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Local Mode Dependent Output Feedback Control of Uncertain Markovian Jump Large-scale Systems

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Abstract— **This paper is concerned with the output feedback guaranteed cost control problem for a class of uncertain stochastic large-scale systems governed by a random parameter. The uncertainties are assumed to satisfy integral quadratic constraints, and the random parameter is a Markov process. A sufficient condition is established for the design of decentralized output feedback guaranteed cost controllers which use local system states and local system operation modes to produce local control inputs, and ensure suboptimal global quadratic performance. The condition is given in terms of a set of rank constrained linear matrix inequalities. A numerical example and simulations are also provided to illustrate the theory.**

I. Introduction

In the recent control literature, much attention has been given to Markovian large-scale systems subject to uncertain perturbations, e.g., see [4], [9], [2]. In particular, [9], [2] derived necessary and sufficient conditions for decentralized stabilization of a class of uncertain Markovian jump parameter systems, in which both local subsystem uncertain perturbations and uncertain interconnections were described in terms of integral quadratic constraints (IQCs). An underlying assumption, required to implement the controllers proposed in many available results including [9], [2], is that the global operation mode of the large-scale system must be known to every controller; we refer to such controllers as global mode dependent controllers. In a global mode dependent control design, the number of controllers for each subsystem is equal to the number of operation modes of the entire system. Also, the controllers have to change their operation modes even if the subsystems they control do not change. Also, to implement such a control algorithm, the system operation modes need to be made known to all subsystems. Such requirement is often impractical and costly.

To remove the dependency of the controllers on the knowledge of the global operation mode, [10] has recently developed a *local mode dependent control* technique where the modes of the decentralized controllers only depend on the modes of the subsystems they control. The results in [10] are a sufficient condition and an algorithm for the design of local mode dependent stabilizing controllers under the assumption of the full state feedback.

This paper extends the results of [10] in several directions. Firstly, in this paper we address the design of output feedback controllers via local mode dependent control. Our result leads to an algorithm for designing dynamic output controllers of full order. Secondly, unlike [10] which focused on the stabilization problem, in this paper, we consider a guaranteed cost control problem that is similar to that in [2]. Our controller design method leads to a set of local mode dependent controllers which are suboptimal with respect to a given quadratic performance cost functional. Thirdly, in [10], the system model was somewhat limited in that the uncertainty outputs, which were employed in the definition of admissible uncertainties used in that paper, did not allow for input feed-through. This paper overcomes this limitation and reintroduces the control input feed-through term in the definition of the uncertainty outputs. This however leads to additional technical difficulties, which render the techniques used in [9], [2], [10] not directly applicable to the problem under consideration in this paper. Hence, the control design technique developed in this paper is different from those developed in the previous work. In particular, a version of the bounded real lemma [11] is adopted to tackle the control input feed-through term, and the projection lemma [1] is used in the derivation of the proposed controller design condition.

Notation: $\mathbb{R}^+$ denotes the set of positive real numbers. $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$, and $\mathbb{S}^+$ denote, respectively, the n-dimensional Euclidean space, the set of $m\times n$ real matrices, and the set of real symmetric positive definite matrices of compatible dimensions. Given a matrix $A\in\mathbb{R}^{m\times n}$ with $r\triangleq \operatorname{rank}(A)<m$, $A^\perp\in\mathbb{R}^{(m-r)\times m}$ is an orthogonal complement of the matrix A if $A^\perp A=0$ and $\operatorname{rank}(A^\perp)=m-r$. Note that $A^\perp$ exists if and only if $r<m$ and is not unique. Also, we have $\left[\begin{smallmatrix} A_1 & * \\ A_2 & A_3\end{smallmatrix}\right]\triangleq\left[\begin{smallmatrix} A_1 & A_2^T \\ A_2 & A_3\end{smallmatrix}\right]$.

II. Problem Formulation

Consider an uncertain Markovian jump large-scale system consisting of N subsystems. The ith subsystem is given by

$$\mathcal{S}_i:\begin{cases}\dot{x}_i(t)=A_i(\eta_i(t))x_i(t)+B_i(\eta_i(t))u_i(t)\\ \qquad\quad +E_i(\eta_i(t))\xi_i(t)+L_i(\eta_i(t))r_i(t),\\ \zeta_i(t)=H_i(\eta_i(t))x_i(t)+G_i(\eta_i(t))u_i(t),\\ y_i(t)=C_i(\eta_i(t))x_i(t)+D_i(\eta_i(t))\xi_i(t),\end{cases}\tag{1}$$

where $i\in\mathcal{N}\triangleq\{1,2,\ldots,N\}$ indicates that $\mathcal{S}_i$ is the ith subsystem, $x_i(t)\in\mathbb{R}^{n_i}$ is the system state of subsystem $\mathcal{S}_i$, $u_i(t)\in\mathbb{R}^{m_i}$ is the control input, $y_i(t)\in\mathbb{R}^{t_i}$ is the measured output which will be used for feedback, $\zeta_i(t)\in\mathbb{R}^{q_i}$ is the uncertainty output, $\xi_i(t)\in\mathbb{R}^{p_i}$ is the local uncertainty input, $r_i(t)\in\mathbb{R}^{s_i}$ is the interconnection input, which describes the effect of other subsystems $\mathcal{S}_j$, $j\neq i$, on $\mathcal{S}_i$ due to the uncertain interconnections between subsystem $\mathcal{S}_i$ and other

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subsystems $\mathcal{S}_j$, $j \neq i$. The random process $\eta_i(t)$ denotes the operation mode of subsystem $\mathcal{S}_i$; it takes values in the finite state space $\mathcal{M}_i \triangleq \{1, 2, \ldots, M_i\}$. It is worth mentioning that the process $\eta_i(t)$ is, in general, not Markovian. The initial condition of subsystem $\mathcal{S}_i$ is given by $x_{i0} \in \mathbb{R}^{n_i}$ and $\eta_{i0} \in \mathcal{M}_i$.

