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Switched Control of Mechanical Systems by Using Musculotendon Actuators*

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Abstract— **This paper addresses the problem of modeling, control, and simulation of a mechanical system actuated by an agonist-antagonist musculotendon subsystem. Contraction dynamics is given by case I of Zajac's model. Saturated semi–positive PD-type controllers with switching as neural excitation inputs are proposed. Linear approaches of nonlinear systems, root locus, switched systems control and SOSTOOLS are used to determine the stability for the obtained closed-loop system. To corroborate the obtained theoretical results numerical simulations have been performed with help of Matlab.**

I. INTRODUCTION

To achieve the motion of body segments, such as a leg or a human arm, a set of forces acting on them need to be known. These forces are produced by skeletal muscles, whose actions are individually controlled by central nervous system through neural excitation [18].

Several mathematical models that try to define the muscle contraction properties can be found in the literature. Two very important references in this topic are the models proposed by Hill [6] and Huxley [7] (see [3]). In this paper we have used the biomechanical model proposed by Zajac in 1989 [18] to model the contraction dynamics. This is a Hill-type dynamical model that describes how muscle and tendon work together to produce and transmit the force to the body segments, and is commonly refered as musculotendon (MT) model. The Zajac's model with some adaptations has been successfully used in other works to model the dynamic equations of mechanical systems (see [5], [10], [11]).

A muscle-skeletal system (MSS) is composed of mechanical structure (skeleton bones) and MT actuators. Such system can be studied from the control theory point of view by defining inputs and outputs, so that by manipulating the input, the output is forced to a desired one.

In a MSS the number of muscles (inputs) is greater than the number of degrees of freedom. The simplest representation of a MSS consists of two MT actuators that interact against a common load in agonist-antagonist relationship (see [4], [10], [17], [16]).

The position and the velocity are the outputs of a MSS, and the input is the neural excitation $u(t)$ of each muscle involved in the motion. The control problem is to design the input $u(t)$, such that the mechanical system tends to a constant desired position as time increases. The Zajac's model considers that the neural excitation is presented in the set $u(t) \in [0, 1]$.

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A few works have presented a solution for the stabilization problem of mechanical systems actuated by MT actuators taking into account the saturated semi–positive constraint of the neural excitation $u(t) \in [0, 1]$. For example, in [11] an optimal control theory approach was suggested. However, unless a problem has special structure (such as the linear, unconstrained models that produce the classic LQ regulator), the evaluation and online implementation of an optimal feedback control presents a difficult challenge.

To the best of the authors' knowledge, the idea that the saturated semi–positive signal $u(t) \in [0, 1]$ can be designed using a switched control systems approach is new. Specifically, in this paper, the saturated semi–positive control inputs are generated by a commutation control law, which converts the closed-loop system in a switched system, that belongs to the class called hybrid systems. Since many of the systems encountered in practice are of hybrid nature, in the last years, this kind of systems have been an interdisciplinary and very active area of research [9]. Hybrid systems theory facilitates the study of complex systems by decompositions of them into simpler systems and by allowing the use of well-known control tools as Lyapunov theory.

The stability of a switched system under arbitrary switching can be achieved by finding a single Lyapunov function whose derivative along solutions of all subsystems satisfies the inequalities of Lyapunov's direct method. Such function is called common Lyapunov function (CLF) [1], [9], [14]. Due to the high order and nonlinearity of the systems studied here, it is extremely difficult to find a CLF in a generalized way. To find a numerical solution, we use SOSTOOLS, which is a free MATLAB toolbox for formulating and solving sums of squares (SOS) optimization programs [13], [14], [15].

The main contributions of the study presented in this paper are:

- An explanation from the point of view of the automatic control to the modeling and control of mechanical systems where MT actuators (using Zajac's model in its iriginal form) take part.
- Such a purpose can be achieved by designing neural excitation controllers using classic control tools and switched systems theory.
- In this way, we obtain an alternative approach to design saturated semi–positive neural excitation control inputs.

