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Stochastic Control of Population Dynamics Using Kalman Filtering with Applications to Artificial Muscle Recruitment

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TABLE I
TERMINOLOGY

Symbol	Meaning
m	The number of finite states each agent has.
$\underline{N}$	A vector describing the number of agents in each state. For example, there are N_i agents in state i.
N^{Total}	The total number of agents in the ensemble being controlled.
$\underline{y}$	A vector of measurements of the system. Each measurement is a weighted sum of the number of agents in each state.
$\underline{y}^{ref}$	A desired measured output for the controlled system.
$\mathbf{A}$	A matrix describing the Markov state transition probabilities broadcast by the central controller. A_{ij} represents the probability of transitioning from state j to state i.
$\mathbf{H}$	A matrix relating the number of agents in each state to the measured output $\underline{y}$, in the form $\underline{y} = \mathbf{H}\underline{N}$.
$\hat{\underline{N}}$	The Kalman filter estimate of the state distribution vector $\underline{N}$.
$\mathbf{Q}$	The covariance of the *a priori* one-step-ahead prediction of $\underline{N}$ conditioned on the present value of $\underline{N}$ and the broadcast command $\mathbf{A}$.
$\mathbf{R}$	The covariance of the measurement $\underline{y}$.
$\mathbf{K}$	The optimal observer gain calculated by the Kalman filter.
$\mathbf{P}$	The estimation covariance calculated by the Kalman filter.
T_{ij}	A discrete random variable describing the number of agents transitioning to state i from state j.

***Abstract*— This paper addresses a problem in distributed control: given a large number of identical hybrid-state agents, control the ensemble behavior of the agents assuming that only limited information is available about the agents' states. This process has relevance to a number of biologically-inspired control problems, such as motor recruitment. In this paper, we describe a stochastic control policy capable of achieving convergent control of the distribution of an ensemble of finite state agents in this way. Using techniques developed for the observation of biological population dynamics, we show that it is possible to observe the state distribution of agents under our control policy using a Kalman filter. Look-ahead control laws based on the Kalman filter estimates are used to achieve a high degree of stability and robustness in systems exhibiting large time delays. An example of control over a hybrid-state, recruitment-like controller for an artificial muscle is presented.**

I. INTRODUCTION

This paper is about a class of control problems for distributed systems that we call recruitment problems, inspired by the example of motor recruitment in nature. Skeletal muscles are an interesting case study in distributed control. They are organized hierarchically into many small subsystems called motor units, connected in parallel between two tendons. To control the amount of force produced by a muscle, the nervous system sends out an excitation that varies in intensity. However, rather than causing all of the motor units to produce a continuous response to this excitation, each unit is "recruited" when the excitation reaches some threshold value and contracts [1]. The active force produced by the muscle is equal to the sum of the forces produced by each motor unit, and consequently is proportional to the number of recruited motor units. This is an example of a natural system that exhibits hybrid-state behavior. The activation dynamics of the motor units can be thought of as the discrete portion of a hybrid-state model for the motor unit. The force-length-velocity relationship of the motor unit in each activation state can be thought of as the continuous-state dynamics.

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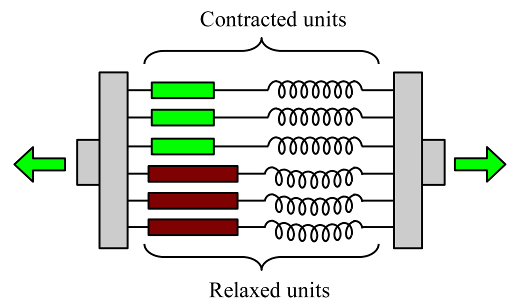

Fig. 1. An SMA actuator designed to function like skeletal muscle. The actuator is made up of many small SMA elements in parallel, which contract to exert force when activated.

