Source Seeking for a Three-Link Model of Fish Locomotion

Jennie Cochran, Eva Kanso, and Miroslav Krstic

Abstract— We present a method of locomotion control for underwater vehicles which are propelled by a periodic deformation of the vehicle body, similar to the way a fish moves. The control law employs extremum seeking, which is a non-model based method that has been used recently in "source seeking" control for nonholonomic mobile robots. In this paper we present a design for a fish model consisting of three rigid body links and employing a two degree of freedom, non-reciprocal, movement which propels the fish through a perfect fluid without the use of a Kutta condition to shed vortices. In a companion paper we present results for a second fish model based on a Joukowski airfoil which has only one degree of freedom in its movement, and thus relies on vortex shedding to move through a perfect fluid. With the use of extremum seeking, we achieve the same results in each case: The fish is capable of performing 'source seeking' in GPS-denied underwater environments, and, if position measurement is available, it is capable of navigating from point A to point B, as well as along a predetermined path.

I. INTRODUCTION

The literature on underwater vehicles has started to address vehicles which are propelled forward not by a traditional motor, but rather by other means - such as mimicking the movement of fish and other aquatic creatures. This paper addresses locomotion control for underwater vehicles which employ periodic shape deformations similar to fish movement to move forward. The motivation for this work comes from our previous work on source seeking for the nonholonomic unicycle with constant forward velocity [3]. In that work we use the extremum seeking method to design a control law which drives the vehicle to the vicinity of a source. We have shown that the scheme is locally exponentially convergent both in 2D [3] and in 3D [4].

Over the course of studying this control law, which employs periodic forcing, we began looking for the most suitable application. We found it in underwater vehicles that employ periodic movement for locomotion. The advantage of using extremum seeking over other control methods is that extremum seeking is a non-model based method — and thus much simpler to employ for systems, such as underwater vehicles and fish models, which do not have simple models.

In this paper (and in a companion paper [2]) we apply the extremum seeking method to two different "fish" models and achieve the same results in each case: The fish is capable of performing 'source seeking' in GPS-denied underwater environments, and, if position measurement is available, it

is capable of navigating from point A to point B, as well as along a predetermined path.

The fish models are distinguished by their respective underlying methods of propulsion. The first fish model, employed in the present paper, was developed in [8], and relies on a two degree of freedom, *non-reciprocal*, movement which propels the fish through a perfect fluid without the use of a Kutta condition to shed vortices. The second fish model, employed in our companion paper [2], developed in [17] (and studied in [11]), has only one degree of freedom, and thus relies on vortex shedding to move through a perfect fluid. In both models, the fish can propel and turn itself by periodically changing its shape. We study how to use a combination of these two "gaits" (i.e., the periodic shape deformations that produce forward and turning motions) to enable the fish to move from an originating point to a desired destination point or along a prespecified path.

Much work has been done in the area of modelling fish movement - both for the understanding of fluid dynamics and for the purpose of building more efficient vehicles that operate underwater. The studies presented in [6], [9], [15], [1], [10], [14], [16] have all examined locomotion by swimming and the role of vortices. References [8], [17], [11], [5] have taken the lessons learned from this previous work and extended it to the development of computational fish models in fluid systems. Unlike earlier methods, [8] uses conservation of circulation and ideas from reduction theory to build a model for a three link fish without the explicit use the fluid variables. This enables [8] to explicitly derive the equations of motion for the fish model and to study the locomotion due solely from body shape changes and not from vorticity. The model developed in [17], [11] spans the gap between studies which look at deformable bodies moving through a fluid without the use of vortex shedding and studies examining systems with rigid bodies and vortices. The Joukowski airfoil fish model relies on only one input and exploits the presence of vortices for both propulsion and steering. Both models were developed with the underlying motivation to build a platform to develop motion-planning algorithms for underwater vehicles.

A common theme in all this work is the periodic movement of the body. This brings the extremum seeking method to mind as this method employs periodic inputs to probe a signal field. We design a source seeking scheme with two channels, one for each of the two joints of the three-link fish. Despite employing two channels of extremum seeking, we use periodic inputs of a single frequency, a sine and a cosine. This choice leads to motions that are not only successful in reaching the source but also natural looking.