The mechanism of mode changes for the large-scale system is described by the random process $\eta(t)$. It depends on (and also determines) the operation modes of the subsystems; e.g., see [10]. It is assumed that the random process $\eta(t)$ is a stationary ergodic continuous-time Markov process defined on a complete probability space $(\Omega, \mathcal{F}, \Pr)$ and takes values in $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$ where $\max_{i\in\mathcal{N}} M_i \leq M \leq \prod_{i=1}^{N} M_i$. The state transition rate matrix of $\eta(t)$ is given by $\mathbf{Q} = (q_{\mu\nu}) \in \mathbb{R}^{M\times M}$, in which $q_{\mu\nu} \geq 0$ if $\nu \neq \mu$, and $q_{\mu\mu} \triangleq -\sum_{\nu=1,\nu\neq\mu}^{M} q_{\mu\nu}$.

The connection between the global operation mode $\eta(t)$ of the large-scale system and the local operation modes $\eta_i(t)$ of the subsystems can be expressed in terms of a bijective function [10]. Let $\mathcal{M}_{\rm p}$ be a set of vectors describing feasible simultaneous operation modes of the subsystems of the large-scale system; it is a subset of the set $\mathcal{M}_1 \times \cdots \times \mathcal{M}_N$ and has M elements. Then there is a bijective function $\Psi : \mathcal{M}_{\rm p} \to \mathcal{M}$ with $\upsilon = \Psi(\upsilon_1, \ldots, \upsilon_N)$ where $\upsilon_i \in \mathcal{M}_i$, $\upsilon \in \mathcal{M}$, and the functions $\Psi_i^{-1} : \mathcal{M} \to \mathcal{M}_i$ with $\upsilon_i = \Psi_i^{-1}(\upsilon) \in \mathcal{M}_i$ for all $i \in \mathcal{N}$, $\upsilon \in \mathcal{M}$.

The uncertainties and interconnections in the large-scale system (1) are described by

$$\begin{aligned}
&\xi_i(t) = \phi_i^{\xi}(t, \zeta_i(t), \eta_i(t)),\\
&r_i(t) = \phi_i^{r}(t, \zeta_1(t), \ldots, \zeta_{i-1}(t), \zeta_{i+1}(t), \ldots, \zeta_N(t), \eta(t)).
\end{aligned}$$

Furthermore, these uncertainties are assumed to satisfy IQCs [7], [9], [2], as described in the following definitions.

Definition 1: Given a set of matrices $\bar{S}_i \in \mathbb{S}^+$, $i \in \mathcal{N}$. A collection of uncertainty inputs $\xi_i(t)$, $i \in \mathcal{N}$, is an admissible local uncertainty for the large-scale system if there exists a sequence $\{t_l\}_{l=1}^{\infty}$ such that $t_l \to \infty$, $t_l \geq 0$, and

$$\mathrm{E}\left(\int_0^{t_l} \left[\|\zeta_i(t)\|^2 - \|\xi_i(t)\|^2\right] dt \mid x_0, \eta_0\right) \geq -x_{i0}^T \bar{S}_i x_{i0} \quad (2)$$

for all l and for all $i \in \mathcal{N}$, where $x_0 = [x_{10}^T, \ldots, x_{N0}^T]^T$ and $\eta_0 = \Psi(\eta_{10}, \ldots, \eta_{N0})$. Here $\Psi(\cdot)$ is the bijective function mapping from $\mathcal{M}_{\rm p}$ to $\mathcal{M}$. The set of admissible local uncertainties is denoted by Ξ^{ξ}.

Definition 2: Given a set of matrices $\tilde{S}_i \in \mathbb{S}^+$, $i \in \mathcal{N}$. The subsystems of the large-scale system are said to have admissible interconnections to other subsystems if there exists a sequence $\{t_l\}_{l=1}^{\infty}$ such that $t_l \to \infty$, $t_l \geq 0$, and

$$\mathrm{E}\left(\int_0^{t_l} \left[\left(\sum_{j=1, j\neq i}^{N} \|\zeta_j(t)\|^2\right) - \|r_i(t)\|^2\right] dt \mid x_0, \eta_0\right) \geq -x_{i0}^T \tilde{S}_i x_{i0} \quad (3)$$

for all l and for all $i \in \mathcal{N}$. The set of admissible interconnection uncertainties is denoted by Ξ^r.

Without loss of generality, we assume that the same sequence $\{t_l\}_{l=1}^{\infty}$ is employed in both definitions.

Consider a decentralized local mode dependent output feedback controller of the form

$$\begin{cases} \dot{x}_{K,i}(t) = A_{K,i}(\eta_i(t))x_{K,i}(t) + B_{K,i}(\eta_i(t))y_i(t), \\ u_i(t) = C_{K,i}(\eta_i(t))x_{K,i}(t) + D_{K,i}(\eta_i(t))y_i(t), \end{cases} \quad (4)$$

where $x_{K,i}(t) \in \mathbb{R}^{n_i}$ is the state of the controller for subsystem $\mathcal{S}_i$. The controller's initial state is set to zero. Note that the controller's initial operation mode is the same as that of the system (1). The matrices $A_{K,i}(\upsilon_i)$, $B_{K,i}(\upsilon_i)$, $C_{K,i}(\upsilon_i)$, $D_{K,i}(\upsilon_i)$, $\upsilon_i \in \mathcal{M}_i$, $i \in \mathcal{N}$, are the parameters of the controller. It is worthwhile to emphasize that for the controllers under consideration of the form (4), these parameters are determined by the state of the local operation mode process $\eta_i(t)$ of the corresponding subsystem $\mathcal{S}_i$, while the controllers proposed in [9], [2] were dependent on the values of the global operation mode process $\eta(t)$.

Definition 3: The closed-loop system corresponding to the uncertain system (1)–(3) with a controller of the form (4) is said to be robustly stochastically stable if there exists a constant $c_1 \in \mathbb{R}^+$ such that $x_i(\cdot) \in \mathbb{L}_2[0, \infty)$, $i \in \mathcal{N}$, and

$$\sum_{i=1}^{N} \mathrm{E}\left(\int_0^{\infty} \|x_i(t)\|^2 \, dt \mid x_0, \eta_0\right) \leq c_1 \sum_{i=1}^{N} \|x_{i0}\|^2$$

for any initial conditions x_0, η_0, any admissible local uncertainty $\xi_i(t)$ and any admissible interconnection $r_i(t)$, $i \in \mathcal{N}$.

