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Resource-aware Quasi-decentralized Control of Nonlinear Plants Over Communication Networks

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Abstract— **This paper presents a quasi-decentralized nonlinear control methodology for multi-unit nonlinear plants whose constituent subsystems communicate over a shared, resource-constrained communication network. The objective is to stabilize the plant while keeping the communication requirements to a minimum in order to reduce the unnecessary utilization of network resources. To this end, an uncertain nonlinear model of the plant is initially used to design, for each unit, a stabilizing nonlinear feedback controller that requires state measurements from the neighboring units for implementation. To reduce the frequency at which the measurements are transmitted over the shared network, a copy of the stable compensated plant model is embedded in each unit to provide estimates of the states of the neighboring units when measurements are not available through the network. The state of the model is then updated at discrete time instances when communication is re-established. By analyzing the behavior of the model estimation error between updates, and exploiting the stability properties of the compensated model, a sufficient condition for practical stability of the networked closed-loop plant is obtained in terms of the update period, the plant-model mismatch and the controller design parameters. The stability condition can be used to obtain estimates of the maximum allowable update period and the size of the achievable residual set. Finally, the implementation of the networked control structure is demonstrated through an application to a chemical plant example.**

I. Introduction

Quasi-decentralized control refers to a distributed control strategy in which most signals used for control are collected and processed locally within each plant unit (typically over dedicated networks), while some signals – the total number of which is kept to a minimum – still need to be transferred between the plant units and their local controllers over a shared (possibly wireless) communication network to account for the interactions between the different units and minimize the propagation of disturbances and process upsets. It represents a compromise solution that aims to overcome the stability and performance limitations of decentralized control approaches while avoiding, at the same time, the complexity and lack of flexibility associated with implementing traditional centralized control structures. While significant research work has explored the benefits and limitations of decentralized controllers (e.g., see [1], [2], [3], [4], [5] and the references therein) and developed various approaches for overcoming some of their limitations (e.g., [6], [7], [8]), most of these studies have focused on plants described by linear systems. For nonlinear plants, on the other hand, results on this problem have been more limited. Examples of recent works include the development of a passivity-based framework for the analysis and stabilization of process networks using concepts from thermodynamics [9], the development of agent-based systems to control reactor networks [10], and the analysis and control of integrated process networks using time-scale decomposition and singular perturbations [11].

One of the key problems in the design of quasi-decentralized control systems for multi-unit plants is the integration of communication issues and limitations in the formulation and solution of the plant-wide control problem. The significance of this problem stems in part from the recent and growing interest in the process industries to augment existing process control systems with low-cost wireless sensor and actuator networks (WSANs)[12], [13]. The low cost, flexibility and ease of installation of WSANs mean that more devices could be deployed and more process variables and devices could be monitored and controlled than is cost-effective with solely wired networks. The availability of more sensor data, more information about the plant and more intercommunication between plant units open up new avenues not only for improving the existing control quality but also for pursuing goals that cannot be attained otherwise, including proactive fault-tolerance and real-time plant reconfiguration to accommodate projected market demand changes [14]. However, unlike dedicated networks, wireless networks are typically resource-constrained due to limited processing and computational capabilities or due to limited power when the WSAN is deployed in harsh environments where continuous power supply is not feasible and the wireless devises have to rely on battery power instead [12]. In this light, the design of a quasi-decentralized control strategy that enforces the desired closed-loop objectives with minimal cross communication between the component subsystems is an appealing goal since it helps conserve network resources and prolong the service life of the network, which are key to enabling the deployment of wireless sensing and control systems in large-scale industrial plants.

In an effort to address the resource constraint problem, we recently developed in [15] a quasi-decentralized model-based networked control framework for multi-unit plants modeled by linear systems of differential equations. A key idea was to reduce the cross communication between

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the plant units by including within each control system a set of linear models that approximate the dynamics of the neighboring units when direct measurements are not transmitted over the plant-wide network. By exploiting the linear structure of the plant and the controllers, both necessary and sufficient conditions for closed-loop stability were obtained leading to an exact characterization of the minimum allowable communication rate. When this architecture is implemented on a nonlinear plant, however, the update period predicted by linearization-based analysis can guarantee stability only for sufficiently small initial conditions. Stabilization from large initial conditions (if at all feasible) requires increasing the frequency of measurement communication (i.e., reducing the update period) substantially which leads to additional network utilization. Since many chemical processes are characterized by strong nonlinear dynamics and need to operated over wide regions of the operating space for economic reasons, it is important to develop networked control approaches that account explicitly for the nonlinearities – both in the control law and in the communication logic designs – and that provide an explicit characterization of the minimum allowable cross-communication frequency in the nonlinear plant.

