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Robust Controller Design for Networked Control Systems with Nonlinear Uncertainties

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Abstract— **We address robust stabilization problem for networked control systems with nonlinear uncertainties and packet losses by modelling such systems as a class of uncertain switched systems. Based on theories on switched Lyapunov functions, we derive the robustly stabilizing conditions for state feedback stabilization and design packet-loss dependent controllers by solving some matrix inequalities. A numerical example and some simulations are worked out to demonstrate the effectiveness of the proposed design method.**

I. INTRODUCTION

Networked control systems (NCSs) are a class of feedback control systems whose sensors, controllers and actuators are connected by using network channels. Due to the use of communication network in the feedback control loop, the analysis and design for an NCS is more complex than that for a traditional control system. In recent years, NCSs have received increasing attentions [1]-[4].

One of the major issues raised in NCSs is the packet loss which is a potential source of instability and poor performance of NCSs. Many results have studied some control problems of NCSs in the presence of packet losses such as [5][6][7]. Generally speaking, two effective approaches have been used to deal with this issue in the existing results: one is delayed system approach [8][9][10] where delayed systems are used to describe the NCSs with packet losses, and the other is switched system approach [5][11][12] where switched systems are obtained by the lifting technique for such systems.

In the field of control, if a single controller fails to solve a control problem, a multiple of controllers might be used in the hope that the problem may be solved by switching among these controllers. Here, based on rich switched system theories [13]-[15], we design the packet-loss dependent controller by the switched system approach. One of the the advantages of the switched system approach is that the controllers can make full use of the previous information to stabilize NCSs when the current state measurements are not available from the network. By the switched system approach, the stabilization problem of NCSs with packet losses have been considered in [7][12]. However, none of them has considered the effects of the uncertainties.

It is necessary to introduce some uncertain parameters in the modelling of NCSs, and study the design of robust stabilizing controllers when it comes to the control synthesis problems. Some existing results have studied the effects of the uncertainties against some robust H_∞ control problems for NCSs. Robust H_∞ output tracking for NCSs was investigated in [6] in the presence of both network-induced delays and packet losses by delayed system approach. [16] discussed robust H_∞ control problems for NCSs with time delays and subject to norm-bounded parameter uncertainties. In [17], a robust H_∞ control problem was considered for a class of networked systems with random communication packet losses and norm-bounded parameter uncertainties. An observer-based feedback controller was designed to robustly exponentially stabilize the system by solving certain linear matrix inequalities (LMIs). However, they only concerned such uncertainties as those in polytopic or norm-bounded frameworks.

Recently, robust stability and stabilization have been studied in [11][18]-[21]. More specially, [11] studied robust stabilization of NCSs subjected to either packet losses or communication delays by modelling such systems as continuous-time nonlinear systems with delayed input. Sufficient conditions on the existence of stabilizing state feedback controllers are established in terms of LMIs. The robust stabilization of NCSs with packet losses was studied in [18], where it has been shown that minimum packet arrival rate and the maximum uncertainty of the system dynamics have a positive correlation. In [19], an improved predictive controller was designed using delayed sensing data, and a compensation scheme was proposed to overcome the negative effects of the network-induced delays and data packet losses. The stability of the closed-loop system was obtained by modelling the system as a time delay system with structural uncertainties. [20] considered delay-dependent stability criteria for a class of multi-input and multi-output (MIMO) NCSs with nonlinear perturbation. Robustly delay-dependent stability criteria were established for such systems with structured uncertainties and external nonlinear perturbation by solving a convex optimization problem. [21] dealt with the problem of delay-dependent stability for NCSs with structured uncertainties and multiple state time-delays, and established a new delay-

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dependent stability criteria for the system in terms of LMIs. But the forementioned results still only have concerned some structured uncertainties.

In this paper, for NCSs in the presence of the packet losses, we introduce a class of unstructured nonlinear uncertainties to the control input, and discuss the robust stabilization problem by using the switched system approach. For state feedback, robustly stabilizing conditions are established by using packet-loss dependent Lyapunov functions. Different from all the forementioned results, we design packet-loss dependent controllers which preserve the robust stability of the closed-loop systems. Moreover, we present some convex optimal problems to derive the robust degree of the stabilization for the uncertain systems and the upper bound of the packet losses which preserves the stabilization of the NCS.