Associated with the large-scale system (1) is the cost functional of the form

$$\begin{aligned} J \triangleq \sum_{i=1}^{N} \mathrm{E}\bigg(\int_0^{\infty} &\left[x_i^T(t) Q_i(\eta_i(t)) x_i(t) \right.\\ &\left. + u_i^T(t) R_i(\eta_i(t)) u_i(t)\right] dt \mid x_0, \eta_0\bigg) \end{aligned} \quad (5)$$

where $Q_i(\upsilon_i) \in \mathbb{S}^+$, $R_i(\upsilon_i) \in \mathbb{S}^+$, $\upsilon_i \in \mathcal{M}_i$, $i \in \mathcal{N}$, are given weighting matrices.

The objective of the paper is to design a dynamic output feedback controller of the form (4) for the uncertain system (1)–(3), such that the resulting closed-loop system is robustly stochastically stable and the corresponding worst-case value of the cost functional (5) subject to the constraints (2)–(3) is upper bounded.

III. Controller Design

This section presents the main results of the paper. Our controller design technique is based on decentralized global mode dependent control with uncertainties. The design methodology involves augmenting the class of uncertainties introduced in the previous section to include effects of mismatch between the global system mode dependent controllers and the local mode dependent controllers. In Section III-A, we show how a local mode dependent controller can be derived from a given global mode controller; the result is a sufficient condition to ensure that such a derivation is possible. The design of a suitable auxiliary global mode dependent controller is described in Section III-B. Here we present a sufficient condition for the existence of such an auxiliary output feedback controller. Then, in Section III-C we propose a controller design technique based on the auxiliary controller presented in Section III-B.

A. Design Methodology

As noted above, the jump processes $\eta_i(t)$ governing the modes of the local controllers (4) are, in general, non-Markovian. In this section, we will study an auxiliary uncertain large-scale system, whose subsystems are governed by the global Markovian operation mode process $\eta(t)$, and hence are easier to deal with.

Consider a class of uncertain large-scale systems consisting of subsystems of the following form

$$\tilde{\mathcal{S}}_i : \begin{cases} \dot{\tilde{x}}_i(t) = \tilde{A}_i(\eta(t))\tilde{x}_i(t) + \tilde{B}_i(\eta(t))\tilde{u}_i(t) \\ \qquad\qquad + \tilde{E}_i(\eta(t))\tilde{\xi}_i(t) + \tilde{L}_i(\eta(t))\tilde{r}_i(t), \\ \tilde{\zeta}_i(t) = \tilde{H}_i(\eta(t))\tilde{x}_i(t) + \tilde{G}_i(\eta(t))\tilde{u}_i(t), \\ \tilde{y}_i(t) = \tilde{C}_i(\eta(t))\tilde{x}_i(t) + \tilde{D}_i(\eta(t))\tilde{\xi}_i(t), \end{cases} \tag{6}$$

where $\tilde{A}_i(\mu) = A_i(\mu_i)$, $\tilde{B}_i(\mu) = B_i(\mu_i)$, $\tilde{E}_i(\mu) = E_i(\mu_i)$, $\tilde{L}_i(\mu) = L_i(\mu_i)$, $\tilde{H}_i(\mu) = H_i(\mu_i)$, $\tilde{C}_i(\mu) = C_i(\mu_i)$, $\tilde{D}_i(\mu) = D_i(\mu_i)$, and $\mu_i = \Psi_i^{-1}(\mu)$, $\mu \in \mathcal{M}$, $i \in \mathcal{N}$. The uncertainty inputs $\tilde{\xi}_i(t)$ and $\tilde{r}_i(t)$ are described, respectively, by the same functions as $\xi_i(t)$ and $r_i(t)$ in (1). So $\tilde{\xi}_i(t) \in \Xi^{\xi}$ and $\tilde{r}_i(t) \in \Xi^r$. The initial condition is given by $\tilde{x}_{i0} = x_{i0}$, $i \in \mathcal{N}$, and $\eta_0 = \Psi(\eta_{10}, \dots, \eta_{N0})$. Note that system (6) and system (1), in fact, have the same sample paths.

Associated with this uncertain system is the following cost functional of the form

$$\tilde{J} \triangleq \sum_{i=1}^{N} \mathrm{E}\left(\int_0^{\infty} \left[\tilde{x}_i^T(t)\tilde{Q}_i(\eta(t))\tilde{x}_i(t) + \tilde{u}_i^T(t)\tilde{R}_i(\eta(t))\tilde{u}_i(t)\right] dt \mid \tilde{x}_0, \eta_0\right) \tag{7}$$

where $\tilde{Q}_i(\mu) = Q_i(\mu_i)$, $\tilde{R}_i(\mu) = R_i(\mu_i)$, and $\mu_i = \Psi_i^{-1}(\mu)$ for $\mu \in \mathcal{M}$, $i \in \mathcal{N}$.

We now consider the problem of guaranteed cost control of the system (6) by means of an uncertain decentralized global mode dependent output feedback controller of the form

$$\begin{cases} \dot{\tilde{x}}_{K,i}(t) = \tilde{A}_{K,i}(\eta(t))\tilde{x}_{K,i}(t) + \tilde{B}_{K,i}(\eta(t))\tilde{y}_i(t) \\ \qquad\qquad + \tilde{\xi}_{1i}(t) + \tilde{\xi}_{2i}(t), \\ \tilde{u}_i(t) = \tilde{C}_{K,i}(\eta(t))\tilde{x}_{K,i}(t) + \tilde{D}_{K,i}(\eta(t))\tilde{y}_i(t) \\ \qquad\qquad + \tilde{\xi}_{3i}(t) + \tilde{\xi}_{4i}(t). \end{cases} \tag{8}$$

The initial state of the controller is zero, and the initial operation mode is the same as that of the system (6). Note that controller dynamics are assumed to be subject to controller uncertainties of the form

$$\tilde{\xi}_{1i}(t) = \phi_{1i}(t, \tilde{x}_{K,i}(t), \eta(t)), \quad \tilde{\xi}_{2i}(t) = \phi_{2i}(t, \tilde{y}_i(t), \eta(t)),$$
$$\tilde{\xi}_{3i}(t) = \phi_{3i}(t, \tilde{x}_{K,i}(t), \eta(t)), \quad \tilde{\xi}_{4i}(t) = \phi_{4i}(t, \tilde{y}_i(t), \eta(t)),$$

which satisfy the following IQCs.