This paper is organized as follows: Section II provides a brief description of the Zajacs's model. In Section III we present the obtained results related to the saturated control

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of a second order system operated by two MT actuators. Finally, concluding remarks are drawn in in Section IV.

II. ZAJAC'S MODEL (1989)

In this model, the input $u(t)$ is the net neural input to the muscle, and the output $F^T(t)$ is the tendon force (see Figure 1).

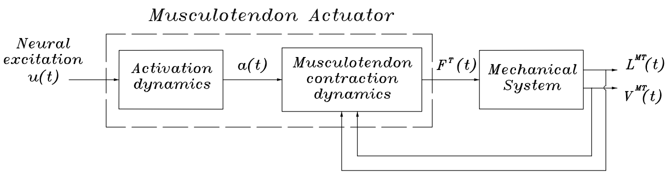

Fig. 1. Block diagram that shows the interaction of tendon and muscle with a mechanical system.

The force $F^T(t)$ developed by the actuator depends on the velocity $V^{MT}(t)$ and the length $L^{MT}(t)$, which are determined from the position of the body segments (mechanical system). At the same time, the dynamics of the body segments depends on the force $F^T(t)$ developed by the MT actuator. The dynamics of the MT actuator is composed of *activation dynamics* and *MT contraction dynamics* (see Figure 1). In the original work, Zajac presents the obtained model in normalized quantities with the optimal muscle length L_0^M, the maximum shortening velocity V_m and the maximum active force F_0^M produced by the muscle in an isometric contraction.

A. Activation dynamics

The activation in the MT model is a variable that affects only the muscle and is given by the following equation:

$$\frac{da(t)}{dt} + \left[\frac{1}{\tau_{act}}\left[\beta + [1-\beta]\,u(t)\right]\right] a(t) = \frac{1}{\tau_{act}} u(t), \quad (1)$$

where

- $a(t)$ is the muscle activation, with constraint
$$0 \le a(t) \le 1,$$
where the value $a(t) = 1$ indicates that the muscle is fully activated, and $a(t) = 0$ fully deactivated,
- $u(t)$ is the neural excitation, which denotes the control input, with constraint
$$0 \le u(t) \le 1,$$
- τ_{act} is the time constant when muscle is fully excited ($u(t) = 1$),
- τ_{dact} is the time constant when muscle is deactivated ($u(t) = 0$),
- and
$$\beta \equiv \frac{\tau_{act}}{\tau_{dact}},$$
where ($0 < \beta < 1$).

Equation (1) can be written in its normalized form as:

$$\frac{da(\tau)}{d\tau} + \left[\frac{1}{\tilde{\tau}_{act}}\left[\beta + [1-\beta]u(\tau)\right]\right] a(\tau) = \frac{1}{\tilde{\tau}_{act}} u(\tau), \quad (2)$$

where

$$\tilde{\tau}_{act} = \frac{\tau_{act}}{\tau_c},$$

where $\tau_c = \frac{L_0^M}{V_m}$, is the time scaling factor.

B. Musculotendon contraction dynamics

The essential part of the Zajac's model is the force dynamics, also named MT contraction dynamics. MT contraction represents the integrated dynamical process of muscle and tendon working together [18].

By assuming for simplicity that the line of action of the muscle fibers is parallel to the line of action of the tendon, the MT contraction dynamics is written in its normalized form as:

$$\frac{d\tilde{F}^T}{d\tau} = \tilde{k}^T \left[\tilde{V}^{MT} - \tilde{V}^M(\tilde{L}^M, \tilde{F}^T, a(\tau))\right], \quad (3)$$

where

- $\tilde{F}^T = \frac{F^T}{F_0^M} = \tilde{F}^M$, is the normalized tendon force, equals the normalized muscle force,
- $\tau = (\frac{1}{\tau_c})t$, is the normalized time,
- $\tilde{k}^T = k^T(\frac{L_0^M}{F_0^M})$ is the normalized tendon stiffness,
- $\tilde{V}^{MT} = V^{MT}(\frac{1}{V_m})$, is the normalized MT velocity,
- $\tilde{V}^M = f(\tilde{L}^M, \tilde{F}^T, a(\tau))$, is the normalized muscle velocity for a given fiber length, muscle force and activation level (called Force-Velocity-Length relationship (FVL) of a Hill-type model).

