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Digital Design of Coefficient Diagram Method

Ö. Öcal, A. Bir and B. Tibken

Abstract— **Coefficient Diagram Method (CDM) is the one of the most effective control design methods in literature. It gives control systems that are very stable and robust, system responses without overshoot and very small settling time. The CDM, resolve the classical control problem after selecting a few meaningful design parameters and by automatically determining a target characteristic polynomial for the closed-loop system. The classical CDM design was originally only performed in *s*-domain. But since, all industrial controllers are digital controllers, it is necessary to improve a digital controller based on CDM. In this paper, the Digitalized Coefficient Diagram method is introduced (D-CDM) and this new digital controller design method applied to different control systems. The very satisfactory results obtained by the new design method are discussed in details.**

I. Introduction

CDM is a relatively new controller design method which is used successful since 90'the in the design of linear time-invariant control systems. In this mainly algebraic approach the coefficients of the controller polynomials are obtained by comparing the coefficients of the characteristic polynomial of the closed loop transfer function with the coefficients of the target characteristic polynomial obtained by using the convenient chosen CDM parameters such as equivalent time constant, stability index, and stability limit index. Using CDM the time response of the controlled closed loop system without overshoot has a small settling time and the system is very robust against the parameter variations. The disturbance effects on the system can be eliminated very successfully. The design procedure is very easy. In addition, by applying the CDM design one obtains good results for all types of control systems. This property is important; since by the way the use of self-tuning controller's increases and this kind of controllers make the best design independent of the human factor. Thus, the CDM design method can be used very effectively in self tuning control applications [1-3].

Furthermore the time-domain performance of a dynamical system is closely related by the closed loop transfer function poles, or the roots of the system characteristic function. In the literature it exist many controller design methods in order to assign the closed-loop system poles to suitable locations [4]. The Coefficient Diagram Method, resolve this classical problem after selecting a few meaningful design parameters, by automatically determining a target characteristic polynomial for the closed-loop system. In the literature there are many applications using the classical design procedure [5-11]. The original CDM design method is basically performed in *s*-domain. Since, all industrial controllers are now digital controllers, the main target of this paper is to improve and develop an equivalent digital controller to the existing CDM.

In this paper the CDM method is first briefly introduced in Section 2. The application of the CDM follows the generalized controller design method developed by [12] and [13]. The Digitalized Coefficient Diagram Method is presented in Section 3 and some applications are section given in Section 4, finally in the last conclusion Section some further researches fields are out pointed.

II. The Coefficient Diagram Method

A. Theory

In this section, only the structure, performance parameters, Target Characteristic Polynomial (TCP) of CDM and the design procedure are shortly given and discussed. The details of the CDM method can be found in [6-8].
The standard SISO block diagram of the CDM is shown in Figure 1. Here, $R(s)$ is the input reference, $Y(s)$ the output, $U(s)$ the control, $Q(s)$ the disturbance, $E(s)$ the error and $M(s)$ the measurement disturbance signal of the control system respectively.

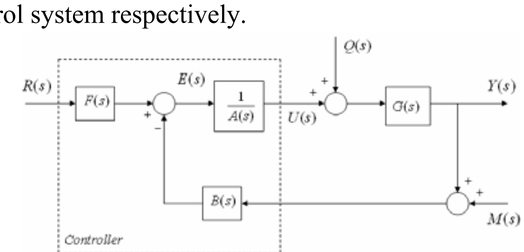

Fig. 1. The standard block diagram of CDM

In Figure 1, $A(s)$, $B(s)$, and $F(s)$ are controller polynomials. The system to be controlled is represented by $G(s)$ transfer function given by