One can imagine artificial systems that function in a similar way. The authors have been working specifically on shape memory alloy (SMA) actuators that mimic this control hierarchy in muscle, as shown in Fig. 1. These artificial muscles are composed of many identical SMA motor units. Each motor unit has a small finite state machine that controls whether its SMA element is heated into its austenite state, or cooled into its martensite state. The idea is to intentionally hybridize the dynamics of each unit. To obtain some desired force or displacement, a central controller must recruit some number of units into the activated state. The phase transition dynamics of SMA, which are difficult to control to produce a continuous range of force and displacement, are dominated by the discrete dynamics of the recruitment process, which

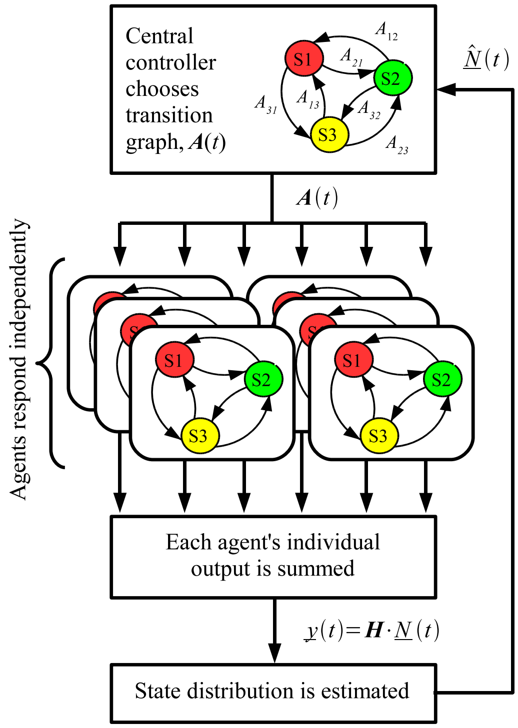

Fig. 2. A schematic diagram of stochastic recruitment control. A central controller chooses a state transition graph based on the observed state distribution. This graph is broadcast to all agents, which respond randomly and independently. Each agent has some small output depending on its state. These outputs are summed to produce an ensemble output.

allow an output resolution equal to the number of motor units [2].

The general problem of recruitment could be posed as the problem of centrally controlling a large number N^{Total} of identical or near-identical m-state agents so the ensemble output of the agents, $\underline{y}$, converges to some desired output $\underline{y}^{ref}$. Each agent generates a discrete output based on its state. The only measurements that the central controller has access to are ensemble outputs, which can be written as a function of $\underline{N}$, the number of agents in each state:

$$\underline{y}(t) = \mathbf{H}\underline{N}(t) \tag{1}$$

The matrix $\mathbf{H}$ defines the outputs of an agent in any state. For example, consider an artificial muscle actuator composed of small parallel mini-actuators controlled by finite state agents. The output of a single agent in an artificial muscle actuator may be the force produced, which takes a value of f_c when contracted and f_r when relaxed. The summed total force could be expressed as a 2×2 matrix $\mathbf{H}$,

$$\begin{bmatrix} \sum F_i(t) \\ \sum N_i(t) \end{bmatrix} = \begin{bmatrix} f_c & f_r \\ 1 & 1 \end{bmatrix} \begin{bmatrix} N_c(t) \\ N_r(t) \end{bmatrix} \tag{2}$$

The other known output of this system is the second row of $\mathbf{H}$, the total number of agents in both states. If the number of agents is fixed and known, this row must always add up to N^{Total}.

A. Stochastic Recruitment

Previously, the authors have shown that one novel and scalable way to control this kind of system is to intentionally randomize the behavior of the finite state agents using a local pseudo-random number generator within each agent. The central controller in this scheme publishes the state transition probability graph with which all agents must respond, as shown in Fig. 2. This broadcast state transition graph could be written as a matrix $\mathbf{A}(t)$,

