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Discrete-time Optimal Reset Control for the Improvement of HDD Servo Control Transient Performance

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Abstract— **This paper proposes a design method of discrete-time reset control systems with the reset time instants pre-specified. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient responses. The design method can guarantee the system stability and the solution of the controller is obtained by solving Riccati equations. The proposed reset controller design method is applied to short-span and long-span track seeking of a hard disk drive servo system. Experiment results show that the proposed design is much more capable of improving transient response than traditional control design methods.**

Index Terms— **Discrete-time reset control systems; reset law; linear quadratic regulation; hard disk drive systems.**

I. INTRODUCTION

Reset control was firstly proposed by Clegg [1] to overcome limitations of linear control. This reset controller, termed as Clegg-integrator, consists of an integrator and a reset law which resets the output of the integrator to zero when its input vanishes. From the basic idea of reset control, one can see that reset control is helpful in reducing windup caused by integration. Moreover, a Clegg integrator has a similar magnitude-frequency response as a pure integrator, but with 51.9^o less phase lag. This favourable property helps to increase the phase margin of a system. In [2], Krishman and Horowitz developed a quantitative control design procedure of Clegg integrator. In [3], Horowitz and Rodenbaum generalized the concept of reset control to higher order systems. More details can be found in [4].

A lot of work has shown the advantages of reset control over linear control. For instance, in [5], an example is presented to show that reset control can achieve some control specifications which cannot be achieved by any ordinary linear control. Besides, [6] shows that reset control can achieve much better sensor noise suppression without degrading disturbance rejection or losing margins. These advantages make reset control an important technique for performance improvement. See [7], [8] and [9] for instance. Recently, reset control was introduced to hard disk drive servo systems [10], [11].

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There are in general two steps in reset control design [12]: linear compensator design and reset element design. Linear compensator is designed to meet all performance specifications other than output overshoot, then reset element is designed to reduce the overshoot. As we know, a reset controller can improve closed-loop performance only when the reset law interacts well with the base linear system. In other words, if the reset controller is not appropriately designed, it may have little contribution to the performance improvement, or even cause performance degradation. For example, reset control may destroy the stability of the closed-loop system if it does not cooperate well with the base linear system.

In reset control system design, there are three basic problems: stability analysis, base linear system design and reset law design.

For the stability analysis, there are many papers in literature addressing this issue, see [13], [14], [15], [16], etc. Most of these existing results require the base linear systems to be stable. However, stability of a reset control system depends on both the base linear system and the reset actions. Each of these two factors can contribute to or destroy the stability of the overall system. Note that reset control systems are also known as impulsive systems and their stability issues have been addressed in [17].

For the reset control law design, most efforts are put on the design of base linear system in existing literatures [4], [6], [10]. The reset law adopted is in general the traditional one, i.e., reset the state of controller to zero when tracking error crosses zero. Base linear system is then designed to interact well with the reset law. We refer to this kind of reset control as the *traditional reset control* in this paper. Actually, reset control can be more general, for instance, the time and the amount of reset can both be designed so that the reset law and the base linear system can cooperate better with each other.

The purpose of this paper is to propose a novel approach to reset control law design for a reset control system and apply the proposed method to improve the transient response of hard disk drive (HDD) systems. We focus on systems of which the base linear systems are already appropriately designed and the reset time instants are pre-specified. The design of the reset law aims to minimize some performance index and its solution is obtained by solving a Riccati equation. In case of equidistant reset control, we show that the resulting reset law is time invariant. Our previous work [18] gives a solution for a continuous form of reset control law, but this form cannot be directly used in digital signal

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control system. This paper shows discrete-time reset control law design method which is suitable for digital sampling control system.

This paper is arranged as follows. In Section II, we set up the problem studied in this paper. In Section III, we investigate optimal reset law design problem. Section IV gives the application of the proposed design method to HDD systems. Some concluding remarks are made in Section V.

II. Problem Setting

A typical reset control system is depicted in Fig. 1. The dynamics of the plant is described by

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^j$ the control input and $y \in \mathbb{R}^p$ the output of the system. The zero-order-hold equivalent description of the plant (1) with the sampling time T_s is given by

$$\begin{cases} x(i+1) &= A_d x(i) + B_d u(i) \\ y(i) &= Cx(i), \end{cases} \tag{2}$$

where $A_d = e^{AT_s}$, $B_d = \int_0^{T_s} e^{A(T_s-\tau)} B d\tau$.

