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Dynamic Feedback Linearization applied to Asymptotic Tracking: generalization about the Turbocharged Diesel Engine outputs choice

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Abstract— **In this paper we apply dynamic feedback linearization to the tracking problem for a turbocharged diesel engine (TDE) equipped with exhaust gas recirculation (EGR) valve and variable geometry turbocharger (VGT). The model used here is the third-order mean-value model, a reduction of the eighth-order mean-value one, see [13], for sake of simplicity. Our goal is to track desired values of suitably chosen outputs. In fact, we first plan to control the input manifold pressure and the compressor mass flow rate instead of the air fuel ratio (AFR) and the EGR fraction. Unfortunately, the former lead to a non-minimum phase system while the latter are not accessible for measurements in a vehicle, see [7]. We thus replace the problem of tracking of desired values of the original output y (input manifold pressure and compressor mass flow rate) by that of tracking a suitably constructed modified output for which the values to be tracked are specifically chosen: namely, when the modified output $\tilde{y}$ approaches them, the original output converges to the desired values. Simulation results are presented.**

I. INTRODUCTION

In order to comply with more constraining European emission regulations, refer to [1] (Euro norms see [16] and [17]), car manufacturers introduce in some diesel engines two actuators: the exhaust gas recirculation (EGR) valve and the variable geometry turbocharger (VGT). The former permits recirculation of exhaust gas into the intake manifold reducing by this way the formation of NO_x while the latter permits the improvement of the relatively low power diesel engine density. But some drawbacks have to be underlined: an important reduction of the amount of fresh air leads to an increase in particulate emissions and possibly visible smoke whereas a low amount of EGR fraction leads to an increase in NO_x emissions, see [7].

To render these two actuators more efficient during combustion, several control design methods have been proposed: polynomial control in [2], dynamic feedback linearization in [12] and [13], optimal nonlinear control in [14], constructive Lyapunov control in [7], indirect passivation in [9], passivation in [8], predictive control in [3], [4] and [11], etc.

The output y of to-be-controlled variables, which consists of the input manifold pressure p_1 and the compressor mass flow rate W_c, leads to a non-minimum phase system. Therefore we propose another choice of output (whose zero dynamics are trivial) such that if that modified output $\tilde{y}$ tracks a suitably chosen value, then the original output $y(t)$ converges asymptotically to its desired value. In [3] we solved the tracking problem for the TDE model using nonlinear predictive control. In [13], Plianos *et al.* apply dynamic feedback linearization based on the property of flatness to a reduced-order TDE model. Pursuing their work, this paper presents a more general result on the application of the dynamic feedback linearization control design method to the simplified TDE model. Indeed, we see, with notions of geometric control that for each suitably chosen vector output, the third-order nonlinear model remains dynamic feedback linearizable with a trivial zero dynamics. To argue this, we chose a different vector output from that of Plianos *et al.* in [13] for this study.

The paper is organized as follows: in Section II a description of TDE is given and its simplified model is presented with a brief remind on the eighth-order one. Vector output is chosen. In Section III we examine the behavior of zero dynamics and propose a modification of the output. A construction of a dynamic extension of the simplified model and a general result about the TDE outputs choice are given in Section IV. Section V presents the control law derived from feedback linearization theory while simulation results are presented in Section VI. Conclusion and some future research directions are briefly given in Section VII.

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II. TURBOCHARGED DIESEL ENGINE: DESCRIPTION, MODEL AND OUPUTS CHOICE

A. Description of TDE functioning

A schematic diagram (see Fig. 1) of TDE is presented below. At the top of this diagram is the turbocharger composed of the turbine and the compressor. During the functioning of TDE, the turbine takes its energy from the exhaust gas. As it is linked to the compressor via a common shaft, the compressor starts rotating, bringing consequently fresh air into the combustion chambers via the intercooler and the intake manifold. A part of the exhaust gas is recirculated into the combustion chambers to reduce NO_x formation refer to [7].