Ö. Öcal; Istanbul Technical University, Istanbul 34230 Turkey (e-mail: oocal@itu.edu.tr).
A. Bir; Istanbul Technical University, Istanbul 34230 Turkey (e-mail: bir@itu.edu.tr)
B. Tibken; The University of Wuppertal, Wuppertal 42119 Germany (e-mail: tibken@uni-wuppertal.de)

$$G(s) = \frac{N(s)}{D(s)} \tag{3}$$

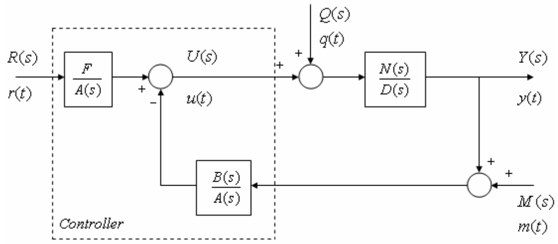

Fig. 2. Equivalent block diagram of CDM

Here $P(s)$ is the characteristic polynomial of the closed-loop system given by

$$P(s) = A(s)D(s) + B(s)N(s) = \sum_{i=0}^{n} a_i s^i \tag{4}$$

1) Performance Parameter and the Target Characteristic Polynomial

CDM needs some design parameters with respect to the characteristic polynomial coefficients such as τ the equivalent time constant, γ_i the stability indices and γ_i^* the stability limits. The relations between these parameters and the coefficients of the characteristic polynomial a_i are given in relations (5);

$$\gamma_i = \frac{a_i^2}{a_{i+1} a_{i-1}} \quad i = 1...(n-1) \tag{5a}$$

$$\tau = \frac{a_1}{a_0} \text{ and } \gamma_i^* = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}} \tag{5b}$$

Using these relations in Equation (5), it is possible to formulate the characteristic polynomial $P(s)$ in terms of the design parameters τ and γ_i as follows:

$$P_T(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{6}$$

$P_T(s)$ is the Target Characteristic Polynomial (TCP). Note that the a_i coefficients of the characteristic polynomial can be expressed as

$$a_i = \frac{\tau^i}{\prod_{j=1}^{i-1} \gamma_j^{i-j}} a_0 \tag{7}$$

This is a relation between equivalent time constant τ and the settling time T_s, since the time constant can be written as $\tau = T_s / \alpha$, where $\alpha \in [2.5, 3]$.

2) CDM Design Procedure

Equating the closed-loop system characteristic polynomial found in section 2 to the target characteristic polynomial given in section 3, one obtains a Diophantine equation witch can be written in the following equivalent Sylvester form

$$[C]_{rxr} \begin{bmatrix} l_i \\ k_i \end{bmatrix}_{rx1} = [a_i]_{rx1} \tag{9}$$

The coefficients of the ***C*** matrix and the a_i parameters are known values. Therefore, the coefficients of the controller polynomials $A(s)$ and $B(s)$ can easily be calculated.
The coefficient F is calculated by using the expression

$$F = \left(\frac{P(s)}{N(s)} \right) \Bigg|_{s=0} \tag{10}$$

Details about the stability analysis of CDM is given in [6, 7, 14, 15]

3) Stability and Robustness

<u>The stability indices and stability</u>
In addition to Routh-Hurwitz criterion, CDM inserts the Lipatov – Sokolov criterion [2-3] the stability and instability conditions are given in [14-15];
<u>Robustness</u>
The robustness of the control system controlled by the CDM is shown in [1-3].

III. THE DIGITALIZED COEFFICIENT DIAGRAM METHOD

A. Introduction

The standard block diagram for digital SISO systems is given in Figure 3. K(z), B(z) and F(z) are the controller polynomials.

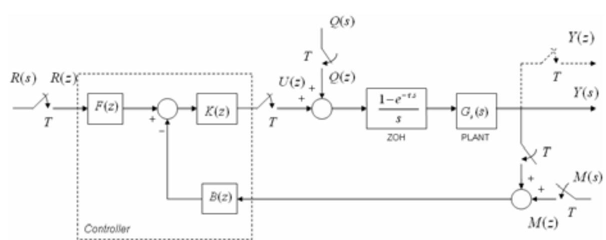

Fig. 3. The Standard block diagram for digital SISO systems

B. Theory

1) The Structure of the D-CDM system

The standard digital block diagram of the CDM for SISO systems is shown in Figure 4. Here, $R(z)$ represent reference input, $Y(z)$ system output, $U(z)$ control signal, $Q(z)$ disturbance signal, $E(z)$ error and $M(z)$ the measurement disturbance signal respectively.