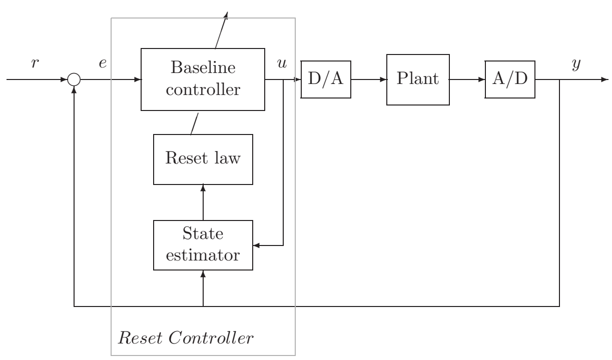

Fig. 1. Block diagram of a reset control system.

Define T_r as the reset period. We assume that the reset interval T_r is at every n sampling intervals, i.e., $T_r = nT_s$, as illustrated below.

$$|\underbrace{\overbrace{---}^{Ts}|\overbrace{---}^{Ts}|\ldots|\overbrace{---}^{Ts}}_{Tr=n\times Ts}|$$

The notations k and i are used to represent the time instant $t = kT_r + iT_s$ with $i = 0, 1, 2, \cdots, n-1$. We then introduce the following discrete reset controller:

$$\begin{cases} z(k,i) = Dz(k,i-1) + Ee(i-1), \\ z(k,0) = \rho_k(x(k-1,n-1), z(k-1,n-1), r), \\ u(k,i) = Gx(k,i) + Hz(k,i) + Me(k,i), \end{cases} \tag{3}$$

where $z \in \mathbb{R}^q$ is the dynamic state in the controller, r is the reference signal which is assumed to be constant, $\rho_k(x,z,r)$ is the reset value at time instant $t = kT_r$ and it is the so-called the reset law. D, E, G, H and M are all constant matrices with compatible dimensions. $e(k,i) = r - y(k,i)$ is the tracking error.

Next we combine (2) and the reset controller (3) as follows.

$$\begin{cases} \bar{x}(k,i) = \bar{A}\bar{x}(k,i-1) + \bar{B}r, \\ z(k,0) = \rho_k(x(k-1,n-1), z(k-1,n-1), r), \\ \bar{y}(k,i) = \bar{C}\bar{x}(k,i), \end{cases} \tag{4}$$

where $\bar{x}(k,i) = (x(k,i)^T \ z(k,i)^T)^T$, $\bar{C} = (C, \ 0_{p\times q})$,

$$\bar{A} = \begin{pmatrix} A_d + B_d G - B_d MC & B_d H \\ -EC & D \end{pmatrix}, \bar{B} = \begin{pmatrix} B_d M \\ E \end{pmatrix}. \tag{5}$$

Before we introduce a cost function for the reset controller design, we need to make an assumption on the steady state of the system (4).

Assumption 1: For any $r \in \mathbb{R}^p$, there exists $\bar{\mathbf{x}}_r = (\mathbf{x}_r^T, \mathbf{z_r}^T)^T \in \mathbb{R}^{n+q}$, such that

$$\bar{A}\bar{\mathbf{x}}_r + \bar{B}r = \bar{\mathbf{x}}_r,$$

$$\bar{C}\bar{\mathbf{x}}_r - r = 0.$$

With the above Assumption, the control input u_r in the steady state is given by

$$u_r = Gx_r + Hz_r. \tag{6}$$

For each time instant $t = kT_r + iT_s$, we define a cost function $J(k,i)$ as

$$J(k,i) = e^T(k,i)Q_{k,i}e(k,i) + (u(k,i) - u_r)^T R_{k,i} (u(k,i) - u_r)$$

Here $Q_{k,i}$ and $R_{k,i}$ are positive semi-definite matrices. The optimal reset law (ORL) design problem considered in this paper is thus formulated as follows.

Problem 1: Design ρ_k, such that the resulting control system is asymptotically stable and meanwhile the cost function $J(\infty)$ is minimized, where

$$J(\infty) := \sum_{k=0}^{\infty}\sum_{i=0}^{n-1} J(k,i) \tag{7}$$

III. Discrete-time Optimal Reset Law Design

In this section, we suppose to solve the ORL problem stated in the previous section. It will be proved that this problem can be equivalently converted into a standard linear quadratic regulation (LQR) problem.