Definition 4: Given $\beta_{1i}(\mu), \beta_{2i}(\mu), \beta_{3i}(\mu), \beta_{4i}(\mu) \in \mathbb{R}^+$, $\mu \in \mathcal{M}$, $i \in \mathcal{N}$. A collection of uncertainty inputs $\tilde{\xi}_{1i}(t)$, $\tilde{\xi}_{2i}(t)$, $\tilde{\xi}_{3i}(t)$, $\tilde{\xi}_{4i}(t)$, $i \in \mathcal{N}$, is an admissible uncertainty input for the dynamic controller in (8) if there exists a sequence $\{t_l\}_{l=1}^{\infty}$ such that $t_l \to \infty$, $t_l \geq 0$ and

$$\begin{aligned} &\mathrm{E}\left(\int_0^{t_l}\left[\beta_{1i}^2(\eta(t))\left\|\tilde{x}_{K,i}(t)\right\|^2 - \left\|\tilde{\xi}_{1i}(t)\right\|^2\right] dt \mid \tilde{x}_0, \eta_0\right) \geq 0, \\ &\mathrm{E}\left(\int_0^{t_l}\left(\beta_{2i}^2(\eta(t))\left\|\tilde{y}_i(t)\right\|^2 - \left\|\tilde{\xi}_{2i}(t)\right\|^2\right) dt \mid \tilde{x}_0, \eta_0\right) \geq 0, \\ &\mathrm{E}\left(\int_0^{t_l}\left[\beta_{3i}^2(\eta(t))\left\|\tilde{x}_{K,i}(t)\right\|^2 - \left\|\tilde{\xi}_{3i}(t)\right\|^2\right] dt \mid \tilde{x}_0, \eta_0\right) \geq 0, \\ &\mathrm{E}\left(\int_0^{t_l}\left(\beta_{4i}^2(\eta(t))\left\|\tilde{y}_i(t)\right\|^2 - \left\|\tilde{\xi}_{4i}(t)\right\|^2\right) dt \mid \tilde{x}_0, \eta_0\right) \geq 0, \end{aligned} \tag{9}$$

for all l and for all $i \in \mathcal{N}$. The set of the admissible controller uncertainty inputs is denoted by Ξ^K.

We can assume that the same sequence $\{t_l\}_{l=1}^{\infty}$ is selected as in Definitions 1, 2, and 4.

The following result gives a sufficient condition for when the controller in (4) to stabilize the uncertain system (1) if the controller (8) can stabilize the uncertain system (6).

Theorem 1: Suppose controller (8) stochastically stabilizes the uncertain large-scale system (6) subject to constraints (2)–(3), (9), and leads to the cost bound $\sup_{\Xi^{\xi},\Xi^r,\Xi^K} \tilde{J} < c$ for some $c \in \mathbb{R}^+$. If the controller matrices in (4) are chosen so that

$$\left\|\tilde{A}_{K,i}(\mu) - A_{K,i}(\mu_i)\right\| \leq \beta_{1i}(\mu), \tag{10}$$
$$\left\|\tilde{B}_{K,i}(\mu) - B_{K,i}(\mu_i)\right\| \leq \beta_{2i}(\mu), \tag{11}$$
$$\left\|\tilde{C}_{K,i}(\mu) - C_{K,i}(\mu_i)\right\| \leq \beta_{3i}(\mu), \tag{12}$$
$$\left\|\tilde{D}_{K,i}(\mu) - D_{K,i}(\mu_i)\right\| \leq \beta_{4i}(\mu), \tag{13}$$

for all $\mu \in \mathcal{M}$, $i \in \mathcal{N}$, where $\mu_i = \Psi_i^{-1}(\mu)$, then the controller in (4) stochastically stabilizes the uncertain large-scale system in (1) subject to constraints (2)–(3) and also leads to the cost bound $\sup_{\Xi^{\xi},\Xi^r} J < c$.

B. Design of Global Mode Dependent Controllers

In this section, a sufficient condition is established for the design of the uncertain guaranteed cost controller of the form (8). This condition, together with Theorem 1, provides us a basis for the design of a local mode dependent guaranteed cost controller of the form (4). We first define the following notation:

$$\hat{A}_i(\mu) = \begin{bmatrix} \tilde{A}_i(\mu) & 0 \\ 0 & 0 \end{bmatrix} = N_i\tilde{A}_i(\mu)N_i^T, \quad N_i = \begin{bmatrix} I_{n_i \times n_i} \\ 0_{n_i \times n_i} \end{bmatrix},$$

$$\hat{B}_i(\mu) = \begin{bmatrix} \tilde{E}_i(\mu) & \tilde{L}_i(\mu) & 0 & 0 & \tilde{B}_i(\mu) & \tilde{B}_i(\mu) \\ 0 & 0 & I & I & 0 & 0 \end{bmatrix},$$

$$\hat{C}_i(\mu) = \begin{bmatrix} \tilde{Q}_i^{\frac{1}{2}}(\mu) & 0 \\ 0 & 0 \\ \tilde{H}_i(\mu) & 0 \\ 0 & \beta_{1i}(\mu)I \\ \beta_{2i}(\mu)\tilde{C}_i(\mu) & 0 \\ 0 & \beta_{3i}(\mu)I \\ \beta_{4i}(\mu)\tilde{C}_i(\mu) & 0 \end{bmatrix},$$

$$\hat{D}_i(\mu) = \begin{bmatrix} 0 & 0\ 0\ 0 & 0 & 0 \\ 0 & 0\ 0\ 0 & \tilde{R}_i^{\frac{1}{2}}(\mu) & \tilde{R}_i^{\frac{1}{2}}(\mu) \\ 0 & 0\ 0\ 0 & \tilde{G}_i(\mu) & \tilde{G}_i(\mu) \\ 0 & 0\ 0\ 0 & 0 & 0 \\ \beta_{2i}(\mu)\tilde{D}_i(\mu) & 0\ 0\ 0 & 0 & 0 \\ 0 & 0\ 0\ 0 & 0 & 0 \\ \beta_{4i}(\mu)\tilde{D}_i(\mu) & 0\ 0\ 0 & 0 & 0 \end{bmatrix},$$