B. Reduced-order model

The following presented nonlinear model (1) is a simplified model from the eighth-order one briefly outlined in [13]:

$$\begin{array}{l} \dot{p_1} = k_1(W_c + W_{egr} - k_e p_1) + \frac{\dot{T_1}}{T_1} p_1 \\ \dot{p_2} = k_2(k_e p_1 - W_{egr} - W_t + W_f) + \frac{\dot{T_2}}{T_2} p_2 \\ \dot{P_c} = \frac{1}{\tau}(\eta_m P_t - P_c) \end{array}, \quad (1)$$

where the compressor (resp. the turbine) mass flow rate is related to the compressor (resp. the turbine) power as follows:

$$W_c = P_c \frac{k_c}{p_1^{\mu} - 1} \quad (2)$$

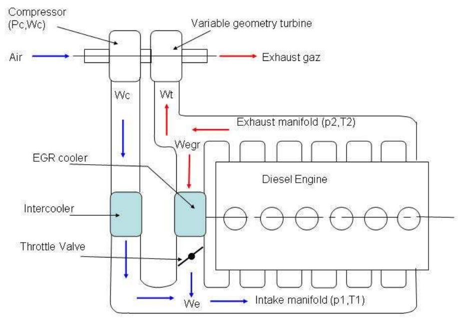

Fig. 1. Turbocharged diesel engine (TDE)

$$\left(\text{resp.}\quad P_t = k_t\left(1 - p_2^{-\mu}\right)W_t\right). \tag{3}$$

Despite of the fact that the real inputs are the EGR valve and VGT openings, the considered inputs, in this study, for sake of simplicity, are $u_1 = W_{egr}$ and $u_2 = W_t$, see [13].

In the sequel, $\dot{T}_1$ and $\dot{T}_2$ are assumed to vanish because their corresponding measured signals T_1 and T_2 have very slow variations, see [7]. The fuel mass flow rate W_f is regarded as an external disturbance and will not be taken into account for the synthesis of the controller. This yields the following system:

$$\begin{array}{l} \dot{p_1} = k_1(W_c + u_1 - k_e p_1) \\ \dot{p_2} = k_2(k_e p_1 - u_1 - u_2) \\ \dot{P_c} = \frac{1}{\tau}(\eta_m P_t - P_c) \end{array} . \tag{4}$$

Replacing W_c and P_t by their expressions (2) and (3) and denoting $K_0 = \frac{\eta_m}{\tau}k_t$ yield the system:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2, \tag{5}$$

where

$$f(x) = \left[\begin{array}{c} k_1 k_c \frac{P_c}{p_1^{\mu} - 1} - k_1 k_e p_1 \\ \\ k_2 k_e p_1 \\ \\ -\frac{P_c}{\tau} \end{array}\right], \tag{6}$$

$$g_1(x) = \left[\begin{array}{c} k_1 \\ -k_2 \\ 0 \end{array}\right] \text{ and } g_2(x) = \left[\begin{array}{c} 0 \\ -k_2 \\ K_0\left(1 - p_2^{-\mu}\right) \end{array}\right] \tag{7}$$

and $x = (p_1, p_2, P_c)$ belong to the set Ω, see [7], defined by

$$\begin{array}{l} \Omega = \{(p_1, p_2, P_c) : 1 < p_1 < p_1^{max}, \\ 1 < p_2 < p_2^{max}, 0 < P_c < P_c^{max}\}, \end{array} \tag{8}$$

with the maximal values $p_1^{max}, p_2^{max}, P_c^{max}$ follow from physical limits of TDE.

All the parameters of the model k_1, k_2, k_c, k_e, k_t, τ and η_m are identified from the eighth-order mean-value nonlinear model at a constant speed of 1600 RPM and a fueling rate of 7.2 kg/h as said in a previous study, see [13]. The full-order model consists of the equations of the change of pressures, masses and fractions of burned gas in the intake and exhaust manifolds. These six equations are completed by two more: the turbocharger speed and the air mass flow rate in the pipe connecting the compressor outlet and the intake manifold, see [13]. For a detailed description of the full-order model see [7] and [15]. The nomenclature of some TDE variables is summarized in TABLE I, see [7] and [13].