Motivated by these considerations, we develop in this work a quasi-decentralized nonlinear control structure for multi-unit nonlinear plants whose constituent subsystems communicate over a shared, resource-constrained communication network. The objective is to stabilize the plant while keeping the communication requirements to a minimum in order to reduce the unnecessary utilization of network resources. The rest of the paper is organized as follows. Following some preliminaries in Section II, the quasi-decentralized control structure is presented in Section III. The structure consists of a family of nonlinear feedback controllers together with a stable model of the closed-loop plant that is embedded in each unit to provide estimates of the states of the neighboring units when communication is suspended over the network. The state of the model is then updated at discrete time instances when communication is re-established. The networked closed-loop plant is then analyzed in Section IV and a sufficient condition for closed-loop stability in terms of the update period is obtained. Finally, the main results are illustrated through an application to a chemical plant example in Section V and concluding remarks are given in Section VI.

II. Preliminaries and problem formulation

We consider a large-scale distributed plant composed of n interconnected processing units, each of which is modeled by a continuous-time nonlinear system, and represented by the following state-space description:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \cdots, x_n) + g_1(u_1) \\ \dot{x}_2 &= f_2(x_1, x_2, \cdots, x_n) + g_2(u_2) \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \cdots, x_n) + g_n(u_n) \end{aligned} \qquad (1)$$

where $x_i := [x_i^{(1)} \; x_i^{(2)} \; \cdots \; x_i^{(p_i)}]^T \in \mathbb{R}^{p_i}$ denotes the vector of process state variables associated with the i-th processing unit, $u_i := [u_i^{(1)} \; u_i^{(2)} \; \cdots \; u_i^{(q_i)}]^T \in \mathbb{R}^{q_i}$ denotes the vector of manipulated inputs associated with the i-th processing unit, x^T denotes the transpose of a column vector x, $f_i(\cdot)$ and $g_i(\cdot)$ are sufficiently smooth nonlinear functions. Note from Eq.1 that each processing unit can in general be connected to all the other units in the plant. Note also that even though each subsystem is referred to as a unit for simplicity, each subsystem can comprise a collection of unit operations depending on how the plant is decomposed. The overall objective is to design a distributed, networked control strategy that stabilizes the individual units (and the overall plant) at or near the origin, and accounts simultaneously for the constrained resources of the plant-wide communication network. To illustrate the main ideas and simplify the presentation of our results, we will focus in this work on the full state feedback problem where the states of all the units are available as measurements. Extensions to the output feedback case are possible and the subject of other research work.

III. Model-based networked control structure

A. Model-based controller synthesis

To realize the desired quasi-decentralized networked control structure, the first step is to synthesize for each unit an appropriate nonlinear controller that enforces closed-loop stability in the absence of communication limitations (i.e., when the sensors of each unit transmit their data continuously using ideal point-to-point connections to the control systems of the other units). To this end, we will consider that a (possibly uncertain) model of the following form is available for each unit for controller synthesis:

$$\dot{\widehat{x}}_i = f_i(\widehat{x}_1, \widehat{x}_2, \cdots, \widehat{x}_n) + g_i(\widehat{u}_i) + w_i(\widehat{x}_1, \widehat{x}_2, \cdots, \widehat{x}_n) \qquad (2)$$

where $\widehat{x}_i$ is the state of the model, and $w_i(\cdot)$ is a sufficiently smooth nonlinear function that represents the plant-model mismatch. The plant model is then given by:

$$\dot{\widehat{\mathbf{x}}} = F(\widehat{\mathbf{x}}) + G(\widehat{\mathbf{u}}) + W(\widehat{\mathbf{x}}) \qquad (3)$$

where $\widehat{\mathbf{x}} = [\widehat{x}_1^T \; \cdots \; \widehat{x}_n^T]^T$, $\widehat{\mathbf{u}} = [\widehat{u}_1^T \; \cdots \; \widehat{u}_n^T]^T$, $F(\cdot) = [f_1^T(\cdot) \; \cdots \; f_n^T(\cdot)]^T$, $G(\cdot) = [g_1^T(\cdot) \; \cdots \; g_n^T(\cdot)]^T$ and $W(\cdot) = [w_1^T(\cdot) \; \cdots \; w_n^T(\cdot)]^T$. Depending on the particular structure of the model, a number of nonlinear controller synthesis techniques can be used to design the desired controllers [16]. Examples include Lyapunov-based control methods, geometric control approaches as well as optimization-based control methods. In the interest of generality, we will not limit the discussion to a particular synthesis method. Instead, we will consider that an appropriate controller has already been designed for each unit. This is formalized in the following assumption.

