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Trajectory Tracking based on Differential Neural Networks for a Class of Nonlinear Systems

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***Abstract*—A very successful scheme to accomplish trajectory tracking of unknown nonlinear systems consists of identifying the unknown dynamics using differential neural networks and on the basis of the so obtained mathematical model to develop an appropriate control law. The purpose of this paper is to present some new results in this sense. In particular, for the neural identifier, a new online learning law which permits to guarantee the boundedness for both the weights and the identification error without using a dead zone function is showed. Likewise, based on this neural identifier, a new control law to guarantee the boundedness of the tracking error is developed. These results are proved using a Lyapunov like analysis. With respect to the approach based on the local optimal control theory, the new approach has a similar performance but its main advantage consists of simplifying considerably the design process. The workability of the suggested approach is illustrated by simulation.**

I. Introduction

TRAJECTORY tracking is an issue of great importance in many applications of the automatic control. Basically, the trajectory tracking problem consists of forcing the states of a given plant in order that they follow a reference trajectory, by applying appropriate control actions to the plant input. An interesting case of this problem it occurs when the physical model of the plant is not available. Due to its practical implications, this case has received increasing attention during the last two decades and different strategies of solution have been proposed. One of these consists of accomplishing the control action based on a mathematical model provided by the online identification of the plant. From this perspective, a key question that should be answered is how to select the proper structure to achieve this identification. Besides, considering the lack of the physical model, it would be very convenient that this structure be general enough. In such cases, due to its properties as a universal approximator, an artificial neural network (ANN) could be a good option.

An ANN is a well posed mathematical model with capabilities of "learning" and which is inspired by biological nervous systems. Roughly speaking, ANNs can be classified as static ones, using the, so called, back-propagation technique [1] or as recurrent ones [2]. In the first kind of networks, a system dynamics is approximated by a static mapping; therefore, the network outputs are uniquely determined by the current inputs and weights. These networks have two major disadvantages: a slow learning rate (which can be inadequate for online training) and a high sensitivity of the function approximation to the training data. On the other hand, the second approach incorporates feedback in its structure. Thus, recurrent neural networks overcome many problems associated with first ones such as global extrema search and consequently they have better approximation properties. Depending on their structure, recurrent neural networks can be classified as (discrete-time) difference ones or (continuous-time) differential ones. In general, recurrent ANNs have demonstrated a good performance in applications for automatic control. In this sense, one of the first works for the continuous-time case was accomplished by Polycarpou and Ioannou. Effectively, in [3] it was employed Lyapunov stability theory to develop stable adaptive laws for identification and control of dynamic systems with unknown nonlinearities. However, the results of this work were restricted to SISO feedback linearizable systems. Afterward, in [4], Rovithakis extended these results to multi input systems simplifying, in addition, the assumptions about the unknown plant. Nonetheless, this work was only applicable to input affine nonlinear systems. On the other hand, in [5] and [6], a more general class of uncertain systems was considered. In that work, the control goal is to force the system states to track a prespecified trajectory. Starting from certain assumptions, such as the existence of a unique solution for a matrix Riccati equation, among others, a learning law with a dead zone function is designed by Lyapunov-like analysis to guarantee the boundedness of both the identification error and the weights of the neural network. Likewise, on the basis of this dynamic neural identifier, a controller is developed using the local optimal control theory. In spite of the effectiveness of this identifier-controller scheme, its main drawbacks are: 1) In order to improve the identification quality, it is necessary to reduce the size of the dead zone. This can be obtained by increasing a certain parameter associated to the Riccati equation. However, this parameter can only be increased up to certain level, beyond which no solution exists to the Riccati equation. 2) The implementation of the suggested control law requires solving another Riccati equation. Consequently, the design process could be awkward. Thereby, in this paper, we present a new learning law by which is possible to guarantee the boundedness of the

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weights of the neural network without using the dead zone function. Likewise, we propose a new control law which has a performance similar to the law based on the optimal control theory but without the need of solving another Riccati equation. Therefore, these results simplify considerably the design process. The workability of suggested approach is illustrated by a simulation example in which a Lorenz model is controlled.