The paper is organized as follows. Section II describes mathematical models of NCSs under study. Section III discusses robust stabilization for the systems via state feedback, and robust state feedback controllers are constructed by using the feasible solutions of some matrix inequalities. A numerical example demonstrating the effectiveness of the proposed design technique are given in Section IV. The conclusion is provided in Section V.

Notations. Throughout this paper, the following notations are used. $\|\cdot\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrix; For any two positive integers i and j satisfying $j \geq i$, $[i,j] = \{i, i+1, \cdots, j\}$; $\lambda_{max}(P)$ denotes the maximum eigenvalue of matrix P.

II. Problem Formulation

Consider the NCS with packet losses illustrated in Fig. 1, where the sensor is clock-driven and the actuator is event-driven, and the packet loss only occurs in the S/C channel (the channel between the sensor and controller). The plant with nonlinear uncertainties is described as

$$x(t+1) = Ax(t) + Bu(t) + g(t, u(t)), \tag{1}$$

and the time-varying controller is given by

$$u(t) = F(t)\bar{x}(t), \tag{2}$$

where $t \in \mathbf{N}$, $x(t) \in \mathbf{R}^n$ is the plant state vector, $u(t) \in \mathbf{R}^m$ is the plant input vector, and $\bar{x}(t) \in \mathbf{R}^n$ is the state measurement that is successfully transmitted over the network. A, B are known real constant matrices with proper dimensions, and $F(t) \in \mathbf{R}^{m\times n}$ is the state feedback gain matrix to be designed. Uncertainty $g(t, u)$ is a known vector-valued nonlinear function and satisfies $g(t, 0) = 0$ for all $t \in \mathbf{N}$ and the following quadratic inequality for all (t, u)

$$g^T(t, u)g(t, u) \leq \alpha^2 u^T G^T G u, \tag{3}$$

where G is a constant matrix with appropriate dimension and α is a nonnegative constant.

Remark 1: Nonlinear uncertainties as defined in (3) have been introduced for a class of discrete-time systems in [22], where output feedback controllers were designed by the unified LMI approach. As shown there, the structure uncertainties [16][18][21] are included in the class of nonlinear uncertainties. Thus, system (1) is more general than the discrete-time linear system with structure uncertainties in control inputs.

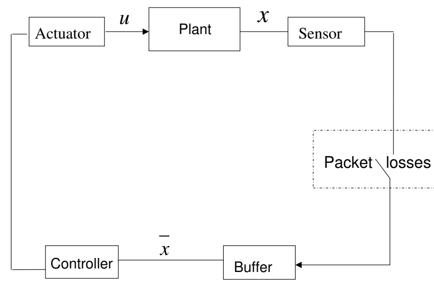

Fig. 1. Structure of an NCS with packet losses in the S/C channel.

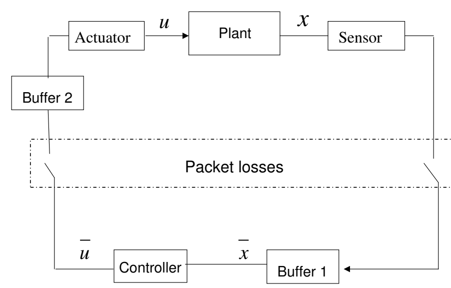

Fig. 2. Structure of an NCS with packet losses in both S/C and C/A channels.

We suppose that a sensor data containing the state information will substitute the old data when it is successfully sent to the controller over the communication channel, and the state information that the controller obtained is denoted by $\bar{x}(t)$. At any time instant t, the controller reads out the content of $\bar{x}(t)$ and utilizes it to compute the new control input. Then $\bar{x}(t)$ is described as

$$\bar{x}(t) = \begin{cases} x(t), & \text{if } x(t) \text{ is transmitted successfully;} \\ \bar{x}(t-1), & \text{otherwise.} \end{cases} \tag{4}$$

Furthermore, we denote the set of successive update instants of $\bar{x}(t)$ as $\{t_0 = 0, t_1, \cdots, t_k, \cdots\}$ which is a subset of $\mathbf{N}$. Here, we consider NCS (1) by using the switched system approach, and in what follows we describe the mathematical model using the switched system approach.