The paper is organized as follows. We review source

J. Cochran and M. Krstic are with the Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093, jcochran@ucsd.edu

E. Kanso is with the faculty of Aerospace and Mechanical Engineering, University of Southern California, 854 Downey Way, RRB 214, Los Angeles, CA 90089

seeking work for a nonholonomic unicycle in Section II. Section III discusses the motion of the fish and fluid systems. Section IV presents the specifics of a three link body moving in a fluid, introduces an extremum seeking control law, and presents the results of applying the control to the fish model.

In a companion paper [2] we develop the same set of results but for a different model of fish locomotion, where a Joukowski airfoil models the fish, a vortex model models the fluid, and only one input ('tail flapping') is used to produce locomotion and perform source seeking.

A note is in order on the notation we employ. The model is quite non-standard as a control theoretic model as it consist of a fifth-order ODE (the fish subsystem, with forces acting on it included), of an infinite-dimensional output map (the fluid potential field), with two inputs. We develop our notation in the paper so that the the original model given in the spirit of geometric mechanics [8] is presented here as a control-oriented (input-state-output) model.

II. REVIEW OF SOURCE SEEKING WITH A NONHOLONOMIC UNICYCLE

In [3] we focus on the problem of seeking the source of a scalar signal using a nonholonomic unicycle with constant forward velocity and no position information. The vehicle relies on locally sensing a scalar signal which emanates from the source it seeks. The strength of the signal is assumed to decay with distance away from the source, though other information about the signal's spatial distribution is unknown. The ES control law estimates the gradient of the signal field and drives the vehicle toward the source.

While various groups have considered source seeking problems this work is different in that the vehicle has no knowledge of its position or the position of the source, there is no communication between it and other entities, and it has nonholonomic dynamics.

The center r_c of the unicycle is governed by $\dot{r}_c = V_c e^{i\theta}$ where θ is its orientation and V_c is its constant forward velocity. The sensor position is $r_s = r_c + R e^{i\theta}$. The control is applied through angular velocity forcing $\dot{\theta} = u$, and the control law is given by

$$\dot{\theta} = u = a\omega\cos(\omega t) + c\xi\sin(\omega t) \qquad (1)$$

$$\xi = \frac{s}{s+h}J(r_s), \qquad (2)$$

where ξ is a washout filter applied to the signal J sensed at the vehicle sensor r_s, located at a distance R away from the center and a, ω, c, h are parameters which affect performance. The control law is made of two terms which serve two different functions. The first term $a\omega\cos(\omega t)$ is a persistent excitation which allows the vehicle to probe the signal space. The second term $c\xi\sin(\omega t)$ is a bias term which jointly estimates the gradient and drives the vehicle to turn up the gradient—in essence maximizing the signal on average. In [3] we consider a measured signal with an (unknown) spatial distribution

$$J = -q_r|r^* - r_s|^2, \qquad (3)$$

where r^* is the location of the signal source, and prove convergence to a small set near the source using averaging.

III. EQUATIONS OF MOTION IN A PERFECT FLUID

The fluid is considered to be inviscid (no viscosity) and incompressible. The fluid particles may slip along the boundaries of the solid but cavities are not allowed to form in the fluid nor at the interface. The fluid is assumed to be at rest at infinity. In [8], the model does not account for *a vortex shedding mechanism*. It is well-known in fluid dynamics that, under these conditions, the equations governing the motion of the body in an irrotational fluid can be written *without explicitly incorporating the ambient fluid* (see [8]). That is, the configuration space of the body-fluid system can be identified with that of the submerged body only.

The fluid velocity field $\mathbf{u}$, in the fluid domain excluding the body and point vortices when accounted for, can be expressed in term of a potential function ϕ:

$$\mathbf{u} = \nabla\phi. \qquad (4)$$

Incompressibility implies that

$$\nabla^2\phi = 0. \qquad (5)$$

The boundary conditions result from the two assumptions that the fluid is at rest at infinity and that fluid particles may slip along the body surface, and are expressed as

$$\nabla\mathbf{u}|_\infty = 0 \qquad (6)$$

$$\mathbf{u}\cdot\mathbf{n} = \dot{\mathcal{B}}|_{\mathcal{S}}\cdot\mathbf{n} \qquad (7)$$

where $\mathcal{B}$ is the fish body and $\mathcal{S}$ is the surface of the body (touching the fluid). In the articulated body model [8], the potential function ϕ is a function of the configuration and velocity of the body and is computed numerically using a boundary element method [13].