TABLE I

NOMENCLATURE OF SOME DIESEL VARIABLES

Variable	Description
EGR	Exhaust gas recirculation
AFR	Air fuel ratio
N	Engine speed
F_1	Intake manifold burned gas fraction
F_2	Exhaust manifold burned gas fraction
m_1	Mass of gas in the intake manifold
m_2	Mass of gas in the exhaust manifold
p_1	Gas pressure in the intake manifold
p_2	Gas pressure in the exhaust manifold
P_c	Compressor power
P_t	Turbine power
W_e	Total mass flow rate into the engine
W_c	Compressor mass flow rate
W_t	Turbine mass flow rate
W_f	Fuel mass flow rate
W_{egr}	EGR mass flow rate
V_1	Intake manifold volume
V_2	Exhaust manifold volume
T_1	Intake manifold temperature
T_2	Exhaust manifold temperature
T_c	Compressor temperature
T_e	Temperature of the exhaust from the engine
T_{egr}	EGR temperature
ω_{tc}	Turbocharger speed
J_{tc}	Turbocharger moment of inertia
η_c	Compressor isentropic efficiency
η_t	Turbine isentropic efficiency
η_m	Turbocharger mechanical efficiency
γ	Specific heat ratio
R	Specific gas constant
μ	$\frac{\gamma-1}{\gamma}$

C. Vector output choice

The output of to-be-controlled variables consists of the input manifold pressure p_1 and the compressor mass flow rate W_c instead of the AFR and EGR fraction because the latter are not accessible for measurements in a vehicle, see [7]. We thus consider the nonlinear system (4) (equivalently, (5)-(7)) with the vector output:

$$y = \left[\begin{array}{c} p_1 \\ W_c \end{array}\right] \tag{9}$$

and the goal is to track desired constant values p_{1d} of p_1 and W_{cd} of W_c. In the sequel, we suppose that all the components (p_1, p_2, P_c) of the state x measured or estimated.

III. ZERO DYNAMICS AND CHANGE OF THE OUTPUT

Consider the square-MIMO $(m \times m)$ nonlinear system

$$\begin{array}{l} \dot{x} = f(x) + \sum_{j=1}^{m} g_j(x)u_j \\ y = (h_1(x), \ldots, h_m(x))^t, \end{array} \tag{10}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^m$ are the vectors state, control, and output, respectively.

To simplify the exposition, the standard geometric notation for Lie derivatives is used in this paper. For a real-valued function h on R^n and a vector field f on R^n, the Lie derivative of h along f at $x \in R^n$ is given by:

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i}(x) f_i(x).$$

Inductively, we define

$$L_f^k h(x) = L_f L_f^{k-1} h(x) = \frac{\partial L_f^{k-1} h}{\partial x}(x) f(x),$$

with $L_f^0 h(x) = h(x)$.

A. Vector relative degree

A system of the form (10) has a vector relative degree $(\rho_1, \cdots, \rho_m)$ if:

(i) for any $x \in R^n$

$$L_{g_j} L_f^k h_i(x) = 0,$$

for all $1 \le i \le m$, all $1 \le j \le m$, and all $0 \le k < \rho_i - 1$;

(ii) the $m \times m$ matrix (decoupling matrix)

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{\rho_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{\rho_1 - 1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{\rho_m - 1} h_m(x) & \cdots & L_{g_m} L_f^{\rho_m - 1} h_m(x) \end{bmatrix} \quad (11)$$

is nonsingular for all $x \in R^n$ (see, e.g., [6]).

For the third-order model (5)-(7) with the output (9), the vector relative degree exists and $(\rho_1, \rho_2) = (1, 1)$ for all $(p_1, p_2, P_c) \in \Omega$. The decoupling matrix is:

$$A(x) = \begin{bmatrix} k_1 & 0 \\ -\frac{\mu k_c k_1 P_c p_1^{\mu - 1}}{(p_1^{\mu} - 1)^2} & -K_0 k_c \frac{p_2^{-\mu} - 1}{p_1^{\mu} - 1} \end{bmatrix}.$$

The sum of the vector relative degree components is equal to 2 which is less than 3, the dimension of the state space of system (5)-(7). Therefore one-dimensional zero dynamics exist. An examination of their stability is necessary before deriving the vector control law (assuring tracking the desired output value).

