

# Path Following of Underactuated Surface Ships

Renxiang Bu, Zhengjiang Liu, Tieshan Li, and Jiangqiang Hu

**Abstract**—The path following problem is concerned for conventional surface ships with second order nonholonomic constraints. A nonlinear feedback algorithm is presented using decoupling control method. The cross track error and heading error are stabilized by means of the rudder alone and the thruster is left to adjust the forward speed. The underactuated following control objective is achieved without a reference orientation generated by a ship model. The estimation of systemic uncertainties and disturbances and the yaw velocity PE (persistent excitation) conditions are not required. Computer simulation results on a full nonlinear hydrodynamic ship model of M.V. YULONG are provided to validate the effectiveness and robustness of the proposed controller.

## I. INTRODUCTION

In recent years, voluminous literature have been presented on the subject of designing trajectory-tracking and/or path-following controllers for underactuated ships (see, e.g., [1]-[6], [8]-[18] and references therein). The trajectory-tracking control problem refers to the case where the vessel must track a reference trajectory generated by a suitable virtual vehicle, and the objective of path following is to force the vessel to follow a given path with a desired forward speed [1]. The underlying difficulties for both problems are that surface ships have fewer actuators than degrees of freedom to be controlled, the constraint on the acceleration is nonintegrable [2] [3], and the system is not transformable into a system without drifts [4]-[6].

To develop an automatic control system for underactuated surface ships, several problems must be solved. Among them the most difficult and challenging is the trajectory or motion planning since the nonholonomic systems cannot track arbitrary trajectories [7]. The uncertain dynamics and drift caused by unknown ocean current make this problem even intractable.

Because of the nonintegrable constraint on accelerations and sideslip, the ship's orientation cannot be regulated to a nominal equilibrium by coordinates transformation for the sake of the path following objective. It must be compensated by a loxodrome, i.e., to maintain a deliberate deviation angle known as "drift angle and leeway", which is, in practice, only available by a trial and error procedure because of the lack of full knowledge about the system and environment. This means that the equilibrium point of the system is not at the origin of transformed coordinates but a drifting point when the wind and current is time and regional variant. In

[8] and [9], integral action is added to the controller by means of a parameter adaptation technique. By introducing sideslip compensation and a dynamic controller state, the results are extended to underactuated vessels [9]. But the desired heading angle must be computed and higher order derivatives need to be generated by a reference model.

However, a firmly established mathematical model of ship maneuvering motions is still not available although studies have been carried out in the last twenty years. The reason is that ship's motions are very complicated, and environmental disturbance such as shallow water and bank effects, wind and currents may have very strong effects on the ship's maneuverability. And when the current is time and regional variant, the ship's dynamics become more complex because the forces and moments involve explicitly current disturbances which are unmeasurable in practice. So that, the hydrodynamic model of a surface ship is highly nonlinear, complex and uncertain, it is infeasible to generate the reference course by an accurate model.

In literature, several methods have also been proposed to deal with the uncertainties of system and external perturbations. Recently, output-feedback trajectory tracking control and stabilization of an underactuated omni-directional intelligent navigator were addressed in [10]. An output-feedback controller for trajectory tracking of underactuated ships was proposed in [11] where a coordinate transformation was introduced to transform the ship dynamics to a system with linear unmeasured velocities. In the above papers, the mass and damping matrices of the ships are assumed to be diagonal, and the nonlinear damping terms are also ignored. Recently, a full state-feedback solution was obtained in [12] which removed the above assumptions but the nonlinear damping terms cannot be included. A continuous time-varying tracking controller is designed by [13] in presence of uncertainty in the hydrodynamic damping coefficients. In [1], a global controller was presented without velocity measurements for feedback, adaptive observer was used to estimate the inaccuracies, and integral actions are added to the controller to compensate for a constant bias of environmental disturbances.

The methodologies for trajectory-tracking and/or path-following in present literature have many connections and all rely on a precise or simplified mathematical model. The saturations and mechanical characteristics on/of actuators are seldom explicitly involved. The control forces are usually calculated based on a precise mathematic model of ship's kinematics and dynamics, which is, as mentioned before, usually unavailable.