Assumption 1: *For each $i \in \{1, 2, \cdots, n\}$, there exists a nonlinear feedback control law of the general form:*

$$\widehat{u}_i = k_i(\widehat{x}_1, \widehat{x}_2, \cdots, \widehat{x}_n) \qquad (4)$$

such that the origin of the closed-loop plant model

$$\dot{\widehat{\mathbf{x}}} = F(\widehat{\mathbf{x}}) + G(K(\widehat{\mathbf{x}})) + W(\widehat{\mathbf{x}}), \tag{5}$$

where $K(\cdot) = [k_1^T(\cdot)\ k_2^T(\cdot)\ \cdots\ k_n^T(\cdot)]^T$*, satisfies:*

$$\| \widehat{\mathbf{x}}(t) \| \leq \alpha \| \widehat{\mathbf{x}}(t_0) \| e^{-\beta(t-t_0)} \tag{6}$$

for some $\alpha \geq 1$, $\beta > 0$*, for all* $\widehat{\mathbf{x}}(t_0) \in \Omega$*, for some compact set* $\Omega \subset \mathbb{R}^m$, $m = \sum_{i=1}^{n} p_i$*.*

Remark 1: Note that in general the model uncertainty can be non-vanishing in the sense that the plant and the model may not share the same equilibrium point. In this case, when the above model-based controller is implemented on the actual plant, asymptotic convergence to the origin will not be possible. Instead, practical stability can be achieved where the controller can be designed to force the states of the closed-loop plant to converge in finite-time to a small neighborhood of the origin (terminal or residual set) whose size can be made arbitrarily small by appropriate selection of the controller.

Remark 2: The requirement that the closed-loop model of the plant be exponentially stable will be used in Section IV to facilitate the analysis of the networked closed-loop system and allow the derivation of an explicit bound on the cross communication frequency between the plant units (see the proof of Theorem 1). However, this requirement is not necessary per se and can be relaxed to allow the use of feedback controllers that enforce asymptotic stability or even bounded stability, as long as the controller synthesis leads to an explicit time-varying bound on the evolution of the closed-loop plant model state that can be used in lieu of Eq.6.

B. Quasi-decentralized implementation over networks

The implementation of each control law in Eq.4 on the nonlinear plant requires the availability of state measurements from both the local subsystem being controlled and the units that are connected to it. Unlike the local measurements which are available continuously through a dedicated network, the measurements from the neighboring units are available only through the shared plant-wide network whose resources are to be conserved. To reduce the transfer of information between the local control systems as much as possible without sacrificing stability, a dynamic model of the plant is included in the local control system of each unit to provide it with an estimate of the evolution of the states of its neighboring units when measurements are not sent over the network. The use of a model allows the sensors of the neighboring units to send their data at discrete time instants since the model can provide an approximation of the plant's dynamics. "Feedforward" from one unit to another is performed by updating the state of each model using the actual states of the corresponding unit provided by its sensors at discrete time instances.

Under this architecture, the local control law for each unit is implemented as follows:

$$\begin{aligned}
u_i(t) &= k_i(\widehat{x}_1(t), \cdots, \widehat{x}_{i-1}(t), x_i(t), \widehat{x}_{i+1}(t), \cdots, \widehat{x}_n(t)) \\
&\quad ,\quad i = 1, 2, \cdots, n \\
\dot{\widehat{x}}_j(t) &= f_j(\widehat{x}_1(t), \widehat{x}_2(t), \cdots, \widehat{x}_n(t)) + g_j(\widehat{u}_j(t)) \\
&\quad + w_j(\widehat{x}_1(t), \widehat{x}_2(t), \cdots, \widehat{x}_n(t)) \\
\widehat{u}_j(t) &= k_j(\widehat{x}_1(t), \widehat{x}_2(t), \cdots, \widehat{x}_n(t)),\ t \in (t_k, t_{k+1}) \\
\widehat{x}_j(t_k) &= x_j(t_k),\ \ j = 1, \cdots, n,\ k = 0, 1, 2, \cdots
\end{aligned} \tag{7}$$

where $\widehat{x}_j$ is an estimate of x_j. Note that the models used by the i-th controller to recreate the behavior of the neighboring units are the same as the models used for the controller synthesis in those units. Note also that, unlike the setup in the linear quasi-decentralized control structure presented in [15], we embed in each control system a dynamic model of the whole plant (instead of models of the neighboring units only) and that in between two consecutive transmission times the evolution of the model states is independent of the plant states (notice, for example, that the j-th model embedded in unit i uses $\widehat{x}_i$ instead of x_i to generate its forecasts). Decoupling the evolution of the model states from the plant states ensures that in between any two consecutive transmission times the closed-loop plant always receives stable and convergent inputs (recall from Assumption 1 that the controllers are designed to ensure that the compensated plant model is stable).

