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A New Reactive Target-tracking Control with Obstacle Avoidance in a Dynamic Environment

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Abstract— **This paper addresses a new reactive control design for point-mass vehicles with limited sensor range to track targets while avoiding static and moving obstacles in a dynamically evolving environment. Towards this end, a multi-objective control problem is formulated and control is synthesized by generating a potential field force for each objective and combining them through analysis and design. Different from standard potential field methods, the composite potential field described in this paper is time-varying and planned to account for moving obstacles and vehicle motion. Basic conditions and key properties are derived using rigorous Lyapunov analysis. Simulation examples are included to illustrate both the design process and performance of proposed control.**

I. Introduction

For most real-world applications, it is desired that mobile robots can explore and move within dynamic environments. In addition, the environment is usually uncertain as complete information and future trajectories of obstacles cannot be assumed *a priori*. In this context, the problem that arising for mobile robots is how to track moving targets where robots have limited sensor range and are simultaneously avoiding static and moving obstacles in real-time.

For stationary environments, standard path planning approaches are classified into three categories: graph method, potential field method and other physical analogies methods. Graph methods [1], [2] are based on a geometrical cell-decomposition of the entire workspace and generate an optimal path with respect to the objective criteria, such as finding the shortest collision-free path. The main criticism of graph methods is they require large computational resources. In the potential field method, the target applies an attractive force to the robot while the obstacles exert a repulsive force onto the robot. The resultant force determines the motion of the robot. The potential field method is particularly useful because of its simplicity, elegance and high efficiency. Some inherent limitations of potential field method have been pointed out in [3], such as trap situations due to local minima. To avoid the drawbacks of the standard potential field method, other physical analogies methods have been proposed using ideas from fluid mechanics [4] or electro magnetics [5] to construct functions free of local minima, but they are generally computationally intensive and therefore inappropriate for dynamic environments.

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For dynamic environments, a common technique is to add a time dimension to the state space and reduce the problem to a static one [6], [7], [8]. The major problem is that it always assumes that the trajectories of the moving obstacles are known *a priori*, which is often not true in real applications.

Another approach was proposed [9], [10], extending the potential field method for moving obstacle avoidance by constructing repulsive potential functions which take into account the velocity information. In [10], the velocity of the obstacle is considered when building the repulsive potential field. But the velocity of the robot is not taken into account. Since the collision between the robot and obstacle depends on the relative position and velocity between them, this method is inadequate. This issue is then addressed in [9] where the repulsive potential function takes full advantage of the velocity information of the robot and the obstacle. However, it was assumed that the relative velocity between the robot and the obstacle is invariant in terms of position of the robot. This assumption is unrealistic as the relative velocity and position are actually time-varying. Thus derivatives of the relative velocity in terms of position cannot be considered zero all the time. In addition, both methods deal with the obstacle avoidance problem applied to stationary targets.

The Ge and Cui method [11] considers repulsive and attractive potentials which take into account the position and velocity of the robot with respect to moving targets and obstacles. Though convergence to the target is proven, there is no rigorous proof of obstacle avoidance.

In the literature [12], [13], [14], potential fields and Lyapunov direct methods are utilized to solve the formation control problem with collision avoidance. Potential fields yield interaction forces between neighboring robots to enforce a desired minimum space for any pair of robots. A virtual leader is a moving reference point that exerts forces on its neighboring robots by means of an additional similar potential field. The purpose of the virtual leader is to introduce the mission: to direct, herd and/or manipulate the vehicle group behavior [12]. A properly designed potential field function yields global asymptotic convergence of a group of mobile robots to a desired formation, and guarantees no collisions among the robots [13]. These two methods do not consider the obstacle avoidance issue. The leader-follower strategy essentially transforms the formation control problem into a tracking problem. Based on this, the decentralized control designed to achieve collision avoidance and target tacking

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for a single robot is proposed. It is then extended to address the problem of coordinated tracking of a group of robots [14]. This method does not consider the moving obstacle and only guarantees the tracking within a bounded error.