In order to avoid the need of explicit knowledge of the

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detailed ship dynamics, application of techniques of neural network [14], fuzzy logic control [15] and other Artificial Intelligence (AI) were also investigated in recent years. But a prior knowledge or a proper treatment is still needed to solve the nonholonomic control problem and the performance of these controllers may rely on a significant amount of experimentation or the expert knowledge of the dynamics of the vessels under consideration.

However, in most of present works, the uncertainty of external perturbation of the ocean current which is not negligible was seldom explicitly involved. In [16], base on feedback linearization and backstepping technique, a control algorithm was developed with an estimation of the uncertain constant ocean current with a known direction to track both line and circumference. A similar estimation of the uncertain constant ocean current is carried out in [17] with relaxed assumptions. However, the assumption and precondition of prior knowledge of current's direction is very restrictive from a practical point of view since the ocean current is time and regional variant.

In the previous study [18], we proposed a iterative non-linear sliding mode (INSM) geometrical path following controller with the control input of ruder deflection alone for an underactuated surface ship under uncertain perturbation of the ocean current. The main thruster was left free. And the reference course generated by accurate model is not needed. Since the cross track error and heading error can be stabilized by means of the rudder alone and a known or desired forward speed is not required, it is possible to adjust the forward speed online by means of the thruster. In the present study, the INSM integrated with simple increment feedback control method is extended to path following and speed adjusting using decoupling control method.

The paper is organized as follows. Section II introduces the marine surface ship model and assumptions. Section III describes a solution to the problem of path following of an underactuated ship with uncertain dynamics under unknown exogenous disturbances. Results of simulations are given in section IV. section V contains the main conclusions and section VI gives the acknowledgements.

## II. PROBLEM FORMULATION

In this section, we give the problem formulation and some assumptions we need in controller design.

### A. Model of Ship's Motion

The kinematics (see Fig.1) and dynamics (MMG model) of an underactuated ship moving in surge, sway, and yaw in the earth-fixed and ship-fixed frames can be described as

$$\begin{cases} \dot{x} = u \cos \varphi - v \sin \varphi + u_c \cos \varphi_c \\ \dot{y} = u \sin \varphi + v \cos \varphi + u_c \sin \varphi_c \\ \dot{\varphi} = r \\ (m + m_x)\dot{u} - (m + m_y)vr = X_S + X_P + X_R + X_E \\ (m + m_y)\dot{v} + (m + m_x)ur = Y_S + Y_P + Y_R + Y_E \\ (I_{zz} + J_{zz})\dot{r} = N_S + N_P + N_R + N_E \end{cases}, \quad (1)$$

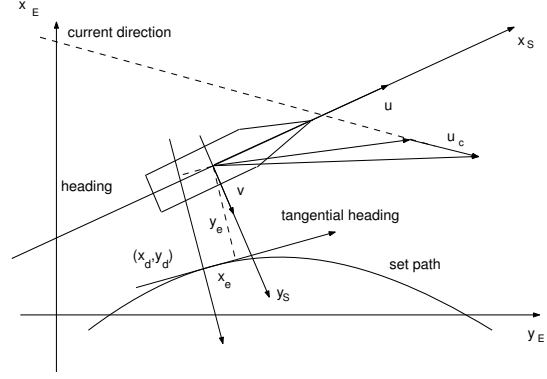


Fig. 1. Interpretation of kinematics and path following errors

where  $x$ ,  $y$  and  $\varphi$  are the surge displacement, sway displacement and heading angle in the earth-fixed frame,  $u$ ,  $v$  and  $r$  denote surge, sway velocities through water and yaw velocity in the ship-fixed frame respectively,  $u_c$  and  $\varphi_c$  denote speed and direction of current in the earth fixed frame,  $m$ ,  $m_x$ ,  $m_y$ ,  $I_{zz}$  and  $J_{zz}$  denote the ship's inertia, added mass and added moment of inertia,  $X$ ,  $Y$ ,  $N$  terms with subscripts  $S$ ,  $P$ ,  $R$ ,  $E$  respectively are forces in longitudinal and lateral directions and moments induced by hydrodynamic damping, propeller(s), rudder(s) and external effects.

