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Stochastic Analysis and Stabilization of Remote Control Systems

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Abstract— **This paper mainly investigated a sampled-data control approach to deal with the stabilization problem of Networked Control Systems (NCSs) with packet losses and bounded time varying delays. A new Lyapunov-Krasovskii functional candidate is constructed to analyze the stability of the overall system with bounded random packet losses and time varying delays. As a result, corresponding stabilizing sampled-data controller is designed based on the stability conditions. A real-time network measurement system has been developed based on MATLAB applications. Instrument Control toolbox was used to implement communications between two computers with MATLAB applications via the internet. Experiments were done to demonstrate the real network properties. A real-time networked control system has been constructed to test the stabilizing ability of the controller design in a real network environment. Experimental results illustrate the effectiveness of the proposed approach, a good combination of the theory and the real applications.**

I. INTRODUCTION

NCSs are feedback control systems with control loops closed via digital communication channels. Advantages of NCSs include low cost, high reliability, less wiring, easy system set-up and maintenance [1]. The study of NCSs raises new interesting and challenging problems such as time delays, packet losses and communication bandwidth: [3]-[8].

In [3], the issue of data packet loss is modelled as a Markovian process, it dealt with the delay which is less than one sampling time interval. In [1], the maximum packet-loss rate under which the overall system remains stable was investigated. In [5], the NCS has been formulated as a Markovian jump system with known packet loss rate, the techniques developed for Markovian jump systems are applied in the work. [4] presented a solution to stabilization of NCSs with the effect of one sampling delay and arbitrary switching packets dropout. In [8], the packet loss process has been defined as the sequence of the time intervals between consecutively successfully transmitted data. In their design one-step time delay has been considered, analysis and synthesis methods are provided based on pure discrete-time model. The literatures reviewed above have not pay enough attention to incorporate the real network properties into the works.

In this paper, the effects of both Markovian packet loss and time varying delays occurring in both channels are considered. In many practical systems, such as computer-based control systems, the continuous-time system is controlled by a sampled-data controller with sample and hold devices [12], the control objective of this work is to design a sampled-data controller to stabilize the system via communication channels with Markovian packet losses and bounded time varying delays. A real-time network induced delay and packet loss measurement system was built to study the real network property. With the experimental measurement, we figure out the characters of time varying delays and packet losses in the real network, which was applied to the stochastic stabilization analysis. The delays and packet losses that were measured during the network experiments are firstly replayed for the simulation. A real-time networked control system has been built based on MATLAB application to test the stabilizing ability of the controller. The experimental results illustrate that the controller works well under real situation.

Notations: $\lambda(D)$ denotes the eigenvalue of the matrix D, where $D \in R^{n\times n}$. $E(\cdot)$ is the expectation. $\rho(A) = \sqrt{\lambda_{max}(A^T A)}$.

II. PROBLEM FORMULATION

NCSs with pure Markovian packet-loss are considered first in this section. Specially, a linear continuous-time system is studied,

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t), \tag{1}$$

where $\mathbf{x}(t) \in \Re^n$ and $\mathbf{u}(t) \in \Re^m$ represent the system state and control input, respectively. $\mathbf{x}_0 = \mathbf{x}(0)$ is the initial state. A and B are two known constant matrices of appropriate dimensions, A is invertible.

Let $\ell = \{n_1, n_2, \ldots\}$ be a subsequence of $1, 2, 3, \ldots$, which denote the sequence of time points of successful data transmissions from the sampler to the actuator. $S = \max_{n_J \in \ell}(n_{J+1} - n_J)$ is the maximum packet-loss upper bound. The following mathematical models are first introduced.

Definition 1: Packet-loss process in the communication channel is modelled as

$$\eta(n_J) = n_{J+1} - n_J : n_J \in \ell, \tag{2}$$

which takes values in the finite state space $\zeta = \{1, 2, \ldots S\}$ where S is a positive integer.

Definition 2: [8] Packet-loss process (2) is said to be Markovian if it is a discrete-time homogeneous Markov chain on a complete probability space, and takes values in ζ with known transition probability matrix $\Pi = (\pi_{ij}) \in \Re^{S\times S}$, where

$$\pi_{ij} = Pr(\eta(n_{J+1}) = j|\eta(n_J) = i) \geq 0 \tag{3}$$

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for all $i, j \in \zeta$, and $\sum_{j=1}^{S} \pi_{ij} = 1$ for each $i \in \zeta$.

