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Stabilization of Multimachine Power Systems via Hybrid Control

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Abstract— **An energy-based hybrid control framework for stabilization of multimachine power systems is proposed as a means to enhance transient stability of power systems. This approach is based on energy representation of power systems by using port-controlled Hamiltonian forms. The controllers utilize logic-based switching to enhance energy dissipation for the synchronous generators. This paper develops general energy dissipating excitation control design and stabilization results for such controllers.**

I. INTRODUCTION

The highly complex, dynamic behavior and nonlinearity of power systems, together with their almost continuously time-varying nature, have posed a great challenge to power system control engineers for decades. A crucial issue encountered at the generating plant level is to maintain stability or synchronism of synchronous generators when subjected to severe disturbances at various operating conditions. An effective and economical means to enhance stability of synchronous generators is excitation control.

Conventional excitation controllers, called the automatic voltage regulators (AVRs), are mainly designed by using linear control theory [1]. These linear excitation controllers can ensure stability following a small disturbance. However, when a large disturbance occurs, protective relaying may change the power system configuration and the post-disturbance steady-state operating condition, if reached, may be quite different from the predisturbance steady-state operating condition. In this case, nonlinearities begin to have significant effects; and a linear controller may not be able to maintain stability of the system. To overcome this flaw, several forms of adaptive control have been proposed to address the problem of performance variation [2].

Moreover, the AVR, which reacts only to the voltage error, always weakens the damping introduced by damper windings of the synchronous generator. This detrimental effect of AVR can be compensated by using a supplementary control loop, which is known as the power system stabilizer (PSS). The PSSs introduce additional system damping signals derived from the machine speed or power through the excitation system in order to improve the damping of power swings [3]. Conventional PSSs work reasonably well over medium range of operating conditions. However, stabilization effect of PSSs may diminish as the generator load changes or the network configuration is altered by faults or other switching conditions, which leads to deterioration in the PSS performance. Thus, remarkable efforts have been devoted to the design of appropriate power system stabilizers using various tools such as root locus, eigenvalue techniques, pole placement, adaptive control, and so on. But among all these methods, model uncertainty cannot be addressed explicitly at the design stage [4]. Henceforth, attention has been focused on the application of nonlinear controllers, which are independent of the equilibrium point and are taking into account the crucial nonlinearities of the power system model.

The application of nonlinear control techniques to solve the transient stabilization problem has been gained much attention [5]–[7]. Most of these controllers are based on feedback linearization technique [8], [9]. It was shown in the literature that the dynamics of the power system could be exactly linearized by employing nonlinear state feedback. The essence of this technique is to first transform a nonlinear system into a linear form by a nonlinear feedback, and then use the well-known linear design techniques to complete the controller design. Consequently one can use conventional linear control to give acceptable performance [6], [10]. Nevertheless, in many cases the feedback linearization method requires precise plant parameters and often cancels some useful nonlinearities. On the other hand, we are frequently faced with uncertainty in practical power systems. In this case, it is difficult to exactly linearize the system with nominal parameters. Adaptive versions of the feedback linearizing controls are then developed in [10]–[12]. Feedback linearization is recently enhanced by using robust control designs such as H_2 control and L_2 disturbance attenuation [7], [13].

Lyapunov theory has been an important tool in linear as well as nonlinear control for a long time [14]. Similar approach has been adopted in power system analysis and control via Lyapunov-like functions [15], [16]. However, its application within nonlinear control has been hampered by the difficulty of finding a Lyapunov function for a given system. The task of finding such a function has often been left to the imagination and experience of the designer.

More recently, a passivity-based control framework for port-controlled Hamiltonian systems is established in [17] and [18]. Specifically, the authors in [18] develop a controller design methodology that achieves stabilization via system passivation. In particular, the interconnection and damping matrix functions of the port-controlled Hamiltonian system are shaped so that the physical (Hamiltonian) system structure is preserved at the closed-loop level, and the closed-

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loop energy function is equal to the difference between the physical energy of the system and the energy supplied by the controller. Since a power system is an energy producing system, it is natural to model the power system as a port-controlled Hamiltonian system. It has been shown in the literature that the Hamitonian function method has some advantages [19], [20].

The aim of this paper is to design an energy-based nonlinear hybrid excitation control, which replaces the conventional AVR and PSS, to enhance the transient stability of synchronous generators. This energy-based hybrid controller can be viewed as a feedback control technique that exploits the coupling between a physical power system and an energy-based controller to efficiently remove energy from the power system. Specifically, if a dissipative power system is at high-energy level, and a lossless feedback controller at low energy level is attached to it, then energy will generally tend to flow from the power system into the controller, decreasing the power system energy and increasing the controller energy [21]. Of course, emulated energy, and not physical energy, is accumulated by the controller. Conversely, if the attached controller is at high energy level and a power system is at low energy level, then energy can flow from the controller to the power system, since a controller can generate real, physical energy to effect the required energy flow. Hence, if and when the controller states coincide with a high-emulated energy level, then we can reset these states to remove the emulated energy so that the emulated energy is not returned to the power system. In this case, the overall closed-loop system consisting of the power system and the controller possesses discontinuous flows since it combines logical switchings with continuous dynamics, leading to impulsive differential equations [22], [23].

The remaining of the paper is organized as follows. In Section II, we describe the multimachine power system model in a state space form. In Section III, we introduce the energy-based hybrid control framework for dissipative dynamical systems. Some further results with weaker conditions are presented. In Section IV, we rewrite the single-machine-infinite-bus power system model as a port-controlled Hamiltonian system and design an energy-based, fixed-order dynamic compensator to enhance energy dissipation of the power system. Moreover, we extend this energy-based hybrid control framework to design a hybrid decentralized controller for multimachine power systems in Section V. Some concluding remarks are mentioned in the final section.