C. Case I: Flat region of the Force-Length curve

The *case I* presented in [18] is the model of the contraction dynamics (3) when the muscle operates at the flat region of its Force-Length curve. Thus, the linear relationship FVL is given by:

$$-\tilde{V}^M = a(\tau) - \tilde{F}^M, \quad (4)$$

where $a(\tau)$ is defined in (2).

The *case I* of the MT contraction dynamics is obtained by replacement of (4) in (3):

$$\frac{d\tilde{F}^T}{d\tau} + \tilde{k}^T \tilde{F}^T = \tilde{k}^T[\tilde{V}^{MT} + a(\tau)]. \quad (5)$$

Equation (5) is normalized with L_0^M, F_0^M and V_m. To work with absolute quantities, in this paper, this equation is rewritten as follows:

$$\frac{dF^T}{dt} = -\frac{30V_m}{L_s^T}F^T + \frac{30F_0^M}{L_s^T}V^{MT} + \frac{30F_0^M V_m}{L_s^T}a(t), \quad (6)$$

where F^T is the MT actuator force, $a(t)$ is the activation level defined in (1). The linear approximation for $\tilde{k}^T$, included in (6), has been defined in [18] as follows:

$$\tilde{k}^T = 30\frac{L_0^M}{L_s^T},$$

where L_s^T is the tendon slack length.

In the remainder of this work, (1) and (6) are used to determine the level of muscle activation and to determine the force F^T, respectively.

III. Control of a Second Order System Operated by Two Musculotendon Actuators

This section provides a solution to the problem of position control of a mechanical system moved in the horizontal plane by two MT actuators in an agonist-antagonist relationship by using controllers to satisfy the condition of saturation of the Zajac's model:

$$u(t) \in [0, 1]. \tag{7}$$

The stability analysis is carried out using switched systems stability theory and classic tools as linear approaches of nonlinear systems and root locus. The system studied is presented in Figure 2.

In the agonist-antagonist relationship, the net force that acts on the load is given by $F_1^T - F_2^T$ (see [4]).

The dynamics of the mechanical system is given by:

$$M\ddot{x}_1 + F_v\dot{x}_1 = F_1^T - F_2^T, \tag{8}$$

where M $[Kg]$ is the mass, F_v $\left[Kg\frac{m}{s}\right]$ is the viscosity, $x_1 = L_1^{MT}$ $[m]$ is the length of actuator 1, $\dot{x}_1 = V_1^{MT}$ is the velocity of actuator 1, F_1^T $[N]$ is the force developed by actuator 1 and F_2^T $[N]$ is the force developed by actuator 2.

Let us notice from Figure 2 that the control inputs are defined by the neural excitation inputs $u_1(t)$ and $u_2(t)$. The total length of the system is given by $L = constant$ $[m]$ and $x_1' = L_2^{MT}$ $[m]$ is the length of actuator 2.

The control problem can be written in terms of limit:

$$\lim_{t\to\infty} \tilde{x}_1 = 0, \tag{9}$$

where $\tilde{x}_1 = x_d - x_1$ is the position error of the mass, $0 < x_d < L$ is a desired position respect to the origin of actuator 1, and x_1 is the actual position of the mass.