An illustrative example of data flow with packet loss is shown in Fig.1.

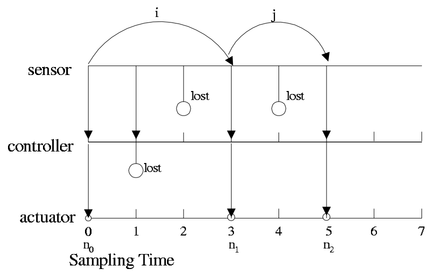

Fig. 1. Data flow diagram of the Markovian Packet loss

Definition 3: For system (1) with Markovian packet-loss process (2), the equilibrium point 0 of $\mathbf{x}$ is stochastically stable if, for every initial state x_0, the following holds:

$$E\{\sum_{J=0}^{\infty} \mathbf{x}^T(n_J)\mathbf{x}(n_J)|\eta(n_0)\} < \infty. \quad (4)$$

Throughout this paper, the sampled-data controller is designed as a state-feedback controller

$$\mathbf{u}(t) = \mathbf{u}(n_J T_s) = K\mathbf{x}(n_J T_s), \quad (5)$$

where $K \in \Re^{m\times n}$ is designed as constant matrix with suitable dimension. The initial control input is set to zero: $\mathbf{u}(0) = 0$. Then the closed-loop system becomes

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + BK\mathbf{x}(n_J T_s), \quad n_J \in \ell. \quad (6)$$

Definition 4: System (1) with Markovian packet-loss process (2) is stochastically stable if, for every initial condition $\mathbf{x}_0$ and $\mathbf{u}_0$, there exists a sampled-data linear feedback control law $\mathbf{u}(t) = \mathbf{u}(n_J T_s) = K\mathbf{x}(n_J T_s)$ such that the closed-loop system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + BK\mathbf{x}(n_J T_s), \quad n_J \in \ell, \quad (7)$$

is stochastically stable.

Lemma 1: - Jensen Inequality [9] For any constant matrix $E \in \mathcal{R}^{n\times n}$, $E = E^T > 0$, vector function $\boldsymbol{\omega} : [0, \tau] \to \mathcal{R}^n$ such that the integrations concerned are well defined, then,

$$\tau \int_0^{\tau} \boldsymbol{\omega}^T(s)E\boldsymbol{\omega}(s)ds \geq \left[\int_0^{\tau} \boldsymbol{\omega}(s)ds\right]^T E \left[\int_0^{\tau} \boldsymbol{\omega}(s)ds\right]. \quad (8)$$

The control objective is to design the controller (5) so that the system (6) with Markovian packet-loss process (2) is stochastically stable.

III. Controller Design and Stability Analysis

A. NCSs with Markovian Packet Losses

Now we analyze the stability property of NCSs. For NCSs with Markovian packet-loss process, the stability condition is established by analyzing the theory from Markovian jump linear systems. The conditions are given in the following theorem.

Theorem 1: Consider the NCS (6) with Markovian packet-loss process (2), $\mathbf{u}(0) = 0$ and $\mathbf{x}(n_0 T_s) = \mathbf{x}(0)$. If there exist symmetric positive definition matrices $P_i = P = P^T$, $i \in \zeta$, matrices $Q > 0$, $S > 0$, X and Y, scalar $\beta > 0$ such that

$$\begin{bmatrix} X^T A^T + AX & BY \\ Y^T B^T & -R \end{bmatrix} < 0, \quad (9)$$

and

$$\begin{bmatrix} -X & * & * & * \\ \Phi & X - G & * & * \\ \Phi & 0 & -X & * \\ X & 0 & 0 & -\beta^{-1}I \end{bmatrix} < 0, \quad (10)$$

with $\Phi = \sum_{i=1}^{S} \pi_{ij} \cdot j \cdot BY + X$, $X = P^{-1}$, $R = X^T SX$, $G = X^T A^{-T} QA^{-1}X$, hold, then the system is stochastically stable with the controller gain designed as $K = YX^{-1}$.