II. Model of Power Systems

Consider an n-machine power system given by the three dimensional flux decay model [16], [26], [27]

$$\dot{\delta}_i = \omega_{i0}\omega_{Mi}, \tag{1}$$

$$\begin{aligned} M_i\dot{\omega}_{Mi} = {} & -D_{Mi}\omega_{Mi} + P_{mi} - V_{qi} \\ & \times \sum_{j=1,j\neq i}^{n} V_{qj}[G_{Mij}\cos(\delta_i-\delta_j) \\ & + B_{Mij}\sin(\delta_i-\delta_j)], \end{aligned} \tag{2}$$

$$\begin{aligned} T_{di}\dot{V}_{qi} = {} & -[1 - B_{Mii}(x_{di}-x'_{di})]V_{qi} - (x_{di}-x'_{di}) \\ & \times \sum_{j=1,j\neq i}^{n} V_{qj}[G_{Mij}\sin(\delta_i-\delta_j) \\ & - B_{Mij}\cos(\delta_i-\delta_j)] \\ & + E_{fsi} + u_{fi}, \quad i = 1,2,\ldots,n, \end{aligned} \tag{3}$$

where δ_i represents the rotor angle, ω_{Mi} represents the rotor speed, V_{qi} represents the quadrature axis internal voltage. Furthermore, the control input is the field excitation signal u_{fi}. The parameters $G_{Mij} = G_{Mji}$ and $B_{Mij} = B_{Mji}$ are, respectively, the conductance and susceptance of the generator i. E_{fsi} represents the constant component of the field voltage and P_{mi} the mechanical power, which is assumed to be a constant. The parameters x_{di}, x'_{di}, ω_{i0}, and D_{Mi} represent the direct axis synchronous reactance, the direct axis transient reactance, the synchronous speed, and damping coefficient, respectively. Note that all the parameters are positive and $x_{di} > x'_{di}$.

To simplify the model, we introduce the parameters $k_i \triangleq E_{fsi}/T_{di}$, $a_i \triangleq D_{Mi}/M_i$, $c_i \triangleq (P_{mi}\omega_{i0})/M_i$, $d_{ij} \triangleq (G_{Mij}\omega_{i0})/M_i$, $b_{ij} \triangleq (B_{Mij}\omega_{i0})/M_i$, $Z_{ij} \triangleq \sqrt{d_{ij}^2 + b_{ij}^2}$, $\alpha_{ij} \triangleq \arctan(d_{ij}/b_{ij})$, $h_i \triangleq [1 - B_{Mij}(x_{di}-x'_{di})]$, and $r_i \triangleq (x_{di}-x'_{di})/T_{di}$. Furthermore, we define the state variables as $x_{1i} \triangleq \delta_i$, $x_{2i} \triangleq \omega_i$, and $x_{3i} \triangleq V_{qi}$, and the control input as $u_i \triangleq u_{fi}/T_{di}$. Then (1)–(3) can be rewritten as the form

$$\dot{x}_{1i} = x_{2i}, \tag{4}$$

$$\begin{aligned} \dot{x}_{2i} = {} & -a_i x_{2i} + c_i - x_{3i}\sum_{j=1,j\neq i}^{n} x_{3j}Z_{ij} \\ & \times \sin(x_{1i} - x_{1j} + \alpha_{ij}), \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{x}_{3i} = {} & -h_i x_{3i} + r_i \sum_{j=1,j\neq i}^{n} x_{3j}Z_{ij} \\ & \times \cos(x_{1i} - x_{1j} + \alpha_{ij}) + k_i + u_i, \\ & \qquad i = 1,2,\ldots,n. \end{aligned} \tag{6}$$

It is important to note that if $d_{ij} = 0$ then $\alpha_{ij} = 0$.

III. Energy-Based Hybrid Control

In this section, we give some further results of energy-based hybrid control design framework developed in [28], [29]. Specifically, we consider nonlinear dynamical systems $\mathcal{G}_\mathrm{p}$ of the form given by

$$\dot{x}_\mathrm{p}(t) = f_\mathrm{p}(x_\mathrm{p}(t), u(t)), \quad x_\mathrm{p}(0) = x_{\mathrm{p}0}, \quad t \geq 0, \tag{7}$$

$$y(t) = h_\mathrm{p}(x_\mathrm{p}(t)), \tag{8}$$

where $t \geq 0$, $x_\mathrm{p}(t) \in \mathcal{D}_\mathrm{p} \subseteq \mathbb{R}^{n_\mathrm{p}}$, $\mathcal{D}_\mathrm{p}$ is an open set with $0 \in \mathcal{D}_\mathrm{p}$, $u(t) \in \mathbb{R}^m$, $f_\mathrm{p} : \mathcal{D}_\mathrm{p} \times \mathbb{R}^m \to \mathbb{R}^{n_\mathrm{p}}$ is smooth (i.e., infinitely differentiable) on $\mathcal{D}_\mathrm{p} \times \mathbb{R}^m$ and satisfies $f_\mathrm{p}(0,0) = 0$, and $h_\mathrm{p} : \mathcal{D}_\mathrm{p} \to \mathbb{R}^l$ is smooth and satisfies $h_\mathrm{p}(0) = 0$.

Next, we consider hybrid resetting dynamic controllers $\mathcal{G}_\mathrm{c}$

of the form

$$\dot{x}_{\mathrm{c}}(t) = f_{\mathrm{cc}}(x_{\mathrm{c}}(t), y(t)), \quad x_{\mathrm{c}}(0) = x_{\mathrm{c}0}, \quad (x_{\mathrm{c}}(t), y(t)) \notin \mathcal{Z}_{\mathrm{c}}, \tag{9}$$

$$\Delta x_{\mathrm{c}}(t) = \eta(y(t)) - x_{\mathrm{c}}(t), \quad (x_{\mathrm{c}}(t), y(t)) \in \mathcal{Z}_{\mathrm{c}}, \tag{10}$$

$$y_{\mathrm{c}}(t) = h_{\mathrm{cc}}(x_{\mathrm{c}}(t), y(t)), \tag{11}$$

where $x_{\mathrm{c}}(t) \in \mathcal{D}_{\mathrm{c}} \subseteq \mathbb{R}^{n_{\mathrm{c}}}$, $\mathcal{D}_{\mathrm{c}}$ is an open set with $0 \in \mathcal{D}_{\mathrm{c}}$, $y(t) \in \mathbb{R}^l$, $y_{\mathrm{c}}(t) \in \mathbb{R}^m$, $f_{\mathrm{cc}} : \mathcal{D}_{\mathrm{c}} \times \mathbb{R}^l \to \mathbb{R}^{n_{\mathrm{c}}}$ is smooth on $\mathcal{D}_{\mathrm{c}} \times \mathbb{R}^l$ and satisfies $f_{\mathrm{cc}}(0,0) = 0$, $\eta : \mathbb{R}^l \to \mathcal{D}_{\mathrm{c}}$ is continuous and satisfies $\eta(0) = 0$, and $h_{\mathrm{cc}} : \mathcal{D}_{\mathrm{c}} \times \mathbb{R}^l \to \mathbb{R}^m$ is smooth and satisfies $h_{\mathrm{cc}}(0,0) = 0$.