We make a coordinate transformation as follows.

$$\begin{cases} \xi_x(k,i) = x(k,i) - \mathbf{x}_r \\ \xi_z(k,i) = z(k,i) - \mathbf{z}_r \end{cases} \tag{8}$$

where the steady-state values $\mathbf{x}_r$ and $\mathbf{z}_r$ are defined in Assumption 1. From (4), we have

$$\begin{cases} \xi(k,i+1) = \bar{A}\xi(k,i) \\ \xi_z(k,0) = \bar{\rho}(k,0) \\ e(k,i) = -\bar{C}\xi(k,i) \end{cases} \tag{9}$$

where $\xi(k) = (\xi_x(k)^T\ \xi_z(k)^T)^T$ and $\bar{\rho}(k,0) = \rho(k,0) - z_r$. From (3 and (6), it follows that

$$u(k) - u_r = [\ G - MC \quad H\ ] \left(\begin{array}{c} \xi_x(k) \\ \xi_z(k) \end{array} \right). \tag{10}$$

Let $N = \left[\ G - MC \quad H\ \right]$. For a reset interval $(iT_r, (i+1)T_r]$, the cost function should be

$$\begin{aligned} \textstyle\sum_{i=0}^{n-1} J(k,i) &= \textstyle\sum_{i=0}^{n-1} e^T(k,i) Q_{k,i} e(k,i) \\ &\quad + (u(k,i) - u_r)^T R_{k,i} (u(k,i) - u_r) \\ &= \textstyle\sum_{i=0}^{n-1} \xi_x^T(k,i) C^T Q_{k,i} C \xi_x(k,i) \\ &\quad + \xi(k,i)^T N^T R_{k,i} N \xi(k,i) \\ &= \textstyle\sum_{i=0}^{n-1} \xi^T(k,i)(\bar{C}^T Q_{k,i} \bar{C} + \\ &\quad N^T R_{k,i} N) \xi(k,i) \\ &= \textstyle\sum_{i=0}^{n-1} \xi^T(k,0)(\bar{A}^T)^i (\bar{C}^T Q_{k,i} \bar{C} + \\ &\quad N^T R_{k,i} N)(\bar{A})^i \xi(k,0) \\ &= \xi^T(k,0) \textstyle\sum_{i=0}^{n-1} [(\bar{A}^T)^i (\bar{C}^T Q_{k,i} \bar{C} + \\ &\quad N^T R_{k,i} N)(\bar{A})^i] \xi(k,0) \\ &= \xi^T(k,0) \Theta_k \xi(k,0). \end{aligned} \tag{11}$$

with $\Theta_k = \sum_{i=0}^{n-1} [(\bar{A}^T)^i (\bar{C}^T Q_{k,i} \bar{C} + N^T R_{k,i} N)(\bar{A})^i]$.

Hence $J(\infty)$ is the sum of $\xi^T(k,0)\Theta_k \xi(k,0)$ at each reset time instant kT_r, and given by

$$J(\infty) = \sum_{k=0}^{\infty} \xi^T(k,0) \Theta_k \xi(k,0) \tag{12}$$

On the other hand, with the reset interval T_r as sampling interval the system (9) can be written as

$$\left(\begin{array}{c} \xi_x(k+1,0) \\ \xi_z(k+1,0) \end{array} \right) = (\bar{A})^n \left(\begin{array}{c} \xi_x(k,0) \\ \bar{\rho}(k,0) \end{array} \right) \tag{13}$$

Partition $(\bar{A})^n$ as follows,

$$(\bar{A})^n = \left(\begin{array}{cc} \Gamma_A & \Gamma_B \\ * & * \end{array} \right), \tag{14}$$

then we have

$$\xi_x(k+1,0) = \Gamma_A \xi_x(k,0) + \Gamma_B \bar{\rho}(k,0) \tag{15}$$

Note that system (15) is simply a linear discrete system with state variable ξ_x and control input $\bar{\rho}$. Consider the following linear quadratic regulation (LQR) problems of system (15).