II. Differential Neural Network

A. Uncertain Dynamics and Basic Assumptions

Since no physical model is available, at least certain assumptions about the unknown system must be accomplished. Consider that the uncertain measurable dynamics of this system can be described, in general, as

$$\dot{x}_t = f(x_t, u_t, t) \tag{1}$$

where $x_t \in \Re^n$ is the system state vector at time $t \in \Re^+ := \{t : t \geq 0\}$, $u_t \in \Re^n$ is a given control action, and $f : \Re^n \times \Re^n \times \Re^+ \to \Re^n$ is an unknown nonlinear vector function. It is important to notice that (1) can always be represented alternatively by a known term plus an unknown term in the following way:

$$\dot{x}_t = Ax_t + W_1^* \sigma(x_t) + W_2^* \phi(x_t) u_t + \Delta f(x_t, u_t, t) \tag{2}$$

where $A \in \Re^{n\times n}$ is a Hurwitz matrix, $W_1^* \in \Re^{n\times m}$ and $W_2^* \in \Re^{n\times n}$ are constant matrices, $\sigma(\cdot) : \Re^n \to \Re^m$ is the activation vector-function with sigmoidal components, that is, $\sigma(\cdot) := [\sigma_1(\cdot), \ldots, \sigma_m(\cdot)]^{\mathsf{T}}$

$$\sigma_j(z) := \frac{a_{\sigma j}}{1 + \exp\left(-\sum_{j=1}^{n} c_{\sigma j} z_j\right)} \quad \text{for} \quad j = 1, \ldots, m$$

$\phi(\cdot) : \Re^n \to \Re^n$ is a diagonal matrix function with sigmoidal components, that is, $\phi(\cdot) := \operatorname{diag}(\phi_1(\cdot), \ldots, \phi_n(\cdot))$

$$\phi_j(w) := \frac{a_{\phi j}}{1 + \exp\left(-\sum_{j=1}^{n} c_{\phi j} w_j\right)} \quad \text{for} \quad j = 1, \ldots, n$$

and $\Delta f : \Re^n \times \Re^n \times \Re^+ \to \Re^n$ is an error term which may include both unmodelled dynamics and deterministic disturbances. $\Delta f(x_t, u_t, t)$ is simply defined as

$$\Delta f(x_t, u_t, t) := f(x_t, u_t, t) - Ax_t - W_1^* \sigma(x_t) - W_2^* \phi(x_t) u_t$$

Hereafter we consider that the following assumptions are complied:

A.1) System (1) satisfies the (uniform on t) Lipschitz condition, that is,

$$\begin{gathered} \| f(x,u,t) - f(z,v,t) \| \leq L_1 \| x - z \| + L_2 \| u - v \| \\ x, z \in \Re^n; \; u, v \in \Re^n; \; 0 \leq L_1, L_2 < \infty \end{gathered} \tag{3}$$

A.2) The function $\sigma(\cdot)$ and $\phi(\cdot)$ satisfy sector conditions [8]:

$$\tilde{\sigma}_t^T \Lambda_\sigma \tilde{\sigma}_t \leq \Delta_t^T D_\sigma \Delta_t,$$

$$u_t^T \tilde{\phi}_t^T \Lambda_\phi \tilde{\phi}_t u_t \leq \Delta_t^T D_\phi \Delta_t \| u_t \|^2$$

where

$$\Delta_t := \hat{x}_t - x_t \tag{4}$$

$$\begin{aligned} \tilde{\sigma}_t &:= \sigma(\hat{x}_t) - \sigma(x_t) \\ \tilde{\phi}_t &:= \phi(\hat{x}_t) - \phi(x_t) \end{aligned} \tag{5}$$

and $\Lambda_\sigma \in \Re^{m\times m}$, $D_\sigma \in \Re^{n\times n}$, $\Lambda_\phi \in \Re^{n\times n}$, $D_\phi \in \Re^{n\times n}$ are known constant positive definite matrices.