$$\Pr\{\,state(t+1) = i|\,state(t) = j\} = A_{ij}(t) \tag{3}$$

The number of agents that transition in response to such a command is random, but approaches a central limit as the number of agents becomes large. Intentionally stochastic behavior of this kind is convenient because it does not involve any communication between agents, nor does it require much computation on the part of any agent. This makes it suitable for integration into a micro-fabrication process, such as lithographic production of many motor units in an artificial muscle [3]. We have shown previously that for systems with accurate and low-latency measurement, this kind of control is possible using a variety of feedback policies: linear feedback policies [3], dynamic programming-based optimal policies [4], and one step look-ahead policies [5]. The simplest and most effective control law the authors have found is the one step look-ahead control law, which chooses the state transition matrix $\mathbf{A}(t)$ so that the expected output error one step ahead is zero:

$$\mathbf{A}(t) = \mathbf{A} : \mathrm{E}\{\underline{y}(t+1)|\hat{\underline{N}}(t), \mathbf{A}\} = \underline{y}^{ref} \tag{4}$$

This expectation is conditioned on the estimated state distribution at the present time, $\hat{\underline{N}}(t)$, and the state transition graph represented by $\mathbf{A}$,

$$\mathrm{E}\{\underline{y}(t+1)|\hat{\underline{N}}(t), \mathbf{A}\} = \mathbf{HA}\hat{\underline{N}}(t) \tag{5}$$

The main topic of this paper is providing good estimates of $\hat{\underline{N}}$ for formulating control laws. If the output data obtainable from the system is rich enough, that $\mathbf{H}$ has rank equal to or greater than m, then the estimated state distribution $\hat{\underline{N}}(t)$ can be calculated using an inverse or pseudo-inverse,

$$\hat{\underline{N}}(t) = \mathbf{H}^{-1}\underline{y}(t) \tag{6}$$

In the case described above in (2), a simple system made up of two-state agents having one measured output, $\mathbf{H}$ is full rank and this kind of algebraic estimate can be made. However, it is unlikely in general that this is the case. It is more likely for a higher number of states that a dynamic observer must be used to produce a credible estimate $\hat{\underline{N}}$ for control. This is similar to ecological applications of the Kalman filter to the problem of computing data-driven estimates of fish populations and other animals [6] [7].

This paper outlines the process of approximating the behavior of a system undergoing stochastic recruitment as a linearized Gauss-Markov process, to which a Kalman filter can be applied. First, a probabilistic model for the time evolution of the agents' state distribution and the measured outputs of the ensemble is introduced in Section II. Section III discusses the application of the Kalman filter to this linearized model, and the assumptions about covariance that need to be made in order to produce a conservative estimate of system behavior. Section IV shows how the one step look-ahead policies of the authors' prior work can be extended using the Kalman filter to better compensate for time delays and other difficult transient behaviors, with the specific example of time delays in artificial muscle recruitment. Section V displays computational results evaluating the performance of the observer-based stochastic recruitment control policies.

II. Approximate Models for Recruitment Behavior

The derivation of the Kalman filter is well known, and will not be repeated here [8]. It will suffice to state that the Kalman filter can be used to produce correct Bayesian state estimates of any linear, time-varying system with additive, normally-distributed noise. Specifically we are interested in a system with the following form:

$$\begin{aligned} \underline{x}(t+1) &= \mathbf{F}(t)\underline{x}(t) + \underline{w}(t) \\ y(t) &= \mathbf{H}(t)\underline{x}(t) + \underline{v}(t) \end{aligned} \tag{7}$$

Here $\underline{w}(t)$ and $\underline{v}(t)$ are IID multivariate normal random variables having zero mean. To obtain a model in this form for the dynamics of recruitment, some modeling approximations must be made for both the time evolution and the output of the recruitment process.