Problem 2: Design a control sequence $\rho(i)$, $i = 0, 1, ...$, for system (15) such that the resulting system is asymptotically stable and the the following quadratic performance index is minimized,

$$J(\infty) = \sum_{k=0}^{\infty} \left(\begin{array}{c} \xi_x(k,0) \\ \bar{\rho}(k,0) \end{array} \right)^T \Theta_k \left(\ \xi_x(k,0)^T \quad \bar{\rho}(k,0)^T\ \right)^T.$$

Proposition 1: Suppose that there exist positive numbers $\varepsilon > 0$ such that

$$\lambda_{min}(\Gamma_B^T \Gamma_B) \geq \varepsilon > 0. \tag{16}$$

Then the ORL Problem 1 is equivalent to LQR Problem 2.

Proof: We only need to prove that under the above condition, system (15) is asymptotically stable if and only if system (9) is asymptotically stable. It is obvious that if system (9) is asymptotically stable, then system (15) is asymptotically stable. In the following, we assume that system (15) is asymptotically stable. Then we have

$$\begin{aligned} \lim_{k\to\infty} \xi_x(k,0) = 0, \\ \lim_{k\to\infty} \Gamma_B \bar{\rho}(k,0) = 0. \end{aligned}$$

According to condition (16), we have

$$\lim_{k\to\infty} \bar{\rho}(k,0) = 0.$$

Note that $\xi_z(k,0) = \bar{\rho}(k,0)$, we have

$$\lim_{k\to\infty} \xi(k,0) = 0,$$

and thus system (9) is asymptotically stable. □

According the above proposition, to solve the ORL problem we only need to solve the corresponding LQR problem 2, which is standard linear quadratic regulation problems, and thus we can obtain the optimal solutions directly by solving some Riccati equations. Partition Θ_k as

$$\Theta_k = \left(\begin{array}{cc} \bar{Q} & \bar{T} \\ \bar{T}^T & \bar{R} \end{array} \right) \tag{17}$$

Define

$$\begin{aligned} \bar{\Gamma}_A = \Gamma_A - \Gamma_B \bar{R}^{-1} \bar{T}^T, \\ \tilde{Q} = \bar{Q} - \bar{T} \bar{R}^{-1} \bar{T}^T \end{aligned} \tag{18}$$

The solution of the LQR problem can be obtained immediately as follows [20].

Proposition 2: Assume that $(\Gamma_A,\ \Gamma_B)$ is controllable and the matrices $\bar{R}$ and $\tilde{Q}$ are positive definite. Then the optimal reset law which stabilizes system (1) and minimizes $J(\infty)$ is given by

$$\rho^*(k,0) = -K(x(k,0) - \mathbf{x}_r) + \mathbf{z}_r, \tag{19}$$

where K is determined by

$$K = (\Gamma_B^T S \Gamma_B + \bar{R})^{-1} (\Gamma_B^T S \Gamma_A + \bar{T}^T) \tag{20}$$

and S is the solution of the Riccati equation

$$S = \tilde{Q} + \bar{\Gamma}_A^T S \bar{\Gamma}_A - \bar{\Gamma}_A^T S \bar{\Gamma}_B (\Gamma_B^T S \Gamma_B + \bar{R})^{-1} \Gamma_B^T S \bar{\Gamma}_A. \tag{21}$$

Furthermore, the minimum of $J(\infty)$ is given by

$$J^*(\infty) = (x(0) - \mathbf{x}_r)^T S (x(0) - \mathbf{x}_r). \tag{22}$$

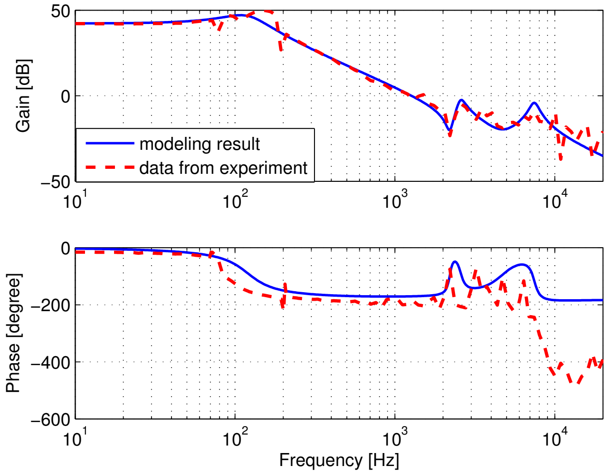

Fig. 2. Frequency response of the VCM actuator.