Without loss of generality, we assume that the packet containing $x(0)$ is transmitted to the controller successfully, that is $\bar{x}(0) = x(0)$, then $x(1) = (A + BF(0))x(0) + g(0, F(0)x(0))$. In the next time instant, if the data packet containing $x(1)$ is transmitted to the controller successfully, then

$$x(2) = (A + BF(1))x(1) + g(1, F(1)x(1)),$$

otherwise,

$$\begin{aligned} x(2) &= Ax(1) + BF(1)x(0) + g(1, F(1)x(0)) \\ &= (A(A + BF(0)) + BF(1))x(0) \\ &\quad + Ag(0, F(0)x(0)) + g(1, F(1)x(0)). \end{aligned}$$

In this pattern of transmission, the states of NCS (1) at the successive update time instants can be described as follows:

$$\begin{aligned} x(t_{k+1}) =& (A^{t_{k+1}-t_k} + \sum_{l=0}^{t_{k+1}-t_k-1} A^l BF(t_{k+1} - l - 1)) \\ & \times x(t_k) + \sum_{l=0}^{t_{k+1}-t_k-1} A^l g(t_{k+1} - l - 1, \\ & F(t_{k+1} - l - 1)x(t_k)), \ \forall k \in \mathbf{N}. \end{aligned}$$

Define $z(0) = x(0)$, $z(1) = x(t_1), \cdots, z(k) = x(t_k), \cdots$,

$$A(k) = A^{t_{k+1}-t_k} + \sum_{l=0}^{t_{k+1}-t_k-1} A^l BF(t_{k+1} - l - 1),$$

$$\begin{aligned} G(k, z(k)) \ =& \sum_{l=0}^{t_{k+1}-t_k-1} A^l g(t_{k+1} - l - 1, \\ & F(t_{k+1} - l - 1)x(t_k)), \end{aligned}$$

we can obtain the following state equation

$$z(k) = A(k)z(k-1) + G(k, z(k)), \ \forall k \in \mathbf{N}. \quad (5)$$

Thus, we obtain a new system (5) by the lifting technique for NCS (1), which presents the state evolution at the update instants. Notice that nonlinear uncertainty $G(k, z(k))$ is a function of the time and the state and satisfies

$$\begin{aligned} & G^T(k, z(k))G(k, z(k)) \\ & \leq (t_{k+1} - t_k)x^T(t_k) \sum_{l=0}^{t_{k+1}-t_k-1} \lambda_{max}((A^T)^l A^l) \\ & \quad \times F^T(t_{k+1} - l - 1)G^T GF(t_{k+1} - l - 1)x(t_k). \end{aligned}$$

In order to keep the stability or good performances of NCSs, it is always being avoided to occur mass of packet losses in the application of NCSs. Thus, it is reasonable to assume that the packet loss is bounded in discussing the control problem of NCSs. Without loss of generality, we assume that the maximum transmission interval is d, that is, $t_{k+1} - t_k \leq d$ for all $k \in \mathbf{N}$. Therefore, the upper bound of the dropped data packets is $d-1$. Here, we refer to the time between t_k and t_{k+1} as one transmission interval. Furthermore, we assume that the time-varying gain $F(t)$ is a piecewise function which takes its values in a finite set $\{F_1, F_2, \cdots, F_d\}$. Thus, with a set of candidate gains $\{F_1, F_2, \cdots, F_d\}$ to be designed, we propose the following design algorithm to stabilize NCS (1):