A. State variables of the open-loop control system

This case study considers that $L = constant$, i.e., one muscle lengthens as the agonist while the other contracts as the antagonist. In this way, we can obtain the relationship between lengths (L_1^{MT} and L_2^{MT}) and velocities (V_1^{MT} and V_2^{MT}) as follows. The length of actuator 2 is given by:

$$x_1' = L - x_1. \tag{10}$$

The desired position of the mass respect to the origin of actuator 2 is:

$$x_d' = L - x_d. \tag{11}$$

Using (10) and (11), we obtain the following relationships:

$$\tilde{x}_1' = x_d' - x_1' = [L - x_d] - [L - x_1] = -x_d + x_1 = -\tilde{x}_1, \tag{12}$$

where $\tilde{x}_1'$ is the error position of the mass respect to the actuator 2.

$$\dot{x}_1' = -\dot{x}_1, \tag{13}$$

where $\dot{x}_1' = V_2^{MT}$ is the velocity of actuator 2. Let us notice from (13) that the shortening velocities V_1^{MT} and V_2^{MT} have opposite signs.

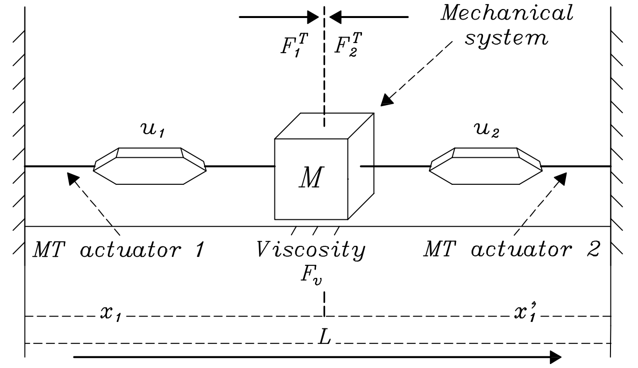

Fig. 2. Agonist-antagonist musculotendon pair acting on a common mass-damper system.

By using (1), (6), (8), (12) and (13), the open-loop system can be written as:

$$\frac{d}{dt}\begin{bmatrix}\tilde{x}_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} = \begin{bmatrix} -x_2 \\ \frac{1}{M}[x_3 - x_4 - F_v x_2] \\ \frac{30F_{01}^M}{L_{s1}^T}x_2 + \frac{30F_{01}^M V_{m1}}{L_{s1}^T}x_5 - \frac{30V_{m1}}{L_{s1}^T}x_3 \\ -\frac{30F_{02}^M}{L_{s2}^T}x_2 + \frac{30F_{02}^M V_{m2}}{L_{s2}^T}x_6 - \frac{30V_{m2}}{L_{s2}^T}x_4 \\ \left[c_{31} - c_{21}x_5\right]u_1(t) - c_{11}x_5 \\ \left[c_{32} - c_{22}x_6\right]u_2(t) - c_{12}x_6 \end{bmatrix}, \tag{14}$$

where

$$\begin{aligned} x_2(t) &= \dot{x}_1(t), & \beta_p &= \frac{\tau_{actp}}{\tau_{dactp}}, \\ x_3(t) &= F_1^T(t), & c_{1p} &= \frac{1}{t_{actp}}\beta_p, \\ x_4(t) &= F_2^T(t), & c_{2p} &= \frac{1}{t_{actp}}(1-\beta_p), \\ x_5(t) &= a_1(t), & c_{3p} &= \frac{1}{t_{actp}}, \\ x_6(t) &= a_2(t), \end{aligned}$$

with $p = 1, 2$. Note that the dynamics of the mass (8) is synthesized in state variables $\tilde{x}_1(t)$ and $x_2(t)$, while the dynamics of the musculotendon actuator 1 and 2 is described by $x_3(t)$–$x_5(t)$ and $x_4(t)$–$x_6(t)$, respectively.

B. Switching law for the controllers $u_1(t)$ and $u_2(t)$

In this section we explain how our solution satisfies the condition (7) and accomplishes the control objective (9). Let us recall the well-known proportional-derivative (PD) controller:

$$PD = k_p\tilde{x}_1 - k_v x_2, \tag{15}$$

where $\tilde{x}_1$ is the error position of the mass, x_2 is the velocity and $k_p, k_v > 0$.