Other than potential field methods, there are other results [15], [16], [17], [18], [19]. [15] proposed a method combining path following with a Deformable Virtual Zone (DVZ)-based reactive obstacle avoidance control. [16] utilized harmonic potential functions along with the panel method for obstacle avoidance in dynamic environment. [17] presented the dynamic window approach to obstacle avoidance in an unknown environment. With a few changes to the basic scheme, convergence to the target position is proved. [18] proposed a method to compute the probability of collision in time for linear velocities of the robot and a reactive algorithm to perform obstacle avoidance in dynamic uncertain environment. [19] gave a preliminary but fairly in-depth study of the novel collision cone approach as a viable collision detection and avoidance tool in a 2-D dynamic environment. Many of the methods are heuristic and the lack of analytical design guidelines can be problematic in applications. In addition, most of these methods increased complexity and computational requirements.

In this paper, we propose a new reactive control to achieve target-tracking in the presence of moving obstacles and with a limited sensing range. The control here developed incorporates planned potential field method and nonlinear damping. A desired trajectory is introduced to resolve the potential conflict between target-tracking and collision avoidance. The planned potential functions are proposed based upon relative positions among the robot, the desired trajectory, and obstacles. Meanwhile, the nonlinear damping is designed to ensure stability and damp oscillation. Generalized potential functions are proposed which have no stable local minima. We put forward a theorem to analyze the stability property of the equilibrium point of the potential functions. More importantly, rigorous Lyapunov proof of target tacking and obstacle avoidance is given.

II. Problem Formulation

Consider a single point-mass agent whose dynamics is given by

$$\dot{q}_r = v_r, \quad \dot{v}_r = u, \tag{1}$$

where $q \triangleq [x,y]^T$ denotes the center position, $v \triangleq [v_x, v_y]^T$ represents the velocity, and u is the control input. Thus we can define the states $S(t) = (q(t), v(t))$. Subscripts r, g and o indicate the vehicle, target and obstacle respectively.

Given the initial configurations $S_r(t_0) = (q_r(t_0), v_r(t_0))$, as shown in Figure 1, the objective of this paper can be summarized as follows:

- tracking the specified target $S_g(t) = (q_g(t), v_g(t))$;
- avoiding the n obstacles $S_{oi} = (q_{oi}(t), v_{oi}(t))$ $(i = 1, 2, \cdots n)$.

We make the following choices without loss of generality:

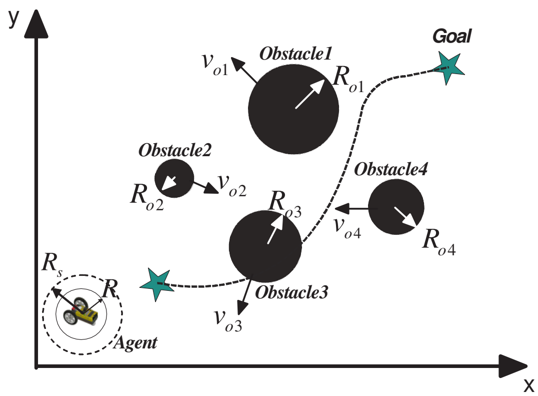

Fig. 1. Illustration of target-tracking with collision avoidance in a 2D dynamic environment

- The agent is represented by a 2-D circle centered at $q_r(t)$ and of radius R. The range of its sensors is also described by a circle centered at $q_r(t)$ and of radius R_s;
- The ith static/moving obstacle is represented by a circle centered at $q_{oi}(t)$ and of radius R_{oi}.

III. Target-tracking and Collision Avoidance for a Single Agent

In this section we derive a nonlinear reactive control using Lyapunov-type analysis that guarantees collision avoidance and tracking of a target for a single robot. To achieve these two design objectives, two potential field functions are used to generate reactive forces. Specifically, consider the following composite potential function:

$$P(q_r - q_o, q_r - q_g) = P_a(q_r - q_g) + P_r(q_r - q_o), \tag{2}$$

where $P_a(\cdot)$ is the attractive potential function and $P_r(\cdot)$ is the repulsive potential function. Intuitively and necessarily,

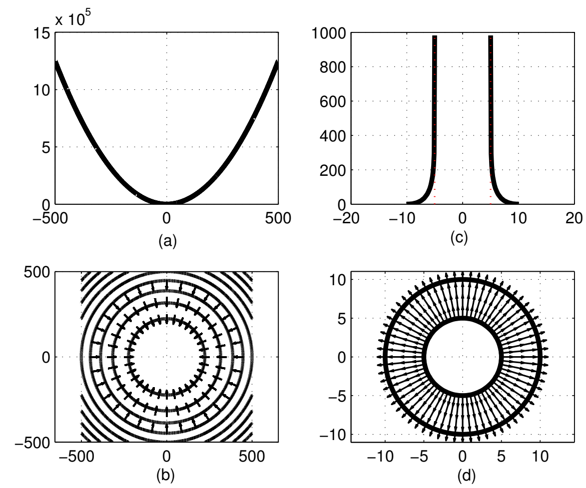

Fig. 2. Typical attractive potential function versus repulsive potential function (a:attractive potential field function; b:contour lines of attractive potential field function; c:repulsive potential field function; d:contour lines of repulsive potential field function)