IV. Closed-loop stability analysis

A key parameter in the analysis of the control law of Eq.7 is the update period $h := t_{k+1} - t_k$, which determines the frequency at which a given unit receives measurements from the other units through the network to update the corresponding model estimates. To simplify the analysis, we consider the case when the update periods are constant and the same for all the units, i.e., we require that all units communicate their measurements concurrently every h seconds (extensions to the case of time-varying updates are the subject of other work).

A. Formulation of the networked closed-loop plant

To formulate the networked closed-loop system, we define the following estimation errors $e_j = x_j - \widehat{x}_j$, $j = 1, 2, \cdots, n$, where e_j represents the difference between the state of the j-th unit and the state of its model embedded in the local control systems of its neighbors (note that all units share the same model). Introducing the augmented vectors $\mathbf{x} := [x_1^T\ x_2^T\ \cdots\ x_n^T]^T$, $\mathbf{e} := [e_1^T\ e_2^T\ \cdots\ e_n^T]^T$, it can be shown that the overall closed-loop plant of Eq.1 subject to the control and update laws of Eq.7 can be formulated as a switched system of the following form:

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{F}(\mathbf{x}(t), \mathbf{e}(t)) \\
\dot{\mathbf{e}}(t) &= \widetilde{\mathbf{F}}(\mathbf{x}(t), \mathbf{e}(t)),\ t \in (t_k, t_{k+1}) \\
\mathbf{e}(t_k) &= \mathbf{0},\ k = 0, 1, 2, \cdots,\ h = t_{k+1} - t_k
\end{aligned} \tag{8}$$

where the process states evolve continuously in time and the model estimation errors are reset to zero at each transmission instance since the state of each model in each unit is updated every h seconds. Referring to Eq.8, $\mathbf{F}(\cdot)$ and $\widetilde{\mathbf{F}}(\cdot)$ are nonlinear functions that depend on the plant dynamics, the models, and the control laws of the different units. The explicit forms of these matrices can be obtained by substituting Eq.7 into Eq.1 and are omitted for brevity.

B. Estimating the maximum allowable update periods

Substituting the controller of Eq.7 into the plant of Eq.1, the closed-loop dynamics of the i-th unit are given by:

$$\dot{x}_i = f_i(x_1, \cdots, x_n) + m_i(\widehat{x}_1, \cdots, \widehat{x}_{i-1}, x_i, \widehat{x}_{i+1}, \cdots, \widehat{x}_n)$$

where $m_i := g_i(k_i(\widehat{x}_1, \cdots, \widehat{x}_{i-1}, x_i, \widehat{x}_{i+1}, \cdots, \widehat{x}_n))$ and $\widehat{x}_j$ is given by:

$$\dot{\widehat{x}}_j = f_j(\widehat{x}_1, \cdots, \widehat{x}_n) + m_j(\widehat{x}_1, \cdots, \widehat{x}_n) + w_j(\widehat{x}_1, \cdots, \widehat{x}_n)$$

Since the functions $f_i(\cdot)$, $w_i(\cdot)$ and $m_i(\cdot)$, $i = 1, 2, \cdots, n$, are sufficiently smooth, it follows from their local Lipschitz properties that there exist positive real constants L_{f_i}, L_{w_i} and L_{m_i} such that the following estimates hold for all $x, y \in \Omega$ where Ω (introduced in Assumption 1) is a ball centered around the origin:

$$\begin{aligned} \| f_i(x) - f_i(y) \| &\leq L_{f_i} \| x - y \| \\ \| w_i(x) - w_i(y) \| &\leq L_{w_i} \| x - y \| \\ \| m_i(x, s) - m_i(y, s) \| &\leq L_{m_i} \| x - y \| \end{aligned} \quad (9)$$

Note that if the plant model is accurate, the constants L_{w_i}, $i = 1, 2, \cdots, n$, will be small. The following theorem provides a sufficient condition for stability of the networked closed-loop plant in terms of the update period, the model uncertainty and the controller design parameters.

Theorem 1: *Consider the closed-loop plant of Eq.1 subject to the control and update laws of Eq.7, and the compensated plant model of Eq.5 for which Assumption 1 holds, with* $\mathbf{x}(t_0) = \widehat{\mathbf{x}}(t_0) \in \Omega$. *Then, if*

$$F_1(h) := 1 - \alpha \left(e^{-\beta h} + (e^{L_e h} - e^{-\beta h}) \frac{L_w}{\beta + L_e} \right) > 0 \quad (10)$$

the states of the networked closed-loop system are bounded and satisfy

$$\| \mathbf{x}(t_{k+1}^-) \| < \| \mathbf{x}(t_k) \| \text{ for all } \| \mathbf{x}(t_k) \| > r(h) \quad (11)$$

for $k = 0, 1, 2, \cdots$, *where* $r(h) = F_2(h)/F_1(h)$,

$$F_2(h) = \frac{L_0}{L_e}(e^{L_e h} - 1) \quad (12)$$

and $L_e = \sum_{i=1}^n (L_{f_i} + L_{m_i})$, $L_w = \sum_{i=1}^n L_{w_i}$, *and* $L_0 = \sum_{i=1}^n \| w_i(0) \|$.