B. Zero dynamics

Since the goal is to track a reference signal, which consists of desired fixed values p_{1d} and W_{cd} of the respective components of the output $y = (p_1, W_c)^t$, define the error

$$e = \begin{bmatrix} p_1 - p_{1d} \\ W_c - W_{cd} \end{bmatrix}, \quad (12)$$

between the to-be-controlled variables (p_1, W_c) and their desired values. The zero dynamics of the error are obtained by applying the control annihilating identically the error $e(t)$ and thus are given by

$$\dot{p_2} = k_2 W_{cd} \left[1 - \frac{(p_{1d}^{\mu} - 1)}{\eta_m k_t k_c (1 - p_2^{-\mu})} \right], \quad (13)$$

which are unstable. Indeed, (13) has a single equilibrium $p_{2e} = \left[1 - \frac{p_{1d}^{\mu} - 1}{\eta_m k_t k_c} \right]^{-\frac{1}{\mu}} \cong 1.7$ bar and the eigenvalue $\lambda \cong 25$ of the linearization of (13) at p_{2e} is positive, see Fig. 2 and Fig. 3. These numerical values are given for the values $p_{1d} \cong 1.6$ bar and $W_{cd} \cong 0.07$ kg/s that are, indeed, natural for applications. It can be noticed, however, that the instability of the zero dynamics (13) depends neither on the choice of p_{1d} nor of W_{cd}. The system is I-O decouplable

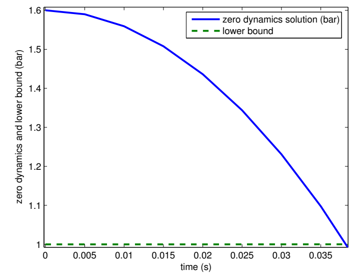

Fig. 2. Unstable zero dynamics with initial condition of $p_{20} = 1.6$ bar

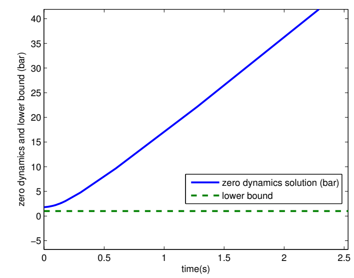

Fig. 3. Unstable zero dynamics with initial condition of $p_{20} = 1.8$ bar

(via static feedback). So using I-O linearization we can easily find a control law that steers the error $e(t)$, given by (12), asymptotically to zero (see, e.g., [6]). Unstable zero dynamics will result, however, in an undesired property: the internal variable p_2 will not remain bounded, it will either reach the limit value $p_2 = 1$ in finite time (if $p_{20} < p_{2e}$, see Fig. 2) or will go to plus infinity with an asymptotically constant velocity (if $p_{20} > p_{2e}$, see Fig. 3). To avoid dealing with unstable zero dynamics, we propose another choice of vector output and a dynamic extension.

C. Change of the vector output

We will overcome the problem of unstable zero dynamics by changing the output (9) such that the modified system has trivial zero dynamics, that is, consisting of a single equilibrium point. This can be achieved by keeping the first component as p_1 and replacing the second output component by a function $\tilde{h}_2(x)$, where $x = (p_1, p_2, P_c)$, such that, indeed, $L_{g_2} \tilde{h}_2 = 0$. Resolving this equation gives

$$\tilde{h}_2(x) = P_c + \frac{K_0}{k_2} \left(p_2 - \frac{1}{1 - \mu} p_2^{1 - \mu} \right),$$

and thus we consider the new output $\tilde{y}(t)$ of to-be-controlled variables defined by

$$\tilde{y} = \tilde{h}(x) = \begin{bmatrix} p_1 \\ P_c + \frac{K_0}{k_2} \left(p_2 - \frac{1}{1 - \mu} p_2^{1 - \mu} \right) \end{bmatrix}. \quad (14)$$

In the next subsection we will show that the system (5)-(7) with the output (14) has, indeed, trivial zero dynamics.