A. Approximating Time Evolution

The discrete dynamics of the ensemble of agents can be put into a usable form by assuming that the distribution of agents' state transitions is well-described by the first two moments,

$$\underline{N}(t+1) \approx \mathrm{E}\{N(t+1)\} + \underline{w}(t) \sim MVN(0, \mathbf{Q}(t)) \tag{8}$$

Here $\mathbf{Q}(t)$ is the covariance of $\underline{N}(t+1)$. When (8) is evaluated, the result is a linear time-varying system with Gaussian noise, defined in terms of $\underline{N}(t)$ and $\mathbf{A}(t)$:

$$\underline{N}(t+1) = \mathbf{A}(t)\underline{N}(t) + \underline{w}(t) \sim MVN(0, \mathbf{Q}(t)) \tag{9}$$

This can be shown by calculating $\underline{N}(t+1)$ as a random variable conditioned on the prior state distribution $\underline{N}(t)$ and the prior command $\mathbf{A}(t)$. The one-step-ahead value of $\underline{N}$ is equal to the sum of the number of agents transitioning into each state,

$$N_i(t+1) = \sum_{k=1}^{m} T_{ik}(t) \tag{10}$$

The number of agents transitioning from some state j is distributed multinomially. The probability of any set of transitions away from state j, expressed as a vector $\underline{T}_j(t)$, can be directly evaluated:

$$\Pr\{\underline{T}_j(t) = \underline{X}|\underline{N}(t), \mathbf{A}(t)\} = N_j! \prod_{k=1}^{m} \frac{A_{kj}^{X_k}}{X_k!} \tag{11}$$

The expected value of $T_{ij}(t)$ can be calculated based on this distribution:

$$\mathrm{E}\{T_{ij}(t)|\underline{N}(t), \mathbf{A}(t)\} = N_j(t)A_{ij}(t) \tag{12}$$

The expected future value of $\underline{N}(t+1)$ can consequently be computed using (10),

$$\mathrm{E}\{N_i(t+1)|\underline{N}(t), \mathbf{A}(t)\} = \sum_{j=1}^{m} A_{ij}(t)N_j(t) \tag{13}$$

This implies that the expected future value of $\underline{N}(t+1)$ is equal to $\mathbf{A}(t)$ multiplied by $\underline{N}(t)$, as shown in (9).

B. Time Evolution Covariance

The covariance of transitions $T_{ij}(t)$ and $T_{kj}(t)$ can be calculated using well-known formulas for the variance of a multinomial distribution,

$$\mathrm{Cov}\{T_{ik}(t), T_{jk}(t)|\underline{N}(t), \mathbf{A}(t)\} = N_k(t)A_{ik}(t)(\delta_{ij} - A_{jk}(t)) \tag{14}$$

By summing the covariance between transitions to any two states i and j, the covariance between the total number transitioning into any particular state can be computed,

$$\begin{aligned} \mathrm{Cov}\{N_i(t+1), N_j(t+1), |\underline{N}(t), \mathbf{A}(t)\} &= Q_{ij}(t) \\ &= \sum_{k=1}^{m} \mathrm{Cov}\{T_{ik}(t), T_{jk}(t)|\underline{N}(t), \mathbf{A}(t)\} \\ &= \sum_{k=1}^{m} N_k(t)A_{ik}(t)(\delta_{ij} - A_{jk}(t)) \end{aligned} \tag{15}$$

This covariance matrix $\mathbf{Q}(t)$ can be used to approximate a normally distributed additive "noise" in (9).

C. Approximating Output Variance

In addition to a dynamic model for recruitment, an estimator will need a model for the uncertainty in the measured output of the ensemble of agents, of the form:

$$\underline{y}(t) = \mathbf{H}\underline{N}(t) + \underline{v}(t) \sim MVN(0, \mathbf{R}(t)) \tag{16}$$

This variability could be the result of many different effects. Sensor noise is one plausible source of variability in measurement. More variation may result from individual variability in the output of each agent. The i-th measured output for an agent in state j is equal to the element H_{ij} from the output matrix $\mathbf{H}$. If the output for each agent has some variance r_{ij} due to manufacturing processes, then the i-th measurement y_i will have some corresponding variance,

$$\mathrm{Var}\{y_i\} = \mathbf{R}_{ii} = \sum_{j=1}^{m} N_j r_{ij} \tag{17}$$

Recall that one "output" used to algebraically estimate the state from the output in (6) was the sum of all the agents in each state. This sum, known *a priori*, can also be used in an observer. The variance assigned to this output is zero.