As mentioned in section I, the hydrodynamic models of a surface ship are highly nonlinear, complex and uncertain. It is infeasible to calculate the control forces based on a precise mathematic model of ship's kinematics and dynamics. To design a robust controller capable of dealing with the saturations and mechanical characteristics on/of actuators, the algorithm should not rely on a fixed model. For Details of ship's dynamic model, readers are referred to the references relevant to this topic.

### B. Control Objective and Assumptions

Our goal is to design a robust controller which can force an underactuated surface ship to follow a desired path with the only control input of rudder angle and adjust its forward speed by tuning the revolution rate of her main thruster. The prior knowledge required is not more than a surface ships' basic steering feature.

*Assumption 1:* The set path is feasible for forward following and the path-following errors are measurable.

*Assumption 2:* The ship's dynamics and desired accelerations are assumed to be bounded and differentiable.

*Assumption 3:* The rudder induced forces  $X_R$ ,  $Y_R$  and their partial derivatives with respect to rudder angle  $\delta$  are trivial compared to that of yawing moment  $N_R$ . And  $N_R$  is a monotonic function of  $\delta$ .

*Assumption 4:* The thruster induced forces  $Y_P$ ,  $N_P$  and their partial derivatives with respect to thruster revolution rate  $n$  are trivial compared to longitudinal thrust  $X_P$ . And  $X_P$  is a monotonic function of  $n$ .

### III. CONTROLLER DESIGN

#### A. Model of Path-Following Errors

We interpret the path-following errors in a frame attached to the path as follows (see Fig. 1):

$$\begin{cases} \varphi_r = \text{atan2}[y'_d(t), x'_d(t)] \\ \theta_r = \text{atan2}[y(t) - y_d(t), x(t) - x_d(t)] \\ \rho_r = \sqrt{[y(t) - y_d(t)]^2 + [x(t) - x_d(t)]^2} \\ x_e = \rho_r \cos(\theta_r - \varphi_r) \\ y_e = \rho_r \sin(\theta_r - \varphi_r) \\ \varphi_e = \varphi - \varphi_r \end{cases}, \quad (2)$$

where  $x_d(t)$ ,  $y_d(t)$  and  $\varphi_r$  are the desired surge displacement, sway displacement and tangential heading angle to the path in the earth-fixed frame,  $\theta_r$ ,  $\rho_r$  denote the azimuth angle and the radial coordinate in a polar coordinate attached to the path such that the polar axe is parallel to the tangential heading, and therefore,  $x_e$ ,  $y_e$ , and  $\varphi_e$  can be referred to as tangential, cross and heading errors respectively.

#### B. Nonlinear Sliding Mode Schemes

For system (1) with aforementioned uncertainties and constraints, and path-following errors defined in (2), our dynamic nonlinear sliding mode controller is designed using decoupling control method. The cross track error  $y_e$  and heading error  $\varphi_e$  can be stabilized by means of the rudder alone [18], and the thruster is left to adjust the speed and fulfill the control objective of stabilization of tangential track error.

Firstly, to stabilize the cross track error  $y_e$  and heading error  $\varphi_e$ , nonlinear sliding surfaces are designed as

$$\begin{cases} \sigma_1^1(y_e) = k_1^1 \tanh(k_0^1 y_e) + y_e \\ \sigma_2^1(\sigma_1^1, \varphi_e) = \varphi_e + k_2^1 \int \tanh(\sigma_1^1) dt \\ \sigma_3^1(\sigma_2^1) = k_3^1 \tanh(\sigma_2^1) + \dot{\sigma}_2^1 \\ \sigma_4^1(\sigma_3^1) = k_4^1 \tanh(\sigma_3^1) + \dot{\sigma}_3^1 \end{cases} \quad (3)$$

where  $k_0^1, k_1^1, k_2^1, k_3^1, k_4^1 \in \mathbb{R}^+$ . Thus the control objective of cross track error and heading error are iteratively transformed into stabilization of  $\sigma_4^1$ .