$$\underline{B}_i(\mu) = \begin{bmatrix} 0 & \tilde{B}_i(\mu) \\ I & 0 \end{bmatrix}, \quad \underline{C}_i(\mu) = \begin{bmatrix} 0 & I \\ \tilde{C}_i(\mu) & 0 \end{bmatrix},$$

$$\underline{D}_i(\mu) = \begin{bmatrix} 0 & 0\ 0\ 0\ 0\ 0 \\ \tilde{D}_i(\mu) & 0\ 0\ 0\ 0\ 0 \end{bmatrix},$$

$$\underline{E}_i(\mu) = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{R}_i^{\frac{1}{2}}(\mu) \\ 0 & \tilde{G}_i(\mu) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \begin{bmatrix} V_{1i}(\mu) & V_{2i}(\mu) \end{bmatrix} = \begin{bmatrix} \tilde{B}_i(\mu) \\ \tilde{R}_i^{\frac{1}{2}}(\mu) \\ \tilde{G}_i(\mu) \end{bmatrix}^{\perp},$$

$$\begin{aligned} Q_{1i}(\mu) &= N_i^T X_i(\mu) N_i \tilde{A}_i^T(\mu) + \tilde{A}_i(\mu) N_i^T X_i(\mu) N_i \\ &\quad + q_{\mu\mu} N_i^T X_i(\mu) N_i, \\ Q_{2i} &= -\operatorname{diag}(\tau_i I, \theta_i I, \tau_{1i} I, \tau_{2i} I, \tau_{3i} I, \tau_{4i} I), \\ Q_{3i} &= -\operatorname{diag}(I, I, \bar{\tau}_i I, \bar{\tau}_{1i} I, \bar{\tau}_{2i} I, \bar{\tau}_{3i} I, \bar{\tau}_{4i} I), \\ Q_{4i}(\mu) &= -\operatorname{diag}(X_i(1), \ldots, X_i(\mu-1), \\ &\quad X_i(\mu+1), \ldots, X_i(M)), \\ Q_{5i}(\mu) &= \tilde{A}_i^T(\mu) N_i^T P_i(\mu) N_i + N_i^T P_i(\mu) N_i \tilde{A}_i(\mu) \\ &\quad + \sum_{\nu=1}^{M} q_{\mu\nu} N_i^T P_i(\nu) N_i, \\ \Gamma_{1i}(\mu) &= \left[\sqrt{q_{\mu,1}} N_i^T X_i(\mu) \ \cdots \ \sqrt{q_{\mu,\mu-1}} N_i^T X_i(\mu) \right. \\ &\quad \left. \sqrt{q_{\mu,\mu+1}} N_i^T X_i(\mu) \ \cdots \ \sqrt{q_{\mu,M}} N_i^T X_i(\mu) \right], \end{aligned}$$

$$\Gamma_{2i}(\mu) = \left[\begin{array}{c|c|c c c c c c|c} V_{1i}(\mu) & 0 & 0 & V_{2i}(\mu) & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{array}\right],$$

$$\tilde{K}_i(\mu) = \begin{bmatrix} \tilde{A}_{K,i}(\mu) & \tilde{B}_{K,i}(\mu) \\ \tilde{C}_{K,i}(\mu) & \tilde{D}_{K,i}(\mu) \end{bmatrix},$$

$$\Theta_{1i}(\mu) = \hat{A}_i^T(\mu) P_i(\mu) + P_i(\mu) \hat{A}_i(\mu) + \sum_{\nu=1}^{M} q_{\mu\nu} P_i(\nu),$$

$$\Theta_{2i}(\mu) = \begin{bmatrix} Q_{1i}(\mu) & * & * & * \\ \hat{B}_i^T(\mu) N_i & Q_{2i} & * & * \\ \hat{C}_i(\mu) X_i(\mu) N_i & \hat{D}_i(\mu) & Q_{3i} & * \\ \Gamma_{1i}^T(\mu) & 0 & 0 & Q_{4i}(\mu) \end{bmatrix},$$

$$\Theta_{3i}(\mu) = \begin{bmatrix} Q_{5i}(\mu) & * & * \\ \hat{B}_i^T(\mu) P_i(\mu) N_i & Q_{2i} & * \\ \hat{C}_i(\mu) N_i & \hat{D}_i(\mu) & Q_{3i} \end{bmatrix},$$

$$\Phi_i(\mu) = \begin{bmatrix} \Theta_{1i} & * & * \\ \hat{B}_i^T(\mu) P_i(\mu) & Q_{2i} & * \\ \hat{C}_i(\mu) & \hat{D}_i(\mu) & Q_{3i} \end{bmatrix}, \tag{14}$$

$$\Psi_{li}(\mu) = \begin{bmatrix} P_i(\mu) \underline{B}_i(\mu) \\ 0 \\ \underline{E}_i(\mu) \end{bmatrix}, \tag{15}$$

$$\Psi_{ri}(\mu) = \begin{bmatrix} \underline{C}_i(\mu) & \underline{D}_i(\mu) & 0 \end{bmatrix}. \tag{16}$$

Theorem 2: Given $\beta_{1i}(\mu)$, $\beta_{2i}(\mu)$, $\beta_{3i}(\mu)$, $\beta_{4i}(\mu) \in \mathbb{R}^+$. Suppose there exist matrices $P_i(\mu) \in \mathbb{S}^+$, $X_i(\mu) \in \mathbb{S}^+$, scalars τ_i, θ_i, τ_{1i}, τ_{2i}, τ_{3i}, τ_{4i}, $\bar{\tau}_i$, $\bar{\tau}_{1i}$, $\bar{\tau}_{2i}$, $\bar{\tau}_{3i}$, $\bar{\tau}_{4i} \in \mathbb{R}^+$, $\mu \in \mathcal{M}$, $i \in \mathcal{N}$, satisfying, for all $\mu \in \mathcal{M}$, $i \in \mathcal{N}$, the coupled LMIs

$$\Gamma_{2i}(\mu) \Theta_{2i}(\mu) \Gamma_{2i}^T(\mu) < 0, \tag{17}$$

$$\begin{bmatrix} \begin{bmatrix} \tilde{C}_i^T(\mu) \\ \tilde{D}_i^T(\mu) \end{bmatrix}^{\perp} & 0 \\ 0 & I \end{bmatrix} \Theta_{3i}(\mu) \begin{bmatrix} \begin{bmatrix} \tilde{C}_i^T(\mu) \\ \tilde{D}_i^T(\mu) \end{bmatrix}^{\perp} & 0 \\ 0 & I \end{bmatrix}^T < 0, \tag{18}$$