potential functions should have the properties that

$$\begin{cases} P_a(0)=0, \quad \nabla P_a(q_r-q_g)\Big|_{(q_r-q_g)=0}=0, \\ 0<P_a(q_r-q_g)<\infty \ \text{ if } \|q_r-q_g\| \text{ is nonzero and finite}, \\ \left\|\nabla P_a(q_r-q_g)\right\|<+\infty \ \text{ if } \|q_r-q_g\| \text{ is finite}, \end{cases} \tag{3}$$

and

$$\begin{cases} P_r(q_r-q_o)=+\infty \ \text{ if } (q_r-q_o)\in\Omega_o, \\ P_r(q_r-q_o)=0 \ \text{ if } (q_r-q_o)\notin\overline{\Omega}_o, \\ P_r(q_r-q_o)\in(0,\infty) \ \text{ if } (q_r-q_o)\in\overline{\Omega}_o \text{ but } (q_r-q_o)\notin\Omega_o, \\ \lim\limits_{(q_r-q_o)\to\Omega_o}\|\nabla P_r(q_r-q_o)\|=+\infty \ \text{ if } (q_r-q_o)\notin\Omega_o, \end{cases} \tag{4}$$

where $\Omega_o\subset\Re^2$ is a compact set representing the 2-dimensional shape of the obstacle, $\overline{\Omega}_o$ is the compact set which is an enlarged version of Ω_o and in which repulsive force becomes active. Ω_o and $\overline{\Omega}_o$ will move with the center q_o. The above defined attractive potential function and repulsive potential function are exemplified by Figure 2.

Should sets $\overline{\Omega}_{oj}$ and $\overline{\Omega}_{ok}$ overlap for some $j\neq k$; the two obstacles can be combined into one obstacle. Thus, without loss of generality, we can assume the following throughout the paper:

Assumption 1: $\overline{\Omega}_{oj}\cap\overline{\Omega}_{ok}$ be empty for $j\neq k$.

Let the vehicle control be a reactive control of the form

$$\begin{aligned} u \;=\; & -\nabla P_a(q_r-q_g')-\nabla P_r(q_r-q_{oi}) \\ & -\xi(q_r-q_g')(v_r-v_g')+\dot{v}_g', \end{aligned} \tag{5}$$

where the terms $\nabla P_a(\cdot)$ and $\nabla P_r(\cdot)$ are the standard reactive control components, $\xi(\cdot)>0$ is a uniformly bounded function designed to ensure stability and damp oscillations. As shown in (5), we introduce a desired trajectory denoted by $q_g'(t)$ to resolve the potential conflict between target-tracking and collision avoidance, which is given as follows:

- $(q_r-q_{oi})\notin\overline{\Omega}_{oi}$

  $$\lim_{t\to\infty}q_g'=q_g, \quad \lim_{t\to\infty}v_g'=v_g, \quad \text{and} \quad \lim_{t\to\infty}\dot{v}_g'=\dot{v}_g.$$

  To meet the above requirements, an obvious choice for q_g', v_g' and $\dot{v}_g'$ is that,

  $$q_g'=q_g, \quad v_g'=v_g, \quad \text{and} \quad \dot{v}_g'=\dot{v}_g.$$

- $(q_r-q_{oi})\in\overline{\Omega}_{oi}$

  $$q_g'=\begin{cases} q_g & \text{if } (q_g-q_{oi})\notin\overline{\Omega}_{oi}, \\ q^*+\varepsilon\cdot(q_g-q_{oi}) & \text{otherwise}, \end{cases}$$