Proof: We begin by analyzing the behavior of the norm of the closed-loop plant state in between consecutive model updates. The stability of the closed-loop system can be established if $\| \mathbf{x}(t) \|$ decreases such that $\| \mathbf{x}(t_k) \| > \| \mathbf{x}(t_{k+1}^-) \|$, where t_k and t_{k+1} are update times with $t_{k+1} - t_k = h$. From the triangular inequality, we have that for any period of time $[t_k, t_{k+1})$:

$$\| \mathbf{x}(t) \| \leq \| \widehat{\mathbf{x}}(t) \| + \| \mathbf{e}(t) \| \quad (13)$$

and therefore $\| \mathbf{x}(t) \|$ will decrease over the period $[t_k, t_{k+1})$ if $\| \widehat{\mathbf{x}}(t) \| + \| \mathbf{e}(t) \|$ decreases. We now establish a bound on the norm of the error as a function of the update period h. To this end, we have from the definition of the estimation error that:

$$\dot{e}_i = \dot{x}_i - \dot{\widehat{x}}_i = f_i(\mathbf{x}) - f_i(\widehat{\mathbf{x}}) - \omega_i(\widehat{\mathbf{x}}) + m_i(\bar{\mathbf{x}}_i, x_i) - m_i(\bar{\mathbf{x}}_i, \widehat{x}_i)$$

where $\bar{\mathbf{x}}_i$ is a vector made up of all $\widehat{x}_j$, $j \neq i$. Solving for e_i, we have that $\forall t \in [t_k, t_{k+1})$:

$$\begin{aligned} e_i(t) = e_i(t_k) &+ \int_{t_k}^t (f_i(\mathbf{x}(s)) - f_i(\widehat{\mathbf{x}}(s)) - w_i(\widehat{\mathbf{x}}(s))) \, ds \\ &+ \int_{t_k}^t (m_i(\bar{\mathbf{x}}_i(s), x_i(s)) - m_i(\bar{\mathbf{x}}_i(s), \widehat{x}_i(s))) \, ds \end{aligned}$$

Taking the norm of both sides and using the fact that $e_i(t_k) = 0$ (since at t_k the plant model state is updated and the error is reset to zero), we have that $\forall t \in [t_k, t_{k+1})$:

$$\begin{aligned} \| e_i(t) \| \leq &\int_{t_k}^t \| f_i(\mathbf{x}(s)) - f_i(\widehat{\mathbf{x}}(s)) \| ds + \int_{t_k}^t \| w_i(\widehat{\mathbf{x}}(s)) \| ds \\ &+ \int_{t_k}^t \| m_i(\bar{\mathbf{x}}_i(s), x_i(s)) - m_i(\bar{\mathbf{x}}_i(s), \widehat{x}_i(s)) \| ds \end{aligned} \quad (14)$$

Substituting the estimates of Eq.9 into Eq.14 and noting that $\| w_i(x) - w_i(0) \| \leq L_{w_i} \| x \| \implies \| w_i(x) \| \leq L_{w_i} \| x \| + \| w_i(0) \|$, the following bound can be obtained $\forall t \in [t_k, t_{k+1})$:

$$\begin{aligned} \| e_i(t) \| \leq &\int_{t_k}^t (L_{f_i} \| \mathbf{e}(s) \| + L_{w_i} \| \widehat{\mathbf{x}}(s) \| + \| w_i(0) \|) \, ds \\ &+ \int_{t_k}^t L_{m_i} \| e_i(s) \| ds \end{aligned}$$