*Proof:*

From the definition of  $\sigma_4^1$ ,  $\sigma_3^1$  and  $\sigma_2^1$  we can easily conclude

$$\sigma_4^1(\sigma_3^1) \rightarrow 0, \quad \dot{\sigma}_3^1 \rightarrow -k_4^1 \tanh(\sigma_3^1) \quad (4)$$

and

$$\sigma_3^1(\sigma_2^1) \rightarrow 0, \quad \dot{\sigma}_2^1 \rightarrow -k_3^1 \tanh(\sigma_2^1) \quad (5)$$

and

$$\sigma_2^1(\sigma_1^1, \varphi_e) \rightarrow 0, \quad \varphi_e \rightarrow -k_2^1 \int \tanh(\sigma_1^1) dt \quad (6)$$

Consider (1), (2) and (3) we can get

$$\sigma_1^1(y_e) = k_1^1 \tanh(k_0^1 y_e) + u \sin \varphi_e + v \cos \varphi_e + u_c \sin(\varphi_c - \varphi_r) \quad (7)$$

In navigational context, if the ship is proceeding forward ( $|\varphi_e| \ll \pi/2$  and  $u > u_c \gg v$ ), and thanks to the boundedness of functions of hyperbola tangent, sine, and cosine, we know

that there exists a  $\varphi_e^*(t) \in (-\pi/2, \pi/2)$  which satisfying  $\sigma_1^1 = 0$ , and

$$\frac{\partial \sigma_1^1}{\partial \varphi_e} = u \cos \varphi_e - v \sin \varphi_e > 0. \quad (8)$$

From (6), we can get

$$\dot{\varphi}_e \rightarrow -k_2^1 \tanh(\sigma_1^1). \quad (9)$$

Suppose at  $t = t_1$ ,  $\sigma_1^1 > 0$ , thanks to (8) we can draw a conclusion that  $\varphi_e^*(t_1) < \varphi_e(t_1)$ . From (9) we know that the heading error is continuously decreasing owing to the monotony of hyperbolic tangent function, Since the kinematics are smooth and  $\varphi_e^*(t)$  is bounded, if  $\varphi_e^*(t)$  is periodical, there must exist  $t = t_2, t = t_3, \dots$ , at which  $\varphi_e = \varphi_e^*$ , and  $\sigma_1^1(y_e) = 0$ , so a practical stability can be achieved. Particularly, if  $\varphi_e^*(t) \rightarrow \alpha_c$  as  $t \rightarrow \infty$ , where  $\alpha_c$  is a constant loxodrome, we can conclude that  $\varphi_e(t) \rightarrow \alpha_c$ , then

$$\dot{y}_e \rightarrow -k_1 \tanh(k_0^1 y_e) \quad (10)$$

which means the cross track error exponentially converges to zero with an maximum converging rate determined by  $k_1^1$ . This is possible in a straight path following under constant disturbances or a circumference following without disturbance (both with fixed forward speed).

*Remark 1:* Form the recursive designing procedure including coordinates transformation, we can see that it is favorable to select parameters satisfying following inequalities:

$$k_0^1 k_1^1 \leq k_2^1 \leq k_3^1 \leq k_4^1. \quad (11)$$

These parameters can be easily valued according to ship's maneuverability because of their clear functional meaning. For example,  $k_2^1$  denotes the upper bound of yawing rate the ship is forced to follow.

*Remark 2:* Note that the ship's dynamics involve Coriolis Force since the reference point  $(x_d, y_d)$  is a floating point (see Fig.1) and the transformed coordinate is not a inertial frame. The aims of application of iterative nonlinear sliding mode are to guarantee the invariability of the controller and to avoid uncertainties estimation, see next subsection.

*Remark 3:* To prevent ships from steering a wrong (backward) way, the integration in (3) can be saturated with a  $\alpha_{\max}$  (the maximum of leeway or drift angle) evaluated according to ship's navigational and hydrometeorological conditions.