A.3) Admissible controls are bounded, to be precise, $U^{adm} := \{u_t : \|u_t\|^2 \leq \bar{u} < \infty\}$. Besides, u_t does not violate the existence of the solution to ODE (1).

A.4) Error term is bounded by

$$\|\Delta f(x_t, u_t, t)\|_{\Lambda_f}^2 \leq \bar{\eta}$$

where $\Lambda_f \in \Re^{n\times n}$ is a constant positive definite matrix.

A.5) The matrices W_1^* and W_2^* are bounded in the following sense

$$W_1^* \Lambda_\sigma^{-1} W_1^{*T} \leq \overline{W}_1$$

$$W_2^* \Lambda_\phi^{-1} W_2^{*T} \leq \overline{W}_2$$

where $\overline{W}_1 \in \Re^{n\times n}$, $\overline{W}_2 \in \Re^{n\times n}$ are known positive definite matrices.

A.6) There exits a strictly positive definite matrix Q_0 such that if the matrices R and Q are defined as

$$\begin{aligned} R &:= \overline{W}_1 + \overline{W}_2 + \Lambda_f^{-1} \\ Q &:= D_\sigma + \bar{u} D_\phi + Q_0 \end{aligned} \tag{6}$$

then the following matrix Riccati equation

$$A^T P + PA + PRP + Q = 0 \tag{7}$$

has a positive solution $P^T = P > 0$ (in [6] there are given conditions for matrices A, R and Q which guarantees the existence of $P>0$).

It is worth mentioning that *the preceding assumptions are not unrealistic*. On the contrary, they are generally met for physically meaningful dynamic systems.

B. New Differential Learning Law

Consider the neural identifier with the following structure

$$\frac{d}{dt} \hat{x}_t = A\hat{x}_t + W_{1,t} \sigma(\hat{x}_t) + W_{2,t} \phi(\hat{x}_t) u_t \tag{8}$$

where $\hat{x}_t \in \Re^n$ is the state of the neural network and $W_{1,t}$ and $W_{2,t}$ are the weight matrices. This neural network can be classified as generalized Hopfield-type one [9]. In particular, to adjust online the weights of (8) and reduce the identification error $\Delta_t := \hat{x}_t - x_t$, we propose the following learning law:

$$\dot{W}_{1,t} = -K_1 P \Delta_t \sigma(\hat{x}_t)^T - \frac{\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right)}{2} \widetilde{W}_{1,t}$$
$$\dot{W}_{2,t} = -K_2 P \Delta_t u_t{}^T \phi(\hat{x}_t)^T - \frac{\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right)}{2} \widetilde{W}_{2,t} \quad (9)$$

where K_1, K_2 are positive definite matrices which are selected by the designer, P is the solution of matrix Riccati equation given by (7) and

$$\begin{aligned} \widetilde{W}_{1,t} &:= W_{1,t} - W_1^* \\ \widetilde{W}_{2,t} &:= W_{2,t} - W_2^* \end{aligned} \quad (10)$$

Next, the basic result on the identification process of unknown measurable dynamics (1) by the neural network (8) is formulated:

Theorem 1: If the assumptions **A.1- A.6** are satisfied and the weight matrices $W_{1,t}$ and $W_{2,t}$ of the neural network (8) are adjusted by the differential learning law (9) then

a) both the identification error and the weights are bounded:

$$\Delta_t, W_{1,t}, W_{2,t} \in L_\infty \quad (11)$$

b) the identification error has the following upper bound:

$$\limsup_{t\to\infty} \Delta_t^T P \Delta_t \le \frac{\overline{\eta}}{\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right)} \quad (12)$$