As we specified, our problem is to track desired constant values p_{1d} of p_1 and W_{cd} of W_c. A natural question is thus how to reformulate the problem in terms of the components of the new output $\tilde{y}$, given by (14), in order to achieve a solution of the original tracking goal. Notice that W_c, P_c, and p_1 are linked via the relation $W_c = P_c \frac{k_c}{p_1^{\mu}-1}$ and hence the desired tracking values p_{1d} and W_{cd} determine uniquely the desired value P_{cd} of P_c as

$$P_{cd} = W_{cd} \frac{p_{1d}^{\mu} - 1}{k_c}.$$

Notice that given any fixed p_{1d} and P_{cd}, there exists a unique point $x_e = (p_{1e}, p_{2e}, P_{ce})$, satisfying $p_{1e} = p_{1d}$ and $P_{ce} = P_{cd}$, and unique control values $u_e = (u_{1e}, u_{2e})$ such that the right hand side of (5)-(7) has an equilibrium at x_e when the controls are evaluated at u_e. To see this, recall that the equilibrium set of (5) consists of the points at which we can create an equilibrium by a suitable feedback and thus $E = \{x : \ f(x) \in \operatorname{span}\{g_1(x), g_2(x)\}\}$. Observe that the zero dynamics manifold of the error e, given by (12), is $Z_d^* = \{p_1 = p_{1d}, \ W_c = W_{cd}\}$ and is thus transversal to the equilibrium set E. More precisely, the intersection $\{x_e\} = E \cap Z_d^*$ consists of a single equilibrium point $x_e = (p_{1d}, p_{2e}, P_{cd})$, with uniquely defined p_{2e}, and there exist unique control values $u_e = (u_{1e}, u_{2e})^t \in \mathbb{R}^2$ such that

$$f(x_e) + u_{1e} g_1(x_e) + u_{2e} g_2(x_e) = 0.$$

We will define the desired tracking value $\tilde{h}_{2d}$ of $\tilde{h}_2$, the second component of the new output $\tilde{y}$ (given by (14)), as

$$\tilde{h}_{2d} = \tilde{h}_2(x_e) = P_{cd} + \frac{K_0}{k_2}\left(p_{2e} - \frac{1}{1-\mu} p_{2e}^{1-\mu}\right).$$

Notice that the zero dynamics manifold $\tilde{Z}_d^*$ of the error

$$\tilde{e} = \begin{bmatrix} p_1 - p_{1d} \\ \tilde{h}_2(x) - \tilde{h}_{2d} \end{bmatrix} \tag{15}$$

passes through the equilibrium point x_e. Now a crucial observation is that when the new output $\tilde{y}(t)$ tracks asymptotically the constant value $(p_{1d}, \tilde{h}_{2d})$ and the overall system approaches the equilibrium point x_e, then the original output $y(t)$ tracks asymptotically the desired values (p_{1d}, W_{cd}). It follows that in order to solve the tracking problem, it is enough to show that the zero dynamics corresponding to the new error (15) are asymptotically stable (for instance, trivial consisting of x_e only). The main idea of changing the output is illustrated in Fig. 4. The zero dynamics, evolving on Z_d^*, of the error (between the original output $y(t)$ and its desired value) are unstable. We look for a new output $\tilde{y}$ and for its desired value such that the zero dynamics of the new error are asymptotically stable and their manifold $\tilde{Z}_d^*$ intersect Z_d^* at an equilibrium point x_e. A solution can be either asymptotically stable dynamics on $\tilde{Z}_d^*$ (as illustrated by Fig. 4) or trivial zero dynamics reduced to the equilibrium x_e (which will be the case of our TDE model).

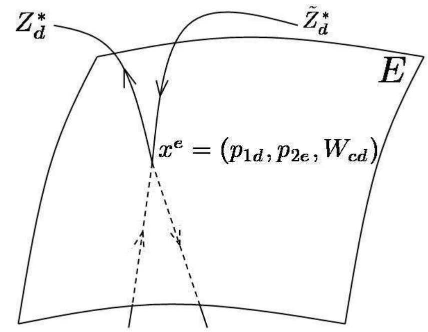

Fig. 4. Equilibrium set E and zero dynamics

IV. Dynamic Extension of the Simplified Model and Generalization About the TDE Outputs Choice

In this section we will follow notions of geometric nonlinear control (see, e.g., [6] and [10]). The system (5)-(7) with the output (14) has the decoupling matrix

$$\tilde{A}(x) = \begin{bmatrix} k_1 & 0 \\ -\frac{\mu k_c k_1 P_c p_1^{\mu-1}}{(p_1^{\mu}-1)^2} & 0 \end{bmatrix}, \tag{16}$$