Proof: Given that $\mathbf{x}(n_0 T_s) = \mathbf{x}(0)$ and $n_1 - n_0 = i$, from the system (6) we have

$$\mathbf{x}(n_1 T_s) - \mathbf{x}(n_0 T_s) = \int_{n_0 T_s}^{n_1 T_s} \dot{\mathbf{x}}(s)ds = A\int_{n_0 T_s}^{n_1 T_s} \mathbf{x}(s)ds + BK \cdot \mathbf{x}_0 \cdot iT_s, \quad (11)$$

then we can get the difference between the system state and its expectation as,

$$E[\mathbf{x}(n_J T_s)|n_{J-1} - n_{J-2} = i] - \mathbf{x}(n_{J-1} T_s) = A\int_{n_{J-1} T_s}^{n_J T_s} \mathbf{x}(s)ds + \Sigma_{j=1}^{S} \pi_{ij} \cdot jT_s BK\mathbf{x}(n_{J-1} T_s). \quad (12)$$

Now take the packet-loss dependent Lyapunov functional candidate as

$$V_1(t) = \mathbf{x}^T(t)P_i\mathbf{x}(t) + (n_{J+1}T_s - t)\mathbf{x}^T(n_J T_s) S\mathbf{x}(n_J T_s), \quad (13)$$

where $P_i = P \in \Re^{n\times n}$ are singular positive definition matrices, and $S \in \Re^{n\times n} > 0$. Then the derivative of $V_1(t)$ becomes

$$\dot{V}_1(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}(n_J T_s) \end{bmatrix}^T \begin{bmatrix} A^T P_i + P_i A & P_i BK \\ K^T B^T P_i & -S \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}(n_J T_s) \end{bmatrix} < 0. \quad (14)$$

The inequality above holds if the following inequality is satisfied:

$$\begin{bmatrix} A^T P_i + P_i A & P_i BK \\ K^T B^T P_i & -S \end{bmatrix} < 0, \quad (15)$$

for symmetric positive definite matrices $P_i = P, P = P^T$. Define $X = P^{-1}$, $M = diag(X, X)$ and $Y = KX$. Then by pre-multiplying the inequality in (15) by M^T and post-multiplying by M, we can obtain the following inequality,

$$\begin{bmatrix} XA^T + AX & BY \\ Y^T B^T & -R \end{bmatrix} < 0, \quad (16)$$

where $R = X^T SX$. If (16) holds, then the closed loop system (6) is asymptotically stable.

In order to achieve the condition for stochastic stability of the closed loop system (6), here take another packet-loss dependent Lyapunov function as

$$V_2(t) = \mathbf{x}^T(t)P_i\mathbf{x}(t) + (n_{J+1}T_s - t)\int_{n_J T_s}^{n_{J+1}T_s} \mathbf{x}^T(s)Q\mathbf{x}(s)ds, \quad (17)$$

where P_i are the same matrices as in $V_1(t)$, and $Q \in \Re^{n\times n}$ is a symmetric positive definiton matrix. Then from (12) and Lemma 1, if the following inequality holds,

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}^T \begin{bmatrix} \Delta + \beta I & \Xi^T P_j A \\ A^T P_j \Xi & A^T P_j A - Q \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} < 0, \quad (18)$$

where

$$\begin{aligned} \Xi &= \sum_{j=1}^{S} \pi_{ij}\cdot j \cdot BK + I \\ \Delta &= [\sum_{j=1}^{S} \pi_{ij}\cdot j \cdot BK + I]^T P_j [\sum_{j=1}^{S} \pi_{ij}\cdot j \cdot BK + I] \\ &\quad -P_i \\ Z_1 &= \mathbf{x}(n_J T_s), \quad Z_2 = [\int_{n_J T_s}^{n_{J+1}T_s} \mathbf{x}(s)ds], \end{aligned} \quad (19)$$

then it means

$$E[V_2(n_{J+1}T_s)|n_J - n_{J-1} = i] - V_2(n_J) < -\beta \mathbf{x}^T(n_J T_s)\mathbf{x}(n_J T_s). \quad (20)$$

Then the inequality (18) can be represented as the form in (10) by Schur complement.