In order to satisfy the constraint (7) in $u_1(t)$ and $u_2(t)$, the PD controller (15) can be used in the switching control law:

$$u_1(t) = \begin{cases}\tanh(\mu_1 PD) & \text{if } PD \geq 0, \\ 0 & \text{if } PD < 0,\end{cases} \tag{16}$$

where $u_1(t) \in [0,1]$ is the input for the actuator 1, and $\mu_1 > 0$, and

$$u_2(t) = \begin{cases} 0 & \text{if } PD \geq 0, \\ -\tanh(\mu_2 PD) & \text{if } PD < 0, \end{cases} \tag{17}$$

where $u_2(t) \in [0,1]$ is the input for the actuator 1, and $\mu_2 > 0$.

The switching between the control inputs depends on the sign of the PD controller (15), which in turn depends on the state variables $\tilde{x}_1$ and x_2. Thus, (16) and (17) define a switched control law, which accomplishes the constraint (7), for the system (14).

C. Feedback control system

With the control inputs (16) and (17) the open-loop system (14) turns into two feedback subsystems $f_1(x)$ and $f_2(x)$, which commute by the following switching law:

$$\dot{x} = f(x) = \begin{cases} f_1(x) & \text{if } PD \geq 0, \\ f_2(x) & \text{if } PD < 0. \end{cases} \tag{18}$$

System (18) operates in the subsystem $f_1(x)$ for nonnegative values of PD (with $u_1(t) \in [0,1]$ and $u_2(t) = 0$), and operates in the subsystem $f_2(x)$ for negative values of PD (with $u_1(t) = 0$ and $u_2(t) \in [0,1]$). Figure 3 shows a block diagram of the system (18).

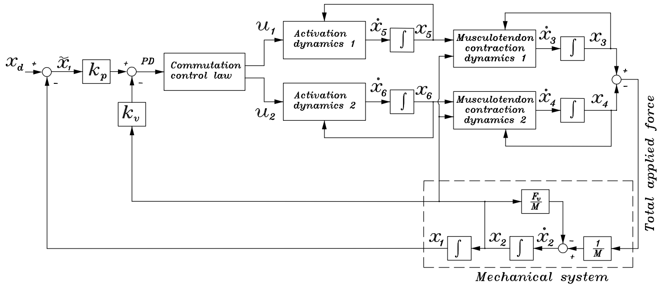

Fig. 3. Block diagram that represents the feedback control system (18).

Despite the complex analysis tools to analyze switched systems, we look to analyze the roll of basic concepts of control systems, like linear approaches of nonlinear systems, and root locus.

D. Definition of hybrid and switched systems

Systems that have dynamics that are described by a set of continuous time differential equations in conjunction with a discrete event process are usually referred to as **switched** or **hybrid systems**. Such systems are of the following form:

$$\dot{x} = f_i(x), \qquad i \in I = \{1, ..., N\}, \tag{19}$$

where $x \in \mathbb{R}^n$ is the continuous state, i is the discrete state, $f_i(x)$ is the vector field describing the dynamics of the i-th mode/subsystem, and I is the index set. The difference between switched and hybrid systems is that in the former only one $i \in I$ is possible for each $x \in \mathbb{R}^n$, and in the later multiple i are possible for some $x \in \mathbb{R}^n$. Without loss of generality, we assume that the state space origin $x = 0 \in \mathbb{R}^n$ is an equilibrium point [14].

This study focuses on switched systems, where switching events can be classified into *state-dependent* or *time-dependent* (see [9]).

Given the control inputs $u_1(t)$ and $u_2(t)$, the obtained feedback system (18) is a state-dependent switched system, where switching depends on the state variables $\tilde{x}_1$ and x_2.

It is noteworthy that the system (18) has the form of (19), with $i \in I = \{1, 2\}$, i.e., the system consist of two operating regions: region 1 if $PD \geq 0$, and region 2 if $PD < 0$.