  $$v_g'=v_{oi}, \quad \text{and} \quad \dot{v}_g'=\dot{v}_{oi},$$

q_g' is reset when the robot reaches the $\text{bd}(\overline{\Omega}_{oi})$. At time t', the robot reaches the $\text{bd}(\overline{\Omega}_{oi})$. Then $q_g'(t')=q_g(t')$, if $(q_g(t')-q_{oi}(t'))\notin\overline{\Omega}_{oi}$. Otherwise, we first draw a line connecting $q_{oi}(t')$ to $q_g(t')$. q^* denotes the crosspoint of the extension line $q_{oi}(t')q_g'(t')$ and $\text{bd}(\overline{\Omega}_{oi})$. ε is a very small positive constant. Then we pick $q_g'(t')=q^*+\varepsilon\cdot(q_g-q_{oi})$ to ensure $(q_g'(t')-q_{oi}(t'))\notin\overline{\Omega}_{oi}$. Furthermore, $v_g'=v_{oi}$, $\dot{v}_g'=\dot{v}_{oi}$ as long as $(q_r-q_{oi})\in\overline{\Omega}_{oi}$. This strategy is depicted in Figure 3. In the same logic, we can specify the initial configuration $q_g'(0)$, $v_g'(0)$, and $\dot{v}_g'(0)$.

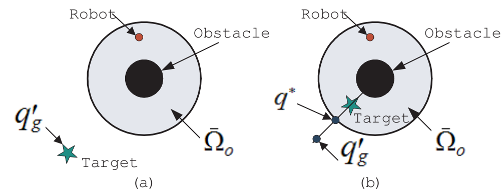

Fig. 3. Strategy to choose desired trajectory when $(q_r-q_{oi})\in\overline{\Omega}_{oi}$ (a:$(q_g(t')-q_{oi}(t'))\notin\overline{\Omega}_{oi}$; b:$(q_g(t')-q_{oi}(t'))\in\overline{\Omega}_{oi}$)

A. Generalized Differentiable Potential Functions

In this section, generalized potential functions will be proposed whose gradients exist everywhere.

The attractive potential function $P_a(q_r-q_g)$ is given by

$$P_a(q_r-q_g)=\frac{k_a}{2}\|q_r-q_g\|^2. \tag{6}$$

Therefore, the attractive force can be given as follows,

$$-\nabla P_a(q_r-q_g)=k_a(q_g-q_r). \tag{7}$$

The repulsive potential function $P_r(q_r-q_{oi})$ is given by

$$P_r(q_r-q_o)=\begin{cases} +\infty & \text{if } d\le 0, \\ 0 & \text{if } d\ge D, \\ k_r\left(\ln\left(\frac{D}{d}\right)-\frac{D-d}{D}\right) & \text{otherwise}, \end{cases} \tag{8}$$

where $d=(\|q_r-q_{oi}\|-R-R_{oi})$, which is the minimum distance between the agent and the ith obstacle. And $D>0$, defining the confined set $\overline{\Omega}_{oi}$. The repulsive force is "active" only if $d<D$, which is formularized as follows,

$$-\nabla P_r(q_r-q_o)=\begin{cases} +\infty & \text{if } d\le 0, \\ 0 & \text{if } d\ge D, \\ k_r\left(\frac{1}{d}-\frac{1}{D}\right)\frac{q_r-q_{oi}}{\|q_r-q_{oi}\|} & \text{otherwise}. \end{cases} \tag{9}$$

Remark 3.1: It is apparent from (9) that, once k_r is given, the smaller the value of D is chosen, the steeper $\nabla P_r(q_r-q_o)$ becomes. As such, an effective way to prevent large acceleration inputs is to increase D. On the other hand, a smaller D means less chance of entering into $\overline{\Omega}_o$, which is beneficial for target-tracking.

B. Stability Analysis of Equilibrium Point

In this section, Theorem 1 is proposed, providing a geometrical method to analyze the stability property of equilibrium points yielded by the composite potential function.

Definition 3.1: *A point in the composite potential function (2), point $q^*\in\Re^2$ is defined to be a stationary point if and only if it satisfies the following equation,*

$$-\nabla P_a(q^*-q_g)=\nabla P_r(q^*-q_o).$$

Definition 3.2: *Curves $C_a(K_a)$ and $C_r(K_r)$ are said to be the level curves of potential functions defined by,*

$$C_a(K_a)\triangleq\left\{q\in\Re^2\,\middle|\,P_a(q-q_g)=K_a\right\} \quad (K_a>0),$$

and

$$C_r(K_r)\triangleq\left\{q\in\Re^2\,|\,P_r(q-q_o)=K_r\right\} \quad (K_r>0).$$

Theorem 1: *Upon the attractor-repeller form potential function (2), at the stationary point q^*, let K_{aq} to be the curvature of the level curve $C_a(P_a(q^*-q_g))$ and K_{rq} to be the curvature of the level curve $C_r(P_r(q^*-q_o))$. The level curves $C_a(P_a(q^*-q_g))$ and $C_r(P_r(q^*-q_o))$ are convex at the stationary point q^*. Suppose the straight line connecting q_g to q^* is normal to the level curves $C_a(K_a)$ and $C_r(K_r)$. Then q^* is saddle point if and only if $K_{aq} < K_{rq}$.*