Design Algorithm 1. For any t_k, assuming that there is a counter which records the length of the last transmission interval $[t_{k-1}, t_k)$, we take the packet-loss dependent feedback gain as $F_{t_k - t_{k-1}}$ in the following transmission interval $[t_k, t_{k+1})$, i.e.,

$$u(t) = F_{t_k - t_{k-1}} \bar{x}(t), t \in [t_k, t_{k+1}). \quad (6)$$

Notice that (6) is packet-loss dependent. In what follow we aim to design controller as defined in (6) to stabilize (1). Applying *Design Algorithm 1* to NCS (1), we get the state evolution equation of NCS (1) at the update instants as follows

$$z(k+1) = \bar{A}_{\eta(k)} z(k) + \bar{G}_{\eta(k)}(k, z(k)), \ k \in \mathbf{N}, \quad (7)$$

where

$$\bar{A}_{\eta(k)} = A^{r(k)} + \sum_{l=0}^{r(k)-1} A^l BF_{r(k-1)},$$

$$\bar{G}_{\eta(k)}(k, z(k)) = \sum_{l=0}^{r(k)-1} A^l g(t_{k+1} - l - 1, F_{r(k-1)} z(k)),$$

and $r(k) = t_{k+1} - t_k$, $\eta(k) = (r(k), r(k-1)) \in [1, d] \times [1, d]$ with $\eta(1) = (r(1), 1)$, which means that $x(0)$ is transmitted to the controller successfully.

Without loss of generality, for any t_k, let $t_{k+1} - t_k = i, t_k - t_{k-1} = j$, and denote

$$\begin{aligned} \bar{A}_{ij} &= A^i + \sum_{l=0}^{i-1} A^l BF_j, \\ \bar{G}_{ij}(k, z(k)) &= \sum_{l=0}^{i-1} A^l g(t_{k+1} - l - 1, F_j z(k)). \end{aligned} \quad (8)$$

Consequently, we have

$$\bar{A}_{\eta(k)} \in \bar{\Omega} := \{\bar{A}_{11}, \bar{A}_{12}, \cdots, \bar{A}_{1d}, \cdots, \bar{A}_{d1}, \bar{A}_{d2}, \cdots, \bar{A}_{dd}\},$$

$$\begin{aligned} \bar{G}_{\eta(k)}(k, z(k)) \in \bar{\Psi} := \{& \bar{G}_{11}(k, z(k)), \bar{G}_{12}(k, z(k)), \\ & \cdots, \bar{G}_{1d}(k, z(k)), \cdots, \bar{G}_{d1}(k, z(k)), \\ & \bar{G}_{d2}(k, z(k)), \cdots, \bar{G}_{dd}(k, z(k))\}. \end{aligned}$$

Remark 2: Let $g(t, u(t)) \equiv 0$, and then the problem would reduce to the normal stabilization problem. Furthermore, let $F_1 = F_2 = \cdots = F_d$, then the stabilization problem via a multiple of controllers under study reduces to the stabilization via a single controller discussed in [7][12]. Thus, the stabilization problems discussed in [7][12] are special cases of that addressed here.

Now, we make some discussions for NCSs with the bounded packet loss in both the S/C channel and the C/A channel (the channel between the actuator and the controller) illustrated in Fig. 2, where the sensor is clock-driven and the actuator is event-driven. We suppose that the set of successive update instants of plant input $u(t)$ is $\{t_0 = 0, t_1, \cdots, t_k, \cdots\}$, which is a subset of $\mathbf{N}$. We suppose that $u(t)$ is defined as

$$u(t) = \begin{cases} \bar{u}(t), & \text{if } \bar{u}(t) \text{ is transmitted successfully;} \\ u(t-1), & \text{otherwise.} \end{cases}$$

Note that the design of multiple controllers in *Scheduling Algorithm 1* depends on packet losses. However, the information of packet losses in the forward channel at present time is not available when we design the controller. Hence, *Scheduling Algorithm 1* is not applied to such NCSs. We do not provide the details on this problem due to the limitation of the length.

III. Stabilization via State Feedback

In this section, we study the robust stabilization problem of NCSs with nonlinear uncertainties via state feedback. Sufficient conditions for the robust stabilization are derived by using a packet-loss dependent Lyapunov function, and stabilizing state feedback controllers are designed by solving some matrix inequalities. Here, we only present the results for the robust stabilization problem of NCSs with S/C channel.