E. Stability analysis

According to switched systems theory, local stability of system (19) under arbitrary switching can be studied using the following theorems:

Theorem 1: [Theorem 1 in [14]] *Suppose that for the set of vector fields $\{f_i(x)\}$ there exists a polynomial $V(x)$ such that $V(0) = 0$ and*

$$V(x) > 0 \; \forall x \neq 0, \tag{20}$$

$$\frac{\partial V}{\partial x} f_i(x) < 0 \; \forall x \neq 0, i \in I, \tag{21}$$

then the origin of the state space of the system (19) *is globally asymptotically stable under arbitrary switchig.*

△

Theorem 2: [Theorem 1 in [1]] *If the differential equations corresponding to the linearization of system* (19) *are (asymptotically) stable in x_0 and have the same quadratic Lyapunov function, then the system* (19) *is (asymptotically) stable in x_0.*

△

According to the previous Theorems, the state space origin $x = 0 \in \mathbb{R}^6$ of the nonlinear system (18) is a locally (asymptotically) stable equilibrium point if the linearized system is (asymptotically) stable.

The problem is to find a CLF $V(x)$ that satisfies the conditions (20) and (21) for the linearized system (18).

The state space origin $x = 0 \in \mathbb{R}^6$ is a common equilibrium point of the subsystems $f_1(x)$ and $f_2(x)$. By linearizing the system (18) around $x = 0 \in \mathbb{R}^6$, we obtain the following switched system:

$$\dot{x} = \begin{cases} A_1 x & \text{if } PD \geq 0, \\ A_2 x & \text{if } PD < 0, \end{cases} \tag{22}$$

where PD is given in (15),

$$A_1 = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{M}F_v & \frac{1}{M} & -\frac{1}{M} & 0 & 0 \\ 0 & \frac{30F_{01}^M}{L_{s1}^T} & -\frac{30V_{m1}}{L_{s1}^T} & 0 & \frac{30F_{01}^M V_{m1}}{L_{s1}^T} & 0 \\ 0 & -\frac{30F_{02}^M}{L_{s2}^T} & 0 & -\frac{30V_{m2}}{L_{s2}^T} & 0 & \frac{30F_{02}^M V_{m2}}{L_{s2}^T} \\ c_{31}\mu_1 k_p & -c_{31}\mu_1 k_v & 0 & 0 & -c_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_{12} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{M}F_v & \frac{1}{M} & -\frac{1}{M} & 0 & 0 \\ 0 & \frac{30F_{01}^M}{L_{s1}^T} & -\frac{30V_{m1}}{L_{s1}^T} & 0 & \frac{30F_{01}^M V_{m1}}{L_{s1}^T} & 0 \\ 0 & -\frac{30F_{02}^M}{L_{s2}^T} & 0 & -\frac{30V_{m2}}{L_{s2}^T} & 0 & \frac{30F_{02}^M V_{m2}}{L_{s2}^T} \\ 0 & 0 & 0 & 0 & -c_{11} & 0 \\ -c_{32}\mu_2 k_p & c_{32}\mu_2 k_v & 0 & 0 & 0 & -c_{12} \end{bmatrix}.$$

A numeric CLF is obtained with the purpose of proving that the system (22) is asymptotically stable.

1) Numerical case study: First, let us notice that a necessary condition for (asymptotic) stability under arbitrary switching is that all of the individual subsystems are (asymptotically) stable [8]. Thus, using the root locus of the characteristic equation of each subsystem

$$\dot{x} = A_i x, \ i = 1, 2,$$

we have computed numerical values of μ_1, μ_2, k_p y k_v such that A_i, $i = 1, 2$, is Hurwitz.

The roots of a characteristic equation are obtained by solving

$$\det \left[\lambda I_6 - A_i \right] = 0. \tag{23}$$

The linear subsystem $\dot{x} = A_i x$ is asymptotically stable if:

$$\Re(\lambda_j\{A_i\}) < 0 \quad \forall j = \{1, 2, ..., 6\}, i \in I = \{1, 2\}. \tag{24}$$