The kinetic energy of the fluid, T_f, is defined

$$T_f = \frac{1}{2}\int_{\mathcal{D}}\mathbf{u}^2 dv, \qquad (8)$$

where $\mathcal{D}$ is the fluid domain (excluding singularities present in the form of point vortices) and dv is the standard volume element. Using Green's theorem, T_f can be rewritten as

$$T_f = -\frac{1}{2}\int_{\partial S}\phi\nabla\phi\cdot\mathbf{n}ds \qquad (9)$$

where ∂S is the surface of the fish body and $\mathbf{n}$ is the unit normal into the fluid. The expression for T_f for in terms of the body variables is presented in Sections IV. The total kinetic energy of the body-fluid system (equal to the kinetic energy T_f of the fluid plus the kinetic energy T_b of the fish), is also presented in Sections IV.

The equations governing the net locomotion of the fish in both models can be viewed as variations of Kirchhoff's equations for the motion of a rigid body in ideal fluid [12]

$$\frac{d\mathbf{L}}{dt} + \Omega\mathbf{k}\times\mathbf{L} = 0 \qquad (10)$$

$$\frac{dA}{dt} + \mathbf{k}\cdot([U\ V]^T\times\mathbf{L}) = 0 \qquad (11)$$

where $\mathbf{L}$ and A are the linear and angular momenta of the body-fluid system and U, V, and Ω are the translational and rotational velocities associated with a net locomotion of the

body. The variables $\mathbf{L}$, A, U, V, Ω are expressed in a body frame moving with the fish. The linear and angular momenta $\mathbf{L}$ and A are obtained by differentiating the kinetic energy,

$$\mathbf{L} = \left[\begin{array}{cc} \dfrac{\partial T}{\partial U} & \dfrac{\partial T}{\partial V} \end{array} \right]^T, \qquad A = \dfrac{\partial T}{\partial \Omega}. \qquad (12)$$

The theme of the derivations is conservation of momentum and starting the system from rest, which in turn implies $\mathbf{L} = 0$ and $A = 0$ for all time. One solves for U, V, and Ω at each time step and then integrates to derive the locomotion of the fish.

IV. LOCOMOTION AND SOURCE SEEKING

Consider the articulated fish model discussed in [8] and formed by three identical rigid links connected via hinge joints that allow the links to rotate relative to each other. Each link $\mathcal{B}_i$ of the fish is an ellipse with semi-major axis of length δ and semi-minor axis of length μ. The joints which connect the links are located at a distance σ away from the tips of the ellipses and the relative angles are denoted by θ_α, $\alpha = 1, 2$. The two inputs to the system are the angular velocities of the two joints $\dot{\theta}_1, \dot{\theta}_2$. Certain prescribed inputs, inducing desired shape deformations, allow the fish to move forward or turn, thus achieving a net locomotion in the plane.

A. ODE Model with an Infinite Dimensional Output Map for a Three Link Fish in a Potential Flow

The equations of motion (10) are derived in [8] for the articulated three-link fish *without the explicit use of the fluid variables*. Further, the net locomotion is formulated as a sum of geometric and dynamic phases over closed curves in shape space parameterized by the shape variables θ_1 and θ_2.

One has five state variables that completely describe the shape as well as the position and orientation of the three link fish relative to an inertial frame, namely,

$$\begin{align} \Xi &= [\theta_1 \; \theta_2 \; g_3^T]^T \qquad (13) \\ g_3 &= [\theta_f \; f_x \; f_y]^T \qquad (14) \end{align}$$

where θ_1 and θ_2 are the relative angles between the links as mentioned above and (f_x, f_y) and θ_f are, respectively, the location of the center of the middle ellipse and the orientation of the middle ellipse with respect to the fixed inertial frame. The inputs are the angular velocities of the joints,

$$\Psi = [\Psi_1 \; \Psi_2]^T, \qquad (15)$$

i.e., $\dot{\theta}_1 = \Psi_1$, $\dot{\theta}_2 = \Psi_2$, while the infinite dimensional output map is the potential field

$$\phi(x, y) = \eta[\Xi, \Psi](x, y), \qquad (16)$$

which is governed by the Laplace equation (5) with boundary conditions (6) and (7), and where the solution operator $\eta[\Xi, \Psi]$ is defined below. To complete this description we explain the governing ODE for $\dot{g}_3$. Reference [8] develops the motion of the three link fish using geometric phases, holonomy, and symmetry. Here we present only a summary into the equations which drive the fish, and do not redo the derivations in [8].