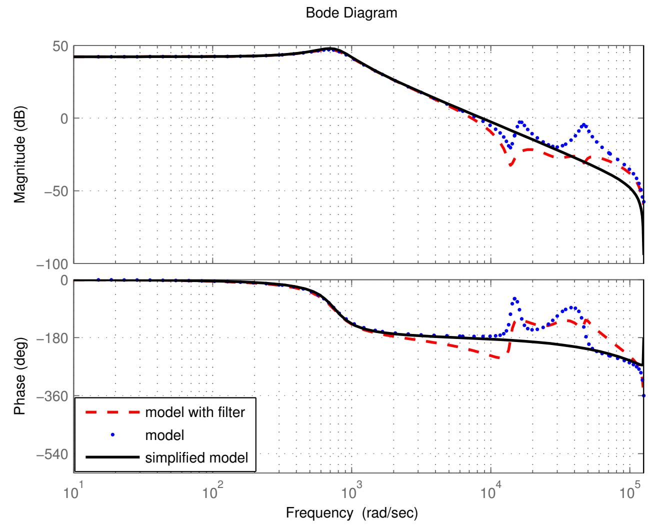

Fig. 3. Frequency response of plant model, plant model with notch filters and simplified plant.

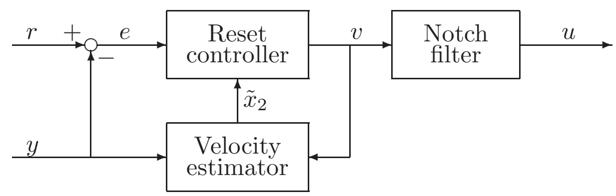

Fig. 4. The structure of reset controller.

IV. Application to HDD servo system

A. System modeling

The frequency responses of a HDD voice coil motor (VCM) actuator have been measured using a Laser Doppler Vibrometer (LDV) and a Dynamic Signal Analyzer (DSA). Based on the measured frequency responses of the actuator, its transfer function is obtained as follows

$$P(s) = \frac{2.4819\times 10^8}{s^2+452.4s+5.685\times 10^5} \times \frac{s^2+1109s+1.922\times 10^8}{s^2+1936s+2.603\times 10^8} \times \frac{s^2+14830s+8.795\times 10^8}{s^2+5931s+2.188\times 10^9} \tag{23}$$

whose frequency responses have been plotted in Fig. 2. We apply notch filters $F_{N1}(s)$ and $F_{N2}(s)$ to compensate the resonance modes.

$$F_{N1}(s) = \frac{s^2+1936s+2.603\times 10^8}{s^2+1.936\times 10^4 s+2.603\times 10^8}$$
$$F_{N2}(s) = \frac{s^2+3181s+2.188\times 10^9}{s^2+59310s+2.188\times 10^9}$$

The system is then simplified as a 2nd order system, which is called the nominal plant and whose frequency response can be seen in Fig 3. The model of the nominal plant is described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -ax_1 - bx_2 + cu, \\ y = x_1, \end{cases} \tag{24}$$

where $a = 5.6849\times 10^5$, $b = 407.1$, $c = 7.3647\times 10^7$. Our target is to design a seeking controller for the VCM actuator to have fast positioning without overshoot.

B. Reset control law design

The controller structure is depicted in Fig. 4.

The reset controller is given by

$$\begin{cases} z(k,i) = Dz(k,i-1) + Ee(k,i-1), \\ z(k,0) = \rho_k(x(k-1,n-1), z(k-1,n-1), r), \\ v(k,i) = \alpha e(k,i) + \beta x_2(k,i) + \gamma z(k,i), \end{cases} \tag{25}$$

with $D = 1$, $E = T_s$, $\alpha = 0.5$, $\beta = 4\times 10^{-4}$, and $\gamma = 6000$.

The base linear controller is given by the first and the third equations in (25). For the reset law design, we assume $R_{k,i}$ and $Q_{k,i}$ are time-invariant. The reset law is given by

$$\rho_k^* = -k_1(x_1 - r) - k_2 x_2 + \frac{a}{c\gamma} r. \tag{26}$$

In the reset controller (26), the velocity x_2 is replaced with its estimated value $\hat{x}_2$. An state observer is needed to obtain the estimated velocity.