Proof: Since the straight line connecting q_g to q^* is normal to the level curves $C_a(K_a)$ and $C_r(K_r)$, let us introduce the coordinate system (see Fig. 4) in which the origin is q_g and $q_g q^*$ represents the positive y axis.

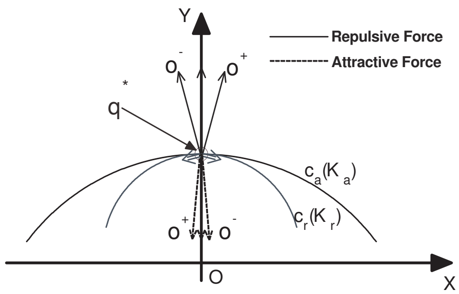

Fig. 4. Level curves tangency at the equilibrium point

Therefore, the stationary point has the following properties(we do not consider the trivial case $-\nabla P_a(\cdot) = \nabla P_r(\cdot) = 0$, in which the stationary point is the target),

$$\begin{cases} \frac{\partial P_a}{\partial x} = \frac{\partial P_r}{\partial x} = 0, \\ \frac{\partial P_a}{\partial y} = -\frac{\partial P_r}{\partial y} > 0, \\ \frac{\partial^2 P_a}{\partial x \partial y} = \frac{\partial^2 P_r}{\partial x \partial y} = 0, \\ -\frac{\partial^2 P_a}{\partial y^2} - \frac{\partial^2 P_r}{\partial y^2} < 0. \end{cases} \tag{10}$$

For the implicit function $P_a(q-q_g) = K_a$, we have

$$\frac{dy}{dx}\Big|_{q^*} = -\frac{\frac{\partial P_a}{\partial x}}{\frac{\partial P_a}{\partial y}}\Big|_{q^*} = 0. \tag{11}$$

And

$$\begin{aligned} \frac{d^2y}{dx^2}\Big|_{q^*} &= -\frac{\frac{\partial^2 P_a}{\partial x^2} + \left(\frac{\partial^2 P_a}{\partial x \partial y} + \frac{\partial^2 P_a}{\partial y \partial x}\right)\frac{dy}{dx} + \frac{\partial^2 P_a}{\partial y^2}\left(\frac{dy}{dx}\right)^2}{\frac{\partial P_a}{\partial y}}\Big|_{q^*} \\ &= -\frac{\frac{\partial^2 P_a}{\partial x^2}}{\frac{\partial P_a}{\partial y}}\Big|_{q^*} \end{aligned}. \tag{12}$$

In addition, $C_a(P_a(q-q_g))$ is convex, which implies $\frac{d^2y}{dx^2} < 0$. Hence combining (10) and (12) yields

$$\frac{\partial^2 P_a}{\partial x^2}\Big|_{q^*} > 0. \tag{13}$$

Moreover, it follows from (11), (12), and (13) that

$$K_{aq} = \frac{\frac{\partial^2 P_a}{\partial x^2}}{\frac{\partial P_a}{\partial y}}\Big|_{q^*}. \tag{14}$$

Similarly, we have

$$\frac{\partial^2 P_r}{\partial x^2}\Big|_{q^*} < 0, \tag{15}$$

and

$$K_{rq} = \frac{\frac{\partial^2 P_r}{\partial x^2}}{\frac{\partial P_r}{\partial y}}\Big|_{q^*}. \tag{16}$$

Now considering the following system model,

$$\begin{aligned} \dot{x} &= -\frac{\partial P_a}{\partial x} - \frac{\partial P_r}{\partial x} \\ \dot{y} &= -\frac{\partial P_a}{\partial y} - \frac{\partial P_r}{\partial y} \end{aligned}.$$