Proof: Before beginning the analysis, we determine the dynamics of the identification error. From (4), the first derivative of Δ_t is

$$\dot{\Delta}_t = \dot{\hat{x}}_t - \dot{x} \quad (13)$$

substituting (8) and (2) into (13) yields

$$\dot{\Delta}_t = A\Delta_t + W_{1,t}\sigma(\hat{x}_t) - W_1^*\sigma(x_t) + W_{2,t}\phi(\hat{x}_t)u_t - W_2^*\phi(x_t)u_t - \Delta f \quad (14)$$

adding and subtracting the terms $W_1^*\sigma(\hat{x}_t)$ and $W_2^*\phi(\hat{x}_t)u_t$ and taking into account equations (5) and (10), (14) can be expressed as

$$\dot{\Delta}_t = A\Delta_t + \widetilde{W}_{1,t}\sigma(\hat{x}_t) + W_1^*\tilde{\sigma}_t + \widetilde{W}_{2,t}\phi(\hat{x}_t)u_t + W_2^*\tilde{\phi}_t u_t - \Delta f \quad (15)$$

To begin the analysis, we select the following non-negative function

$$V_t := \Delta_t^T P \Delta_t + tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] + tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] \quad (16)$$

where P is the positive solution for the matrix Riccati equation given by (7). The first derivative of V_t is

$$\dot{V}_t = \frac{d}{dt}\left(\Delta_t^T P \Delta_t\right) + \frac{d}{dt} tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] + \frac{d}{dt} tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] \quad (17)$$

Each term of (17) will be calculated separately. For $\frac{d}{dt}\left(\Delta_t^T P \Delta_t\right)$, we have

$$\frac{d}{dt}\Delta_t^T P \Delta_t = 2\Delta_t^T P \dot{\Delta}_t \quad (18)$$

substituting (15) into (18) yields

$$\begin{aligned} \frac{d}{dt}\Delta_t^T P \Delta_t = {} & 2\Delta_t^T P A \Delta_t + 2\Delta_t^T P \widetilde{W}_{1,t}\sigma(\hat{x}_t) \\ & + 2\Delta_t^T P W_1^* \tilde{\sigma}_t + 2\Delta_t^T P \widetilde{W}_{2,t}\phi(\hat{x}_t)u_t \\ & + 2\Delta_t^T P W_2^* \tilde{\phi}_t u_t - 2\Delta_t^T P \Delta_f \end{aligned} \quad (19)$$

On the other hand, for $\frac{d}{dt} tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right]$, using several properties of the trace of a matrix, we obtain

$$\begin{aligned} \frac{d}{dt} tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] &= tr\left[\frac{d}{dt}\left(\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right)\right] \\ &= tr\left[\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t} + \widetilde{W}_{1,t}^T K_1^{-1} \dot{\widetilde{W}}_{1,t}\right] \\ &= tr\left[\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] + tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \dot{\widetilde{W}}_{1,t}\right] \\ &= 2tr\left[\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \end{aligned} \quad (20)$$

if (10) is differentiated then

$$\dot{\widetilde{W}}_{1,t} = \dot{W}_{1,t}$$

but $\dot{W}_{1,t}$ is given by the learning law (9). Thus, substituting (9) into the last term of (20), $\frac{d}{dt} tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right]$ can be calculated as

$$\begin{aligned} \frac{d}{dt} tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] &= -2tr\left[\sigma(\hat{x}_t)\Delta_t^T P K_1 K_1^{-1} \widetilde{W}_{1,t}\right] \\ & \quad - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \\ &= -2\Delta_t^T P \widetilde{W}_{1,t}\sigma(\hat{x}_t) \\ & \quad - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \end{aligned} \quad (21)$$

proceeding in a similar way for $tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right]$

$$\begin{aligned} \frac{d}{dt} tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] &= -2\Delta_t^T P \widetilde{W}_{2,t}\phi(\hat{x}_t)u_t \\ & \quad - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] \end{aligned} \quad (22)$$

finally, substituting (19), (21), and (22) into equation (17), $\dot{V}_t$ can be expressed as