Secondly, to stabilize tangential following error  $x_e$  using thruster, nonlinear sliding surfaces are decentralized and designed in a similar procedure as

$$\begin{cases} \sigma_1^2(x_e) = k_1^2 \tanh(k_2^2 x_e) + x_e \\ \sigma_2^2(\sigma_1^2) = k_3^2 \tanh(k_4^2 \sigma_1^2) + \dot{\sigma}_1^2 \end{cases}, \quad (12)$$

where  $k_1^2, k_2^2, k_3^2, k_4^2 \in \mathbb{R}^+$ . Thus the control objective of  $x_e$  is transformed into stabilization of  $\sigma_2^2$ .

### C. Feed Back Control Laws

To stabilize  $\sigma_4^1$  and  $\sigma_2^2$ , robust control laws are needed because of the ship's complicated and uncertain dynamics and exogenous disturbances. We employ simple increment feedback control laws

$$\begin{cases} \dot{\delta} = -k_5^1 \sigma_4^1 + k_6^1 \text{sgn}(\sigma_4^1) \\ \dot{n} = -k_5^2 \sigma_2^2 + k_6^2 \text{sgn}(\sigma_2^2) \end{cases}, \quad (13)$$

where  $k_5^1, k_6^1, k_5^2, k_6^2 \in \mathbb{R}^+$ . Without uncertainties estimation, the control laws in (13) can asymptotically stabilize  $\sigma_4^1$  and  $\sigma_2^2$ .

*Proof for stability of  $\sigma_4^1$ :*

Expanding  $\sigma_4^1$  yields

$$\begin{aligned} \sigma_4^1 &= k_4^1 \tanh(\sigma_3^1) + \dot{r} - \dot{\varphi}_r + \\ &k_3^1 (r - \dot{\varphi}_r + k_2^1 \tanh(\sigma_1^1)) / (\cosh(\sigma_2^1))^2 + \\ &k_2^1 ((k_1^1 k_0^1 \dot{y}_e / (\cosh(k_0^1 y_e^1))^2 + \dot{y}_e) / (\cosh(\sigma_1^1))^2). \end{aligned} \quad (14)$$

Considering (1), (2) and (14), we can get

$$\begin{aligned} \frac{\partial \sigma_4^1}{\partial \delta} &= \frac{\partial \dot{r}}{\partial \delta} + \frac{\partial}{\partial \delta} (k_2^1 \dot{y}_e / (\cosh(\sigma_1^1))^2) \\ &= \frac{\partial N_R}{\partial \delta} / (I_{ZZ} + J_{ZZ}) + \\ &k_2^1 \frac{\partial X_R}{\partial \delta} \sin \varphi_e / (m + m_x) / (\cosh(\sigma_1^1))^2 + \\ &k_2^1 \frac{\partial Y_R}{\partial \delta} \cos \varphi_e / (m + m_y) / (\cosh(\sigma_1^1))^2. \end{aligned} \quad (15)$$

From assumption 3 it can be concluded that

$$\frac{\partial \sigma_4^1}{\partial \delta} > 0. \quad (16)$$

Since the dynamics and disturbance are bounded and continuous, when the parameters  $k_i^1$  ( $i=0, 1, 2, 3, 4$ ) of controller are properly selected, form (1), (2) and (14), we can conclude that there must exists a  $\delta^*(t) \in [-\delta_{max}, \delta_{max}]$  which satisfies  $\sigma_4^1(t) = 0$ . (Note that  $k_i^1 \rightarrow 0$  means  $\sigma_4^1 \rightarrow \dot{r} - \dot{\varphi}_r$ , if such a  $\delta^*(t)$  does not exist, the closed-loop system must be uncontrollable, the reason may be that the desired path is infeasible.)

Thus a proof of the closed-loop system's stability similar with that of  $\sigma_1^1(y_e)$  in last subsection can be given. The function of parameter  $k_6^1$  in (13) is to guarantee the asymptotical stability of  $\sigma_4^1(t)$  under oscillatory disturbances.

