transmissions of two consecutive sensor suites in a given execution of the schedule, while the expression in Eq.8 provides the closed-loop response for the time period between the transmission of the last sensor suite in a given execution and the transmission of the first sensor suite in the next execution round (see Fig.2). As expected the responses are parameterized by the transmission sequence (which determines the structure of the matrices $I_s^{s_j}$) as well as the transmission times (which are determined by Δt_j). Note that the term M^k captures the repetitive nature of the transmission schedule execution.

C. Characterizing the maximum allowable update period

The following theorem provides a sufficient condition for stability of the closed-loop plant under the quasi-decentralized networked control structure with scheduled sensor transmissions in the WSN.

Theorem 1: *Referring to the system of Eq.5 with a transmission schedule $\{s_1, s_2, \cdots, s_n\}$ and the initial condition $\xi(t_0^{s_1}) = [\mathbf{x}^T(t_0^{s_1})\ \ \mathbf{e}_1^T(t_0^{s_1}),\ \cdots,\ \mathbf{e}_n^T(t_0^{s_1})]^T = \xi_0$ with $\mathbf{e}_{s_1}(t_0^{s_1}) = \mathbf{0}$, the zero solution, $\xi = [\mathbf{x}^T\ \mathbf{e}^T]^T = [\mathbf{0}\ \mathbf{0}]^T$, is globally exponentially stable if the eigenvalues of the test matrix given in Eq.9 are strictly inside the unit circle.*

Proof: Evaluating the norm of the response of the scheduled closed-loop system described in Proposition 1, we have from Eq.6 that for $t \in [t_k^{s_j},\ t_k^{s_{j+1}}),\ j = 1, 2, \cdots, n-1,\ k = 0, 1, 2, \cdots$:

$$\|\xi(t)\| \leq \|e^{\Lambda(t-t_k^{s_j})}\|\|I_s^{s_j}\|\|\Gamma_j(\Delta t_j, I_s^{s_j})\|\|M^k\|\|\xi_0\|$$

where $\|\Gamma_j(\Delta t_j, I_s^{s_j})\| \leq \prod_{j-1-\mu=0}^{j-2}\|e^{\Lambda\Delta t_\mu}\|\|I_s^{s_\mu}\|$. Since $\|I_s^{s_j}\| = 1,\quad j = 1, 2, \cdots, n-1$, and $\Delta t_j < h$, we can write:

$$\|\xi(t)\| \quad\leq\quad k_1\|e^{\Lambda(t-t_k^{s_j})}\|\cdot\|M^k\|\|\xi_0\| \qquad (15)$$

where $k_1 = \prod_{j-1-\mu=0}^{j-2}\|e^{\Lambda\Delta t_\mu}\|$ is a positive constant. Analyzing the first term on the right hand side of Eq.15:

$$\begin{aligned}\|e^{\Lambda(t-t_k^{s_j})}\| &\leq \textstyle\sum_{i=0}^{\infty}\|\frac{1}{i!}\Lambda^i(t-t_k^{s_j})^i\|\\ &= \textstyle\sum_{i=0}^{\infty}\|\frac{1}{i!}(t-t_k^{s_j})^i\sigma^i\|\\ &= e^{\sigma(t-t_k^{s_j})} \leq e^{\sigma\Delta t_j} := k_2\end{aligned} \qquad (16)$$

where σ is the largest singular value of Λ. In general this term can always be bounded since the time difference $t-t_k^{s_j}$ is always smaller than Δt_j. In other words, even if Λ has eigenvalues with positive real parts, $\|e^{\Lambda(t-t_k^{s_j})}\|$ can only grow a certain amount, and this growth is independent of k. The second term on the right hand side of Eq.15 is bounded if and only if all the eigenvalues of M lie inside the unit circle, i.e.,:

$$\|M^k\| \leq k_3 e^{-\alpha k} = k_3 e^{\alpha}e^{-\alpha\frac{t_{k+1}^{s_1}}{h}} \qquad (17)$$

for some $k_3, \alpha > 0$, where we have used the fact that $k = t_k^{s_1}/h$ to establish the equality. Since $t \in [t_k^{s_j},\ t_k^{s_{j+1}})$ and $t_{k+1}^{s_1} > t$, we obtain:

$$\|M^k\| \leq k_3 e^{\alpha}e^{-\alpha\frac{t_{k+1}^{s_1}}{h}} \leq k_3 e^{\alpha}e^{-\alpha\frac{t}{h}} := \bar{k}_3 e^{-\bar{\alpha}t} \qquad (18)$$

where $\bar{k}_3 = k_3 e^{\alpha} > 0$ and $\bar{\alpha} = \alpha/h > 0$. Combining Eq.15 with Eqs.(16) and (18), we finally arrive at the following bound on the augmented state:

$$\|\xi(t)\| \leq k_4\|\xi_0\|e^{-\bar{\alpha}t},\ t \in [t_k^{s_j},\ t_k^{s_{j+1}}) \qquad (19)$$

where $k_4 = k_1k_2\bar{k}_3 > 0$. In a similar fashion, one can show that on the interval $t \in [t_k^{s_n},\ t_{k+1}^{s_1}),\quad k = 0, 1, 2, \cdots$, the closed-loop response satisfies a bound of the form $\|\xi(t)\| \leq k_5\|\xi_0\|e^{-\bar{\beta}t}$ for some $k_5, \bar{\beta} > 0$. This, together with Eq.19, implies that the origin of the networked closed-loop system is globally exponentially stable. This completes the proof.

Remark 3: The requirement that the eigenvalues of the matrix M lie strictly inside the unit circle ensures stability by limiting the growth of the closed-loop state within each sub-interval of the transmission schedule (see Fig.2) as the schedule is executed repeatedly over time (i.e., as k increases).

Remark 4: By examining the structure of the test matrix M in Eq.9, it can be seen that its eigenvalues depend on the update period h, the closed-loop matrix Λ (which depends on the plant-model mismatch as well as the controller gains), the time intervals between sensor transmissions $\Delta t_1, \Delta t_2, \cdots, \Delta t_{n-1}$, as well as the sensor transmission sequence $\{s_1, s_2, \cdots, s_n\}$. The stability criteria in Theorem 1 can therefore be used to compare different schedules (by varying the transmission sequence as well as the transmission times) to determine the schedules that require the least communication rate between the sensors and the target controllers and therefore produce the biggest savings in the WSN resource utilization (e.g., battery power). For a fixed schedule, the stability criteria can also be used to compare different models and different controllers in terms of their robustness with respect to communication suspension (i.e., which ones require measurement updates less frequently than others). Note that choosing $\Delta t_1 = \Delta t_2 = \cdots = \Delta t_{n-1} = 0$ reduces the problem to one where all nodes in the WSN transmit their measurements simultaneously. As expected, in this case stability of the networked closed-loop system depends only on Λ and h.

V. Simulation study: Application to chemical reactors with recycle

We consider a plant composed of three cascaded non-isothermal continuous stirred-tank reactors (CSTRs) with recycle. The output of CSTR 3 is passed through a separator that removes the products and recycles unreacted A to CSTR 1. The reactant species A is consumed in each reactor by three parallel irreversible exothermic reactions. Due to the non-isothermal nature of the reactions, a jacket is used to remove/provide heat to each reactor. Under standard modeling assumptions, a plant model of the following form can be derived from material and energy balances:

$$\begin{aligned}\frac{dT_j}{dt} &= \frac{F_j^0}{V_j}(T_j^0 - T_j) + \frac{F_{j-1}}{V_j}(T_{j-1} - T_j)\\ &+ \textstyle\sum_{i=1}^{3}\frac{(-\Delta H_i)}{\rho c_p} R_i(C_{Aj}, T_j) + \frac{Q_j}{\rho c_p V_j}\\ \frac{dC_{Aj}}{dt} &= \frac{F_j^0}{V_j}(C_{Aj}^0 - C_{Aj}) + \frac{F_{j-1}}{V_j}(C_{A(j-1)} - C_{Aj})\\ &- \textstyle\sum_{i=1}^{3} R_i(C_{Aj}, T_j),\ j = 1, 2, 3\end{aligned}$$