Now, we present the following definitions for later use.

Definition 1: [12] A packet-loss process $\{r(k) \in \mathbf{N} : r(k) = t_{k+1} - t_k\}$ is said to be arbitrary if it takes its values in the interval $[1, d]$ arbitrarily.

Definition 2: A function $\phi : \mathbf{R}^n \to \mathbf{R}_+$ is of class K if it is continuous, strictly increasing, and $\phi(0) = 0$.

Definition 3: Let $x(t)$ be the trajectory of system (1) without control inputs and any uncertainty, and then the system is said to be stable, if for any $\varepsilon > 0$, there exists a number $\delta > 0$ such that $\|x(0)\| < \delta$ implies $\|x(t)\| < \varepsilon$ for all $t \in \mathbf{N}$. Furthermore, it is said to be asymptotically stable if it is stable and $\lim_{t\to\infty} x(t) = 0$ for any initial state $x(0) \in \mathbf{R}^n$.

Definition 4: NCS (1) is robustly stabilizable with degree α if there exists a controller as defined in (6) such that the close-loop system is asymptotically stable for all $g(t, u)$.

Without loss of generality, we assume that 0 is an unique equilibrium of NCS (1), and the state response starts at $t_0 = 0$ with an initial condition $x(0)$. The following results shows us that the asymptotic stability of (7) implies the robustly asymptotic stability of NCS (1).

Lemma 1: Let $x(t)$ be the trajectory of the closed-loop system (1) with (6). If there exists a piecewise continuous function $V : \mathbf{R}^n \to \mathbf{R}_+$ taking its values in a continuous function set $\Omega = \{V_1, V_2, \cdots, V_d\}$ with $V_l(0) = 0$ for all $l \in [1, d]$, and functions α, β, γ of class K such that for all $x \in B_r = \{x : \|x\| \le r\}$,

$$\alpha(\|x\|) \le V_l(x) \le \beta(\|x\|), \ \forall l \in [1, d], \tag{9}$$

and

$$\begin{aligned} \Delta V(x(t_k)) &= V(x(t_{k+1})) - V(x(t_k)) \\ &\le -\gamma(\|x(t_k)\|), \ \forall k \in \mathbf{N}, \end{aligned} \tag{10}$$

then the closed-loop system is robustly asymptotically stable. The proof can be obtained by referring that of Lemma 1 in [7].

Theorem 1: If there exist symmetric positive definite matrices $X_1, X_2, \cdots, X_d$, matrices $Y_1, Y_2, \cdots, Y_d$ and scalars γ, d, such that

$$\begin{bmatrix} -X_j & (A^i X_j + \sum_{l=0}^{i-1} A^l B Y_j)^T & (GY_j)^T \\ A^i X_j + \sum_{l=0}^{i-1} A^l B Y_j & I - X_i & 0 \\ GY_j & 0 & -\gamma \frac{1}{\lambda i^2} I \end{bmatrix} < 0,$$
$$\forall (i, j) \in [1, d] \times [1, d], \tag{11}$$

where $\lambda = \max_{l\in[0,d-1]} \lambda_{max}((A^T)^l A^l)$, then NCS (1) is robustly stabilizable with degree α and largest transmission interval d via the state feedback control law

$$u(t) = Y_{r(k-1)} X_{r(k-1)}^{-1} \bar{x}(t), \ t \in [t_k, t_{k+1}), \ k \in \mathbf{N}.$$

Proof: Based on Lemma 1, we only need to prove that there exists a feedback gain set $\{F_1, F_2, \cdots, F_d\}$ guaranteeing the robustly asymptotical stability of switched system (7) for arbitrary switching.

It is obvious that the switched system (7) can be represented equivalently by

$$z(k+1) = \sum_{j=1}^{d} \sum_{i=1}^{d} \xi_{ij}(k)(\bar{A}_{ij} z(k) + \bar{G}_{ij}(z, z(k))). \tag{12}$$

From (12), we know that there are $d \times d$ subsystems and each of them is denoted by a number pair $(i, j) \in [1, d] \times [1, d]$. More specifically, $\xi_{ij}(k) = 1$ if the (i, j) subsystem is active and $\xi_{ij}(k) = 0$ otherwise. Now, for switched system (12), we adopt the following switched Lyapunov function

$$V(k, z(k)) = z^T(k) \sum_{i=1}^{d} \xi_i(k) P_i z(k), \tag{13}$$

where P_i's are the parameters to be designed.