*Proof for stability of  $\sigma_2^2$ :*

Expanding  $\sigma_2^2$  yields

$$\sigma_2^2(\sigma_1^2) = k_3^2 \tanh(k_4^2 \sigma_1^2) + k_1^2 k_2^2 \dot{x}_e / \cosh^2(\sigma_1^2) + \dot{x}_e. \quad (17)$$

Consider (1), (2) and (17), we can get

$$\frac{\partial \sigma_2^2}{\partial n} = \frac{\partial \dot{x}_e}{\partial n} = \frac{\partial}{\partial n} (\dot{u} \cos \varphi_e - \dot{v} \sin \varphi_e). \quad (18)$$

According to assumption 3, the following approximation holds

$$\frac{\partial \sigma_2^2}{\partial n} \approx \frac{\partial}{\partial n} \dot{u} \cos \varphi_e = \frac{\partial X_P}{\partial n} (\cos \varphi_e) / (m + m_x). \quad (19)$$

Since the ship's inertia and added mass are positive definite, so we can conclude

$$\frac{\partial \sigma_2^2}{\partial n} > 0. \quad (20)$$

Thanks to the boundedness of hyperbolic tangent function and its derivative in (17), there exists a  $n^*(t) \in [-n_{max}, n_{max}]$  which satisfying  $\sigma_2^2 = 0$  if the parameters  $k_1^2, k_2^2, k_3^2$  and  $k_4^2$  are properly selected, where  $n_{max}$  denotes maximum rotates rate of the ship's main engine. Thus a proof of the stability of  $\sigma_2^2$  and  $x_e$  similar with that of  $\sigma_4^1$  and  $y_e$  can be given.

*Remark 4:* Theoretically speaking, the larger the increment feedback gains are valued, the better the controller performs. However, there exist operational limits on actuators determined by their mechanical nature. Moreover, due to the time lags during the transmission of measure and control signals, an excess of feedback gain may cause over-sensitiveness and oscillating of the actuator(s).

*Remark 5:* High frequency (HF) components of disturbances can cause unnecessary wear and tear of the actuator(s) and must be removed by lowpass filter from the vessel measurements before they can enter the control loop [19]. In the present study, the necessary filtering of HF is assumed to have been taken care of in the output measurements.

## IV. SIMULATION RESULTS

To demonstrate the practicality of our designing, we performed some simulations using a full nonlinear dynamic model of an underactuated marine surface vessel. The vessel data come from a training ship "YULONG", which has a displacement of 14635 tons at full loaded condition, a length overall of 139.8m, a molded breadth of 20.8m and a block coefficient of 0.681. Details of ship's data and dynamic model are given in [20] and references therein.

### A. Circumference Following

In circumference following simulation case 1 and 2, the reference path was chosen as a circle centered at (0, 0) with a radius of 500 m. The initial vessel states were chosen to be  $x=0\text{m}$ ,  $y=-550\text{m}$ ,  $u=3\text{m/s}$  and  $\varphi = 020^\circ$ , the ship's forward speed was required to be adjusted on line. The rotate rate of main engine was set to be 120 rpm (rotates per minute). The controller gains were chosen as  $k_0^1=0.01$ ,  $k_1^1=1.5$ ,  $k_2^1=0.02$ ,  $k_3^1=0.02$ ,  $k_4^1=0.02$ ,  $k_5^1=100$ ,  $k_6^1=0$ ,  $k_1^2=1$ ,  $k_2^2=0.01$ ,  $k_3^2=0.1$ ,  $k_4^2=1$ ,  $k_5^2=1$ , and  $k_6^2=0$ . Simulation case 1 was carried out with environmental disturbances which were set as a  $240^\circ$ -going sinusoidal current (amplitude 1kn, bias 1kn, and period 12h.) and a  $90^\circ$ -coming sinusoidal wind (amplitude 10m/s, bias 10m/s, and period 1min). In simulation case 2, circumference following was carried out without disturbance. The planar trajectories are shown in Fig.2 and Fig.3 respectively, where dashed lines are desired paths and markers "o" denote the desired positions at time 0s, 200s, 400s, 600s, 800s and 1000s respectively. Fig.4 and Fig.5 demonstrate outputs and inputs of simulation results.