$$\begin{aligned} \dot{V}_t = {} & 2\Delta_t^T P A \Delta_t + 2\Delta_t^T P W_1^* \tilde{\sigma}_t \\ & + 2\Delta_t^T P W_2^* \tilde{\phi}_t u_t - 2\Delta_t^T P \Delta_f \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] \end{aligned} \quad (23)$$

now, we need to find an upper bound for $\dot{V}_t$. To accomplish this task, first we consider the term $2\Delta_t^T P W_1^* \tilde{\sigma}_t$. Since this term is an scalar, it is possible to express it alternatively as

$$2\Delta_t^T P W_1^* \tilde{\sigma}_t = \Delta_t^T P W_1^* \tilde{\sigma}_t + \tilde{\sigma}_t^T W_1^{*T} P \Delta_t$$

using the matrix inequality proved in [6]

$$X^T Y + Y^T X \le X^T \Gamma^{-1} X + Y^T \Gamma Y \qquad (24)$$

which is valid for any $X, Y \in \Re^{n\times k}$ and for any positive definite matrix $0 < \Gamma = \Gamma^T \in \Re^{n\times n}$, $2\Delta_t^T P W_1^* \tilde{\sigma}_t$ can be bounded by

$$2\Delta_t^T P W_1^* \tilde{\sigma} \le \Delta_t^T P W_1^* \Lambda_\sigma^{-1} W_1^{*T} P \Delta_t + \tilde{\sigma}_t^T \Lambda_\sigma \tilde{\sigma}_t$$

but, from the assumptions A.2 and A.5, we can conclude

$$2\Delta_t^T P W_1^* \tilde{\sigma}_t \le \Delta_t^T P \overline{W}_1 P \Delta_t + \Delta_t^T D_\sigma \Delta_t \qquad (25)$$

likewise, using the inequality (24) in $2\Delta_t^T P W_2^* \tilde{\phi}_t u_t$, we have

$$\begin{aligned} 2\Delta_t^T P W_2^* \tilde{\phi}_t u_t &= \Delta_t^T P W_2^* \tilde{\phi}_t u_t + u_t^T \tilde{\phi}_t^T W_2^{*T} P \Delta_t \\ &\le \Delta_t^T P W_2^* \Lambda_\phi^{-1} W_2^{*T} P \Delta_t + u_t^T \tilde{\phi}_t^T \Lambda_\phi \tilde{\phi}_t u_t \\ &\le \Delta_t^T P \overline{W}_2 P \Delta_t + \overline{u} \Delta_t^T D_\phi \Delta_t \qquad (26) \end{aligned}$$

This last inequality is concluded by the assumptions A.2, A.3, and A.5. On the other hand, the following inequality is a corollary from (24):

$$-Z^T Y - Y^T Z \le Z^T \Gamma^{-1} Z + Y^T \Gamma Y$$

which is valid for any $Z, Y \in \Re^{n\times k}$ and for any positive definite matrix $0 < \Gamma = \Gamma^T \in \Re^{n\times n}$. Using this result to bound $-2\Delta_t^T P \Delta f$, we find that

$$-2\Delta_t^T P \Delta_f \le \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \Delta_f^T \Lambda_f \Delta_f$$

but, in accordance with the assumption A.4

$$-2\Delta_t^T P \Delta_f \le \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \overline{\eta} \qquad (27)$$

substituting (25), (26), and (27) into (23), the following bound for $\dot{V}_t$ can be determined

$$\begin{aligned} \dot{V}_t \le\; & 2\Delta_t^T P A \Delta_t + \Delta_t^T P \overline{W}_1 P \Delta_t + \Delta_t^T D_\sigma \Delta_t \\ & + \Delta_t^T P \overline{W}_2 P \Delta_t + \overline{u} \Delta_t^T D_\phi \Delta_t \\ & + \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \overline{\eta} \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] \end{aligned}$$