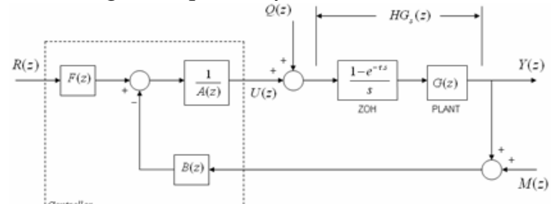

Fig. 4. The standard block diagram of D-CDM

In Figure 4, $A(z)$, $B(z)$, and F are controller polynomials and $GH(z)$ is the digital transfer function of the system to be controlled, where $GH(z)$ is defined as

$$HG(z) = Z\left\{\frac{(1-e^{-\tau s})}{s}.G(s)\right\} = \frac{N(z)}{D(z)} \qquad (11)$$

Identical block diagrams are given in Figure 5-6.

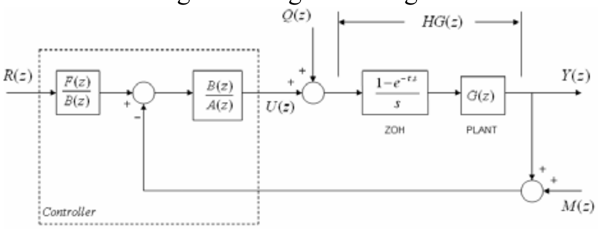

Fig. 5 The standard block diagram of D-CDM

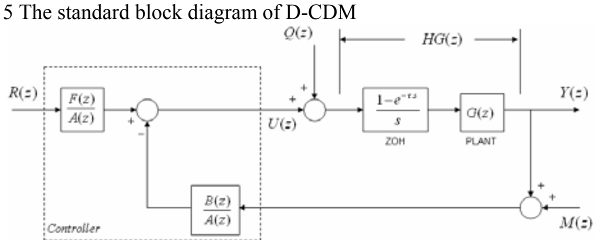

Fig. 6 The standard block diagram of D-CDM

In the following it is assumed that $G(z)$ is a strictly proper system (thus $\deg\{D(z)\} \geq \deg\{N(z)\}$). It is possible to show that

$$Y(z) = \frac{A(z)N(z)}{P(z)}Q(z) + \frac{F(z)N(z)}{P(z)}R(z) - \frac{B(z)N(z)}{P(z)}M(z)$$

$$U(z) = \frac{F(z)D(z)}{P(z)}R(z) - \frac{B(z)N(z)}{P(z)}Q(z) - \frac{B(z)D(z)}{P(z)}M(z)$$

and

$$E(z) = \frac{A(z)F(z)D(z)}{P(z)}R(z) - \frac{A(z)B(z)N(z)}{P(z)}Q(z) - \frac{A(z)B(z)D(z)}{P(z)}M(z)$$

where $P(z)$ is the characteristic polynomial of the closed-loop system given by

$$P(z) = A(z)D(z) + B(z)N(z) = \sum_{i=0}^{n} a_i z^i .$$

2) Performance Parameters and the Target Characteristic Polynomial

The CDM target characteristic polynomial can be expressed by the characteristic polynomial coefficients which are the equivalent time constant τ , the stability indices γ_i , and the stability limits γ_i^* by the following expression

$$P_T(s) = a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\} + \tau s + 1\right]$$

(12)