From Lemma 1, we only need to show that Lyapunov function (13) proves the robustly asymptotical stability of system (12). Notice that

$$\bar{G}_{ij}^T(k, z(k)) \bar{G}_{ij}(k, z(k)) \le \lambda i^2 \alpha^2 z^T(k) F_j^T G^T G F_j z(k).$$

Hence, we have

$$\begin{aligned} \Delta \quad & V(k, z(k)) - [\bar{G}_{ij}^T(k, z(k)) \bar{G}_{ij}(k, z(k)) \\ & -\lambda i^2 \alpha^2 z^T(k) F_j^T G^T G F_j z(k)] \\ = \quad & z^T(k+1) \sum_{i=1}^{d} \xi_i(k+1) P_i z(k+1) \\ & -z^T(k) \sum_{i=1}^{d} \xi_i(k) P_i z(k) \\ & -[\bar{G}_{ij}^T(k, z(k)) \bar{G}_{ij}(k, z(k)) \\ & -\lambda i^2 \alpha^2 z^T(k) F_j^T G^T G F_j z(k)] \\ = \quad & \sum_{j=1}^{d} \sum_{i=1}^{d} \xi_{ij}(k)(\bar{A}_{ij} z(k) + \bar{G}_{ij}(k, z(k))^T \\ & \times \sum_{i=1}^{d} \xi_i(k+1) P_i \sum_{j=1}^{d} \sum_{i=1}^{d} \xi_{ij}(k)(\bar{A}_{ij} z(k) \\ & +\bar{G}_{ij}(k, z(k)) - z^T(k) \sum_{i=1}^{d} \xi_i(k) P_i z(k) \\ & -[\bar{G}_{ij}^T(k, z(k)) \bar{G}_{ij}(k, z(k)) \\ & -\lambda i^2 \alpha^2 z^T(k) F_j^T G^T G F_j z(k)] \\ \le \quad & z^T(k)[-P_j + \lambda i^2 \alpha^2 F_j^T G^T G F_j + \bar{A}_{ij}^T P_i \bar{A}_{ij} \\ & +\bar{A}_{ij}^T P_i (I - P_i)^{-1} P_i \bar{A}_{ij}] z(k) \\ & -[(I - P_i)^{-\frac{1}{2}} P_i \bar{A}_{ij} z(k) - (I - P_i)^{\frac{1}{2}} \bar{G}_{ij}(k, z(k))]^T \\ & \times[(I - P_i)^{-\frac{1}{2}} P_i \bar{A}_{ij} z(k) - (I - P_i)^{\frac{1}{2}} \bar{G}_{ij}(k, z(k))]. \end{aligned}$$

Then robust stability of system (12) is proved by the Lyapunov function (13) if

$$\begin{aligned} & -P_j + \lambda i^2 \alpha^2 F_j^T G^T G F_j + \bar{A}_{ij}^T P_i \bar{A}_{ij} \\ & +\bar{A}_{ij}^T P_i (I - P_i)^{-1} P_i \bar{A}_{ij} < 0. \end{aligned} \tag{14}$$

Notice that

$$(P_i^{-1} - I)^{-1} = P_i + P_i(I - P_i^{-1})P_i,$$

and then (14) is equivalent to

$$-P_j + \lambda i^2\alpha^2 F_j^T G^T G F_j + \bar{A}_{ij}^T (P_i^{-1} - I)^{-1}\bar{A}_{ij} < 0.$$

Furthermore, based on Schur complement lemma, the inequality above holds if and only if the following condition holds

$$\begin{bmatrix} -P_j & \bar{A}_{ij}^T & F_j^T G^T \\ \bar{A}_{ij} & I - P_i^{-1} & 0 \\ GF_j & 0 & \frac{1}{-\lambda i^2\alpha^2} I \end{bmatrix} < 0, \ \forall [i, j] \in [1, d]\times[1, d], \tag{15}$$