Adding and subtracting $\Delta_t^T Q_0 \Delta_t$ into the right-hand side of the last inequality, the expression $A^T P + PA + P\left(\overline{W}_1 + \overline{W}_2 + \Lambda_f^{-1}\right)P + D_\sigma + D_\phi \overline{u} + Q_0$ is formed. However, this expression in accordance with the assumption A.6 is equal to zero. Then

$$\begin{aligned} \dot{V}_t \le\; & -\Delta_t^T Q_0 \Delta_t + \overline{\eta} \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] \qquad (28) \end{aligned}$$

Now, we consider that

$$\Delta_t^T Q_0 \Delta_t = \Delta_t^T P^{\frac{1}{2}} \left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) P^{\frac{1}{2}} \Delta_t$$

using Rayleigh inequality [7] , we obtain

$$\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) \Delta_t^T P \Delta_t \le \Delta_t^T Q_0 \Delta_t$$

or else

$$-\Delta_t^T Q_0 \Delta_t \le -\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) \Delta_t^T P \Delta_t$$

consequently,

$$\begin{aligned} \dot{V}_t \le\; & -\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) \Delta_t^T P \Delta_t \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] \\ & - \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) tr\left[\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right] + \overline{\eta} \end{aligned}$$

from the definition (16) for the non-negative function V_t, finally $\dot{V}_t$ can be bounded as

$$\dot{V}_t \le -\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right) V_t + \overline{\eta}$$

which implies that

$$V_t \le V_0 \exp(-\xi t) + \frac{\overline{\eta}}{\xi}\left(1 - \exp(-\xi t)\right) \qquad (29)$$

where $\xi = \lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right)$. Since V_t is an upperly bounded non-negative function then $\Delta_t, W_{1,t}, W_{2,t} \in L_\infty$ and the first part of the theorem 1 has been proved. On the other hand, from definition of V_t (16) is evident that

$$\Delta_t^T P \Delta_t \le V_t$$

but from (29)

$$\Delta_t^T P \Delta_t \le V_0 \exp(-\xi t) + \frac{\overline{\eta}}{\xi}\left(1 - \exp(-\xi t)\right)$$

finally, taking $\limsup_{t\to\infty}$ for both sides of the last inequality, we can conclude that

$$\limsup_{t\to\infty} \Delta_t^T P \Delta_t \le \frac{\overline{\eta}}{\lambda_{\min}\left(P^{-\frac{1}{2}} Q_0 P^{-\frac{1}{2}}\right)}$$

and the last part of the theorem 1 has been proved. ■

III. Trajectory Tracking based on the Neural Identifier

In this section, on the basis of the neural network identifier (8), we will design a controller to force the nonlinear system (1) to track a reference trajectory

$$x_t^* \in \Re^n$$

which is assumed to be smooth enough. This trajectory is regarded as a solution of a nonlinear reference model given by

$$\dot{x}_t^* = \varphi(x_t^*, t) \tag{30}$$

with a known fixed initial condition. In other words, we would like to *synchronize* the dynamics (1) with a given reference dynamics given by (30). If the trajectory has points of discontinuity in some fixed moments, we can use any approximating trajectory which is smooth. In the case of regulation problem, we have

$$\varphi(x_t^*, t) = 0, \quad x^*(0) = k$$

where *k* is a known constant vector.

Theorem 2: If the following control law is used

$$u_t = \left[W_{2,t}\phi(x_t)\right]^{-1}\left\{\dot{x}_t^* - Ax_t - W_{1,t}\sigma(x_t) - C(x_t - x_t^*)\right\} \tag{31}$$

where C is a positive definite matrix which is selected by the designer such that $\lambda_{\min}(C) > \frac{\lambda_{\max}(\Lambda_f^{-1})}{2}$, then the tracking error which is defined as $e_t := x_t - x_t^*$ has the following upper bound:

$$\|e_t\| \le \sqrt{\|e_0\|^2 + \frac{\overline{\eta}}{2\lambda_{\min}(C) - \lambda_{\max}(\Lambda_f^{-1})}}$$