Recall that for the dynamical system $\mathcal{G}_{\mathrm{p}}$ given by (7) and (8), a function $s_{\mathrm{p}}(u,y)$, where $s_{\mathrm{p}} : \mathbb{R}^m \times \mathbb{R}^l \to \mathbb{R}$ is such that $s_{\mathrm{p}}(0,0) = 0$, is called a *supply rate* [24] if it is locally integrable for all input-output pairs satisfying (7) and (8), that is, for all input-output pairs $u \in \mathcal{U}$ and $y \in \mathcal{Y}$ satisfying (7) and (8), $s_{\mathrm{p}}(\cdot,\cdot)$ satisfies $\int_t^{\hat{t}} |s_{\mathrm{p}}(u(\sigma), y(\sigma))| \mathrm{d}\sigma < \infty$, $t, \hat{t} \geq 0$. Here, $\mathcal{U}$ and $\mathcal{Y}$ are input and output spaces, respectively, that are assumed to be closed under the shift operator. Furthermore, we assume that $\mathcal{G}_{\mathrm{p}}$ is *dissipative with respect to the supply rate* $s_{\mathrm{p}}(u,y)$ with a continuously differentiable nonnegative-definite *storage function* $V_{\mathrm{s}} : \mathcal{D}_{\mathrm{p}} \to \overline{\mathbb{R}}_+$ such that $V_{\mathrm{s}}(0) = 0$ and

$$V_{\mathrm{s}}(x_{\mathrm{p}}(t)) = V_{\mathrm{s}}(x_{\mathrm{p}}(t_0)) + \int_{t_0}^{t} [s_{\mathrm{p}}(u(\sigma), y(\sigma)) - d(x_{\mathrm{p}}(\sigma))] \mathrm{d}\sigma, \quad t \geq t_0, \tag{12}$$

for all $t_0, t \geq 0$, where $x_{\mathrm{p}}(t)$, $t \geq t_0$, is the solution to (7) with $u \in \mathcal{U}$ and $d : \mathcal{D}_{\mathrm{p}} \to \overline{\mathbb{R}}_+$ is a continuous, nonnegative-definite dissipation rate function. In addition, we assume that the nonlinear dynamical system $\mathcal{G}_{\mathrm{p}}$ is *completely reachable* [24] and *zero-state observable* [24], and there exists a function $\kappa : \mathbb{R}^l \to \mathbb{R}^m$ such that $\kappa(0) = 0$ and $s_{\mathrm{p}}(\kappa(y), y) < 0$, $y \neq 0$, so that all storage functions $V_{\mathrm{s}}(x_{\mathrm{p}})$, $x_{\mathrm{p}} \in \mathcal{D}_{\mathrm{p}}$, of $\mathcal{G}_{\mathrm{p}}$ are positive definite [25].

Assume that there exists a continuously differentiable function $V_{\mathrm{c}} : \mathcal{D}_{\mathrm{c}} \times \mathbb{R}^l \to \overline{\mathbb{R}}_+$ such that $V_{\mathrm{c}}(x_{\mathrm{c}}, y) \geq 0$, $x_{\mathrm{c}} \in \mathcal{D}_{\mathrm{c}}$, $y \in \mathbb{R}^l$, and $V_{\mathrm{c}}(x_{\mathrm{c}}, y) = 0$ if and only if $x_{\mathrm{c}} = \eta(y)$ and

$$\dot{V}_{\mathrm{c}}(x_{\mathrm{c}}(t), y(t)) = s_{\mathrm{c}}(u_{\mathrm{c}}(t), y_{\mathrm{c}}(t)), \quad (x_{\mathrm{c}}(t), y(t)) \notin \mathcal{Z}, \quad t \geq 0, \tag{13}$$

where $s_{\mathrm{c}} : \mathbb{R}^l \times \mathbb{R}^m \to \mathbb{R}$ is such that $s_{\mathrm{c}}(0,0) = 0$.

Consider the negative feedback interconnection of $\mathcal{G}_{\mathrm{p}}$ and $\mathcal{G}_{\mathrm{c}}$ given by $y = u_{\mathrm{c}}$ and $u = -y_{\mathrm{c}}$. In this case, the closed-loop system $\mathcal{G}$ is given by

$$\dot{x}(t) = f_{\mathrm{c}}(x(t)), \quad x(0) = x_0, \quad x(t) \notin \mathcal{Z}, \quad t \geq 0, \tag{14}$$

$$\Delta x(t) = f_{\mathrm{d}}(x(t)), \quad x(t) \in \mathcal{Z}, \tag{15}$$

where $t \geq 0$, $x(t) \triangleq [x_{\mathrm{p}}^{\mathrm{T}}(t), x_{\mathrm{c}}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\mathcal{Z} \triangleq \{x \in \mathcal{D} : (x_{\mathrm{c}}, h_{\mathrm{p}}(x_{\mathrm{p}})) \in \mathcal{Z}_{\mathrm{c}}\}$,

$$f_{\mathrm{c}}(x) = \begin{bmatrix} f_{\mathrm{p}}(x_{\mathrm{p}}, -h_{\mathrm{cc}}(x_{\mathrm{c}}, h_{\mathrm{p}}(x_{\mathrm{p}}))) \\ f_{\mathrm{cc}}(x_{\mathrm{c}}, h_{\mathrm{p}}(x_{\mathrm{p}})) \end{bmatrix}, \quad f_{\mathrm{d}}(x) = \begin{bmatrix} 0 \\ \eta(h_{\mathrm{p}}(x_{\mathrm{p}})) - x_{\mathrm{c}} \end{bmatrix}. \tag{16}$$