with rank constraints

$$\operatorname{rank}\left(\begin{bmatrix} P_i(\mu) & I \\ I & X_i(\mu) \end{bmatrix} \right) \le 2n, \tag{19}$$

$$\operatorname{rank}\left(\begin{bmatrix} \tau_i + \bar{\theta}_i & 1 \\ 1 & \bar{\tau}_i \end{bmatrix} \right) \le 1, \ \operatorname{rank}\left(\begin{bmatrix} \tau_{1i} & 1 \\ 1 & \bar{\tau}_{1i} \end{bmatrix} \right) \le 1, \tag{20}$$

$$\operatorname{rank}\left(\begin{bmatrix} \tau_{2i} & 1 \\ 1 & \bar{\tau}_{2i} \end{bmatrix} \right) \le 1, \ \operatorname{rank}\left(\begin{bmatrix} \tau_{3i} & 1 \\ 1 & \bar{\tau}_{3i} \end{bmatrix} \right) \le 1, \tag{21}$$

$$\operatorname{rank}\left(\begin{bmatrix} \tau_{4i} & 1 \\ 1 & \bar{\tau}_{4i} \end{bmatrix} \right) \le 1. \tag{22}$$

Consider the matrix inequality

$$\Phi_i(\mu) + \Psi_{li}(\mu) \tilde{K}_i(\mu) \Psi_{ri}(\mu) + \Psi_{ri}^T(\mu) \tilde{K}_i^T(\mu) \Psi_{li}^T(\mu) < 0, \tag{23}$$

in which the matrices $\Phi_i(\mu)$, $\Psi_{li}(\mu)$, $\Psi_{ri}(\mu)$ are obtained by substituting the solution of (17)–(22) to (14)–(16). Then an output feedback controller of form (8) can be obtained by solving the coupled LMIs in (23) for $\tilde{K}_i(\mu)$. In addition, with this controller, the corresponding closed-loop value of the cost functional (7) satisfies

$$\sup_{\Xi^\xi, \Xi^r, \Xi^K} \tilde{J} < \sum_{i=1}^{N} \tilde{x}_{i0}^T \left[N_i^T P_i(\eta_{i0}) N_i + \tau_i \bar{S}_i + \theta_i \tilde{S}_i \right] \tilde{x}_{i0}. \tag{24}$$

Remark 1: The introduction of τ_i, θ_i, τ_{1i}, τ_{2i}, τ_{3i} and τ_{4i} allows us to apply the following optimization procedure to minimize the upper bound of the cost functional (7):

$$\inf_{\substack{P_i(\mu), X_i(\mu), \tau_i, \theta_i, \tau_{1i}, \tau_{2i}, \\ \tau_{3i}, \tau_{4i}, \bar{\tau}_i, \bar{\tau}_{1i}, \bar{\tau}_{2i}, \bar{\tau}_{3i}, \bar{\tau}_{4i}, \\ \text{subject to (17)–(22)}}} \sum_{i=1}^{N} \tilde{x}_{i0}^T \begin{bmatrix} N_i^T P_i(\eta_{i0}) N_i \\ + \tau_i \bar{S}_i + \theta_i \tilde{S}_i \end{bmatrix} \tilde{x}_{i0}. \tag{25}$$

C. The Main Result: Design of Local Mode Dependent Controllers

In this section, we select the dynamic local mode dependent controller (4) as the expectation of a controller (8) conditioned on the subsystem operation modes as time approaches infinity; see [10] for details. That is,

$$K_i(\upsilon_i) = \begin{bmatrix} A_{K,i}(\upsilon_i) & B_{K,i}(\upsilon_i) \\ C_{K,i}(\upsilon_i) & D_{K,i}(\upsilon_i) \end{bmatrix} \triangleq \frac{\sum_{\mu=1}^{M} \{\tilde{K}_i(\mu)\pi_{\infty\mu}\mathbb{I}_i(\mu,\upsilon_i)\}}{\sum_{\mu=1}^{M}\{\pi_{\infty\mu}\mathbb{I}_i(\mu,\upsilon_i)\}} \quad (26)$$

for all $\upsilon_i \in \mathcal{M}_i$, $i \in \mathcal{N}$, where $\pi_{\infty\mu}$ is the μ-th component of vector $\pi_\infty = \mathbf{e}(\mathbf{Q}+\mathbf{E})^{-1}$, $\mathbf{e} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{1\times M}$, $\mathbf{E} = \begin{bmatrix} \mathbf{e}^T & \mathbf{e}^T & \cdots & \mathbf{e}^T \end{bmatrix}^T \in \mathbb{R}^{M\times M}$, and $\mathbb{I}_i(\mu,\upsilon_i) = 1$ if $\upsilon_i = \Psi_i^{-1}(\mu)$, $\mathbb{I}_i(\mu,\upsilon_i) = 0$ otherwise. We also have

$$\Delta_i(\mu) \triangleq \tilde{K}_i(\mu) - K_i(\mu_i) = \frac{\sum_{\nu=1,\nu\neq\mu}^{M}\{\mathbb{I}_i(\nu,\mu_i)\pi_{\infty\nu}\left[\tilde{K}_i(\mu)-\tilde{K}_i(\nu)\right]\}}{\sum_{\nu=1}^{M}\{\mathbb{I}_i(\nu,\mu_i)\pi_{\infty\nu}\}} \quad (27)$$

where $\mu_i = \Psi_i^{-1}(\mu)$. Note that $\Delta_i(\mu)$ is a linear matrix function of $\tilde{K}_i(\mu)$, $\mu \in \mathcal{M}$.