Proof: We can see that another alternative representation for (1) is given by

$$\dot{x}_t = Ax_t + W_{1,t}\sigma(x_t) + W_{2,t}\phi(x_t)u_t + \delta_t \tag{32}$$

where δ_t is an error term. Since $Ax_t + W_{1,t}\sigma(x_t) + W_{2,t}\phi(x_t)u_t$ with the weights $W_{1,t}$ and $W_{2,t}$ adjusted by (9) is a better approximator than simply $Ax_t + W_1^*\sigma(x_t) + W_2^*\phi(x_t)u_t$ then

$$\|\delta_t\|_{\Lambda_f}^2 \le \|\Delta f(x_t, u_t, t)\|_{\Lambda_f}^2$$

However, from the assumption A.4, we have

$$\|\delta_t\|_{\Lambda_f}^2 \le \overline{\eta} \tag{33}$$

On the other hand, substituting the control law (31) into (32) and after some operations, yields

$$\dot{x}_t = \dot{x}_t^* - C(x_t - x_t^*) + \delta_t$$

considering that in accordance with the definition $e_t = x_t - x_t^*$ and consequently $\dot{e}_t = \dot{x}_t - \dot{x}_t^*$, we can obtain

$$\dot{e}_t = -Ce_t + \delta_t \tag{34}$$

which is the dynamics of the tracking error. To analyze the behavior of this dynamics, we use the following Lyapunov function candidate

$$V_t = e_t^T e_t$$

The first derivative of V_t is

$$\dot{V}_t = 2e_t^T \dot{e}_t \tag{35}$$

substituting (34) into (35) yields

$$\dot{V}_t = -2e_t^T Ce_t + 2e_t^T \delta_t$$

Now, in accordance with Rayleigh inequality

$$-2e_t^T Ce_t \le -2\lambda_{\min}(C)e_t^T e_t$$

Likewise, using the inequality (24), the term $2e_t^T\delta_t$ can be estimated by

$$2e_t^T\delta_t \le e_t^T\Lambda_f^{-1}e_t + \delta_t^T\Lambda_f\delta_t$$

However, using again Rayleigh inequality and in accordance with (33), we obtain

$$2e_t^T\delta_t \le \lambda_{\max}(\Lambda_f^{-1})e_t^T e_t + \overline{\eta}$$

then, using these results, $\dot{V}_t$ can be bounded as

$$\dot{V}_t \le -\left(2\lambda_{\min}(C) - \lambda_{\max}(\Lambda_f^{-1})\right)V_t + \overline{\eta}$$

which implies that

$$V_t \le V_0 \exp(-\gamma t) + \frac{\overline{\eta}}{\gamma}(1 - \exp(-\gamma t))$$

where $\gamma = 2\lambda_{\min}(C) - \lambda_{\max}(\Lambda_f^{-1})$. Since, by hypothesis, $\lambda_{\min}(C) > \frac{\lambda_{\max}(\Lambda_f^{-1})}{2}$ then

$$V_t \le V_0 + \frac{\overline{\eta}}{2\lambda_{\min}(C) - \lambda_{\max}(\Lambda_f^{-1})}$$

Finally, we can conclude that

$$\|e_t\| \le \sqrt{V_0 + \frac{\overline{\eta}}{2\lambda_{\min}(C) - \lambda_{\max}(\Lambda_f^{-1})}}$$

■

IV. Simulation Results

In this section, the results proposed in this work are applied to control a Lorenz model. This model is used for the fluid conviction description especially for some feature of atmospheric dynamics [10]. The uncontrolled model is given by

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= \rho x_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= -\beta x_3 + x_1x_2 \end{aligned} \tag{36}$$

where x_1, x_2 and x_3 represent measures of fluid velocity, horizontal and vertical temperature variations, correspondingly. The parameters σ, ρ and β are positive parameters that represent the Prandtl number, Rayleigh number and geometric factor, correspondingly. We select $\sigma = 10$, $\beta = 8/3$, and $\rho = 28$. On the other hand, the Lorenz system subjected to a control can be expressed as [11]