We refer to the differential equation (14) as the *continuous-time dynamics*, and we refer to the difference equation (15) as the *resetting law*. Note that although the closed-loop state vector consists of plant states and controller states, it is clear from (16) that only those states associated with the controller are reset. Sufficient conditions that guarantee that wellposedness of the impulsive dynamical system (14) and (15) in the sense of [28], [29] are given by the following proposition. For this result, the following definition is needed. First, however, recall that the *Lie derivative* of a continuously differentiable function $\mathcal{X} : \mathcal{D} \to \mathbb{R}$ along the vector field of the continuous-time dynamics $f_{\mathrm{c}}(x)$ is given by $L_{f_{\mathrm{c}}}\mathcal{X}(x) \triangleq \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{X}(\psi(t,x))|_{t=0} = \frac{\partial \mathcal{X}(x)}{\partial x} f_{\mathrm{c}}(x)$.

Definition 3.1: Let $\mathcal{Q} \triangleq \{x \in \mathcal{D} : \mathcal{X}(x) = 0\}$, where $\mathcal{X} : \mathcal{D} \to \mathbb{R}$ is a continuously differentiable function. A point $x \in \mathcal{Q}$ such that $f_{\mathrm{c}}(x) \neq 0$ is *transversal almost everywhere* to (14) if

$$L_{f_{\mathrm{c}}}\mathcal{X}(x) \neq 0 \quad \text{a.e.}, \tag{17}$$

where "a.e." denotes almost everywhere in the sense of the Lebesgue measure.

Proposition 3.1: Consider the impulsive dynamical system $\mathcal{G}$ given by (14) and (15). Let $\mathcal{X} : \mathcal{D} \to \mathbb{R}$ be a continuously differentiable function such that $\overline{\mathcal{Z}} = \{x \in \mathcal{D} : \mathcal{X}(x) = 0\}$, and assume that every $x \in \overline{\mathcal{Z}}$ is transversal almost everywhere to (14). Then $\mathcal{G}$ is well defined in the sense of [28], [29].

We associate with the plant a positive-definite, continuously differentiable function $V_{\mathrm{p}}(x_{\mathrm{p}}) \triangleq V_{\mathrm{s}}(x_{\mathrm{p}})$, which we will refer to as the *plant energy*. Furthermore, we associate with the controller a nonnegative-definite, infinitely differentiable function $V_{\mathrm{c}}(x_{\mathrm{c}}, y)$ called the controller *emulated energy*. Finally, we associate with the closed-loop system the function $V(x) \triangleq V_{\mathrm{p}}(x_{\mathrm{p}}) + V_{\mathrm{c}}(x_{\mathrm{c}}, h_{\mathrm{p}}(x_{\mathrm{p}}))$, called the *total energy*.

Next, we construct the resetting set for the closed-loop system $\mathcal{G}$ in the following way

$$\mathcal{Z} = \{(x_{\mathrm{p}}, x_{\mathrm{c}}) \in \mathcal{D}_{\mathrm{p}} \times \mathcal{D}_{\mathrm{c}} : L_{f_{\mathrm{c}}}V_{\mathrm{c}}(x_{\mathrm{c}}, h_{\mathrm{p}}(x_{\mathrm{p}})) = 0 \text{ and } V_{\mathrm{c}}(x_{\mathrm{c}}, h_{\mathrm{p}}(x_{\mathrm{p}})) > 0\}. \tag{18}$$

The resetting set $\mathcal{Z}$ is thus defined to be the set of all points in the closed-loop state space that correspond to the instant when the controller is at the verge of decreasing its emulated energy. By resetting the controller states, the plant energy can never increase after the first resetting event. This energy dissipating hybrid controller effectively enforces a one-way energy transfer between the plant and the controller after the first resetting event. For practical implementation, knowledge of x_{c} and y is sufficient to determine whether or not the closed-loop state vector is in the set $\mathcal{Z}$.

The next theorem gives sufficient conditions for asymptotic stability of the closed-loop system $\mathcal{G}$ using state-dependent hybrid controllers.

Theorem 3.1: Consider the closed-loop impulsive dynamical system $\mathcal{G}$ given by (14) and (15). Assume that $\mathcal{D}_{\mathrm{ci}} \subset \mathcal{D}$ is a compact positively invariant set with respect to $\mathcal{G}$ such that 0 is in the interior of $\mathcal{D}_{\mathrm{ci}}$, assume that $\mathcal{G}_{\mathrm{p}}$ is dissipative with respect to the supply rate $s_\mathrm{p}(u,y)$ and with a positive definite, continuously differentiable storage function $V_\mathrm{p}(x_\mathrm{p})$, $x_\mathrm{p} \in \mathcal{D}_\mathrm{p}$, and assume there exists a smooth (i.e., infinitely differentiable) function $V_\mathrm{c} : \mathcal{D}_\mathrm{c} \times \mathbb{R}^l \to \overline{\mathbb{R}}_+$ such that $V_\mathrm{c}(x_\mathrm{c}, y) \geq 0$, $x_\mathrm{c} \in \mathcal{D}_\mathrm{c}$, $y \in \mathbb{R}^l$, and $V_\mathrm{c}(x_\mathrm{c}, y) = 0$ if and only if $x_\mathrm{c} = \eta(y)$ and (13) holds. Furthermore, assume that every $x_0 \in \overline{\mathcal{Z}}$ is transversal almost everywhere to (14) and

$$s_\mathrm{p}(u,y) + s_\mathrm{c}(u_\mathrm{c}, y_\mathrm{c}) \leq 0, \quad x \notin \mathcal{Z}, \tag{19}$$

where $y = u_\mathrm{c} = h_\mathrm{p}(x_\mathrm{p})$, $u = -y_\mathrm{c} = -h_\mathrm{cc}(x_\mathrm{c}, h_\mathrm{p}(x_\mathrm{p}))$, and $\mathcal{Z}$ is given by (18). If the largest invariant set contained in $\mathcal{R} \triangleq \{(x_\mathrm{p}, x_\mathrm{c}) \in \mathcal{D}_\mathrm{ci} : d(x_\mathrm{p}) = 0\}$ is $\mathcal{M} = \{(0,0)\}$, then the zero solution $x(t) \equiv 0$ to the closed-loop system $\mathcal{G}$ is asymptotically stable.