III. Using a Kalman Filter to Predict the State Distribution

Now that a probabilistic state evolution and output model have been formulated, it is possible to construct a Kalman filter estimator to predict $\underline{\hat{N}}(t)$, the number of agents in each state at time t. The update model from (9) and the covariance matrix from (15) can be used as a model for computing the *a priori* expectation $\underline{\hat{N}}(t+1|t)$ and the covariance $\mathbf{P}(t+1|t)$:

$$\begin{aligned} \underline{\hat{N}}(t+1|t) &= A(t)\underline{\hat{N}}(t) \\ \mathbf{P}(t+1|t) &= \mathbf{A}(t)\mathbf{P}(t|t)\mathbf{A}^T(t) + \mathbf{Q}(t) \end{aligned} \tag{18}$$

The output uncertainty model from (16) can be used to compute the observer gain, $\mathbf{K}$:

$$\begin{aligned} &\underline{\tilde{y}}(t+1) = \underline{y}(t+1) - \mathbf{H}\underline{\hat{N}}(t+1|t) \\ &\mathbf{K}(t+1) = \mathbf{P}(t+1|t)\mathbf{H}^T[\mathbf{HP}(t+1|t)\mathbf{H}^T + R(t)]^{-1} \\ &\underline{\hat{N}}(t+1|t+1) = \underline{\hat{N}}(t+1|t) + \mathbf{K}(t)\underline{\tilde{y}}(t+1) \\ &\mathbf{P}(t+1|t+1) = (\mathbf{I} - \mathbf{K}(t+1)\mathbf{H})P(t+1|t) \end{aligned} \tag{19}$$

If the total number of agents is known, the measurement vector $\underline{y}(t)$ will always contain at least one element, corresponding to the total number of agents N^{Total}. This prior knowledge is incorporated into the estimator as an assumed measurement, and serves a very interesting purpose. In a population model having a finite number of agents, the actual output of the system will always be bounded. As a consequence, the covariance of any estimate should also be bounded if the linearized random process model accurately describes the agents' collective behaviors. Without any measurements, one would normally expect a Kalman filter's covariance to grow without bound. However, by assuming that the summed number of agents in all states is constant and representing this knowledge as a measurement, the Kalman filter covariance is much more likely to remain bounded.

A. Conservative Covariance Estimates

The Kalman filter just derived has one moderate implementation difficulty. The covariance of the dynamic model, derived in (15), is a function of the present state, $\underline{N}(t)$. This information is not available to a Kalman filter. This difficulty could be addressed either by assuming that $\underline{\hat{N}}$ is a reasonable surrogate for $\underline{N}$, or by attempting to find an upper bound on covariance. Substitution yields the following result:

$$Q_{ij}(t) \approx \sum_{k=1}^{m} \hat{N}_k(t)A_{ik}(t)(\delta_{ij} - A_{jk}(t)) \tag{20}$$

This method may work, but it is dissatisfying on a number of levels. The filter is not guaranteed to have a good initial value of $\underline{\hat{N}}(t)$, so the covariance produced in transient state may bear little resemblance to the true covariance matrix. The alternative would be to seek to bound the covariance. The trace of the covariance matrix predicted in (15) strictly grows in magnitude with the assumed number of agents in each state. One solidly conservative estimate would be to assume that the estimated covariance is equal to a strict upper bound. Such an estimate could be produced by substituting N^{Total} for the number of agents in all states,

$$Q_{ij}(t) = N^{Total} \sum_{k=1}^{m} A_{ki}(t)(\delta_{ij} - A_{ki}(t)) \tag{21}$$

This method will significantly overestimate the estimation covariance, but this is not necessarily a problem. It simply means that the filter will rely less heavily on the dynamic system model than it theoretically could.