Correspondingly, the Jacobian matrix $[J]_{2\times 2}$ is given by,

$$[J]_{2\times 2} = \begin{bmatrix} -\frac{\partial^2 P_a}{\partial x^2} - \frac{\partial^2 P_r}{\partial x^2} & -\frac{\partial^2 P_a}{\partial x \partial y} - \frac{\partial^2 P_r}{\partial x \partial y} \\ -\frac{\partial^2 P_a}{\partial y \partial x} - \frac{\partial^2 P_r}{\partial y \partial x} & -\frac{\partial^2 P_a}{\partial y^2} - \frac{\partial^2 P_r}{\partial y^2} \end{bmatrix}.$$

It follows from (10), at the stationary point q^*, we can obtain the eigenvalues as follows,

$$\lambda_1 = -\frac{\partial^2 P_a}{\partial x^2} - \frac{\partial^2 P_r}{\partial x^2} \text{ and } \lambda_2 = -\frac{\partial^2 P_a}{\partial y^2} - \frac{\partial^2 P_r}{\partial y^2} < 0. \tag{17}$$

Substituting (14) and (16) into λ_1, we can rewritten λ_1 as

$$\lambda_1 = \frac{\partial P_a}{\partial y}\left(K_{rq} - K_{aq}\right). \tag{18}$$

Following form (10), (17), and (18), we can conclude that q^* is saddle point if and only if $K_{aq} < K_{rq}$. ■

Under the potential functions (6) and (8), level curves $C_a(K_a)$ and $C_r(K_r)$ are concentric circles with centers q_g and q_o respectively. From the geometric viewpoint, the obstacle is closer to the local minimum than the target, which implies $K_{aq} < K_{rq}$. Invoked by Theorem 1, it is the saddle point. Thus we can assume the following throughout the paper:

Assumption 2: Composite potential field function (2) has only one stable local minimum, which is the target.

C. Tracking of a Target and Obstacle Avoidance

The tracking problem is to ensure that the agent will converge to the target position q_g. Meanwhile, the obstacle avoidance problem is to ensure that the agent will not enter the given compact set Ω_{oi} provided that its initial position is not in the set. The following theorem provides the basic result.

Theorem 2: *Suppose that potential field function (2) satisfies properties (3) and (4). If assumptions 1 and 2 hold, as long as $(q_r(t_0) - q_{oi}(t_0)) \notin \Omega_{oi}$ and the initial conditions $S_r(t_0) = (q_r(t_0), v_r(t_0))$ are finite, then system (1) under control (5) is collision-free provided that $v_g(t)$ and $v_{oi}(t)$ are uniformly bounded. Furthermore, after a finite time instant t^*, if $[q_g(t) - q_{oi}(t)] \notin \overline{\Omega}_{oi}$ for all $t \geq t^*$, $q_r(t)$ converges asymptotically to $q_g(t)$. If $[q_g(t) - q_{oi}(t)]$ stays in or intermittently returns to $\overline{\Omega}_{oi}$, there is no convergence of $[q_r(t) - q_g(t)] \to 0$.*

Proof: Let us choose the following Lyapunov function candidate:

$$V_1(t) = \frac{1}{2}\left\|v_r - v_g'\right\|^2 + P(q_r - q_g', q_r - q_{oi}).$$

Considering the case $[q_r(t) - q_{oi}(t)] \in \overline{\Omega}_{oi}$, under assumption 1, it follows from (1) and (5) that

$$\begin{aligned}\dot{V}_1 &= \left(v_r - v_g'\right)^T \left(\dot{v}_r - \dot{v}_g'\right) + \left(v_r - v_g'\right)^T \\ &\quad \nabla P_a\left(q_r - q_g'\right) + \left(v_r - v_{oi}\right)^T \nabla P_r\left(q_r - q_{oi}\right) \\ &= \left(v_r - v_g'\right)^T \left(-\nabla P_a\left(q_r - q_g'\right) - \nabla P_r\left(q_r - q_{oi}\right)\right. \\ &\quad \left. -\xi\left(q_r - q_g'\right)\left(v_r - v_g'\right)\right) \\ &\quad + \left(v_r - v_g'\right)^T \nabla P_a\left(q_r - q_g'\right) \\ &\quad + \left(v_r - v_{oi}\right)^T \nabla P_r\left(q_r - q_{oi}\right) \\ &= -\xi\left(q_r - q_g'\right)\left\|v_r - v_g'\right\|^2 \\ &\quad + \left(v_g' - v_{oi}\right)^T \nabla P_r\left(q_r - q_{oi}\right) \\ &= -\xi\left(q_r - q_g'\right)\left\|v_r - v_g'\right\|^2. \end{aligned} \tag{19}$$