Each link $\mathcal{B}_i$ is defined by an orientation and position $g_i = [\theta_{\mathcal{B}_i} \; \mathcal{B}_{ix} \; \mathcal{B}_{iy}]^T$ with respect to a fixed inertial frame. The angular and translational velocities are expressed with respect to the fixed inertial frame as $\dot{g}_i$ or with respect to their own body frame as $\xi_i = \left[\begin{array}{c} \Omega_i \\ U_i \\ V_i \end{array} \right]$, $i = 1, 2, 3$. As the derivation of $\dot{g}_3$ is made simpler when considering the movement with respect to the $\mathcal{B}_3$ fixed frame, instead of the inertial frame, we make use of ξ_1 and ξ_2, not g_1 and g_2. The relationship between $\dot{g}_3$ and ξ_3 is defined by

$$\dot{g}_3 = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \cos(\theta_f) & -\sin(\theta_f) \\ 0 & \sin(\theta_f) & \cos(\theta_f) \end{array} \right] \xi_3, \qquad (17)$$

where we still must explain the relationship between ξ_3 and (Ξ, Ψ). [8] shows that the entire configuration can be defined through the movement of one link (the middle link $\mathcal{B}_3$ is the link of choice) plus the movement of the joints, i.e. the entire system can be defined by the state variables Ξ. With this in mind, the velocities of the other two links relative to the third link, but expressed with respect to their respective fixed frames, are

$$\begin{align} \zeta_1 &= \xi_1 - \mathrm{Ad}_{x_1^{-1}} \xi_3 \qquad (18) \\ \zeta_2 &= \xi_2 - \mathrm{Ad}_{x_2^{-1}} \xi_3, \qquad (19) \end{align}$$

where

$$\mathrm{Ad}_{x_1^{-1}}(\theta_1) = \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ (\delta + \sigma)\sin(\theta_1) & \cos(\theta_1) & \sin(\theta_1) \\ (\delta + \sigma)(1 + \cos(\theta_1)) & -\sin(\theta_1) & \cos(\theta_1) \end{array} \right] \qquad (20)$$

$$\mathrm{Ad}_{x_2^{-1}}(\theta_2) = \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ -(\delta + \sigma)\sin(\theta_2) & \cos(\theta_2) & \sin(\theta_2) \\ -(\delta + \sigma)(1 + \cos(\theta_2)) & -\sin(\theta_2) & \cos(\theta_2) \end{array} \right] \qquad (21)$$

denote the matrices that transform ξ_3 from the $\mathcal{B}_3$-fixed frame to the respective $\mathcal{B}_1$-fixed and $\mathcal{B}_1$-fixed frames. The variables ζ_1, ζ_2 are given by

$$\begin{align} \zeta_1 &= \Pi_1 \dot{\theta}_1 = \Pi_1 \Psi_1 \qquad (22) \\ \zeta_2 &= \Pi_2 \dot{\theta}_2 = \Pi_2 \Psi_2 \qquad (23) \\ \Pi_1 &= \left[\begin{array}{ccc} 1 & 0 & +(\delta + \sigma) \end{array} \right]^T \qquad (24) \\ \Pi_2 &= \left[\begin{array}{ccc} 1 & 0 & -(\delta + \sigma) \end{array} \right]^T. \qquad (25) \end{align}$$

To continue the summary, we examine the kinetic energy of the fluid (9) where $\partial S = \sum_1^3 \partial \mathcal{B}_i$ is the boundary over all three bodies, which can be expressed in terms of "added inertias" $\mathbb{M}_{ij}^f$ and ξ_i as

$$T_f = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \xi_i^T \mathbb{M}_{ij}^f(\Xi) \xi_j. \qquad (26)$$

The "added inertias" $\mathbb{M}_{ij}^f$ depend on the configuration of the three link body Ξ and are derived in [8]. This is a

consequence of being able to express ϕ solely in terms of the body configuration and velocities:

$$\phi \;=\; \sum_{i=1}^{3}\left(\Omega_i\mathcal{X}_i + \begin{bmatrix} U_i \\ V_i \end{bmatrix}\cdot\varphi_i\right) \;=\; \sum_{i=1}^{3}[\mathcal{X}_i\ \varphi_i^T]\xi_i \quad (27)$$