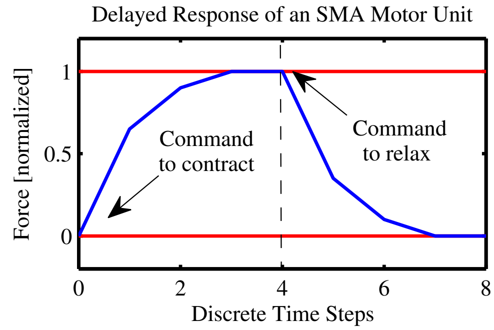

Fig. 3. An example of unmodeled continuous dynamics. A controller that assumes that each motor unit has two states will not account for the physical delay associated with activating the SMA.

IV. K Step Look-Ahead Control

We will now demonstrate how state estimation can be used to solve higher-order, complex recruitment problems, using the example of a shape memory alloy actuator. One cardinal assumption made in recruitment-based control of hybrid-state agents is that the output due to any continuous-state dynamics can be predicted by looking only at the value of the discrete states, $\underline{\hat{N}}(t)$, using an output matrix $\mathbf{H}$. This may not be true if the continuous state behavior of some system includes large time delays or long settling times. For example, a SMA actuator may produce as an output a summed force from each motor unit, which takes one of two states, state R (relaxed) and state C (contracted). The predicted output $\underline{y}(t)$ consists of the summed force and the total number of agents,

$$\begin{bmatrix} \sum F_i(t) \\ \sum N_i(t) \end{bmatrix} = \mathbf{H}\underline{N}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \underline{N}(t) \tag{22}$$

The state transition matrix $\mathbf{A}(t)$ is a function of a probability of contracting, A_{RC}, and a probability of relaxing, A_{CR}:

$$\mathbf{A}(t) = \begin{bmatrix} 1 - A_{CR}(t) & A_{RC}(t) \\ A_{CR}(t) & 1 - A_{RC}(t) \end{bmatrix} \tag{23}$$

This model assumes that the force produced by an SMA unit is 0 in the relaxed state and 1 in the contracted state. However, once the motor unit begins to contract, the phase change in the SMA associated with this contraction will take some time, as shown in Fig. 3. As a result, the output may change in time although the discrete state has not changed. Systems with this kind of behavior may need to be augmented so that the discrete-state model better reflects

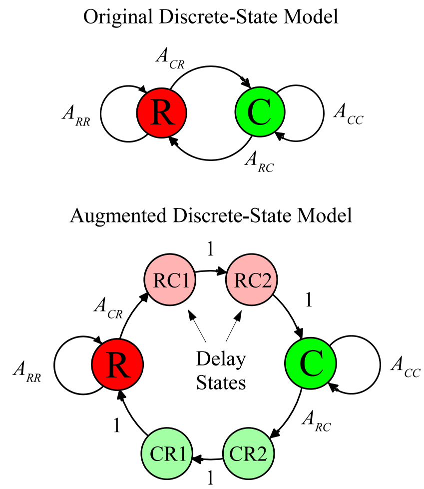

Fig. 4. One way of addressing unmodeled dynamics is to introduce additional states into each agent that can be used to produce a more fine-grained model of the agent's continuous-state behavior. This has the effect of reducing the rank of the output matrix relative to the number of states, m.

the continuous-state dynamics. Figure 4 shows a model of the two-state recruitment dynamics for an SMA motor unit, and an augmented state machine that introduces refractory delays into the state transition dynamics. The augmented output can assign partial output values based on the rate of phase transition. For the 6-state augmented model, the output could be defined based on the measured output of a single agent:

$$\begin{bmatrix} \sum F_i(t) \\ \sum N_i(t) \end{bmatrix} = \begin{bmatrix} 0 & 0.65 & 0.9 & 1 & 0.35 & 0.1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \underline{N}(t) \tag{24}$$

$$\mathbf{A}(t) = \begin{bmatrix} 1-A_{RC}(t) & 0 & 0 & 0 & 0 & 1 \\ A_{RC}(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1-A_{CR}(t) & 0 & 0 \\ 0 & 0 & 0 & A_{CR}(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{25}$$