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) + u_1 \\ \dot{x}_2 &= \rho x_1 - x_2 - x_1x_3 + u_2 \\ \dot{x}_3 &= -\beta x_3 + x_1x_2 + u_3 \end{aligned} \tag{37}$$

In particular, we consider that (37) has the following initial condition: $x_0 = [0, -0.5, 3]^T$. Now we will manage to force this system into a desirable periodic trajectory. The nonlinear reference model to be followed is selected as:

$$\begin{aligned} \dot{x}_1^* &= x_2^* \\ \dot{x}_2^* &= \sin(x_1^*) \\ x_3^* &= 4 \end{aligned} \tag{38}$$

with initial conditions equal to $x_1^*(0) = 1, x_2^*(0) = 0$. The main parameters for the control law (31) and the learning law (9) are selected as

$$A = diag(-2, -1, -3.5),\ P = diag(15, 10, 17)$$
$$C = diag(20, 18, 11),\ K_1 = diag(4, 8, 7)$$
$$K_2 = diag(10, 9, 5),\ Q_0 = diag(0.5, 0.5, 0.5)$$

The trajectory tracking results are shown in Fig. 1 and Fig. 2 for $x_{1,t}$ and $x_{2,t}$, respectively. The solid lines correspond to the states of the reference dynamics (38), the dashed lines are the states of Lorenz system (37).

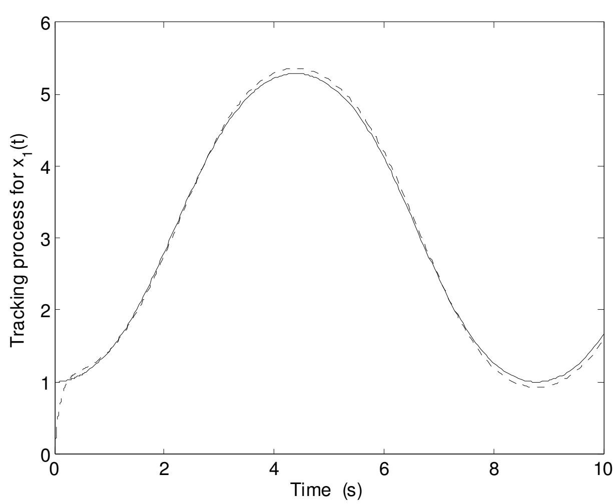

Fig. 1. Tracking process result for $x_{1,t}$

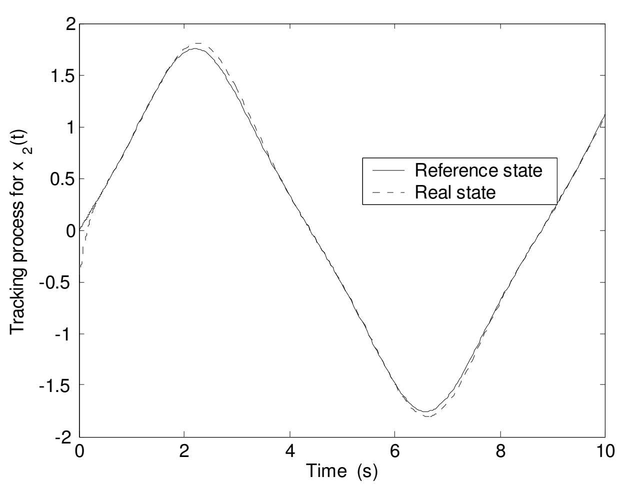

Fig. 2. Tracking process result for $x_{2,t}$

V. CONCLUSIONS

In this work, we have proposed a new learning law for a neural identifier. The boundedness of both the weights and the identification error has been guaranteed without resorting to any dead zone method. Likewise, a control law has been developed on the basis of this neural identifier without resorting to optimal local control theory. Therefore, our main contribution has been to simplify the design process of controllers based on differential neural networks in order to accomplish trajectory tracking

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