Using the fact that $\| e_i \| \leq \| \mathbf{e} \|$, $i = 1, 2, \cdots, n$ and $\| \mathbf{e} \| \leq \sum_{i=1}^n \| e_i \|$, it can then be verified that $\forall t \in [t_k, t_{k+1})$:

$$\| \mathbf{e}(t) \| \leq L_e \int_{t_k}^t \| \mathbf{e}(s) \| ds + L_w \int_{t_k}^t \| \widehat{\mathbf{x}}(s) \| ds + L_0 (t - t_k)$$

where $L_e = \sum_{i=1}^n (L_{f_i} + L_{m_i})$, $L_w = \sum_{i=1}^n L_{w_i}$, and $L_0 = \sum_{i=1}^n \| w_i(0) \|$. Substituting the bound of Eq.6 for $\| \widehat{\mathbf{x}}(s) \|$ into the above equation and applying the Gronwall-Bellman inequality [17] yield:

$$\begin{aligned} \| \mathbf{e}(t) \| \leq &\| \widehat{\mathbf{x}}(t_k) \| \frac{\alpha L_w}{\beta + L_e} (e^{L_e (t - t_k)} - e^{-\beta (t - t_k)}) \\ &+ \frac{L_0}{L_e} (e^{L_e (t - t_k)} - 1), \ \forall t \in [t_k, t_{k+1}) \end{aligned} \quad (15)$$

From Eq.15 we note that the error signal will be zero if the update period $h = t_{k+1} - t_k$ is zero and also if the model and the plant dynamics are the same ($L_0 = L_w = 0$). With this bound on the estimation error and the bound on the state of the model given in Eq.6, we can proceed to calculate a bound on the plant state using Eq.13, where it can be shown after some algebraic manipulations that $\forall \ t \in [t_k, t_{k+1})$:

$$\| \mathbf{x}(t) \| \leq \| \widehat{\mathbf{x}}(t_k) \| (1 - F_1(t - t_k)) + F_2(t - t_k) \quad (16)$$

where $F_1(\cdot)$ and $F_2(\cdot)$ are given by Eq.10 and Eq.12, respectively. Using the above estimate to calculate $\|\mathbf{x}(t_{k+1}^-)\|$ and noting that $\| \widehat{\mathbf{x}}(t_k) \| = \| \mathbf{x}(t_k) \|$, we finally obtain:

$$\| \mathbf{x}(t_{k+1}^-) \| - \| \mathbf{x}(t_k) \| \leq F_2(h) - F_1(h) \| \mathbf{x}(t_k) \|$$

$\forall \ t \in [t_k, t_{k+1})$. Clearly, if $F_1(h) > 0$ and $\| \mathbf{x}(t_k) \| > F_2(h)/F_1(h)$, then $\| \mathbf{x}(t_{k+1}^-) \| - \| \mathbf{x}(t_k) \| < 0$ and Eq.11 holds. This completes the proof of the theorem.

Remark 3: Theorem 1 establishes that if the update period is chosen such that Eq.10 is satisfied, the norm of the networked closed-loop plant state is guaranteed to decrease at successive update times as long as the closed-loop trajectory is outside some terminal neighborhood of the origin (the size of which is fixed by the choice of the update period). This implies that the closed-loop state is guaranteed to converge in finite-time to the terminal set where it remains confined for all future times. Note from Eq.16 that in between consecutive model updates the closed-loop plant state always remains bounded and can grow only a certain amount (since $t - t_k < h$), and this growth is independent of k.

Remark 4: The expressions in Eq.10 and Eq.12 capture intuitively the dependence of the size of the terminal set on the update period (h), the plant-model mismatch (L_0, L_w) and the controller design parameters (α, β). For example, as h increases, the size of the terminal set increases. Similarly, a larger plant-model mismatch results in a larger terminal set. Note that in the special case when the model uncertainty is vanishing (i.e., $w_i(0) = 0$ and $L_0 = 0$), we have $F_2(h) = 0$ and the terminal set collapses to the origin. In this case, and as long as h is chosen such that $F_1(h) > 0$, the origin of the networked closed-loop plant is guaranteed to be asymptotically stable.

V. Simulation Study: Application to chemical reactors with recycle

We consider a plant composed of two cascaded non-isothermal continuous stirred-tank reactors (CSTRs) with recycle. The output of CSTR 2 is passed through a separator that removes the products and recycles the unreacted material to CSTR 1. The reactant species A is consumed in each reactor by three parallel irreversible exothermic reactions; and a jacket is used to remove/provide heat to each reactor. Under standard modeling assumptions, a plant model of the following form can be derived:

$$\dot{T}_1 = \frac{F_0}{V_1}(T_0 - T_1) + \frac{F_r}{V_1}(T_2 - T_1) + \sum_{i=1}^{3} G_i(T_1)C_{A1} + \frac{Q_1}{\rho c_p V_1}$$

$$\dot{C}_{A1} = \frac{F_0}{V_1}(C_{A0} - C_{A1}) + \frac{F_r}{V_1}(C_{A2} - C_{A1}) - \sum_{i=1}^{3} R_i(T_1)C_{A1}$$