This state evolution and output model will predict the continuous-state behavior of the SMA with much greater accuracy. However, it also increases the number of finite states, m, in each agent, so that $\text{rank}(\mathbf{H}) < m$. A Kalman filter will be needed to estimate the state distribution of the agents. Also, it is now impossible to specify all of the state transition probabilities between all states, because the delay states are constrained. This means that even if the central controller commands all state transitions to cease, agents in delay states will keep transitioning for two more time steps. It is not enough in this case to choose $\mathbf{A}(t)$ such that the one-step-ahead error is zero as in (4); it is necessary to set the expected error to zero *after the system has settled, when a command to stop,* $\mathbf{A}_0$*, is given.* For an augmented model of the kind shown in Fig. 4, this means that the time horizon for the look-ahead prediction must be extended to $K-1$ steps, where $K-1$ is the number of time steps it takes all agents' outputs to settle to a steady state value. Using (9), this K step look-ahead predictor can be written for the estimated state distribution $\hat{\underline{N}}(t+K)$,

$$\mathrm{E}\{\hat{\underline{N}}(t+K)|\hat{\underline{N}}(t), \mathbf{A}(t)\} = \mathbf{A}_0^{K-1}\mathbf{A}(t)\hat{\underline{N}}(t) \tag{26}$$

Using (26) and (1), the expected output K steps ahead can be predicted,

$$\mathrm{E}\{\underline{y}(t+K)|\hat{\underline{N}}(t), \mathbf{A}(t)\} = \mathbf{H}\mathbf{A}_0^{K-1}\mathbf{A}(t)\hat{\underline{N}}(t) \tag{27}$$

Notice that this can be written in the form similar to (5) with a different output matrix, $\mathbf{H}^*$, which is equal to $\mathbf{H}\mathbf{A}_0^{K-1}$:

$$\mathrm{E}\{\underline{y}(t+K)-\underline{y}^{ref}|\hat{\underline{N}}(t), \mathbf{A}(t)\} = \mathbf{H}^*\mathbf{A}(t)\hat{\underline{N}}(t)-\underline{y}^{ref} \tag{28}$$

The output matrix $\mathbf{H}^*$ can be calculated offline. For example, the augmentation proposed in the earlier problem of a delayed SMA was to add two delay states, as depicted in Fig. 4. If all active transitions cease ($\mathbf{A}_{RC} = \mathbf{A}_{CR} = 0$ in $\mathbf{A}_0$), the agents will cease transitioning after 2 time steps:

$$\mathbf{H}^* = \mathbf{H}\mathbf{A}_0^2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{29}$$

The value obtained for $\mathbf{H}^*$ makes sense. The controller assumes that all agents in the intermediate states will eventually converge to the relaxed or the contracted state. Thus, all agents in these intermediate states are assigned the output of the relaxed or the contracted state, based on their eventual destination.

V. Computational Performance Comparison

To demonstrate the effect that the Kalman filter-based, K step look-ahead control policy has on performance and stability, the time-delayed SMA model was implemented, having a true output response equal to the $\mathbf{H}$ matrix from (24). Two control policies were compared. The first policy utilized the two state model whose output is defined in (22). This model estimated $\hat{\underline{N}}$ by inverting the 2×2 output matrix as in (6), then chose $\mathbf{A}(t)$ based on (4).

The second policy utilized the six state augmented discrete model, whose output is defined in (24). Under this model, the state estimates are not algebraically computable from the output, so a Kalman filter was used. These two models were simulated with $N^{Total} = 50$ agents, starting from identical initial conditions, with all agents in the relaxed state. The initial conditions on the estimator were set to assume that all agents were relaxed. No simulated noise was added to the feedback measurements. Both control systems were commanded to hold the output force at 30.