IV. Hybrid Control for Single-Machine-Infinite-Bus Power Systems

In the case where $n = 1$, the model (4)–(6) reduces to the single machine infinite bus power system given by

$$\dot{x}_1 = x_2, \tag{20}$$
$$\dot{x}_2 = -a x_2 + c - x_3 Z \sin(x_1 + \alpha), \tag{21}$$
$$\dot{x}_3 = -h x_3 + r\cos(x_1 + \alpha) + k + u, \tag{22}$$

where we have introduced some obvious simplifying notation. Let

$$\mathcal{H}(x) \triangleq \frac{r}{2hZ} x_2^2 - \frac{rc}{hZ} x_1 - \frac{r}{h} x_3 \cos(x_1 + \alpha) - \frac{k}{h} x_3 + \frac{1}{2} x_3^2, \tag{23}$$

where $x \triangleq [x_1, x_2, x_3]^\mathrm{T}$. Note that $\mathcal{H}(\cdot)$ is bounded from below since $x_1 \in [-\pi, \pi]$. Then we have

$$\dot{x} = \begin{bmatrix} 0 & \frac{hZ}{r} & 0 \\ -\frac{hZ}{r} & -\frac{ahZ}{r} & 0 \\ 0 & 0 & -h \end{bmatrix} \left(\frac{\partial \mathcal{H}(x)}{\partial x}\right)^\mathrm{T} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u. \tag{24}$$

We define the system output y given by

$$y = x_3 - \frac{r}{h}\cos(x_1 + \alpha) - \frac{k}{h}. \tag{25}$$

Then (24) and (25) can be rewritten as a port-controlled Hamiltonian system given by

$$\dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)] \left(\frac{\partial \mathcal{H}(x)}{\partial x}\right)^\mathrm{T} + G(x) u, \tag{26}$$
$$y = G^\mathrm{T}(x) \left(\frac{\partial \mathcal{H}(x)}{\partial x}\right)^\mathrm{T}, \tag{27}$$

where

$$\mathcal{J}(x) = \begin{bmatrix} 0 & \frac{hZ}{r} & 0 \\ -\frac{hZ}{r} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{R}(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{ahZ}{r} & 0 \\ 0 & 0 & -h \end{bmatrix},$$
$$G(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{28}$$

Assume that the operation point of the single-machine-infinite-bus system (20)–(22) is given by $x_\mathrm{e}^* = [x_{1\mathrm{e}}^*, 0, x_{3\mathrm{e}}^*]^\mathrm{T}$, where $x_{1\mathrm{e}}^*$ and $x_{3\mathrm{e}}^*$ satisfy

$$k + u_\mathrm{e}^* = h x_{3\mathrm{e}}^* - r\cos(x_{1\mathrm{e}}^* + \alpha), \tag{29}$$
$$c = Z x_{3\mathrm{e}}^* \sin(x_{1\mathrm{e}}^* + \alpha), \tag{30}$$

where $u_\mathrm{e}^* \in \mathbb{R}$ is a constant.

Next, consider the fixed-order, energy-based hybrid dynamic controller of the form

$$\dot{x}_\mathrm{c}(t) = \mathcal{J}_\mathrm{c}(x_\mathrm{c}(t)) \left(\frac{\partial \mathcal{H}_\mathrm{c}}{\partial x_\mathrm{c}}(x_\mathrm{c}(t))\right)^\mathrm{T} + G_\mathrm{c}(x_\mathrm{c}(t)) \left(y(t) - \frac{1}{h} u_\mathrm{e}^*\right), \quad x_\mathrm{c}(0) = x_{\mathrm{c}0},$$
$$(x(t), x_\mathrm{c}(t)) \notin \mathcal{Z}, \quad t \geq 0, \tag{31}$$
$$x_\mathrm{c}(t^+) = 0, \quad (x(t), x_\mathrm{c}(t)) \in \mathcal{Z}, \tag{32}$$
$$u(t) = u_\mathrm{e}^* - G_\mathrm{c}^\mathrm{T}(x_\mathrm{c}(t)) \left(\frac{\partial \mathcal{H}_\mathrm{c}}{\partial x_\mathrm{c}}(x_\mathrm{c}(t))\right)^\mathrm{T}, \tag{33}$$

where $x_\mathrm{c}(t) \in \mathcal{D}_\mathrm{c} \subseteq \mathbb{R}^{n_\mathrm{c}}$, $\mathcal{D}_\mathrm{c}$ is an open set with $0 \in \mathcal{D}_\mathrm{c}$, $\mathcal{H}_\mathrm{c} : \mathcal{D}_\mathrm{c} \to \mathbb{R}$ is an infinitely differentiable Hamiltonian function for (31), $\mathcal{J}_\mathrm{c} : \mathcal{D}_\mathrm{c} \to \mathbb{R}^{n_\mathrm{c} \times n_\mathrm{c}}$ is such that $\mathcal{J}_\mathrm{c}(x_\mathrm{c}) = -\mathcal{J}_\mathrm{c}^\mathrm{T}(x_\mathrm{c})$, $x_\mathrm{c} \in \mathcal{D}_\mathrm{c}$, $\mathcal{J}_\mathrm{c}(x_\mathrm{c})(\frac{\partial H_\mathrm{c}}{\partial x_\mathrm{c}}(x_\mathrm{c}))^\mathrm{T}$, $x_\mathrm{c} \in \mathcal{D}_\mathrm{c}$ is smooth on $\mathcal{D}_\mathrm{c}$, $G_\mathrm{c} : \mathcal{D}_\mathrm{c} \to \mathbb{R}^{n_\mathrm{c} \times 1}$, and the resetting set $\mathcal{Z} \subset \mathcal{D} \times \mathcal{D}_\mathrm{c}$ is given by