where T_j, C_{Aj}, Q_j, and V_j denote the temperature, the reactant concentration, the rate of heat input, and the volume of the j-th reactor, respectively, $R_i(C_{Aj}, T_j) = k_{i0}\exp\left(\frac{-E_i}{RT_j}\right) C_{Aj}$ is the reaction rate of the i-th reaction, F_j^0 denotes the flow rate of a fresh feed stream associated with the j-th reactor, F_j is the flow rate of the outlet stream of the j-th reactor, with $F_0 = F_r, T_0 = T_3, C_{A0} = C_{A3}$ denoting the flow rate, temperature and reactant concentration of the recycle stream, $\Delta H_i,\ k_i,\ E_i,\ i = 1, 2, 3$, denote the enthalpies, pre-exponential constants and activation energies of the three reactions, respectively, c_p and ρ denote the heat capacity and density of fluid in the reactor. Using typical values for the process parameters, the plant with $Q_j = 0$, $C_{Aj}^0 = C_{Aj}^{0s}$ and a recycle ratio of $r = 0.5$, has an unstable steady state. The control objective is to stabilize the plant at the (open-loop) unstable steady-state. Operation at this point is desired to avoid high temperatures, while simultaneously achieving reasonable conversion. The manipulated variables for each reactor are chosen to be Q_j and C_{Aj}^0, $j = 1, 2, 3$. A plant-wide WSN composed of 3 wireless sensor suites is deployed. Each sensor suite collects measurements of the local state variables in a given unit and broadcasts it to the rest of the plant. It is desired to stabilize the plant with minimal data exchange over the WSN to conserve battery power for the wireless devices.

Linearizing the plant around the unstable steady state yields a system of the form of Eq.1 to which the networked control and scheduling architecture described in the previous section is applied. The synthesis details are omitted due to space limitations. As mentioned in Section III, we focus on scheduling configurations where at each transmission time, only the wireless sensor suite of one reactor is allowed to transmit its measurement updates to the local controllers of the other two reactors. This configuration is represented in Fig. 2, where the time intervals between the transmission times of consecutive transmitting nodes are defined as $\Delta t_1 := t_k^{s_2} - t_k^{s_1}$ and $\Delta t_2 := t_k^{s_3} - t_k^{s_2}$, for $k = 0, 1, 2, \cdots$, where $\{s_1, s_2, s_3\}$ is the transmission sequence, $s_j \in \{1, 2, 3\}$, and $s_i \neq s_j$ if $i \neq j$. Note that the sequence is executed repeatedly every h seconds and that $\Delta t_1 + \Delta t_2 < h$.

It can be verified that the test matrix M in Eq.9 is given by:

$$M(h) = e^{\Lambda(h - \Delta t_1 - \Delta t_2)} I_s^{s_3} e^{\Lambda \Delta t_2} I_s^{s_2} e^{\Lambda \Delta t_1} I_s^{s_1}$$

which shows that the eigenvalues of M depend on the update period h, as well as the time intervals, Δt_1 and Δt_2, and the sensor transmission sequence (determined by $I_s^{s_1}, I_s^{s_2}, I_s^{s_3}$). In the remainder of this section, we will investigate the impact of the choices of the intervals between transmissions and the sensor transmission schedule on the stabilizing update period. Since closed-loop stability requires all eigenvalues of M to lie within the unit circle, it is sufficient to consider only the maximum eigenvalue magnitude, denoted by $\lambda_{\max}(M)$.

TABLE I

Sensor transmission schedules for a 3-unit plant.

Schedule	$s_1, s_2, s_3, s_1, s_2, s_3, \cdots$
1	$1, 2, 3, 1, 2, 3, \cdots$
2	$1, 3, 2, 1, 3, 2, \cdots$
3	$2, 1, 3, 2, 1, 3, \cdots$
4	$2, 3, 1, 2, 3, 1, \cdots$
5	$3, 1, 2, 3, 1, 2, \cdots$
6	$3, 2, 1, 3, 2, 1, \cdots$

We consider first the case when $\Delta t_1 = \Delta t_2 = \Delta t$. Fig.3(a) is a contour plot showing the dependence of the $\lambda_{\max}(M)$ on both the interval between transmissions, Δt, and the update period, h, under the six possible sensor transmission schedules listed in Table I when imperfect models are embedded in the local control systems (each model has a 30% parametric uncertainty in the heat of reaction). For each schedule, the area enclosed by the unit contour line is the stability region of the plant. It can be seen that the update period obtained under any of the six schedules is larger than the one obtained when no scheduling takes place (i.e., with $\Delta t = 0$). Forcing the different sensor suites to transmit their data and update their target models at different times (rather than simultaneously) in this case helps provide a more targeted and timely (though only partial) correction to model estimation errors that helps reduce the rate at which each node in the WSN must collect and transmit data. We also observe that sequences 2, 3 and 6 allow the use of the largest update periods among all possible sequences.