A computational method for the design of the guaranteed cost controller (4) is presented in the following result, which is based upon Theorem 1, Theorem 2 and the selection of the controller parameters in (26). Let

$$N_{1i} = \begin{bmatrix} I_{n_i\times n_i} \\ 0_{m_i\times n_i} \end{bmatrix}, \qquad N_{2i} = \begin{bmatrix} I_{n_i\times n_i} \\ 0_{t_i\times n_i} \end{bmatrix},$$
$$N_{3i} = \begin{bmatrix} 0_{n_i\times t_i} \\ I_{t_i\times t_i} \end{bmatrix}, \qquad N_{4i} = \begin{bmatrix} 0_{n_i\times m_i} \\ I_{m_i\times m_i} \end{bmatrix}.$$

Theorem 3: Given a set of $\beta_{1i}(\mu)$, $\beta_{2i}(\mu)$, $\beta_{3i}(\mu)$, $\beta_{4i}(\mu) \in \mathbb{R}^+$. Suppose a set of solutions $P_i(\mu) \in \mathbb{S}^+$, $X_i(\mu) \in \mathbb{S}^+$, τ_i, θ_i, τ_{1i}, τ_{2i}, τ_{3i}, τ_{4i}, $\bar{\tau}_i$, $\bar{\tau}_{1i}$, $\bar{\tau}_{2i}$, $\bar{\tau}_{3i}$, $\bar{\tau}_{4i} \in \mathbb{R}^+$, $\mu \in \mathcal{M}$, $i \in \mathcal{N}$ is found for (17)–(22).

If there exist matrices $\tilde{K}_i(\mu)$ such that the following LMIs

$$\begin{bmatrix} \beta_{1i}(\mu)I & N_{2i}^T\Delta_i^T(\mu)N_{1i} \\ N_{1i}^T\Delta_i(\mu)N_{2i} & \beta_{1i}(\mu)I \end{bmatrix} \geq 0, \quad (28)$$
$$\begin{bmatrix} \beta_{2i}(\mu)I & N_{3i}^T\Delta_i^T(\mu)N_{1i} \\ N_{1i}^T\Delta_i(\mu)N_{3i} & \beta_{2i}(\mu)I \end{bmatrix} \geq 0, \quad (29)$$
$$\begin{bmatrix} \beta_{3i}(\mu)I & N_{2i}^T\Delta_i^T(\mu)N_{4i} \\ N_{4i}^T\Delta_i(\mu)N_{2i} & \beta_{3i}(\mu)I \end{bmatrix} \geq 0, \quad (30)$$
$$\begin{bmatrix} \beta_{4i}(\mu)I & N_{3i}^T\Delta_i^T(\mu)N_{4i} \\ N_{4i}^T\Delta_i(\mu)N_{3i} & \beta_{4i}(\mu)I \end{bmatrix} \geq 0, \quad (31)$$

and the LMIs in (23) hold for all $\mu \in \mathcal{M}$, $i \in \mathcal{N}$, where $\Delta_i(\mu)$ is the linear function of $\tilde{K}_i(\mu)$ defined in (27), then the local mode dependent controller of the form (4) with the parameters defined in (26) robustly stabilizes the uncertain system (1) subject to the constraints (2)–(3) and leads to the cost bound

$$\sup_{\Xi^\xi,\Xi^r} J < \sum_{i=1}^{N} x_{i0}^T \left[N_i^T P_i(\eta_{i0})N_i + \tau_i\bar{S}_i + \theta_i\tilde{S}_i\right] x_{i0}. \quad (32)$$

IV. An Illustrative Example

In this section, we present a numerical example to illustrate the developed theory. The uncertain large-scale system in the example has 3 subsystems, and each subsystem can operate in 2 different modes. The system data for the system (1) are as follows.

$$A_1(1) = \begin{bmatrix} 1 & 0 \\ -0.5 & -0.5 \end{bmatrix}, B_1(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_1(1) = \begin{bmatrix} 0.6 & 0 \end{bmatrix},$$
$$A_1(2) = \begin{bmatrix} 1 & 0 \\ 0.1 & -0.5 \end{bmatrix}, B_1(2) = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, C_1(2) = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$A_2(1) = \begin{bmatrix} -0.6 & 0.5 \\ 0 & 0.5 \end{bmatrix}, B_2(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_2(1) = \begin{bmatrix} 0.1 & 1 \end{bmatrix},$$
$$A_2(2) = \begin{bmatrix} -1 & 0 \\ -0.5 & 0.5 \end{bmatrix}, B_2(2) = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, C_2(2) = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$A_3(1) = \begin{bmatrix} -1 & 0 \\ -0.1 & 0.1 \end{bmatrix}, B_3(1) = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, C_3(1) = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$A_3(2) = \begin{bmatrix} -0.2 & 0 \\ 0.1 & -0.2 \end{bmatrix}, B_3(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_3(2) = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$E_i(1) = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, L_i(1) = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, H_i(1) = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$
$$E_i(2) = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, L_i(2) = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, H_i(2) = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

and $G_i(\upsilon_i) = 0.1$, $D_i(\upsilon_i) = 0.1$ for $\upsilon_i = 1, 2$, $i = 1, 2, 3$. The weighting matrices in the cost functional are given by

$$Q_i(\upsilon_i) = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}, \quad R_i(\upsilon_i) = 0.01.$$

The initial condition of the system is assumed to be

$$x_{10} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}, \quad x_{20} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}, \quad x_{30} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$
$$\eta_{10} = \eta_{20} = \eta_{30} = 1.$$

The initial state condition of the controller is set to zero.

In this example, we assume that subsystems $\mathcal{S}_2$ and $\mathcal{S}_3$ switch from one mode to another synchronously, and therefore they are governed by a common switching process. However, subsystem $\mathcal{S}_1$ has its own operation regime. According to this, the operating pattern set of the subsystems is $\{(1,1,1),(1,2,2),(2,1,1),(2,2,2)\}$. That is, the large-scale system has 4 operation modes in total. The mode transition rate matrix of the Markov process $\eta(t)$ is assumed to be

$$\mathbf{Q} = \begin{bmatrix} -2 & 0.5 & 0.1 & 1.4 \\ 0.2 & -0.5 & 0.1 & 0.2 \\ 0.4 & 0.8 & -1.3 & 0.1 \\ 0.1 & 0.3 & 0.2 & -0.6 \end{bmatrix}.$$

Note that despite subsystems $\mathcal{S}_2$ and $\mathcal{S}_3$ switch synchronously, they cannot be aggregated into a larger subsystem, since they do not share their measured outputs. Furthermore, augmenting will create an additional problem in that it will be necessary to ensure the decentralized structure of the respective controllers.