If (20) holds, we have

$$\frac{E[V_2(n_{J+1}T_s)|n_J - n_{J-1} = i] - V_2(n_J T_s)}{V_2(n_J T_s)} \le \frac{-\beta \mathbf{x}^T(n_J T_s)\mathbf{x}(n_J T_s)}{V_2(n_J T_s)}. \quad (21)$$

Define $0 < \alpha = 1 - \beta \min\{\frac{1}{\lambda \max(P_i)}\} < 1$, it is obvious

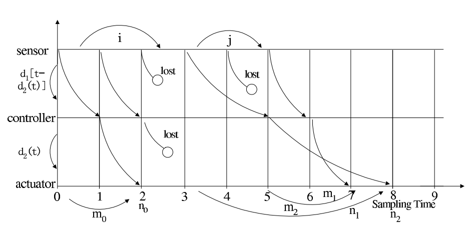

Fig. 2. Data flow diagram of NCSs with time varying delay and packet loss

that

$$\frac{-\beta \mathbf{x}^T(n_J T_s)\mathbf{x}(n_J T_s)}{\mathbf{x}^T(n_J T_s)P_i\mathbf{x}(n_J T_s)} \le -\beta \min\{\frac{1}{\lambda \max(P_i)}\}. \quad (22)$$

From (21), there exist a positive γ with $\alpha \le \gamma < 1$, such that

$$\frac{E[V_2(n_{J+1}T_s)|n_J - n_{J-1} = i] - V_2(n_J T_s)}{V(n_J T_s)} \le \gamma - 1. \quad (23)$$

Then

$$\begin{cases} E[V(n_{J+1}T_s)|n_J - n_{J-1} = i] \le \gamma V(n_J T_s), \\ \quad\vdots \\ E[V(n_1 T_s)|\eta(n_0) = i] \le \gamma V(n_0 T_s). \end{cases} \quad (24)$$

Taking the expectation $E[\cdot|\eta(n_0) = i]$ on both sides of (24) we have

$$E[V_2(n_2 T_s)|\eta(n_0) = i] \le \gamma^2 V_2(n_0 T_s). \quad (25)$$

By iterative derivation,

$$E[V_2(n_{J+1}T_s)|\eta(n_0) = i] \le \gamma E[V_2(n_J T_s)|\eta(n_0) = i] \le \ldots \le \gamma^{J+1} V_2(n_0 T_s). \quad (26)$$

From (26), the summation of $E[V_2(\cdot)|\eta(n_0) = i]$ when $J = 0 \sim N$ becomes

$$E[\sum_{J=0}^{N} V_2(n_{J+1}T_s)|\eta(n_0) = i] \le \frac{1-\gamma^N}{1-\gamma} V_2(n_0 T_s).$$

As a result,

$$\lim_{N\to\infty} E[\sum_{J=0}^{N} V_2(n_{J+1}T_s)|\eta(n_0) = i] \le \frac{1}{1-\gamma} V_2(n_0 T_s). \quad (27)$$

From (27), we obtain that

$$\lim_{N\to\infty} E[\sum_{J=0}^{N} \mathbf{x}^T(n_{J+1}T_s)\mathbf{x}(n_{J+1}T_s)|\eta(n_0) = i] \le \frac{1}{\max \rho(P_j)(1-\gamma)} V_2(n_0 T_s). \quad (28)$$

The limit of the expectation in (28) is bounded, this completed the proof for the stochastic stability of the closed loop system. ■

In the next section, network induced time varying delays are taken into account too.

B. NCSs with Bounded Delays and Stochastic Packet Losses

The system considered here is assumed to be a simple linear continuous-time system with time varying delay of the form,

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t - d_2(t)), \quad (29)$$

$d_2(t)$ is the controller to actuator varying delay with a certain bound. $S = \max_{n_J \in \ell}(n_{J+1} - n_J)$ is the maximum packet-loss upper bound. In Fig.2, the data flow of NCSs with time varying delay and packet loss is shown with $d_1(t - d_2(t))$ being the sampler to controller delay.

The actuator and controller considered in this work are both time driven and $d_1(t - d_2(t)) + d_2(t) = m_J T_s$ where

$m_J \in \{1, 2, ...M_b\}$ with a known positive integer M_b. The sampled-data controller is designed as a static state-feedback controller and it can be represented as

$$\begin{aligned} \mathbf{u}(t - d_2(t)) &= K\mathbf{x}[t - d_2(t) - d_1(t - d_2(t))] \\ &= K\mathbf{x}(n_J T_s - m_J T_s), \end{aligned} \tag{30}$$

where $K \in \Re^{m\times n}$ is to be designed. The initial control input is set to be zero: $\mathbf{u}(0) = 0$. T_s is the sampling time of the sampled-data controller. Then the closed-loop system becomes

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + BK\mathbf{x}(n_J T_s - m_J T_s), \quad n_J \in \ell. \tag{31}$$

The objective is to design the controller (30) so that the closed loop system (31) with Markovian packet-loss process (29) is stochastically stable.