These predictions are further confirmed by the closed-loop state and manipulated input profiles shown in Fig.3(b), which show that the linearized plant is stable under sequence 2 but unstable under sequence 1, when $\Delta t = 0.05$ and $h = 0.05$ (for brevity, only the temperature profiles for

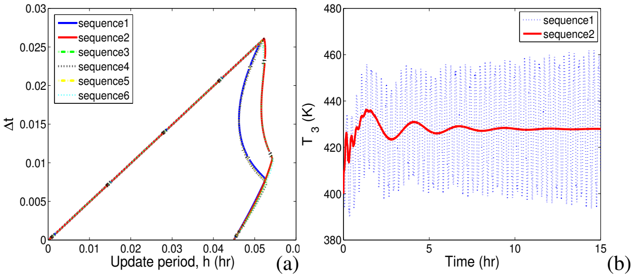

Fig. 3. (a) Dependence of $\lambda_{\max}$ on the sensor transmission interval Δt and the update period for different sensor transmission sequences with a fixed model uncertainty, and (b) Closed-loop temperature profiles for CSTR 3 under the model-based quasi-decentralized control strategy using two different sensor transmission schedules with the same update period.

CSTR 3 are shown; the state and manipulated input profiles for the other reactors exhibit similar behavior).

We consider next the more general case when $\Delta t_1 \neq \Delta t_2$. Fig.4(a) is a contour plot showing the dependence of $\lambda_{\max}$ on Δt_1 and h for different values of Δt_2, when the WSN nodes transmit following sequence 1 and an imperfect model is used (nominal value of the heat of reaction is 30% higher than the actual value). It can be seen that one can get a larger update period (and hence further reduce the wireless network utilization) by carefully scheduling the transmission times for the sensor suites of different units than in the case when $\Delta t_1 = \Delta t_2$. For example, consider the case when $\Delta t_1 = 0.01$ hr and $h = 0.05$ hr. This point lies outside the stability region of schedule 1 with $\Delta t_2 = \Delta t_1 = 0.01$ hr. If we choose $\Delta t_2 = 0.005$ hr, however, the point lies inside the stability region for schedule 1. These observations are further confirmed in Fig.4(b).

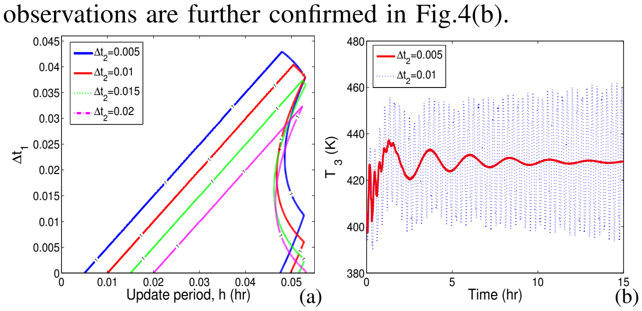

Fig. 4. (a) Dependence of $\lambda_{\max}$ on Δt_1 and h for different values of Δt_2 under sensor transmission schedule 1 with a fixed model uncertainty, and (b) Closed-loop temperature profiles for CSTR 3 when $\Delta t_1 = 0.01$ hr and $h = 0.05$ hr for two different values of Δt_2.

VI. Concluding Remarks

In this work, a model-based framework for networked control and sensor scheduling was developed for plants with quasi-decentralized control systems that communicate using a resource-constrained WSN. An explicit condition for characterizing the minimum allowable rate at which each node in the WSN must transmit its data was provided. It was shown that by judicious selection of the transmission schedule and the models, it is possible to enhance the savings in WSN resource utilization. Finally, the results were illustrated through application to a linearized model of a chemical plant example. Efforts to extend the proposed framework to nonlinear plants are currently underway.

References

[1] P. R. Kumar, "New technological vistas for systems and control: The example of wireless networks," *IEEE Contr. Syst. Mag.*, vol. 21, pp. 24–37, 2001.