which is negative semi-definite. Thus, as long as $(q_r(t_0) - q_{oi}(t_0)) \notin \Omega_{oi}$ and the initial conditions $S_r(t_0) = (q_r(t_0), v_r(t_0))$ are finite, $P(q_r(t) - q_g'(t), q_r(t) - q_{oi}(t))$ will remain finite provided that $v_g(t)$ and $v_{oi}(t)$ are uniformly bounded (As proved subsequently, $v_g(t)$ is required to be uniformly bounded, which ensures $v_r(t)$ remains bounded provided that the initial conditions are finite when $[q_r(t) - q_{oi}(t)] \notin \overline{\Omega}_{oi}$). Thus collision avoidance is guaranteed.

It follows from (19), $[q_r(t) - q_g'(t)] \to 0$ under the assumption 2 invoked by LaSalle's invariant set theorem [20]. Hence, using proof by contradiction, we can conclude that no convergence of $[q_r(t) - q_g(t)] \to 0$ can be achieved if $[q_g(t) - q_{oi}(t)]$ stays in or intermittently returns to $\overline{\Omega}_o$

Moreover, from the geometric viewpoint, the transient process to track the target and avoid collision can be illustrated in Figure 5. As shown in Figure 5, once the robot is in the set

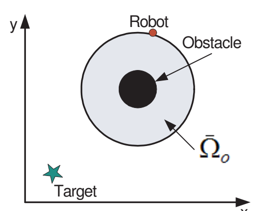

Fig. 5. Illustration of transient process of tracking and obstacle avoidance

$\overline{\Omega}_o$, it will asymptotically converge to its desired trajectory q_g'. Thus, unless $[q_g(t) - q_{oi}(t)]$ stays in or intermittently returns to $\overline{\Omega}_{oi}$ (in the limit of $t \to \infty$), the agent will not stay in or intermittently be in $\overline{\Omega}_{oi}$ which implies $[q_r(t) - q_{oi}(t)] \notin \overline{\Omega}_{oi}$ for all $t \geq \bar{t}^*$ ($\bar{t}^* > t^*$).

To show asymptotic convergence under the condition $[q_r(t) - q_{oi}(t)] \notin \overline{\Omega}_{oi}$ for all $t \geq \bar{t}^*$ ($\bar{t}^* > t^*$), we note that after $\bar{t}^*$, the tracking error dynamics of system (1) under control (5) reduces to

$$\dot{e}_1 = e_2, \quad \dot{e}_2 = -\nabla P_a(e_1) - \xi(e_1)e_2,$$

where $e_1 = q_r - q_g'$ and $e_2 = v_r - v_g'$. Adopting the simple Lyapunov function

$$V_2(t) = P_a(e_1) + \frac{1}{2}\|e_2\|^2.$$

We have

$$\begin{aligned}\dot{V}_2 &= e_2^T \nabla P_a(e_1) + e_2^T\left[-\nabla P_a(e_1) - \xi(e_1)e_2\right] \\ &= -\xi(e_1)\|e_2\|^2, \end{aligned} \tag{20}$$

which implies asymptotic stability of e_1 and e_2 under the assumption 2 invoked by LaSalle's invariant set theorem. Considering $q_g' \to q_g$, $v_g' \to v_g$, and $\dot{v}_g' \to \dot{v}_g$ as $t \to \infty$, asymptotic convergence can be concluded. ■

IV. SIMULATIONS

This section describes the simulation results of a differential drive vehicle.

A. *Model and vehicle control for differential drive vehicle*

Consider the following kinematic and dynamic model of a differential drive vehicle (as shown in Figure 6),

$$\left\{\begin{array}{lcl}\dot{x} &=& V\cos\theta \\ \dot{y} &=& V\sin\theta \\ \dot{\theta} &=& \omega \\ \dot{V} &=& \frac{F}{M}\end{array}\right., \tag{21}$$

where θ is the orientation, V is the linear velocity, ω is the angular velocity, F is the applied force and M is the mass.