$$\mathcal{Z} \triangleq \Big\{(x, x_\mathrm{c}) \in \mathcal{D} \times \mathcal{D}_\mathrm{c} : \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_\mathrm{c}(x_\mathrm{c}) = 0 \text{ and } \mathcal{H}_\mathrm{c}(x_\mathrm{c}) > 0\Big\}, \tag{34}$$

where $\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_\mathrm{c}(x_\mathrm{c}) \triangleq \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_\mathrm{c}(\psi(t, x_\mathrm{c}))|_{t=0}$. Here, we assume that $\mathcal{H}_\mathrm{c}(0) = 0$ and $\mathcal{H}_\mathrm{c}(x_\mathrm{c}) > 0$ for all $x_\mathrm{c} \neq 0$ and $x_\mathrm{c} \in \mathcal{D}_\mathrm{c}$.

Theorem 4.1: Consider the closed-loop dynamical system $\mathcal{G}$ given by (20)–(22), (31)–(33), and the resetting set $\mathcal{Z}$ given by (34). Assume that $\mathcal{D}_\mathrm{i} \subset \mathcal{D} \times \mathcal{D}_\mathrm{c}$ is a compact positively invariant set with respect to $\mathcal{G}$ such that $(x_\mathrm{e}^*, 0)$ is in the interior of $\mathcal{D}_\mathrm{i}$. Furthermore, assume that every $\tilde{x}_0 \in \overline{\mathcal{Z}}$ is transversal almost everywhere to (14) with $\mathcal{X}(\tilde{x}) = \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_\mathrm{c}(x_\mathrm{c})$ and

$$f_\mathrm{c} \triangleq \begin{bmatrix} [\mathcal{J}(x) - \mathcal{R}(x)] \left(\frac{\partial \mathcal{H}(x)}{\partial x}\right)^\mathrm{T} \\ -G(x) G_\mathrm{c}^\mathrm{T}(x_\mathrm{c}) \left(\frac{\partial \mathcal{H}_\mathrm{c}}{\partial x_\mathrm{c}}(x_\mathrm{c})\right)^\mathrm{T}, \\ \mathcal{J}_\mathrm{c}(x_\mathrm{c}) \left(\frac{\partial \mathcal{H}_\mathrm{c}}{\partial x_\mathrm{c}}(x_\mathrm{c})\right)^\mathrm{T} \\ +G_\mathrm{c}(x_\mathrm{c}) G^\mathrm{T}(x) \left(\frac{\partial \mathcal{H}(x)}{\partial x}\right)^\mathrm{T} \end{bmatrix}, \tag{35}$$

where $\tilde{x} = [x^\mathrm{T}, x_\mathrm{c}^\mathrm{T}]^\mathrm{T}$. If the largest invariant set contained in

$$\mathcal{R} \triangleq \left\{(x, x_\mathrm{c}) \in \mathcal{D}_\mathrm{i} : \mathcal{R}(x) \left(\frac{\partial \mathcal{H}(x)}{\partial x}\right)^\mathrm{T} = 0\right\} \tag{36}$$

is $\mathcal{M} = \{(x_\mathrm{e}^*, 0)\}$, then the equilibrium solution $(x(t), x_\mathrm{c}(t)) \equiv (x_\mathrm{e}^*, 0)$ to $\mathcal{G}$ is asymptotically stable.

V. Hybrid Decentralized Control for Multimachine Power Systems

For the n-machine case, we design an energy-based hybrid decentralized controller for multimachine power systems to improve the transient stability. Specifically, we consider an n-machine power system $\mathcal{G}$ of the form given by (4)–(6). Let

$$\mathcal{H}_i(x) \triangleq \frac{r_i}{2h_i}x_{2i}^2 - \frac{r_i c_i}{h_i}x_{1i} - \frac{r_i}{h_i}x_{3i}\sum_{j=1,j\neq i}^{n} Z_{ij}x_{3j} \times \cos(x_{1i}-x_{1j}+\alpha_{ij}) - \frac{k_i}{h_i}x_{3i} + \frac{1}{2}x_{3i}^2, \quad (37)$$

where $i=1,\ldots,n$. Then (4)–(6) can be rewritten as

$$\dot{x}_i = [\mathcal{J}_i(x_i) - \mathcal{R}_i(x_i)]\left(\frac{\partial \mathcal{H}_i(x)}{\partial x_i}\right)^{\mathrm{T}} + G_i(x_i)u_i, \quad (38)$$

where

$$\mathcal{J}_i(x_i) = \begin{bmatrix} 0 & \frac{h_i}{r_i} & 0 \\ -\frac{h_i}{r_i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{R}_i(x_i) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{a_i h_i}{r_i} & 0 \\ 0 & 0 & -h_i \end{bmatrix},$$

$$G_i(x_i) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (39)$$

Here, we define the system output y_i as

$$y_i = G_i^{\mathrm{T}}(x_i)\left(\frac{\partial \mathcal{H}_i(x)}{\partial x_i}\right)^{\mathrm{T}}. \quad (40)$$

Suppose that the operation point of the multimachine power system (4)–(6) is given by $x_{\mathrm{e}2i}^* = 0$ and

$$c_i = x_{\mathrm{e}3i}^* \sum_{j=1,j\neq i}^{n} x_{\mathrm{e}3j}^* Z_{ij}\sin(x_{\mathrm{e}1i}^* - x_{\mathrm{e}1j}^* + \alpha_{ij}), \quad (41)$$

$$k_i + u_{\mathrm{e}i}^* = h_i x_{\mathrm{e}3i}^* - r_i \sum_{j=1,j\neq i}^{n} x_{\mathrm{e}3j}^* Z_{ij} \times \cos(x_{\mathrm{e}1i}^* - x_{\mathrm{e}1j}^* + \alpha_{ij}), \quad i=1,\ldots,n. \quad (42)$$