Next, by using the z-transformation $z = e^{Ts}$ (where T is sample time), the target characteristic polynomial is converted to the digital form. We know that the target characteristic polynomial $P_T(s)$ has s_i ($i = 1,2,...,n$) roots. If $\mathbf{s}$ represent the vector

$$\mathbf{s} = \begin{bmatrix} s_1 & s_2 & \cdots & s_i \end{bmatrix}$$

in which, s_i are the roots of the target characteristic polynomial, the roots in z-plane can be represented by a similar vector $\mathbf{z}$

$$\mathbf{z} = \begin{bmatrix} z_1 & z_2 & \cdots & z_i \end{bmatrix}.$$

Thus the digital target characteristic polynomial can be written as

$$P_T(z) = \prod_{j=1}^{n}\left(z - z_j\right) \qquad (13)$$

Where ($i = 1,2..,n$)

$$P_T(z) = \alpha_n z^n + \alpha_{n-1}z^{n-1} + ... + \alpha_1 z + \alpha_0 = \sum_{i=0}^{n}\alpha_i z^i$$

3) Design Procedure and A(z), B(z) and F Controller Polynomials

The denominator $D(z)$ and numerator $N(z)$ polynomials of the plant can be given by (where $m \leq n$):

$$D(z) = b_n z^n + b_{n-1}z^{n-1} + ... + b_1 z + b_0 = \sum_{i=0}^{n} b_i z^i \qquad (14)$$

$$N(z) = a_m z^m + a_{m-1}z^{m-1} + ... + a_1 z + a_0 = \sum_{i=0}^{m} a_i z^i \qquad (15)$$

Controller polynomials are defined by following expressions (where $q \leq l$):

$$A(z) = l_p z^p + l_{p-1}z^{p-1} + ... + l_1 z + l_0 = \sum_{i=0}^{p} l_i z^i \qquad (16)$$

$$B(z) = k_q z^q + k_{q-1}z^{q-1} + ... + k_1 z + k_0 = \sum_{i=0}^{q} k_i z^i \qquad (17)$$

The characteristic equation can be rewritten as

$$P(z) = A(z)D(z) + B(z)N(z)$$
$$= \left(\sum_{i=0}^{p} l_i z^i\right).\left(\sum_{i=0}^{n} b_i z^i\right) + \left(\sum_{i=0}^{q} k_i z^i\right).\left(\sum_{i=0}^{n-1} a_i z^i\right) \qquad (18)$$

Since, this closed-loop system characteristic polynomial $P(z) = 0$ and the known target characteristic polynomial given as $P_T(z) = 0$ are identical to each other; the following Diophantine equation

$$A(z)D(z) + B(z)N(z) = P_T(z) \qquad \text{or} \qquad (19)$$

$$P_T(z) = \sum_{i=0}^{n+p}\alpha_i z^i = \left(\sum_{i=0}^{p} l_i z^i\right).\left(\sum_{i=0}^{n} b_i z^i\right) + \left(\sum_{i=0}^{q} k_i z^i\right).\left(\sum_{i=0}^{n-1} a_i z^i\right)$$

can be written. The equation (19) is converted to Sylvester Form as

$$[D]_{rxr}\begin{bmatrix} l_i \\ k_i \end{bmatrix}_{rx1} = [\alpha_i]_{rx1} \qquad (20)$$

Here, matrix D and α_i parameters are known values. Therefore the coefficients of the controller polynomials $A(z)$ and $B(z)$ can easily be calculated. Furthermore, $n+p \le n+(n-1) = 2n-1$ so $P_T(z)$ is given by

$$P_T(z) = \sum_{i=0}^{2n-1} \alpha_i z^i \tag{21}$$

Finally, F can be obtained from

$$F = \left. \left(\frac{P(z)}{N(z)} \right) \right|_{z=1} \tag{22}$$

But if zeros are cancelled, which means that $N(z)=1$, it is enough to find the value of $P(z=1)$ to determine F.