[2] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: a survey," *Computer Networks*, vol. 38, pp. 102–114, 2002.

[3] A. Herrera, "Wireless I/O devices in process control systems," in *Proceedings of Sensors for Industry Conference*, New Orleans, LA, 2004, pp. 146–147.

[4] J. Song, A. K. Mok, D. Chen, and M. Nixon, "Challenges of wireless control in process industry," in *Workshop on Research Directions for Security and Networking in Critical Real-Time and Embedded Systems*, San Jose, CA, 2006.

[5] P. D. Christofides, J. F. Davis, N. H. El-Farra, J. N. G. J. K. Harris, and D. Clark, "Smart plant operations: Vision, progress and challenges," *AIChE J.*, vol. 53, pp. 2734–2741, 2007.

[6] B. E. Ydstie, "New vistas for process control: Integrating physics and communication networks," *AIChE J.*, vol. 48, pp. 422–426, 2002.

[7] Y. Sun and N. H. El-Farra, "Quasi-decentralized model-based networked control of process systems," *Comp. & Chem. Eng.*, vol. 32, pp. 2016–2029, 2008.

[8] G. Walsh and H. Ye, "Scheduling of networked control systems," *IEEE Contr. Syst. Mag.*, vol. 21, pp. 57–65, 2001.

[9] M. Branicky, S. M. Phillips, and W. Zhang, "Scheduling and feedback co-design for networked control systems," in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, NV, 2002, pp. 1211–1217.

[10] B. Hamdaoui and P. Ramanathan, "Sufficient conditions for flow admission control in wireless ad-hoc networks," *Mobile Computing and Communications Review*, vol. 9, pp. 15–24, 2004.

[11] L. Zhanga and D. Hristu-Varsakelis, "Communication and control co-design for networked control systems," *Automatica*, vol. 42, pp. 953–958, 2006.

[12] L. A. Montestruque and P. J. Antsaklis, "On the model-based control of networked systems," *Automatica*, vol. 39, pp. 1837–1843, 2003.

[13] T. Nesić and A. R. Teel, "Input-to-state stability of networked control systems," *Automatica*, vol. 40, pp. 2121–2128, 2004.

[14] P. F. Hokayem and C. T. Abdallah, "Inherent issues in networked control systems: A survey," in *Proceedings of the American Control Conference*, Boston, MA, 2004, pp. 329–336.

[15] Y. Xu and J. P. Hespanha, "Optimal communication logics for networked control systems," in *Proceedings of the 43rd IEEE Conference on Decision and Control*, Atlantis, Paradise Island, Bahamas, 2004, pp. 3527–3532.

[16] D. Munoz de la Pena and P. D. Christofides, "Lyapunov-based model predictive control of nonlinear systems subject to data losses," in *Proceedings of American Control Conference*, New York, NY, 2007, pp. 1735–1740.

[17] D. D. Siljak, *Decentralized Control of Complex Systems*. London: Academic Press, 1991.

[18] J. Lunze, *Feedback Control of Large Scale Systems*. U.K.: Prentice-Hall, 1992.

[19] H. Cui and E. Jacobsen, "Performance limitations in decentralized control," *J. Proc. Contr.*, vol. 12, pp. 485–494, 2002.

[20] K. Hangos, A. Alonso, J. Perkins, and B. Ydstie, "Thermodynamic approach to the structural stability of process plants," *AIChE J.*, vol. 45, pp. 802–816, 1999.

[21] E. Camponogara, D. Jia, B. Krogh, and S. Talukdar, "Distributed model predictive control," *IEEE Contr. Syst. Mag.*, vol. 22, pp. 44–52, 2002.

[22] A. N. Venkat, J. B. Rawlings, and S. J. Wright, "Stability and optimality of distributed model predictive control," in *Proceedings of 44th IEEE Conference on Decision and Control*, Seville, Spain, 2005, pp. 6680–6685.

[23] E. Tatara, I. Birol, F. Teymor, and A. Cinar, "Agent-based control of autocatalytic replicators in networks of reactors," *Comp. & Chem. Eng.*, vol. 29, pp. 807–815, 2005.

[24] M. Baldea, P. Daoutidis, and A. Kumar, "Dynamics and control of integrated networks with purge streams," *AIChE J.*, vol. 52, pp. 1460–1472, 2006.