By post- and pre-multiplying both sides of (15) using matrix $diag\{P_j^{-1}, I, I\}$, and letting $X_i = P_i^{-1}$, $Y_i = F_i X_i$ and $\gamma = \frac{1}{\alpha^2}$, we can get (11). Thus, this completes the proof. ■

Remark 3: Based on the analysis in Section II, when we apply *Design Algorithm 1* to NCS (1), we can get the state evolution of the close-loop system as follows:

$$x(t+1) = Ax(t) + BF_{r(k-1)}x(t_k) + g(t, F_{r(k-1)}x(t_k)),$$
$$t \in [t_k, t_{k+1}), \ \forall k = 1, 2, \cdots .$$

For the closed-loop system above, we adopt the following packet-loss dependent Lyapunov function

$$V(t) = x^T(t)P_{r(k)}x(t), t \in [t_k, t_{k+1}), \ \forall k = 1, 2, \cdots . \tag{16}$$

Let $r(k) = t_{k+1} - t_k = i$, then (16) is exactly (13). Thus, (13) implies a packet-loss dependent Lyapunov function for NCS (1). Consequently, we say that the switched controllers designed in Theorem 1 are packet-loss dependent.

Remark 4: If nonlinear uncertainties $g(t, u) \equiv 0$, then the inequalities (11) reduce to the following ones.

$$\begin{bmatrix} -X_j & (A^i X_j + \sum_{l=0}^{i-1} A^l BY_j)^T \\ A^i X_j + \sum_{l=0}^{i-1} A^l BY_j & -X_i \end{bmatrix} < 0,$$
$$\forall (i, j) \in [1, d] \times [1, d], \tag{17}$$

which are LMIs with respect to parameters Z_i's and Y_i's, and preserves the stabilization of (1) without uncertainties.

To end this section, we makes some discussions on inequalities (11). Notice that inequalities (11) are bilinear matrix inequalities (BMIs) in the variables $\frac{1}{\lambda i^2}, \gamma$, and i, λ are depend on the maximum transmission internal d. It follows that inequalities (11) depend on the variables d, γ, the X_i's and the Y_i's. If we fix d, then they are LMIs in γ, the X_i's and the Y_i's; If we fix γ, then they are LMIs in d, the X_i's and the Y_i's. Moreover, if we fix d, an upper bound for the robust degree α can be found by solving the following optimization problem

$$\min_{X_i,\, Y_i} \quad \gamma$$
$$\text{subject to (11) with a given } d. \tag{18}$$

If we fix γ, they are convex in d, the X_i's and the Y_i's. Therefore, an upper bound for the transmission interval d which preserves the stabilization of the system can be found by solving the following optimization problem

$$\max_{X_i,\, Y_i} \quad d$$
$$\text{subject to (11) with a given } \gamma. \tag{19}$$

Notice that all the constraints in these two optimal problems are convex, which can be solved by the Matlab LMI Toolbox.

IV. Numerical examples

In this section, a numerical example and some simulations are given to demonstrate the effectiveness of the proposed design technique.

Example 1: Consider the cart and inverted pendulum problem [10], and the linearized state-space model with nonlinear uncertainties is described as

$$x(t+1) = \begin{bmatrix} 1.00 & 0.100 & -0.0166 & -0.0005 \\ 0 & 1.00 & -0.3374 & -0.0166 \\ 0 & 0 & 1.0996 & 0.1033 \\ 0 & 0 & 2.0247 & 1.0996 \end{bmatrix} x(t)$$
$$+ \begin{bmatrix} 0.0045 \\ 0.0896 \\ -0.0068 \\ -0.1377 \end{bmatrix} u(t) + g(t, u(t)), \tag{20}$$
$$u(t) = F_i \bar{x}(t), \forall i \in [1, d],$$