To prove the effectiveness of the proposed control law, a numerical case is presented. For such purpose, identical actuators are considered. The numerical parameters L_0^M, L_s^T and F_0^M of the *Brachioradialis* muscle presented in [2] are used. The values for τ_c, τ_{act} and τ_{dact} are taken from [18]:

$$\begin{aligned} L_{0p}^M &= 0.2703 \ \left[cm\right], & \tau_c &= 0.1 \ \left[s\right], \\ F_{0p}^M &= 101.58 \ \left[N\right], & \tau_{actp} &= 0.015 \ \left[s\right], \\ L_{sp}^T &= 0.0604 \ \left[cm\right], & \tau_{dactp} &= 0.050 \ \left[s\right], \end{aligned}$$

where $p = 1, 2$. Mass M and viscosity F_v are proposed as:

$$\begin{aligned} M &= 3.8 \ \left[Kg\right], \\ F_v &= 1 \ \left[Kg\tfrac{m}{s}\right]. \end{aligned}$$

With the root locus of the individual characteristic equations in (23), we propose the parameters μ_1, μ_2, k_p y k_v for the control inputs $u_1(t)$ and $u_2(t)$:

$$\begin{aligned} \mu_1 &= 1, & k_p &= 0.003, \\ \mu_2 &= 1, & k_v &= 0.23. \end{aligned}$$

The proposed parameters μ_1, μ_2, k_p and k_v, allow us to satisfy the local stability condition (24). Small values of the mass and the viscosity can be controlled by actuators with small values of the parameter F_0^M.

2) Using SOSTOOLS to find a common Lyapunov function: Once obtained the stability conditions for the individual subsystems $\dot{x} = A_i x$ $(i = 1, 2)$ of (22), a CLF $V(x)$ that satisfies the inequalities (20) and (21) is constructed. Such a function is obtained by using SOSTOOLS [15] and can be written as follows:

$$V(x) = x^T P x \quad x \in \mathbb{R}^6, P = P^T > 0, P \in \mathbb{R}^{6\times 6}. \tag{25}$$

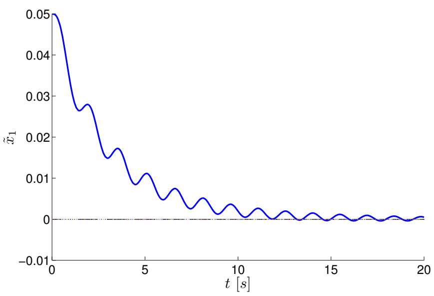

Fig. 4. Position error $\tilde{x}_1(t)$.

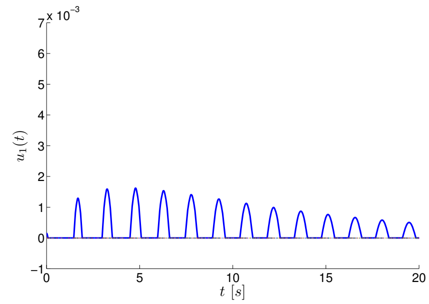

Fig. 5. Control input $u_1(t)$ to actuator 1.

In particular we have computed the CLF given in (26).

Function (26) leads to the conclusion that by Theorem 1, the switched system (22) is asymptotically stable. Therefore, according to Theorem 2, the state space origin $x = 0 \in \mathbb{R}^6$ of the system (18) is locally asymptotically stable.

F. Simulation results

With the obtained local stability of the system (18), a simulation using Matlab is presented. In this simulation the system initial conditions are equal to zero, except for the position error, which starts with a value of $\tilde{x}_1(0) = 0.05$ $[m]$. With these initial conditions the system starts in the region 1.

Figure 4 shows the position error $\tilde{x}_1(t)$. The saturated control inputs $u_1(t)$ and $u_2(t) \in [0, 1]$ are shown in Figures 5 and 6, respectively. The graph to compare the function $V(x)$ evaluated along the systems (18) (dashed line) and (22) (continuous line) is presented in Figure 7. In this Figure, vertical lines indicate the switching between regions 1 and 2 for the systems (18) (dashed line) and (22) (continuous line).