C. Simulation result

The input disturbance, output disturbance and measurement noise are added in the simulation. output the sampling period T_s is 2.5×10^{-5}sec and $Q = 1$, $R = 17$. The reset interval $T_r = 2\times T_s$. The reset law is given by (26). By solving Riccati equation (21) and (20), we have

$$k_1 = -3.6049\times 10^{-5},$$
$$k_2 = -4.6124\times 10^{-8}.$$

Fig. 5 gives the step responses for the base linear control, traditional reset control and the optimal reset control. We can

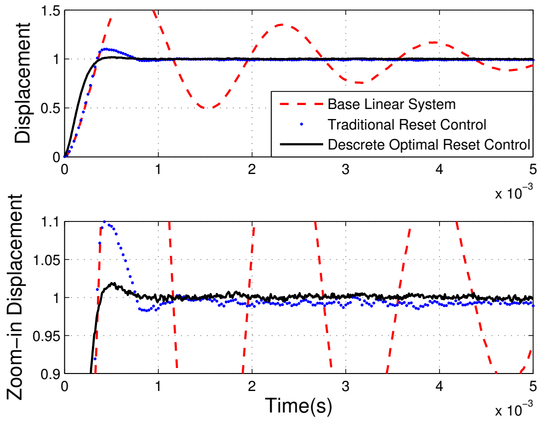

Fig. 5. Step responses for base linear control, traditional reset control and proposed discrete optimal reset control.

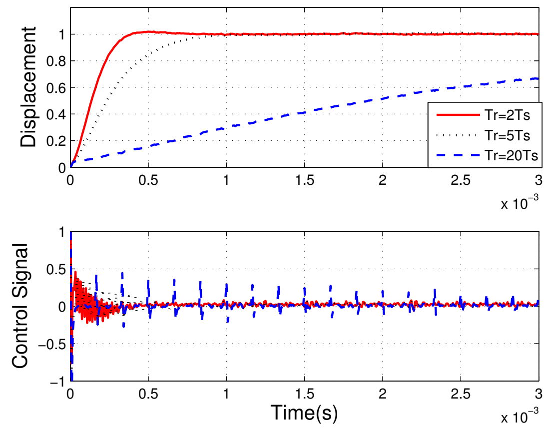

Fig. 6. Step responses for optimal reset control system with different T_r for $r = 1$.

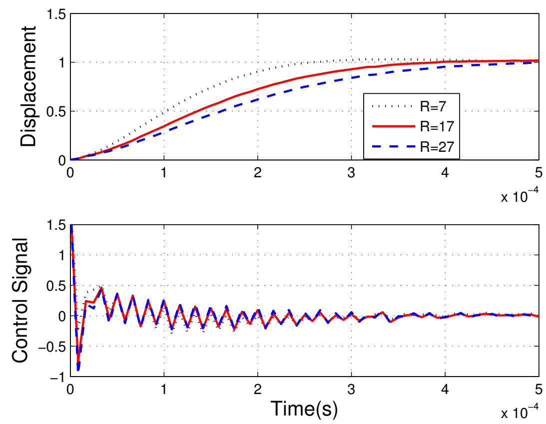

Fig. 7. Step responses for different R/Q.

see that the proposed optimal reset control almost removes the overshoot completely and at the same time reduces the rise time significantly.

Fig. 6 gives the responses for different reset intervals $T_r = 2T_s$, $5T_s$, $20T_s$ respectively. The transient performance is better when reset interval is smaller. In Fig. 7, step responses for different R and fixed value $Q = 1$ are given.

Compared with our previous work in [19], the main advantage of the proposed optimal reset control in this paper is that the design process is more simple. We do not need to worry about the stability since the stability is naturally assured.

Remark 1: In the short-span seeking, we find that the performance is not sensitive to T_r ($T_r = nT_s$) with $n \leq 3$. But if $n > 3$, the performance will become worse. Since the optimal cost function weights R and Q should be set for different input levels, in order to tradeoff the ratio of input and error signal. we need different values of R/Q for different reference levels r so as to achieve the best performance.

Remark 2: For case of $T_r = 20T_s$ in Fig. 6, when reset is injected, sharp changes in the output of the reset element occur, and then the abrupt responses cause big spikes in the control signal.

In the long-span seeking control, we choose $Q = 1$ and $R = 12$. We add the control signal saturation $|u| < 2.5$ as the bounded input effort. We found that in this range, the control input has great efficient during rising stage (see the control input signals in Fig. 8). Compared to the design of optimal time control, we do not need to design a transitional controller to smooth the input in order to avoid chattering, thus the design process is more simple.