Theorem 2: Consider the system (31) with Markovian packet-loss process (2), $\mathbf{u}(0) = 0$ and $\mathbf{x}(n_0) = \mathbf{x}(0)$. If there exists symmetric positive definition matrice $S > 0, P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} > 0$, $Q = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} > 0$, $R = \begin{bmatrix} R_{11} & 0 \\ 0 & R_{22} \end{bmatrix} > 0$, matrices $P_i = \hat{P} = \hat{P^T} \in \Re^+, i \in \zeta, \hat{Q} = \hat{Q}^T > 0, \hat{S} > 0$, X and Y with appropriate dimensions and scalars $\beta > 0$, $M_b > 0$ such that the following two inequalities holds,

$$\begin{bmatrix} H_{11} & * & * & * & * & * & * & * \\ H_{21} & H_{22} & * & * & * & * & * & * \\ H_{31} & H_{32} & H_{33} & * & * & * & * & * \\ H_{41} & H_{42} & H_{43} & H_{44} & * & * & * & * \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} & * & * & * \\ H_{61} & H_{62} & H_{63} & 0 & 0 & H_{66} & * & * \\ H_{71} & H_{72} & H_{73} & H_{74} & H_{75} & H_{76} & H_{77} & * \\ H_{81} & H_{82} & H_{83} & H_{84} & H_{85} & H_{86} & H_{87} & H_{88} \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} H_{11} &= Q_{11} - M_1^T A - A^T M_1 + M_b T_s R_{11}, \\ H_{21} &= P_2^T + N_1 - M_2^T A \quad H_{22} = N_2 + N_2^T + S, \\ H_{31} &= P_1 + M_1 - M_3^T A, \quad H_{32} = N_3^T + M_2 \\ H_{33} &= Q_{22} + M_b T_s R_{22} + M_3^T + M_3, \\ H_{41} &= -M_4^T A - P_2^T \quad H_{42} = N_4^T, \quad H_{43} = M_4, \\ H_{44} &= -Q_{11}, \quad H_{51} = -M_5^T A \quad H_{52} = P_3 + N_5^T, \\ H_{53} &= P_2^T + M_5^T, \quad H_{54} = -P_3, \quad H_{55} = \frac{-1}{M_b T_s} R_{11} \\ H_{88} &= -S - N_8^T - N_8 - M_8 BK - K^T B^T M_8 \\ H_{66} &= \frac{-1}{M_b T_s} R_{22} \quad H_{71} = -N_1 - M_7^T A, \\ H_{72} &= -N_2 + N_7^T, \quad H_{73} = -N_3 + M_7^T \\ H_{74} &= -N_4, \quad H_{75} = -N_5, \quad H_{77} = -N_7^T - N_7 \\ H_{76} &= -N_6, \quad H_{81} = -N_1 - M_8^T A - K^T B^T M_1, \\ H_{82} &= N_8^T - N_2 - K^T B^T M_2 \quad H_{61} = -M_6^T A, \\ H_{83} &= -N_3 + M_8^T - K^T B^T M_3, \\ H_{84} &= -N_4 - K^T B^T M_4 \quad H_{62} = N_6^T, \quad H_{63} = M_6^T, \\ H_{85} &= -N_5 - K^T B^T M_5, \quad H_{86} = -N_6 - K^T B^T M_6, \\ H_{87} &= -N_8^T - N_7 - K^T B^T M_7, \end{aligned} \tag{33}$$

and

$$\begin{bmatrix} -X & * & * & * \\ \sum_{i=1}^{S} \pi_{ij} \cdot j \cdot BY + X & X - \hat{E} & * & * \\ \sum_{i=1}^{S} \pi_{ij} \cdot j \cdot BY + X & 0 & -X & * \\ X & 0 & 0 & -\beta^{-1} I \end{bmatrix} < 0, \tag{34}$$

with $\hat{E} = X^T A^{-T} \hat{Q} A^{-1} X$, then the system is stochastically stable with the controller gain $K = Y X^{-1}$.

Proof: The proving process has been elided due to the page limitation. ■

Note that the LMI condition in (32) is non-convex and hence the following theorem is proposed to be the sufficient condition of (32).