$$\dot{T}_2 = \frac{F_1}{V_2}(T_1 - T_2) + \frac{F_3}{V_2}(T_{03} - T_2) + \sum_{i=1}^{3} G_i(T_2)C_{A2} + \frac{Q_2}{\rho c_p V_2}$$

$$\dot{C}_{A2} = \frac{F_1}{V_2}(C_{A1} - C_{A2}) + \frac{F_3}{V_2}(C_{A03} - C_{A2}) - \sum_{i=1}^{3} R_i(T_2)C_{A2}$$

where $R_i(T_j) = k_{i0}\exp\left(\frac{-E_i}{RT_j}\right)$, $G_i(T_j) = \frac{(-\Delta H_i)}{\rho c_p}R_i(T_j)$, for $j = 1, 2$. T_j, C_{Aj}, Q_j, F_j, and V_j denote the temperature, reactant concentration, rate of heat input, outlet flow rate and volume of the j-th reactor, respectively. F_0 and F_3 denote the flow rates of fresh feed streams to CSTR 1 and 2, respectively, C_{A0} and C_{A03} are the molar concentrations of A in the fresh feed streams, T_0 and T_{03} are the temperatures of those streams, and F_r is the flow rate of the recycle stream. ΔH_i, k_i, E_i, $i = 1, 2, 3$, denote the enthalpies, pre–exponential constants and activation energies of the three reactions, respectively, c_p and ρ denote the heat capacity and density of fluid in the reactor. Using typical values for the process parameters (see [15]), the plant with $Q_1 = Q_2 = 0$, $C_{A0} = C_{A0}^s, C_{A03} = C_{A03}^s$ and recycle rate $r = 0.5$, has three steady states: two locally asymptotically stable and one unstable at $(T_1^s, C_{A1}^s, T_2^s, C_{A2}^s) = (457.9\ K, 1.77\ kmol/m^3, 415.5\ K, 1.75\ kmol/m^3)$. The control objective is to stabilize the plant at the (open-loop) unstable steady-state. Operation at this point is typically sought to avoid high temperatures, while simultaneously achieving reasonable conversion. The manipulated variables for the first reactor are chosen to be Q_1 and C_{A0}, while Q_2 and C_{A03} are used as manipulated variables for the second reactor. The control objective is to be achieved with minimal data exchange between the local control systems of the reactors over the communication network.

Following the methodology proposed in Section III, the plant is initially cast in the following form:

$$\dot{x}_i = f_i(x_1, x_2) + G_i u_i,\ i = 1, 2$$

where x_i and u_i are the (dimensionless) state and manipulated input vectors for the i-th unit, respectively, $f_i(\cdot)$ is a sufficiently smooth nonlinear vector function and G_i is a constant matrix. To synthesize an appropriate feedback controller for each unit, we consider the following uncertain model of the plant:

$$\dot{\widehat{x}}_i = f_i(\widehat{x}_1, \widehat{x}_2) + G_i\widehat{u}_i + \omega_i(\widehat{x}_1, \widehat{x}_2)\delta_1$$

where δ_1 represents parametric uncertainty in the enthalpy of the first reaction ($\delta_1 = (\Delta H_1^m - \Delta H_1)/\Delta H_1$, where ΔH_1^m is a nominal value used in the models). It can be verified that $\omega_i(0,0) \neq 0$ and therefore the uncertainty is non-vanishing. As an example of controllers that exponentially stabilize the above plant model, we consider:

$$\widehat{u}_i = -G_i^{-1}(f_i(\widehat{x}_1, \widehat{x}_2) + \lambda_i\widehat{x}_i + \omega_i(\widehat{x}_1, \widehat{x}_2)\delta_1), i = 1, 2 \quad (17)$$

where $\lambda_i > 0$ is a controller design parameter that places the closed-loop eigenvalue of the i-th model at $-\lambda_i$. Note that the above control law is applied only to the plant model. The quasi-decentralized controllers implemented on the actual plant take the form:

$$u_i = -G_i^{-1}\left(f_i(x_i, \widehat{x}_j) + \lambda_i x_i + \omega_i(x_i, \widehat{x}_j)\delta_1\right),\ j \neq i \quad (18)$$

where $\widehat{x}_j$ is an estimate of x_j provided by the model of Eqs.17-18 which is embedded in both control systems. The model estimate is used by the local controller so long as no measurements are transmitted from the neighboring unit, but is updated using the true measurement whenever it becomes available from the network. Our objective is to determine appropriate update periods that stabilize the networked closed-loop plant near the desired steady-state. To apply the result of Theorem 1, the following Lipschitz constants for the plant were calculated, $L_{f_1} = 289.4$ and $L_{f_2} = 94.3$ (other parameters, such as L_{m_i}, L_w, L_0, α and β, can also be calculated once the plant model and controllers are fixed). Then $F_1(h)$ and $F_2(h)$ can be calculated to characterize the admissible update periods h as well as the achievable terminal set.