The software we use are the Matlab LMIRank toolbox [5] with the YALMIP interface [3] and the underlying SeDuMi solver [8]. Firstly, we treated the optimization problem in (25) as a additional constraint

$$\sum_{i=1}^{N} \tilde{x}_{i0}^T \left[N_i^T P_i(\eta_{i0}) N_i + \tau_i \bar{S}_i + \theta_i \tilde{S}_i \right] \tilde{x}_{i0} \leq \gamma.$$

Secondly, let $\gamma = 60$ and $\beta_{ji}(\mu) = 1$ for $j = 1, \ldots, 4$, $i = 1, \ldots, 3$, $\mu = 1, \ldots, 4$. Thirdly, we solved (17)–(22) and the above additional constraint. Fourthly, we solved (23) and (28)–(31). Finally, we obtained a local mode dependent controller using (26). Surprisingly, the obtained controller had very small values of $B_{K,i}(\upsilon_i)$ and $C_{K,i}(\upsilon_i)$, which indicates that these controllers can be approximated by static output controllers $u_i(t) = D_{K,i}(\eta_i(t)) y_i(t)$. Hence we only provide the values of the static gains $D_{K,i}(\upsilon_i)$:

$$\begin{aligned} D_{K,1}(1) &= -6.9814, & D_{K,1}(2) &= -3.7084, \\ D_{K,2}(1) &= -6.4037, & D_{K,2}(2) &= -3.9084, \\ D_{K,3}(1) &= -3.2025, & D_{K,3}(2) &= -1.7732. \end{aligned}$$

We now present some simulations to illustrate properties of the resulting local mode dependent static output feedback controllers. In our simulations, the admissible uncertainties were chosen to have the form, for $i = 1, 2, 3$,

$$\xi_i(t) = \zeta_i(t), \qquad r_i(t) = -\sum_{j=1, j\neq i}^{3} \zeta_j(t).$$

The reason for this particular uncertainty choice is as follows. With these particular uncertainties and the controller designed using our approach, the stability properties of the open-loop and the closed-loop large-scale systems are easy to verify by substituting the uncertainties into the system and forming a corresponding Markovian jump linear system. It follows that the open-loop system with $u_i(t) \equiv 0$ was not stochastically stable while the closed-loop system was found to be stochastically stable; this confirms the result of Theorem 3.

For a comparison, the algorithm given in [2] was used to design a optimal worst-case global mode dependent output feedback controller. Firstly, the system (1) with the cost functional (5) was regarded as a special class of the system (6) with the cost functional (7), which were studied in [2]. Then following the design procedure in [2], it was found that a optimal global mode dependent controller could be designed which yields a upper bound of 3.4 for the cost functional (7). Simulations showed that in practice our local mode dependent controller may have better transient performance as shown in Figure 1 at least for some uncertainties, although it is not guaranteed to hold for all uncertainties. After calculating the sample of the cost functional along the system trajectory over the first 15 seconds, it was found that for our controller the sample cost was equal to 0.9814, while the optimal controller based on the results of [2] produced a sample cost of 0.9819.

V. Conclusions

This paper has studied the decentralized output feedback guaranteed cost control problem for a class of uncertain Markovian jump large-scale systems. The controllers are entirely decentralized with respect to the subsystems. They use local system states and local operation modes of the subsystems to produce the local control inputs. A sufficient condition in terms of rank constrained LMIs has been developed to construct such controllers. Also, the developed theory has been illustrated by a numerical example and simulations.

References

[1] T. Iwasaki and R. E. Skelton, "All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas," *Automatica*, vol. 30, no. 8, pp. 1307–1317, 1994.

[2] L. Li, V. A. Ugrinovskii, and R. Orsi, "Decentralized robust control of uncertain Markov jump parameter systems via output feedback," *Automatica*, vol. 43, no. 11, pp. 1932–1944, 2007.

[3] J. Löfberg, "YALMIP : A toolbox for modeling and optimization in MATLAB," in *Proceedings of the CACSD Conference*, 2004. [Online]. Available: http://control.ee.ethz.ch/ joloef/yalmip.php

[4] M. S. Mahmoud, P. Shi, and Y. Shi, "H_∞ and robust control of interconnected systems with Markovian jump parameters," *Discrete and Continuous Dynamical Systems — Series B*, vol. 5, no. 2, pp. 365–384, 2005.

[5] R. Orsi, "LMIRank : software for rank constrained LMI problems," 2005. [Online]. Available: http://users.rsise.anu.edu.au/~robert/lmirank/

[6] R. Orsi, U. Helmke, and J. B. Moore, "A Newton-like method for solving rank constrained linear matrix inequalities," *Automatica*, vol. 42, no. 11, pp. 1875–1882, 2006.

[7] I. R. Petersen, V. A. Ugrinovskii, and A. V. Savkin, *Robust Control Design Using H_∞ Methods*. Springer-Verlag, 2000.

[8] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimization Methods and Software*, vol. 11–12, pp. 625–653, 1999. [Online]. Available: http://sedumi.mcmaster.ca/

[9] V. A. Ugrinovskii and H. R. Pota, "Decentralized control of power systems via robust control of uncertain Markov jump parameter systems," *International Journal of Control*, vol. 78, no. 9, pp. 662–677, 2005.

[10] J. Xiong, V. A. Ugrinovskii, and I. R. Petersen, "Local mode dependent decentralized control of uncertain Markovian jump large-scale systems," in *Proceedings of the Conference on Decision and Control*, 2008, pp. 4345–4350.

[11] L. Zhang, B. Huang, and J. Lam, "H_∞ model reduction of Markovian jump linear systems," *Systems & Control Letters*, vol. 50, no. 2, pp. 103–118, 2003.

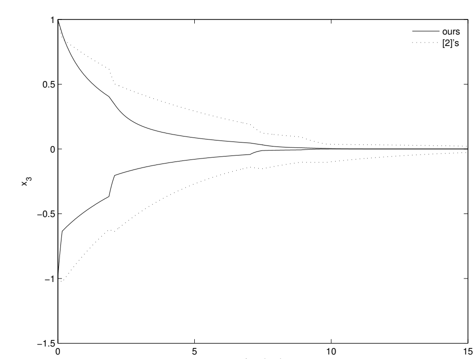

Fig. 1. Initial condition response of x_3