4) *Stability and Robustness*

Stability

For stability, we know that CDM guarantees the stability of CDM control system because of the target characteristic polynomial (Sec. 2.A.3). In D-CDM, the digital target characteristic polynomial is obtained by means of the characteristic polynomial of the CDM [1-3]. So D-CDM also guarantees the stability of the control system automatically.

Robustness

Robustness of the control system is shown in next section.

5) *Disturbance Effects*

$A(z)$, $B(z)$ and $P(z)$ polynomials against disturbance types and degrees are given in Table 1 and 2.

TABLE 1
A(z), B(z) AND P(z) POLYNOMIALS AGAINST DISTURBANCE TYPES

	Unit Step	Ramp	Sinus
$A(z)$	$(z-1)\sum_{i=0}^{p-1} l_i z^i$	$(z-1)^2 \sum_{i=0}^{p-1} l_i z^i$	$(z^2 - 2\cos(\omega T)z + 1)\sum_{i=0}^{p-1} l_i z^i$
$B(z)$	$\sum_{i=0}^{q} k_i z^i$	$\sum_{i=0}^{q+1} k_i z^i$	$\sum_{i=0}^{q+1} k_i z^i$
$P(z)$	$z.P_T(z)$	$z^2.P_T(z)$	$\sin(\omega T).z^2.P_T(z)$

TABLE 2
DEGREES OF THE A(z), B(z) AND P(z) POLYNOMIALS FOR DIFFERENT DISTURBANCE EFFECTS

	Unit Step	Ramp	Sinus
$\deg\{A(z)\}$	n	n+1	n+1
$\deg\{B(z)\}$	n	n+1	n+1
$\deg\{P(z)\}$	2n	2n+1	2n+1

IV. APPLICATION

A. *Third Order Systems*

Consider the third order system

$$G(s) = \frac{1}{(s+1)^3} \tag{23}$$

The digital transfer function $HG(z)$ is calculated as

$$HG(z) = \frac{3.22579z^2 + 6.20235z + 0.718282}{20.0855z^3 - 22.1672z^2 + 8.15485z - 1} \tag{24}$$

For $\tau = 1$, $a_0 = 2$ and $\gamma_i = \begin{bmatrix} 2.6 & 2.2 & 2 & 2 \end{bmatrix}$, the continuous and digital characteristic polynomials are obtained as:

$$P_T(s) = 0.000513782s^5 + 0.0117553s^4 + 0.134481s^3$$
$$+0.769231s^2 + 2s + 2$$
$$P_T(z) = z^5 - 0.496657z^4 + 0.0747057z^3 - 0.00214075z^2$$
$$-0.0000988748z - 0.0000107565$$

According to the target characteristic polynomial $P_T(z)$ the controller polynomials of the D-CDM are calculated as

$$A(z) = 0.0497871z^2 + 0.0243049z + 0.0023516$$
$$B(z) = 0.0368294z^2 - 0.0211393z + 0.00325894$$
$$F = 0.0571709 .$$

With this D-CDM controller the closed loop system transfer functions can easily be obtained. In Figures 7-9 are shown the step responses of the $y(kT)$system output, $u(kT)$ control and $e(kT)$.error signal.

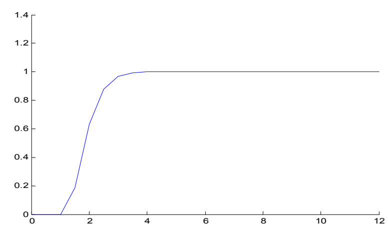

Fig. 7 Step response $y(kT)$ of the closed loop system

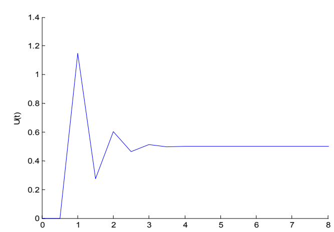

Fig. 8 The step response of the control signal $u(kT)$

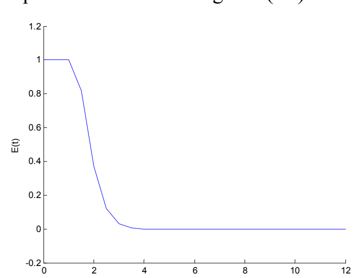

Fig. 9 The error signal $e(kT)$ of the control system

To show how robust the system is, we take the *a*, *b* and *c* parameters of the transfer function free and write:

$$HG(z) = \frac{3.22579z^2 + 6.20235z + 0.718282}{20.0855z^3 + (-22.1672 + c)z^2 + (8.15485 + b)z + (-1 + a)}$$
$$= \frac{a_2 z^2 + a_1 z + a_0}{b_3 z^3 + b_2 z^2 + b_1 z + b_0}$$

We take the nominal values as $a_0 = 0$, $b_0 = 0$ $c_0 = 0$ and choose the limits as:

$-0.3 < a < 0.3$
$-2 < b < 2$
$-3 < c < 3$ or
$-0.7 < b_0 < -1.3$
$6.15485 < b_1 < 10.15485$
$-25.1672 < b_2 < -19.1672$

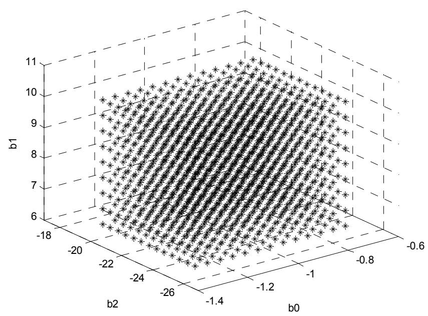

Fig. 10 The coefficient grade of the system for $-0.3 < a < 0.3$ $-2 < b < 2$ and $-3 < c < 3$

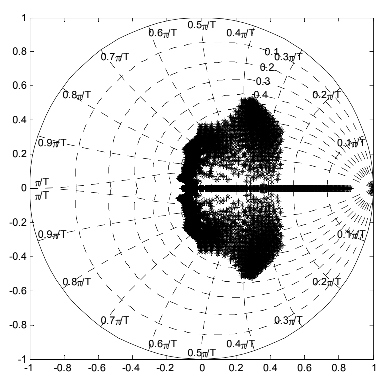

Fig. 11 The pole spread of the closed loop system for $-0.3 < a < 0.3$ $-2 < b < 2$ and $-3 < c < 3$

The coefficients and the poles grade of the system is given in figures 10 and 11.

B. Unstable Systems

Consider the third order system

$$G(s) = \frac{1}{(s+1)^2 (s-1)} \tag{25}$$

The digital transfer function $HG(z)$ is calculated as

$$HG(z) = \frac{-1.03018z^2 - 3.42202z - 0.621018}{-7.38906z^3 + 25.5221z^2 - 15.7781z + 2.71828}$$

For $T = 0.5$, $a_0 = 2$ and $\gamma_i = [2.6 \quad 2.2 \quad 2 \quad 2]$, the continuous

$$P_T(s) = 0.000513782s^5 + 0.0117553s^4 + 0.134481s^3 + 0.769231s^2 + 2s + 2$$

and digital characteristic polynomial is obtained as

$$P_T(z) = z^5 - 0.496657z^4 + 0.0747057z^3 + 0.00214075z^2 - 0.0000988748z - 0.0000107565$$

According to the target characteristic polynomial $P_T(z)$ the controller polynomials of the D-CDM are calculated:

$$A(z) = -0.135335z^2 - 0.224124z - 0.383611$$
$$B(z) = -1.2632z^2 + 0.918932z - 0.167895$$
$$F = -0.114342$$

The closed loop system transfer functions can easily be obtained. In Figures 12-14 the step responses system output $y(kT)$, control signal $u(kT)$ and error signal $e(kT)$ are shown.