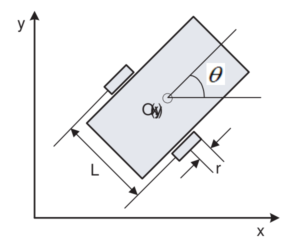

Fig. 6. Relevant variables for the unicycle (top view)

Consider the following dynamic compensator [21]:

$$\left\{\begin{array}{lcl}\omega &=& \frac{u_2\cos\theta - u_1\sin\theta}{V} \\ F &=& M\left(u_1\cos\theta + u_2\sin\theta\right)\end{array}\right.. \tag{22}$$

Substituting (22) into (21) yields the transformed system (1). Note the following facts: (1) the sign of linear velocity V will determine forward or backward motion of the vehicle; (2) transformation (22) is singular at $V = 0$, i.e., when the mobile robot is not moving. We take the following two measures to cope with the singularity problem.

- Set the initial linear velocity to be nonzero;
- Let $V(k+1) = \left\{\begin{array}{ll}\delta & if\ V(k) + \dot{V}(k)T < \delta \\ V(k) + \dot{V}(k)T_s & otherwise\end{array}\right.$, where T_s is the sampling period, $k = 0, 1, 2\cdots$, and δ is a very small positive constant.

B. *Simulation results*

For these simulations, the potential functions are given by (6) and (8). The nonlinear damping function $\xi(\cdot)$ is simply chosen to be a constant function. The parameters used for these simulations are: $R = 1$ m, $R_s = 2$ m, $R_{oi} = 1$ m, $k_a = 100$, $k_r = 20$, $D = 1$ m, $\xi(\cdot) = 80$, $\varepsilon = 0.1$, $\delta = 0.1$ m/s,

$r = 0.6$ m and $L = 1.821$ m. In addition, the initial location of the vehicle is (1,1), $v(0) = 1$ m/s, $\dot{v}(0) = 0$ m/s^2, and $\omega(0) = 0$ rad/s. And the bounds on the angular velocity of both wheels is $\frac{50}{3}$ rad/s.

1) Target-tracking and collision avoidance with static obstacles: There are three static obstacles (2,7,1),[1] (10,10,1), and (22,19,1). The simulation result is shown in Figure 7.

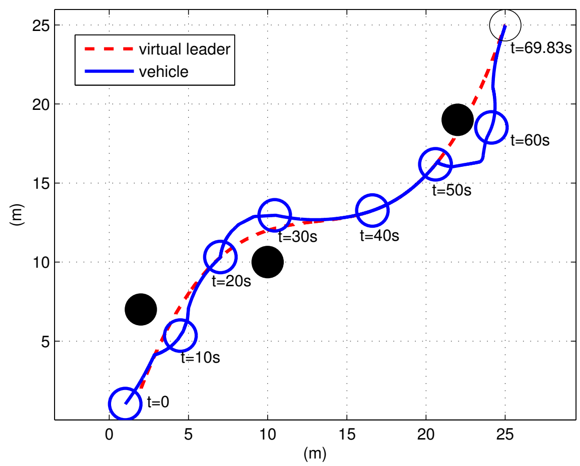

Fig. 7. Collision avoidance with static obstacles

2) Target-tracking and collision avoidance with moving obstacles: Compared with example 1, three moving obstacles of radius being 1 are also considered. The simulation result is shown in Figure 8.

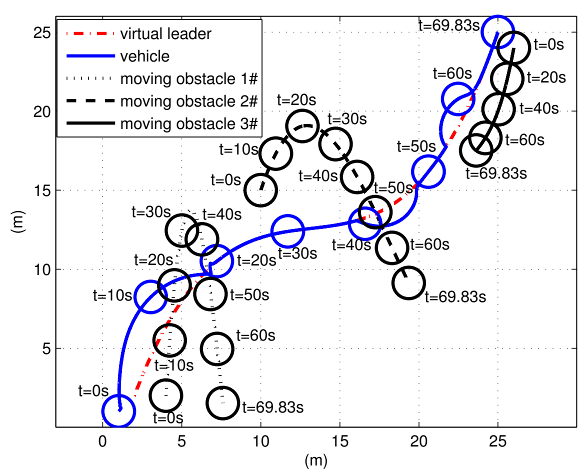

Fig. 8. Collision avoidance with moving obstacles

V. Conclusions

In this paper, we proposed a systematic approach to achieve multiple objectives for tracking virtual command vehicle and collision avoidance. Examples through simulation confirm the effectiveness of Lyapunov design of multi-objective control for the point-mass agent proposed in Section 3. Future research will consider more complex dynamical models to accommodate a larger class of mobile robots. In addition, investigation of the oscillation issues inherent in the potential field method and improving the overall performance will be addressed.

[1]Data format:(center position, radius). For example, (2,7) denotes the center position. The radius is 1 m.

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