Next, we consider hybrid decentralized dynamic controllers $\mathcal{G}_{\mathrm{c}i}$, $i=1,\ldots,n$, of the form

$$\dot{x}_{\mathrm{c}i}(t) = \mathcal{J}_{\mathrm{c}i}(x_{\mathrm{c}i}(t))\left(\frac{\partial \mathcal{H}_{\mathrm{c}i}}{\partial x_{\mathrm{c}i}}(x_{\mathrm{c}i}(t))\right)^{\mathrm{T}} + G_{\mathrm{c}i}(x_{\mathrm{c}i}(t))\left(y_i(t) - \frac{1}{h_i}u_{\mathrm{e}i}^*\right), \quad x_{\mathrm{c}i}(0) = x_{\mathrm{c}i0}, \quad (x_i(t), x_{\mathrm{c}i}(t)) \notin \mathcal{Z}_i, \quad t \geq 0, \quad (43)$$

$$x_{\mathrm{c}i}(t^+) = 0, \quad (x_i(t), x_{\mathrm{c}i}(t)) \in \mathcal{Z}_i, \quad (44)$$

$$u_i(t) = u_{\mathrm{e}i}^* - G_{\mathrm{c}i}^{\mathrm{T}}(x_{\mathrm{c}i}(t))\left(\frac{\partial \mathcal{H}_{\mathrm{c}i}}{\partial x_{\mathrm{c}i}}(x_{\mathrm{c}i}(t))\right)^{\mathrm{T}}, \quad (45)$$

where $x_{\mathrm{c}i}(t) \in \mathcal{D}_{\mathrm{c}i} \subseteq \mathbb{R}^{n_{\mathrm{c}i}}$, $\mathcal{D}_{\mathrm{c}i}$ is an open set with $0 \in \mathcal{D}_{\mathrm{c}i}$, $\mathcal{H}_{\mathrm{c}i} : \mathcal{D}_{\mathrm{c}i} \to \mathbb{R}$ is an infinitely differentiable Hamiltonian function for (43), $\mathcal{J}_{\mathrm{c}i} : \mathcal{D}_{\mathrm{c}i} \to \mathbb{R}^{n_{\mathrm{c}i}\times n_{\mathrm{c}i}}$ is such that $\mathcal{J}_{\mathrm{c}i}(x_{\mathrm{c}i}) = -\mathcal{J}_{\mathrm{c}i}^{\mathrm{T}}(x_{\mathrm{c}i})$, $x_{\mathrm{c}i} \in \mathcal{D}_{\mathrm{c}i}$, $\mathcal{J}_{\mathrm{c}i}(x_{\mathrm{c}i})(\frac{\partial H_{\mathrm{c}i}}{\partial x_{\mathrm{c}i}}(x_{\mathrm{c}i}))^{\mathrm{T}}$, $x_{\mathrm{c}i} \in \mathcal{D}_{\mathrm{c}i}$ is smooth on $\mathcal{D}_{\mathrm{c}i}$, and $G_{\mathrm{c}i} : \mathcal{D}_{\mathrm{c}i} \to \mathbb{R}^{n_{\mathrm{c}i}\times 1}$.

We associate with the n-machine power systems a positive-definite, continuously differentiable function $\mathcal{H}_{\mathrm{p}}(x) \triangleq \sum_{i=1}^{n}[\mathcal{H}_i(x) - \mathcal{H}_i(x_{\mathrm{e}})]$, which we will refer to as the *multimachine system energy*. We call the term $\mathcal{H}_i(x) - \mathcal{H}_i(x_{\mathrm{e}})$ *single machine system energy*. Furthermore, we associate with the controller a nonnegative-definite, infinitely differentiable function $\mathcal{H}_{\mathrm{c}}(x_{\mathrm{c}}) \triangleq \sum_{i=1}^{n}\mathcal{H}_{\mathrm{c}i}(x_{\mathrm{c}i})$ called the controller *emulated energy*. We call the term $\mathcal{H}_{\mathrm{c}i}(x_{\mathrm{c}i})$ *subcontroller emulated energy*. Finally, we associate with the closed-loop system the function $\mathcal{H}(\tilde{x}) \triangleq \mathcal{H}_{\mathrm{p}}(x) + \mathcal{H}_{\mathrm{c}}(x_{\mathrm{c}})$, called the *total energy*, where $x \triangleq [x_1^{\mathrm{T}},\ldots,x_n^{\mathrm{T}}]^{\mathrm{T}}$, $x_{\mathrm{c}} \triangleq [x_{\mathrm{c}1}^{\mathrm{T}},\ldots,x_{\mathrm{c}n}^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{x}_i \triangleq [x_i^{\mathrm{T}}, x_{\mathrm{c}i}^{\mathrm{T}}]^{\mathrm{T}}$, and $\tilde{x} \triangleq [\tilde{x}_1^{\mathrm{T}},\ldots,\tilde{x}_n^{\mathrm{T}}]^{\mathrm{T}}$.

The resetting set $\mathcal{Z}_i \subset \mathcal{D}_i \times \mathcal{D}_{\mathrm{c}i}$ is given by

$$\mathcal{Z}_i \triangleq \left\{(x_i, x_{\mathrm{c}i}) \in \mathcal{D}_i \times \mathcal{D}_{\mathrm{c}i} : \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_{\mathrm{c}i}(x_{\mathrm{c}i}) = 0 \text{ and } \mathcal{H}_{\mathrm{c}i}(x_{\mathrm{c}i}) > 0\right\}. \quad (46)$$

Here, we assume that $\mathcal{H}_{\mathrm{c}i}(0) = 0$ and $\mathcal{H}_{\mathrm{c}i}(x_{\mathrm{c}i}) > 0$ for all $x_{\mathrm{c}i} \neq 0$ and $x_{\mathrm{c}i} \in \mathcal{D}_{\mathrm{c}i}$. The resetting sets $\mathcal{Z}_i$, $i=1,\ldots,n$, are thus defined to be the sets of all points in the closed-loop state space that correspond to decreasing subcontroller emulated energy. By resetting the subcontroller states, the single machine system energy can never increase after the first resetting event. Hence, this approach allows the single machine system energy to flow to the subcontroller, where it increases the subcontroller emulated energy but does not allow the subcontroller emulated energy to flow back to the single machine system after the first resetting event. This energy dissipating hybrid decentralized controller effectively enforces a one-way energy transfer between each single machine power system and corresponding subcontroller.