where $\mathcal{X}_i(x,y,\Xi)$ and $\varphi_i(x,y,\Xi)$ define potential functions which depend *only* on θ_1, θ_2 and g_3 and the spatial coordinates (x,y). The quantities $\mathcal{X}_i$ and φ_i depend on coefficients which are found using a boundary element method [13] and depend only on Ξ, and do not depend on the spatial coordinates. These coefficients are used to find $\mathbb{M}_{ij}^f(\Xi)$. The kinetic energy of the bodies $T_{\mathcal{B}_i} = \frac{1}{2}\left(\mathcal{I}\Omega_i^2 + m\left(U_i^2 + V_i^2\right)\right)$ can also be expressed in terms of ξ_i

$$T_{\mathcal{B}_i} \;=\; \frac{1}{2}\xi_i^T\mathbb{M}_i^b\xi_i \quad (28)$$

$$\mathbb{M}_i^b \;=\; \begin{pmatrix} \mathcal{I} & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \quad (29)$$

where m is the mass of the ellipse and $\mathcal{I} = m(a^2 + b^2)/4$ is the body moment of inertia. The total kinetic energy of the system is then expressed as

$$T \;=\; \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\xi_i^T\mathbb{I}_{ij}\xi_j \quad (30)$$

$$\mathbb{I}_{ij} \;=\; \mathbb{M}_{ij}^f(\Xi) \qquad \text{for } i\neq j \quad (31)$$

$$\mathbb{I}_{ii} \;=\; \mathbb{M}_{ii}^f(\Xi) + \mathbb{M}_i^b. \quad (32)$$

The total effective momentum, $h_s = [A\ \mathbf{L}^T]^T$, expressed with respect to the $\mathcal{B}_3$ fixed frame is

$$h_s \;=\; \sum_{i=1}^{3}\sum_{j=1}^{3}\mathrm{Ad}_{x_i^{-1}}^T\mathbb{I}_{ij}\xi_j \quad (33)$$

where $\mathrm{Ad}_{x_i^{-1}}^T(\theta_i)$ transforms from the $\mathcal{B}_i$-fixed frame to the $\mathcal{B}_3$-fixed frame. (Note that $\mathrm{Ad}_{x_3^{-1}}^T$ is the identity operator.) Equation (33) can now be rewritten as

$$h_s = \sum_{i=1}^{3}\mathrm{Ad}_{x_i^{-1}}^T\mathbb{I}_{i3}\xi_3 + \sum_{i=1}^{3}\sum_{\alpha=1}^{2}\mathrm{Ad}_{x_i^{-1}}^T\mathbb{I}_{i\alpha}\left(\zeta_\alpha + \mathrm{Ad}_{x_\alpha^{-1}}\xi_3\right) \quad (34)$$

The quantity h_s is governed by Kirchhoff-like equations and as we assume that the system starts from rest, h_s remains zero for all time. This leads to an equation for ξ_3

$$\xi_3(\Xi,\Psi) = $$
$$-\left(\sum_{i=1}^{3}\mathrm{Ad}_{x_i^{-1}}^T\mathbb{I}_{i3} + \sum_{i=1}^{3}\sum_{\alpha=1}^{2}\mathrm{Ad}_{x_i^{-1}}^T\mathbb{I}_{i\alpha}\mathrm{Ad}_{x_\alpha^{-1}}\right)^{-1}$$
$$\times\sum_{i=1}^{3}\sum_{\alpha=1}^{2}\mathrm{Ad}_{x_i^{-1}}^T\mathbb{I}_{i\alpha}\Pi_\alpha\Psi_\alpha. \quad (35)$$

Thus the evolution equation governing the system is

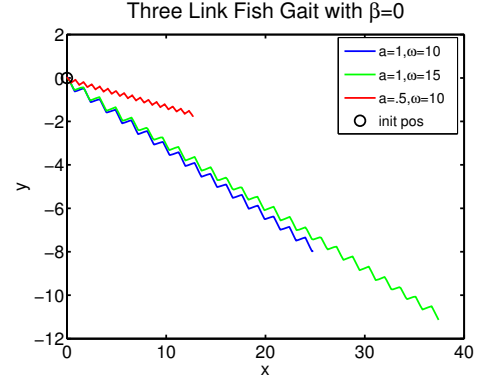$$\dot\Xi \;=\; \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ l\left(\Xi,\Psi\right) \end{bmatrix} \quad (36)$$

Fig. 1. Forward gaits of a three link fish. $\beta = 0$.