where the state feedback gains $F_i, i \in [1, d]$ are to be designed. We consider the case that the maximum transmission interval $d = 4$ and $G = [0.2\ 0.1\ 0.3\ 0.2]^T$. By solving the matrix inequalities in Theorem 1 using the Matlab LMI Toolbox, we can obtain robust degree $\alpha = 2.5750 \times 10^{-4}$ and the feedback gains:

$$F_1 = \begin{bmatrix} 0.1154 & 0.4178 & 19.8963 & 4.5628 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} 0.1141 & 0.4152 & 19.8756 & 4.5582 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} 0.1111 & 0.4055 & 19.8511 & 4.5517 \end{bmatrix},$$
$$F_4 = \begin{bmatrix} 0.1130 & 0.4112 & 20.0332 & 4.5953 \end{bmatrix}.$$

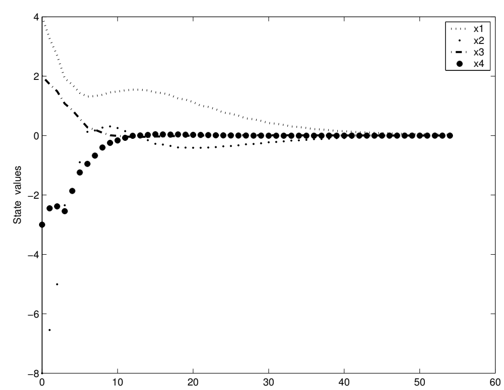

Fig. 3. State response with both packet losses and uncertainties.

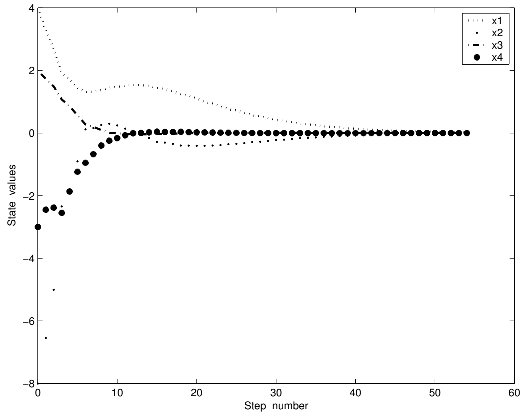

Fig. 4. State response without packet losses.

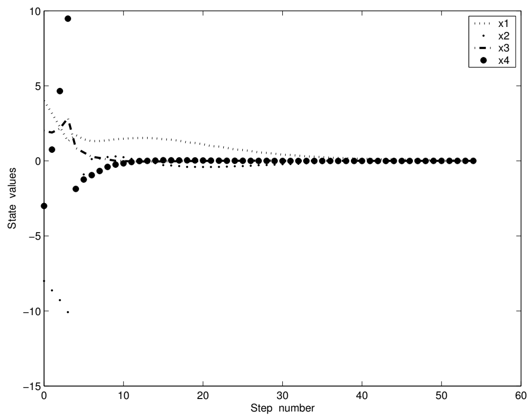

Fig. 5. State response without both packet losses and uncertainties.

For the case that the distribution of transmission interval is $1, 2, 3, 1, 2, 3, 1, 2, 3, \cdots$, the initial state $x_0 = [4\ -8\ 2\ -3]^T$, and uncertain parameter $g(t, u) = \alpha G sin(t)u$, the system state response is shown in Fig. 3. We plot the state responses of the system only with packet losses in Fig. 4, and the system without both the packet losses and the uncertainties in Fig. 5. Comparing the three figures, we can see that the impacts on the stabilization of the packet losses are small when we adopt the switched controllers for system (20).

V. Conclusion

This paper has studied the robust stabilization problem of NCSs with nonlinear uncertainties in the presence of bounded packet losses. The uncertainties NCSs have been modelled as a class of uncertain switched systems by the lifting technique. Furthermore, based on the theories for discrete-time switched systems and the unified LMI approach, we have derived the robustly stabilizing conditions for the state feedback stabilization, and designed the robustly packet-loss dependent controllers by solving some LMIs. The robust degree and the upper bound of the packet loss have been derived by solving some optimal problems. An example and some simulations have been worked out to demonstrate the effectiveness of the proposed design technique.

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