Remark 3: In simulations, the performance is also not sensitive to T_r ($T_r = nT_s$) with $n \leq 10$. But if $n > 10$, the performance will become worse. In the long-span seeking control, the performance is not sensitive to R/Q. The control signal will not be amplified boundless as a result of the saturation part, so the R/Q can be fixed without changing the performance.

D. Experimental Results

The discrete optimal reset control law is verified via implementation in the HDD servo system and the seeking performance is investigated.
Fig. 9 shows the seeking result for $r = 0.25\mu m$ with comparison to the linear PID control. It is seen that the rising time of the linear controller and the reset controller are similar, but when reaching the target, the response with the ORL method is swiftly settled to the target with less overshoot. The settling time is $0.33ms$. Fig. 10 shows the seeking performance for $r = 4\mu m$. The performance of this longer span seeking with the reset control is also much better than that with the PID control. No overshoot is noticed and the settling time is less than 0.5 ms. The control effort of ORL method have much more efficiency since it is minimized via the cost function including the control input term.

V. Conclusion

In this paper, we have studied discrete optimal reset law design of reset control systems. Firstly, we transferred closed-loop reset control system to discrete linear system

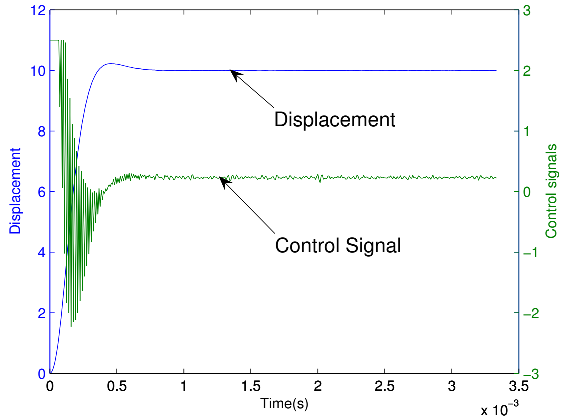

Fig. 8. Displacement and control signal for $r = 10$.

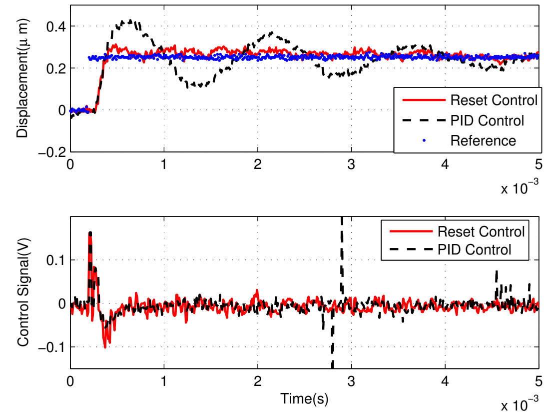

Fig. 9. Displacement and control signal for $r = 0.25\mu m$.

with reset value as control input. Then we proved that ORL design problem is equivalent to LQR problem. Based on this, the optimal reset laws were given in terms of Riccati equations. The design process of the proposed reset law is very simple. The resulting reset law is of feedback form and thus can easily be implemented online.

The proposed design was applied to seeking control of single-stage hard disk drive systems. Both short-span and long-span seeking control were studied. Simulation results show that the proposed design can achieve better seeking performance.

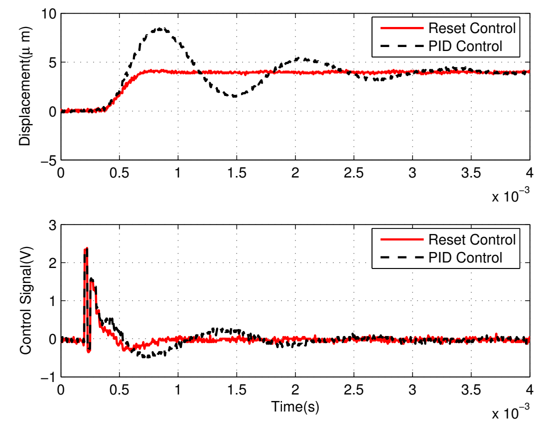

Fig. 10. Displacement and control signal for $r = 4\mu m$.

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