Next, we present a sufficient condition to guarantee stability of the closed-loop system when the proposed hybrid decentrailzed controller is applied. To this end, we need the following definition and lemma.

Definition 5.1: Let $\mathcal{Q} \triangleq \bigcup_{i=1}^{n}\{\tilde{x} \in \tilde{\mathcal{D}} : \mathcal{X}_i(\tilde{x}) = 0\}$, where $\mathcal{X}_i : \tilde{\mathcal{D}} \to \mathbb{R}$, $i=1,\ldots,n$, are continuously differentiable functions and $\tilde{\mathcal{D}} \triangleq \bigcup_{i=1}^{n}(\mathcal{D}_i \times \mathcal{D}_{\mathrm{c}i})$. A point $\tilde{x} \in \mathcal{Q}$ such that $\tilde{f}_{\mathrm{c}}(\tilde{x}) \neq 0$ is *jointly transversal almost everywhere* to

$$\dot{z}(t) = \tilde{f}_{\mathrm{c}}(z(t)), \quad z(0) = \tilde{x}, \quad t \geq 0, \quad (47)$$

if

$$L_{\tilde{f}_{\mathrm{c}}}\mathcal{X}_i(\tilde{x}) \neq 0, \quad i=1,\ldots,n, \quad \text{a.e.} \quad (48)$$

Lemma 5.1: Consider the closed-loop dynamical system $\mathcal{G}$ given by (38), (40), (43)–(45). Let $\mathcal{X}_i = \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_{\mathrm{c}i}(x_{\mathrm{c}i})$ and assume that every $x \in \mathcal{Q}$ is jointly transversal almost everywhere to (47) where $\tilde{f}_{\mathrm{c}} \triangleq [\tilde{f}_{\mathrm{c}1}^{\mathrm{T}},\ldots,\tilde{f}_{\mathrm{c}n}^{\mathrm{T}}]^{\mathrm{T}}$ and

$$\tilde{f}_{\mathrm{c}i} \triangleq \begin{bmatrix} [\mathcal{J}_i(x_i) - \mathcal{R}_i(x_i)]\left(\frac{\partial \mathcal{H}_i(x)}{\partial x_i}\right)^{\mathrm{T}} \\ -G_i(x_i)G_{\mathrm{c}i}^{\mathrm{T}}(x_{\mathrm{c}i})\left(\frac{\partial \mathcal{H}_{\mathrm{c}i}}{\partial x_{\mathrm{c}i}}(x_{\mathrm{c}i})\right)^{\mathrm{T}}, \\ \mathcal{J}_{\mathrm{c}i}(x_{\mathrm{c}i})\left(\frac{\partial \mathcal{H}_{\mathrm{c}i}}{\partial x_{\mathrm{c}i}}(x_{\mathrm{c}i})\right)^{\mathrm{T}} \\ +G_{\mathrm{c}i}(x_{\mathrm{c}i})G_i^{\mathrm{T}}(x_i)\left(\frac{\partial \mathcal{H}_i(x)}{\partial x_i}\right)^{\mathrm{T}} \end{bmatrix}. \quad (49)$$

Then $\mathcal{G}$ is well defined in the sense of [28], [29].

Theorem 5.1: Consider the closed-loop dynamical system $\mathcal{G}$ given by (38), (40), (43)–(45), and the resetting set $\mathcal{Z}_i$

given by (46). Assume that $\mathcal{D}_{\mathrm{n}i} \subset \mathcal{D}_i \times \mathcal{D}_{\mathrm{c}i}$ is a compact positively invariant set with respect to $\mathcal{G}$ such that $(x_{\mathrm{e}i}^*, 0)$ is in the interior of $\mathcal{D}_{\mathrm{n}i}$. Furthermore, assume that every $\tilde{x}_0 \in \overline{\mathcal{Z}}$ is jointly transversal almost everywhere to (47), where $\tilde{f}_{\mathrm{c}}$ is given by (49) and $\mathcal{Z} \triangleq \bigcup_{i=1}^n \{\tilde{x} \in \tilde{\mathcal{D}} : \tilde{x}_i \in \mathcal{Z}_i\}$. If the largest invariant set contained in

$$\mathcal{R} \triangleq \bigcap_{i=1}^{n} \left\{ (x_i, x_{\mathrm{c}i}) \in \mathcal{D}_{\mathrm{n}i} : \mathcal{R}_i(x_i) \left(\frac{\partial \mathcal{H}_i(x)}{\partial x_i} \right)^{\mathrm{T}} = 0 \right\} \tag{50}$$

is $\mathcal{M} = \{(x_{\mathrm{e}1}^*, 0) \times \cdots \times (x_{\mathrm{e}n}^*, 0)\}$, then the equilibrium solution $\tilde{x}(t) \equiv (x_{\mathrm{e}1}^*, 0) \times \cdots \times (x_{\mathrm{e}n}^*, 0)$ to $\mathcal{G}$ is asymptotically stable.

VI. Conclusion

We have presented an energy-based hybrid output feedback controller that ensures enhanced energy dissipation and asympotic stability of the operating equilibrium for multimachine power systems. Unlike standard static controllers in the literature, the proposed controller is a dynamic compensator which is scalable in the sense that it has a decentralized structure for multimachine power systems. Currently, we are working on the development of a realistic simulation example so that the proposed scheme can be compared with the classical two-machine transmission system with power system stabilizers and static var compensator. Future research will include the presence of transfer conductances in transmission lines. This will hamper the assignment of a simple Hamiltonian function for power systems. Next, insteading of using linear Hamiltonian forms in our simulation, designing nonlinear Hamiltonian forms in the controller might further improve transient performance of the power system. Finally, some real experimental tests will be carried out to test the proposed control algorithm.

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