where the vector field $l = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^T$ is defined by the right hand side of (17). The infinite dimensional output map $\eta[\Xi,\Psi](x,y)$ defined in (27), which describes the fluid field throughout the domain is given in a more detailed form as

$$\eta[\Xi,\Psi](x,y) \;=\; \tau_1(\Xi,\Psi)^T\Pi_1\Psi_1 + \tau_2(\Xi,\Psi)^T\Pi_2\Psi_2$$
$$+ \sum_{i=1}^{3}\tau_i(\Xi,\Psi)^T\Gamma_i(\Xi)\xi_3(\Xi,\Psi) \quad (37)$$

where $\tau_i(\Xi,\Psi) = [\mathcal{X}_i(x,y,\Xi)\ \varphi_i^T(x,y,\Xi)]^T$ and $\Gamma_1 = \mathrm{Ad}_{x_1^{-1}}(\theta_1)$, $\Gamma_2 = \mathrm{Ad}_{x_2^{-1}}(\theta_2)$ and $\Gamma_3 = I$. Thus the complete dynamic system is given by the 5-dimensional state equation (36) and the infinite-dimensional output map (37).

B. Basic Gaits for Three Link Fish

We consider two basic gaits: moving forward and turning, first studied by [8]. (Further studies into optimal motion are found in [7].) The angular velocities for both gaits are

$$\dot\theta_1 \;=\; a\omega\sin(\omega t) \quad (38)$$
$$\dot\theta_2 \;=\; a\omega\cos(\omega t) \quad (39)$$

but the initial condition differs. The initial conditions for moving forward are $\theta_1|_{t=0} = -a, \theta_2|_{t=0} = 0$, leading to

$$\theta_1 \;=\; -a\cos(\omega t) \quad (40)$$
$$\theta_2 \;=\; \sin(\omega t). \quad (41)$$

However, the initial conditions for turning are $\theta_1|_{t=0} = \beta - a, \theta_2|_{t=0} = -\beta$, leading to

$$\theta_1 \;=\; -a\cos(\omega t) + \beta \quad (42)$$
$$\theta_2 \;=\; a\sin(\omega t) - \beta. \quad (43)$$