which is not invertible, and thus the system has no a vector relative degree. We can, however, construct a suitable dynamic extension with a well defined relative degree. To this end, put $z = u_1 = W_{egr}$, $\dot{z} = v_1$ and apply the new vector control $[v_1, v_2]^t$, where $v_2 = u_2 = W_t$. This yields the following extended nonlinear system

$$\begin{aligned} \dot{p_1} &= k_1(W_c + z - k_e p_1) \\ \dot{p_2} &= k_2(k_e p_1 - z - v_2) \\ \dot{P_c} &= \tfrac{1}{\tau}(\eta_m P_t - P_c) \\ \dot{z} &= v_1, \end{aligned} \tag{17}$$

which we can rewrite, denoting its extended state by $x^e = (p_1, p_2, P_c, z)^t \in \Omega \times [W_{egr}^{min}; W_{egr}^{max}]$ (where W_{egr}^{min} and W_{egr}^{max} are the minimum and maximum values of the EGR mass flow rate), as

$$\dot{x}^e = f^e(x^e) + g_1^e(x^e) v_1 + g_2^e(x^e) v_2, \tag{18}$$

where

$$f^e(x^e) = \begin{bmatrix} k_1(k_c \frac{P_c}{p_1^{\mu}-1} - k_e p_1 + z) \\ k_2(k_e p_1 - z) \\ -\frac{P_c}{\tau} \\ 0 \end{bmatrix}, \tag{19}$$

$$g_1^e(x^e) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } g_2^e(x^e) = \begin{bmatrix} 0 \\ -k_2 \\ K_0\left(1 - p_2^{-\mu}\right) \\ 0 \end{bmatrix}. \tag{20}$$

The extended system (18)-(20) with the output (14) has the vector relative degree $(\rho_1^e, \rho_2^e) = (2, 2)$ and the invertible decoupling matrix

$$A^e(x^e) = \begin{bmatrix} k_1 & k_1 k_c K_0 \frac{1-p_2^{-\mu}}{p_1^{\mu}-1} \\ K_0(p_2^{-1}-1) & \mu k_2 K_0 (z-k_e p_1) p_2^{-\mu-1} \\ & +\frac{K_0}{\tau}(p_2^{-\mu}-1) \end{bmatrix}. \quad (21)$$

The extended system is thus I-O decouplable and has trivial zero dynamics (since the sum of the components of its vector relative degree is $\rho_1^e + \rho_2^e = 2 + 2 = 4$, the dimension of the state space of the extended system). Notice that the original system (5)-(7), with the output (14), has trivial zero dynamics too because the latter does not depend on invertible endogenous feedback.

A natural question is whether a dynamic extension is necessary. In other words, is it possible to choose another output, say $\bar{y} = \bar{h}(x)$, such that the original system (5)-(7), with the output $\bar{y}$, would have the vector relative degree $(\bar{\rho}_1, \bar{\rho}_2) = (2, 1)$. In this case, the original system would be static feedback I-O decouplable with trivial zero dynamics (with respect to the output $\bar{y}$) and no extension would be needed. The answer is: this is impossible, independently of the choice of the output $\bar{y} = \bar{h}(x)$. Indeed, if such an $\mathbb{R}^2$-valued function $\bar{h}$ exists, then the original system (5)-(7) would be static feedback linearizable. This is not the case, however, because the distribution $\mathcal{D} = \operatorname{span}\{g_1, g_2\}$ is not involutive since the Lie bracket

$$[\ g_1, g_2\ ] = \begin{bmatrix} 0 \\ 0 \\ -\mu k_2 K_0 p_2^{-\mu-1} \end{bmatrix} \quad (22)$$

is independent of g_1 and g_2.

Another way of looking at the extension procedure that we propose is to observe that although the system (5)-(7) is not static feedback linearizable it is, however, flat, refer to [5] and [13] (that is, dynamic feedback linearizable) since any 3-dimensional system with noninvolutive distribution $\mathcal{D} = \operatorname{span}\{g_1, g_2\}$ is so and the components p_1 and $\tilde{h}_2$, given by (14), or any other suitably chosen outputs, see [13], of the new output $\tilde{y}$ are actually flat outputs (linearizing outputs) of the system (5)-(7). Indeed, we can express the state components p_1, p_2, P_c as well as the controls u_1 and u_2 of the system using p_1 and $\tilde{h}_2$ and their time derivatives; when calculating u_2 we will have to differentiate u_1 which confirms that the system is dynamically (but not statically) linearizable.