Figs.1(a)-(b) are contour plots that depict the dependence of $F_1(h)$ and $r(h)$ on both the size of the uncertainty δ_1 and the update period. In plot (a), the area enclosed by the zero contour lines represents an estimate of the region within

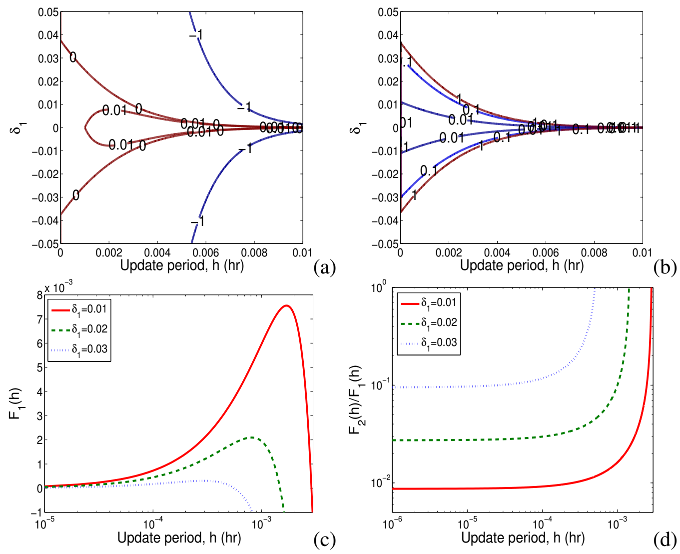

Fig. 1. Dependence of $F_1(h)$ (left) and $r(h) := F_2(h)/F_1(h)$ (right) on the plant-model mismatch for various update periods.

which the closed-loop nonlinear plant can be stabilized, while in plot (b) the value of each contour line represents an upper bound on the size of the terminal set that the closed-loop plant state will converge to when the values for δ_1 and h are chosen within the zone enclosed by that contour line. In obtaining this plot, λ_1 and λ_2 were selected by placing the eigenvalues of the closed-loop models of both reactors at -10. As expected, for a given terminal set, the range of feasible update periods shrinks as the plant-model mismatch increases. Also, for a given plant-model mismatch, the size of the terminal sets grows as the update period is increased. These trends are also depicted in Figs.1(c)-(d). For example, when $\delta_1 = 0.01$ (solid line), a sufficient condition for stability is to choose $h = 0.002$ hr ($F_1(0.002) > 0$); and in this case the plant state will converge to a terminal set with radius less than $F_2(0.002)/F_1(0.002) = 0.0424$. This is further confirmed by the closed-loop state and manipulated profiles in Fig.2 (solid), where the closed-loop plant is stable and converges near the desired steady-state for $h = 0.002$ hr and $\lim_{t\to\infty} \|\mathbf{x}(t)\| = 0.0049 < 0.0424$ (for brevity, only the temperature and rate of heat input profiles for CSTR 1 are shown). Note that since the stability conditions of Theorem 1 are only sufficient, it is possible to achieve the same steady-state offset with larger update periods. The dashed profiles in Fig.2 show that the steady-state offset and closed-loop performance deteriorate substantially as the update period is increased to $h = 0.23$.

VI. Concluding Remarks

In this work, we presented a methodology for the design of a quasi-decentralized networked control system for plants with interconnected nonlinear subsystems that communicate with one another through a resource-constrained (possibly wireless) communication network. To achieve closed-loop stability with minimal cross-communication between the units, a model of the plant was embedded in each control system to recreate the states of the neighboring units when

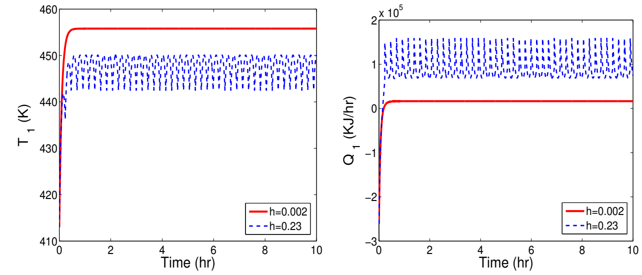

Fig. 2. Closed-loop state and manipulated input profiles for CSTR 1 under the model-based nonlinear quasi-decentralized control strategy with model uncertainty of $\delta_1 = 0.01$ and different update periods.

measurements are not available. The model was updated at discrete time instances to compensate for modeling errors. A sufficient condition for closed-loop stability was obtained leading to an estimate of the maximum allowable update period in terms of model uncertainty and controller design parameters. The results were illustrated using a chemical plant example.

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