Good performance can be seen in both of simulation case 1 and 2. The similar planar trajectories illuminate the invariability and robustness of the proposed controller.

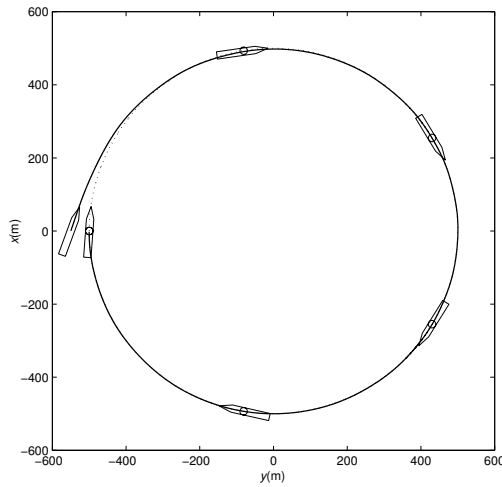


Fig. 2. Simulation case 1: circumference following with disturbances

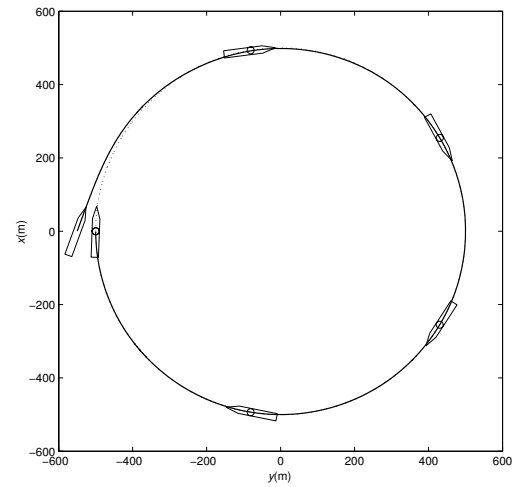


Fig. 3. Simulation case 2: circumference following without disturbance

### B. Sinusoidal Reference Following

Sinusoidal reference following was carried out in simulation case 3. The reference path was chosen as  $x_d(t) = 3t + \sin(0.001\pi t)$ ,  $y_d(t) = 200\sin(0.001\pi t)$ . The initial vessel states were chosen to be  $x=0$ ,  $y=100\text{m}$ ,  $u=2\text{m/s}$  and  $\varphi = 000^\circ$ . The rotate rate of main engine was set to be 120 rpm. The controller gains are chosen as the same as that in simulation case 1 and 2. Environmental disturbances are set as a  $210^\circ$ -going sinusoidal current (amplitude 2kn, bias 2kn, and period 12h.) and a  $150^\circ$ -coming sinusoidal wind (amplitude 10m/s, bias 10m/s, and period 1min). Fig.6 shows the planar trajectory, where dashed line is the desired path and markers "o" denote the desired positions at time 0s, 160s, 320s, 480s, 640s and 800s respectively. The outputs and inputs are given in Fig.7.

The simulation results of sinusoidal reference following demonstrate again the effectiveness of the proposed controller. However, as mentioned in section III, since the loxodrome must be periodical in a zigzag maneuver, and even a slow-varying or constant bias of the current or wind can cause oscillatory disturbances, in such a case, only practical stability can be achieved in the present study.

## V. CONCLUSIONS AND FUTURE WORKS

### A. Conclusions

In the present study, we extend the previous geometrical path following to speed adjusting. The underactuated ship path following objectives were achieved by means of decoupling control method. The cross following error and heading error can be stabilized by the input of rudder angle alone. The forward speed can be adjusted on line. And the reference course generated by an accurate model is not needed. The practicality lies in that the complex and unnecessary estimation of system uncertainties and environmental disturbances is left out. Simulation results successfully demonstrate the capability of the proposed control strategy.