A. Results

Figure 5 shows one simulated response of the two state model without the Kalman filter. The long settling time and the overshoot are symptomatic of the delayed response. The controller keeps commanding agents to contract in response to the error. It has no basis for believing that enough agents are already responding to earlier commands. In contrast, Fig. 6 shows a simulated response of the six state augmented model with a Kalman filter. Notice the relatively small overshoot and the fast settling time. The agents still converge on the correct output over the course of several time steps. However, because the desired output is calculated based on (29), the controller does not command additional agents to transition, because the filter keeps track of the agents currently undergoing the transition.

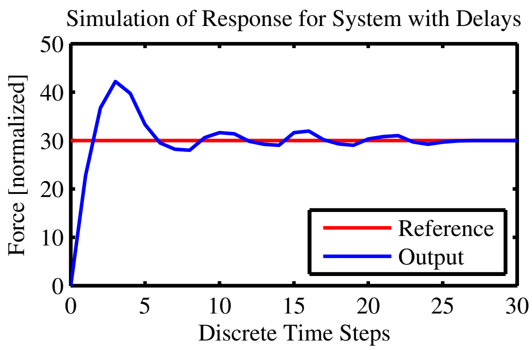

Fig. 5. The two-state model fails to correctly estimate the state of the agents controlling each SMA element. The result is overshoot and long settling times.

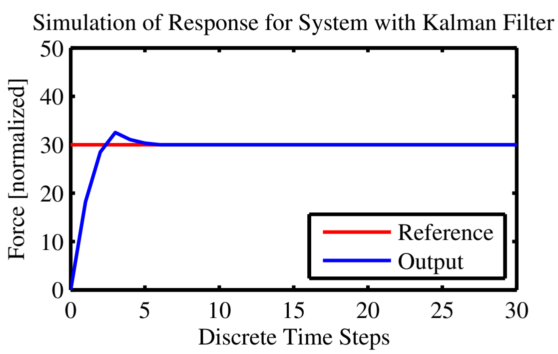

Fig. 6. The six-state model with a Kalman filter has comparatively little overshoot.

Ten thousand simulations of both control policies were run to determine the distribution of the settling times. Settling time was defined empirically as the time at which three consecutive output measurements were recorded within 2 percent of the desired output. A histogram of the results, plotted in Fig. 7 highlights the improvement made by the introduction of the augmented model and the Kalman filter. The peak of the settling time distribution is at a lower value for the augmented model, and, perhaps more importantly, the tail of the distribution approaches zero at a greater rate.

VI. Conclusions

In the future, biologically-inspired control architectures, made up of many individual agents acting according to simple rules, will be increasingly important. Stochastic recruitment control provides a simple framework for obtaining highly convergent ensemble behaviors from many small agents that do not need to perform complex calculations or communicate with each other. This paper has shown

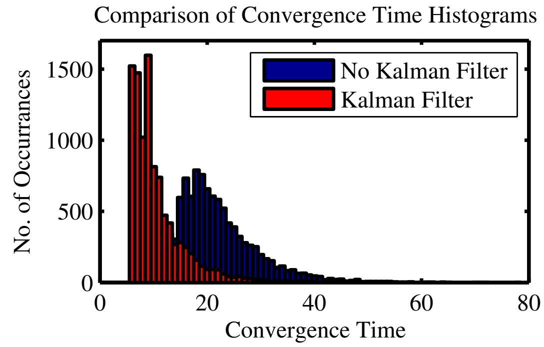

Fig. 7. A histogram of convergence times for many simulated experiments shows the advantage of control based on dynamic estimation to compensate for transient system behaviors.

that many of the informational problems associated with centrally estimating state distributions can be overcome with augmented models and probabilistic estimators. The techniques described in this paper could be used to control the ensemble dynamics of relatively complex, high-order hybrid-state agents.

VII. Future Work

Work is underway to apply these control techniques to a biologically-inspired variable impedance actuator. The authors are also collaborating with biological engineers on the problem controlling the ensemble migration behavior of cells *in vitro*, using a similar observation and broadcast control framework.

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