V. CONTROL LAW

In [3] we solved the tracking problem for the simplified TDE model using nonlinear predictive control. In this section we will linearize the extended system which will allow us to calculate the control law that assures tracking desired values of the modified output (14) which, in turn, guarantees tracking the desired values of the original output (9) (as we explained in Section III). The extension (18)-(20) of the TDE model, together with the output (14), is I-O decouplable and has trivial zero dynamics so the linearizing coordinates and the linearizing feedback can be calculated as follows, see, e.g., [6] and [10]. Introduce new coordinates

$$\begin{aligned} \varphi_1^1 &= \tilde{h}_1(x^e) = p_1 \\ \varphi_2^1 &= L_{f^e}\tilde{h}_1(x^e) = k_1\left(z - k_e p_1 + \frac{k_c}{p_1^{\mu}-1}P_c\right) \\ \varphi_1^2 &= \tilde{h}_2(x^e) \\ \varphi_2^2 &= L_{f^e}\tilde{h}_2(x^e) = -\frac{P_c}{\tau} + K_0(k_e p_1 - z)(1-p_2^{-\mu}) \end{aligned} \quad (23)$$

followed by the feedback

$$v(x) = A^e(x^e)^{-1}[-b(x^e) + w] \quad (24)$$

where $A^e(x^e)$ is the decoupling matrix (21), and $b(x^e)$ is given by:

$$b(x^e) = \begin{bmatrix} k_1^2\left(k_e + \frac{\mu k_c P_c p_1^{\mu-1}}{(p_1^{\mu}-1)^2}\right)\left(k_c\frac{P_c}{p_1^{\mu}-1} + z - k_e p_1\right) \\ +\frac{k_1 k_c}{\tau}\frac{P_c}{p_1^{\mu}-1} \\ \\ -\frac{P_c}{\tau^2} - k_1 k_e K_0(1-p_2^{-\mu})\left(k_c\frac{P_c}{p_1^{\mu}-1} + z - k_e p_1\right) \\ -\mu k_2 K_0 p_2^{-\mu-1}(k_e p_1 - z)^2 \end{bmatrix}.$$

This yields the following decoupled system:

$$\begin{aligned} \dot{\varphi}_1^1 &= \varphi_2^1 \\ \dot{\varphi}_2^1 &= w_1 \\ \dot{\varphi}_1^2 &= \varphi_2^2 \\ \dot{\varphi}_2^2 &= w_2 \end{aligned} \quad \text{with the output} \quad \begin{aligned} \tilde{y}_1 &= \varphi_1^1 \\ \tilde{y}_2 &= \varphi_1^2. \end{aligned} \quad (25)$$

The desired tracking value p_{1d} of φ_1^1 is given while desired tracking value $\tilde{h}_d$ of φ_1^2 is deduced from W_{cd} and from the requirement that $x_e^e = (p_{1d}, p_{2e}, P_{cd}, z_d)$ is an equilibrium point (see Section III). This yields

$$\begin{aligned} z_d &= u_{1d} = k_e p_{1d} - W_{cd} \\ p_{2e} &= \left(1 - \frac{P_{cd}}{\tau K_0 W_{cd}}\right)^{(-\frac{1}{\mu})} \\ h_d &= P_{cd} + \frac{K_0}{k_2}\left(p_{2e} - \frac{1}{1-\mu}p_{2e}^{1-\mu}\right). \end{aligned} \quad (26)$$

The tracking control law is now calculated as

$$\begin{aligned} w &= \begin{bmatrix} K_1^1(\varphi_1^1 - p_{1d}) + K_2^1\varphi_2^1 \\ K_1^2(\varphi_1^2 - \tilde{h}_{2d}) + K_2^2\varphi_2^2 \end{bmatrix} \\ &= \begin{bmatrix} K_1^1\varphi_1^1 + K_2^1\varphi_2^1 \\ K_1^2\varphi_1^2 + K_2^2\varphi_2^2 \end{bmatrix} - \begin{bmatrix} K_1^1 p_{1d} \\ K_1^2\tilde{h}_{2d} \end{bmatrix}, \end{aligned} \quad (27)$$

where $K_j^i \in \mathbb{R}$ are chosen so that, for $i = 1, 2$, the polynomials $\lambda^2 - K_2^i\lambda - K_1^i$ are Hurwitz. This control law guarantees that output $\tilde{y}(t)$ tracks asymptotically the constant values $(p_{1d}, \tilde{h}_d)$ and therefore the original output $y(t)$ tracks asymptotically the desired values (p_{1d}, W_{cd}).