Note that $\beta = a = \omega = 1$ in [8]. Figure 1 shows the fish moving forward for different parameter combinations, while Figure 2 shows the fish turning in circles for different parameter combinations. Figure 3 shows snapshots in time of the fish moving forward.
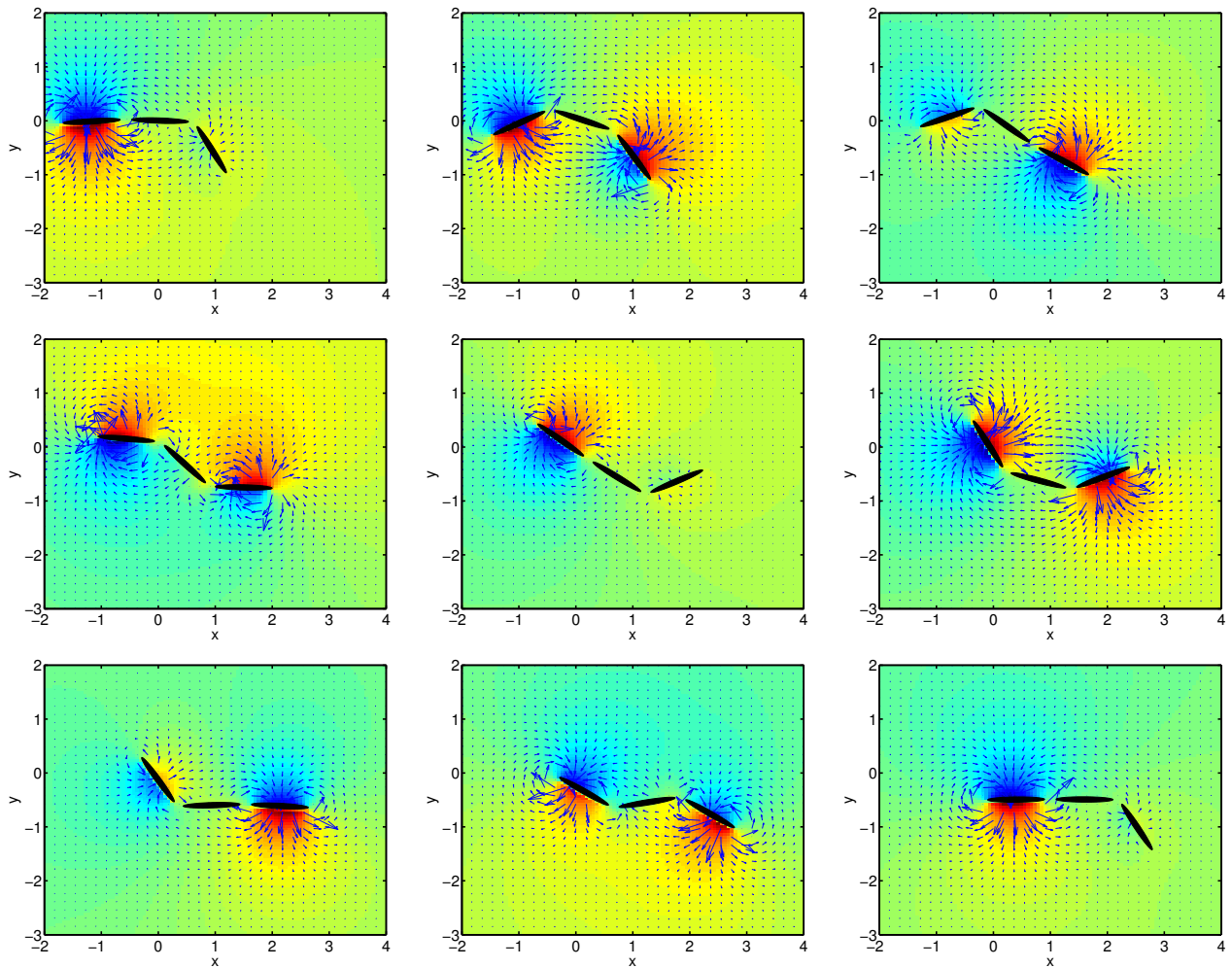
Fig. 3. Snapshots in time of the fish moving forward. The background color field represents the potential field ϕ with red representing positive values and blue representing negative values. $a = 1$, $\theta_1|_{t=0} = -1$, $\theta_2|_{t=0} = 0$
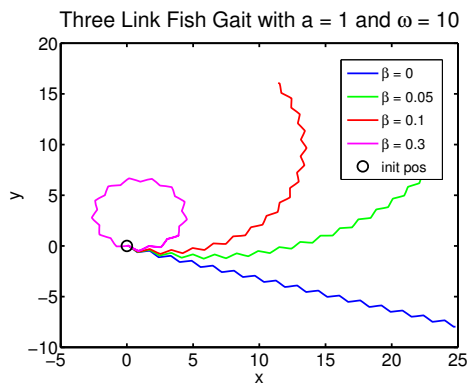

Fig. 2. Turning gaits of a three link fish. $a = 1$, $\omega = 10$.

C. Source Seeking With a Three Link Fish

We take the basic gaits of the three link fish and modify them to enable the fish to move both from an initial location to a desired location in space and along a predetermined path. There are two parts to our control law: 1) how to

apply extremum seeking to the gaits and 2) what function to optimize with extremum seeking.

As explained in Section II, the control law for the non-holonomic unicycle (1) is made up of two parts: a persistent excitation $a\omega\cos(\omega t)$ which allows the entity to probe the space and a feedback term $c\xi\sin(\omega t)$ which allows the entity to turn and move up the gradient. The persistent excitation $a\omega\cos(\omega t)$ is exactly what we see in (38), modulo a phase shift. If we assume β depends on time instead of being constant, then, following (42)–(43) we find

$$\dot{\theta}_1 = a\omega\sin(\omega t) + \dot{\beta} \tag{44}$$
$$\dot{\theta}_2 = a\omega\cos(\omega t) - \dot{\beta}. \tag{45}$$

By equating $\dot{\beta} = -c\xi\cos(\omega t)$ we arrive at our control law

$$\dot{\theta}_1 = a\omega\sin(\omega t) - c\xi\cos(\omega t) \tag{46}$$
$$\dot{\theta}_2 = a\omega\cos(\omega t) + c\xi\cos(\omega t). \tag{47}$$
$$\xi = \frac{s}{s+h}[J] \tag{48}$$