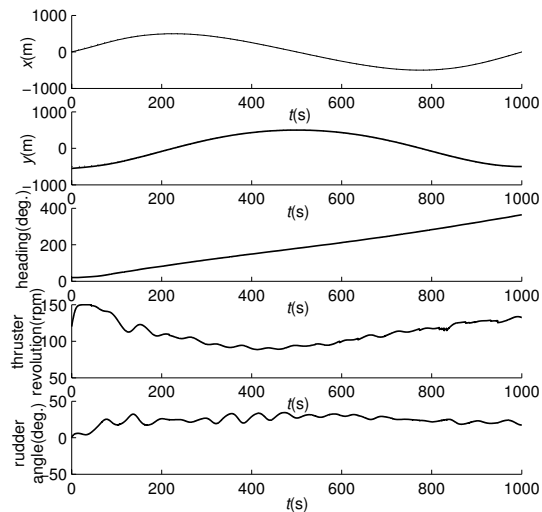


Fig. 4. Outputs and inputs of simulation case 1

### B. Future Works

It's an ideal case that the measurements of the ship velocities and accelerations are available. Since measurements of the position are often corrupted by noise, to overcome the shortcomings resulted from numerical differential action, nonlinear sliding surfaces and integral actions are added to the controller. An observer based output-feedback controller will be studied further.

The capabilities of the iterated sliding mode design will be exploited in underactuated ship stabilization. Future works also include controller parameters optimizing and intelligentizing.

## VI. ACKNOWLEDGMENTS

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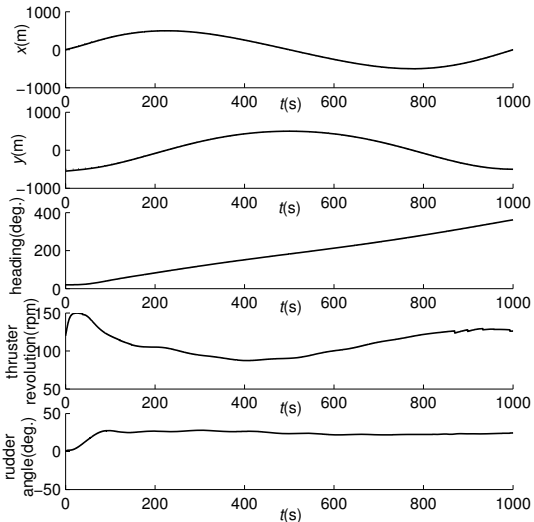


Fig. 5. Outputs and inputs of simulation case 2

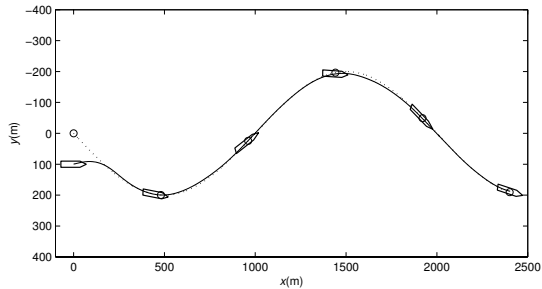


Fig. 6. Simulation case 3: sinusoidal reference following with disturbances

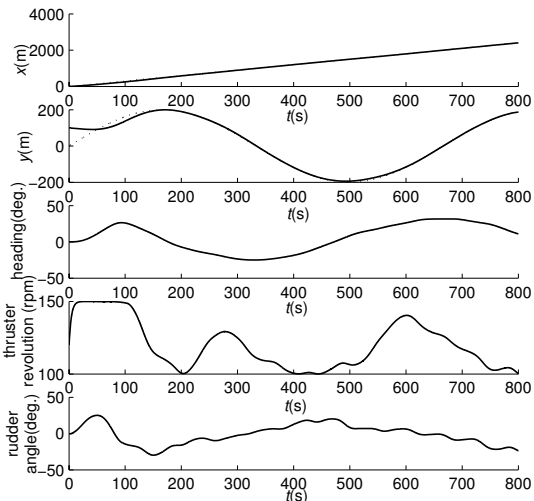


Fig. 7. Outputs and inputs of simulation case 3

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