It is worthwhile to notice that several sets of control parameters μ_1, μ_2, k_p and k_v that rendered the closed–loop system (14)-(17) locally asymptotically stable were found. For each one of those sets of control parameters, a CLF was obtained, while the necessary condition of Hurwitz matrices A_1 and A_2 was also achieved.

For this numerical case study, the numerical value of the control inputs $u_1(t)$ and $u_2(t)$ is small, as seen in Figures 5 and 6. However, if there is a large position error $\tilde{x}_1(t)$, then large forces will be required in the musculotendon actuators, which, at the same time, will imply the application of large

$$V(x)=\begin{bmatrix}\tilde{x}_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix}^T\begin{bmatrix} .20831 & -\frac{.42654}{2} & -\frac{.85872\times10^{-4}}{2} & \frac{.85872\times10^{-4}}{2} & -\frac{.56896}{2} & \frac{.56896}{2}\\ -\frac{.42654}{2} & .46856 & \frac{.18277\times10^{-3}}{2} & -\frac{.18277\times10^{-3}}{2} & \frac{1.229}{2} & -\frac{1.229}{2}\\ -\frac{.85872\times10^{-4}}{2} & \frac{.18277\times10^{-3}}{2} & .58872\times10^{-7} & -\frac{.44717\times10^{-7}}{2} & \frac{.23565\times10^{-3}}{2} & -\frac{.24376\times10^{-3}}{2}\\ \frac{.85872\times10^{-4}}{2} & -\frac{.18277\times10^{-3}}{2} & -\frac{.44717\times10^{-7}}{2} & .58871\times10^{-7} & -\frac{.24376\times10^{-3}}{2} & \frac{.23565\times10^{-3}}{2}\\ -\frac{.56896}{2} & \frac{1.229}{2} & \frac{.23565\times10^{-3}}{2} & -\frac{.24376\times10^{-3}}{2} & .82876 & -\frac{1.6516}{2}\\ \frac{.56896}{2} & -\frac{1.229}{2} & -\frac{.24376\times10^{-3}}{2} & \frac{.23565\times10^{-3}}{2} & -\frac{1.6516}{2} & .82876 \end{bmatrix}\begin{bmatrix}\tilde{x}_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix}. \quad (26)$$

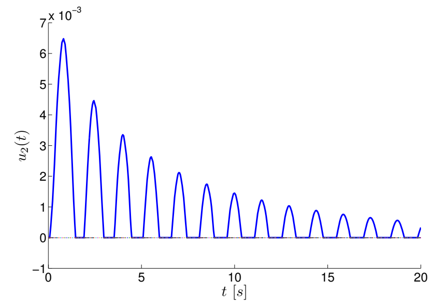

Fig. 6. Control input $u_2(t)$ to actuator 2.

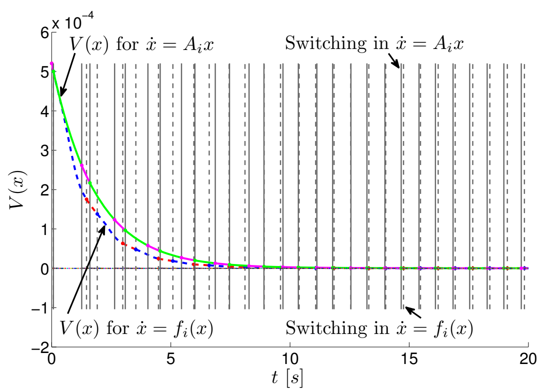

Fig. 7. Common Lyapunov function $V(x)$.

neural excitation inputs $u_1(t)$ and $u_2(t)$. However, thanks to the incorporation of the hyperbolic tangent function in the switched controller (16)-(17), no matter how large is the position error $\tilde{x}_1(t)$, the neural excitation inputs $u_1(t)$ and $u_2(t)$ will be valuated into the set $[0, 1]$.

IV. Conclusions

The switched systems stability theory, the root locus, and SOSTOOLS can be used to design saturated semi–positive neural excitation inputs $u_1(t)$, $u_2(t)$ which stabilize a second order mechanical system actuated by two Zajac's musculotendon subsystems

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