Thus the tuning parameter β depends on the function J which we wish to optimize. The fish moves in such a way to maximize the output value of J.
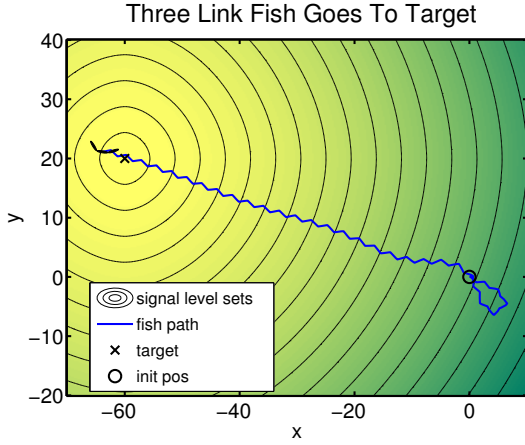
Fig. 4. Source seeking for a three link fish. The background color field represents the "concentration" of the signal field J with yellow representing higher values than green. $a = 1$, $c = 2$, $\omega = 10$, $h = 10$, $q_r = 1/100$, $\delta = 5/6$, $\sigma = 1/6$.

Fig. 5. Three link fish following a predetermined path. The background color field represents the "concentration" of the signal field J with yellow representing higher values than green. $a = 1$, $c = 2$, $\omega = 10$, $h = 10$, $\delta = 5/6$, $\sigma = 1/6$.

The question now becomes, what exactly is J? The ultimate goal is to move the fish either from an initial location in space to a desired one (x^*, y^*) in space or along a path P^*. In the first case, if we assume the fish can somehow sense the distance between itself and the goal location (x^*, y^*), then we wish to minimize that distance and, similar to (3), we can define J as the function

$$J = -q_r\left(\left(f_{sx} - x^*\right)^2 + \left(f_{sy} - y^*\right)^2\right) \tag{49}$$

$$f_s = \left[\begin{array}{c} f_x + \cos(\theta_f)(\delta + \sigma) + \cos(\theta_1 + \theta_f)(2\delta + \sigma) \\ f_y + \sin(\theta_f)(\delta + \sigma) + \sin(\theta_1 + \theta_f)(2\delta + \sigma) \end{array} \right] \tag{50}$$

where $f_s = (f_{sx}, f_{sy})$ is the location of the sensor which we assume to be at the tip of the forward ellipse, i.e. the fish nose. Figure 4 shows a typical simulation of the fish moving to a desired location under the algorithm (46)–(48). This simulation was made while enforcing the constraint that the tuning variable β does not exceed a certain value — the amplitude of the probing signal a. This ensures the links do not cross themselves as the fish moves.

From (49), (50) it may appear that the fish needs the information about the target's position and about its own position. This is not always the case. With measurement of $J(t)$ alone, the fish can be guided by algorithm (46)–(48) to reach a local maximum J^* on physical space, as in [3], [4].

D. Path Following for a Three Link Fish

The function J, optimized by extremum seeking, can be modified so that the fish can not only move from to a desired location, but also so that the fish can follow a predefined path. There are a myriad of ways to construct J for this purpose; However, the one we choose to use here is fairly simple. We define a path parametrically and use the error to define J. For instance, we define the target path $x = a_1 y^3 + a_2 y^2 + a_3 y + a_4$ and define J as a function of the error between f_{sx} and $a_1 f_{sy}^3 + a_2 f_{sy}^2 + a_3 f_{sy} + a_4$. The error can be multiplied by a gain and can be raised to a power to obtain different
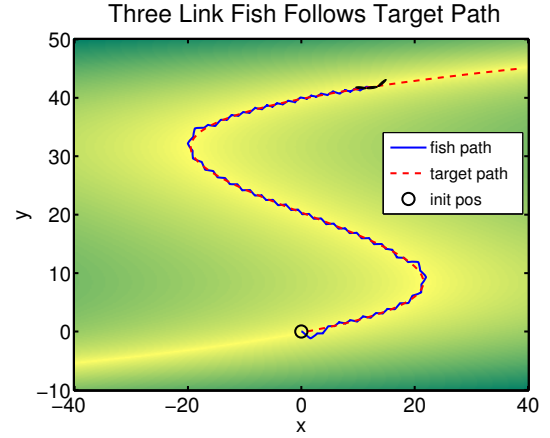
gradient fields. Figure 5 shows the fish following the path defined by $x = 2/300 y^3 - 2/5 y^2 + 16/3 y + 1$. The fish optimizes

$$J = -5\sqrt{|f_{sx} - 2/300 f_{sy}^3 - 2/5 f_{sy}^2 + 16/3 f_{sy} + 1|} \tag{51}$$

to follow this path.

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