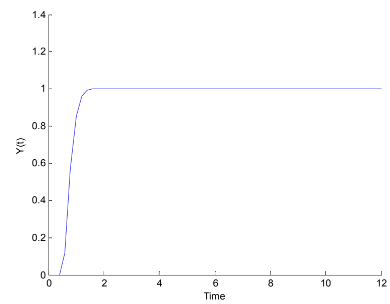

Fig. 12 Step response y(kT) of the closed loop system

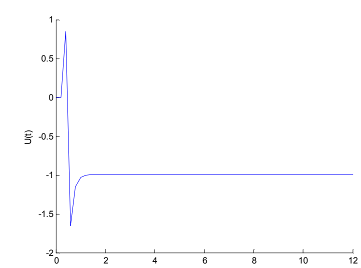

Fig. 13 The step response of the control signal $u(kT)$

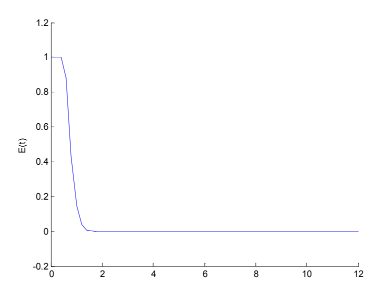

Fig. 14 The error signal $e(kT)$ of the control system

Again to show how robust the system is, we consider the *a*, *b* and *c* parameters of the transfer function free and write:

$$HG(z) = \frac{-1.03018z^2 - 3.42202z - 0.621018}{-7.38906z^3 + (25.5221 + c)z^2 + (-15.7781 + b)z + (2.71828 + a)}$$
$$= \frac{a_2z^2 + a_1z + a_0}{b_3z^3 + b_2z^2 + b_1z + b_0}$$

The coefficient limits are chosen as

$-0.6 < a < 0.6$

$-0.3 < b < 0.3$

$-0.5 < c < 0.5$ or

$2.11828 < b_0 < 3.31828$

$-16.0781 < b_1 < -15.4781$

$25.0221 < b_2 < 26.0221$.

The nominal values are $a_0 = 0$, $b_0 = 0$ and $c_0 = 0$.

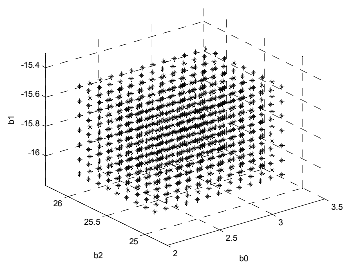

Fig. 15 The coefficient spread of the system for $-0.6 < a < 0.6$ $-0.3 < b < 0.3$ and $-0.5 < c < 0.5$

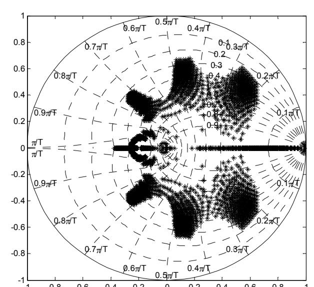

Fig. 16 The pole spread of the closed loop system for $-0.6 < a < 0.6$ $-0.3 < b < 0.3$ and $-0.5 < c < 0.5$

The robustness of the system can be increased by changing the controller parameters by changing the Performance Parameters and Sampling Time.

V. Conclusion

In this paper the CDM design method is briefly introduced. This mainly algebraic approach is derived by comparing the coefficients of the controller polynomials with the coefficients of the characteristic polynomial of the closed loop transfer function and the coefficients of the target characteristic polynomial obtained by using the convenient chosen CDM parameters such as equivalent time constant, stability index, and stability limit index. The time response of the controlled closed loop system has a small settling time without overshoot in the case of using CDM. Secondly; the D-CDM is introduced for the first time to the control literature. The D-CDM is a very powerful tool in controller design.

Forthcoming the D-CDM control method will be applied to real-time control applications. The reaction of the D-CDM procedure against the disturbance effects and the cancellation of the closed loop zeros investigated.

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