VI. SIMULATION RESULTS

We chose as the output $y(t)$ to be tracked a concatenation of constant desired values p_{1d} of p_1 and W_{cd} of W_c, corresponding to specific rate of emission of the engine, in the neighborhood of that chosen by Plianos *et al.*, see [13]. The eigenvalues of the closed loop system are picked up as -3 and -4 for the first subsystem and -2 and -1 for the second. The simulations have been done under Matlab Simulink V 7.0, with the following characteristics: all "step sizes" and "tolerances" are under the position "auto" and the used solver is "Ode 23s (stiff/Mod. Rosenbrock). The time of simulation is 40 s. We show in Figures 5, 6, 7 (zoom of Fig. 6) and 8 the behavior of the to-be-controlled output $y(t) = (p_1(t), W_c(t))$ as well as that of the second component $\tilde{h}_2(x(t))$ of the modified output $\tilde{y}(t)$.

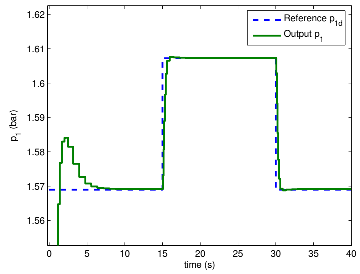

Fig. 5. Extended model (4^{th} order): Output p_1 and its reference signal p_{1d}

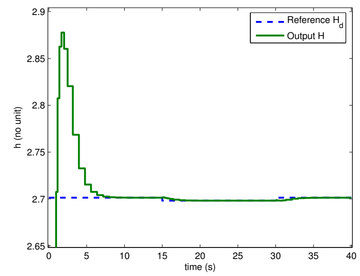

Fig. 6. Extended model (4^{th} order): Output $\tilde{h}_2$ (denoted by H) and its reference signal $\tilde{h}_{2d}$ (denoted by H_d)

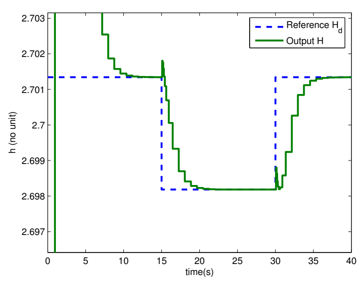

Fig. 7. Extended model (4^{th} order): Zoom in on output $\tilde{h}_2$ (denoted by H) and its reference signal $\tilde{h}_{2d}$ (denoted by H_d)

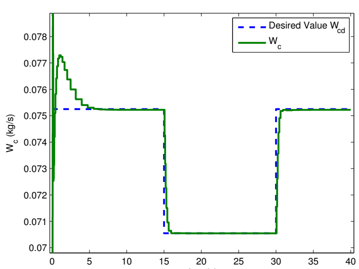

Fig. 8. Extended model (4^{th} order): Output W_c and its desired value W_{cd}

VII. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, a 3-dimensional nonlinear TDE model is considered. In order to avoid dealing with unstable zero dynamics we propose a solution of the tracking problem based on two basic ingredients: change of the output (which yields a system with a trivial zero dynamics) and dynamic extension (which allows a simple calculation of a tracking controller).

B. Future Works

In our study we suppose that all states are accessible for measurements which may not always be the case in practice (for instance, the gas pressure in the exhaust manifold is not accessible for measurements). Therefore in our future works we are planning to use a (nonlinear) observer for that state of the turbocharged diesel engine TDE and to do a comparative study between our controller based dynamic feedback linearization and that of Plianos *et al.*, see [13]. We are planning also to study robustness of the control law with respect to the fuel mass flow rate W_f considered as an external perturbation in this study.

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