Theorem 3: For given scalars θ_i, $i = 1, 2, ..., 8$, and a given upper bound of the time varying delay $M_b T_s$, if there exist symmetric positive definite matrices $\bar{S} > 0, \bar{P} = \begin{bmatrix} \bar{P}_1 & \bar{P}_2 \\ \bar{P}_2^T & \bar{P}_3 \end{bmatrix} > 0$, $\bar{Q} = \begin{bmatrix} \bar{Q}_{11} & 0 \\ 0 & \bar{Q}_{22} \end{bmatrix} > 0$, $\bar{R} = \begin{bmatrix} \bar{R}_{11} & 0 \\ 0 & \bar{R}_{22} \end{bmatrix} > 0$, matrices $\bar{X}$ and $\bar{Y}$ with appropriate dimensions such that the following inequality holds,

$$\begin{bmatrix} \bar{H}_{11} & * & * & * & * & * & * & * \\ \bar{H}_{21} & \bar{H}_{22} & * & * & * & * & * & * \\ \bar{H}_{31} & \bar{H}_{32} & \bar{H}_{33} & * & * & * & * & * \\ \bar{H}_{41} & \bar{H}_{42} & \bar{H}_{43} & \bar{H}_{44} & * & * & * & * \\ \bar{H}_{51} & \bar{H}_{52} & \bar{H}_{53} & \bar{H}_{54} & \bar{H}_{55} & * & * & * \\ \bar{H}_{61} & \bar{H}_{62} & \bar{H}_{63} & 0 & 0 & \bar{H}_{66} & * & * \\ \bar{H}_{71} & \bar{H}_{72} & \bar{H}_{73} & \bar{H}_{74} & \bar{H}_{75} & \bar{H}_{76} & \bar{H}_{77} & * \\ \bar{H}_{81} & \bar{H}_{82} & \bar{H}_{83} & \bar{H}_{84} & \bar{H}_{85} & \bar{H}_{86} & \bar{H}_{87} & \bar{H}_{88} \end{bmatrix} < 0, \tag{35}$$

where

$$\begin{aligned} \bar{H}_{11} &= \bar{Q}_{11} - \theta_1 A\bar{X} - \theta_1 \bar{X}^T A^T + M_b T_s \bar{R}_{11} \\ \bar{H}_{21} &= \bar{P}_2^T + \bar{N}_1 - \theta_2 A\bar{X}, \quad \bar{H}_{22} = \bar{N}_2 + \bar{N}_2^T + \bar{S} \\ \bar{H}_{31} &= \bar{P}_1 + \theta_1 \bar{X}^T - \theta_3 A\bar{X}, \quad \bar{H}_{32} = \bar{N}_3^T + \theta_2 \bar{X}^T \\ \bar{H}_{33} &= \bar{Q}_{22} + M_b T_s \bar{R}_{22} + \theta_3 \bar{X}^T + \theta_3 \bar{X}, \\ \bar{H}_{41} &= -\theta_4 A\bar{X} - \bar{P}_2^T \quad \bar{H}_{42} = \bar{N}_4^T, \quad \bar{H}_{43} = \theta_4 \bar{X}, \\ \bar{H}_{44} &= -\bar{Q}_{11}, \quad \bar{H}_{51} = -\theta_5 A\bar{X} \quad \bar{H}_{52} = \bar{P}_3 + \bar{N}_5^T, \\ \bar{H}_{53} &= \bar{P}_2^T + \theta_5 \bar{X}, \quad \bar{H}_{54} = -\bar{P}_3, \quad \bar{H}_{55} = \frac{-1}{M_b T_s} \bar{R}_{11} \\ \bar{H}_{61} &= -\theta_6 A\bar{X}, \quad \bar{H}_{62} = \bar{N}_6^T, \quad \bar{H}_{63} = \theta_6 \bar{X}, \\ \bar{H}_{66} &= \frac{-1}{M_b T_s} \bar{R}_{22} \quad \bar{H}_{71} = -\bar{N}_1 - \theta_7 A\bar{X} \quad \bar{H}_{76} = -\bar{N}_6 \\ \bar{H}_{72} &= -\bar{N}_2 + \bar{N}_7^T, \quad \bar{H}_{73} = -\bar{N}_3 + \theta_7 \bar{X} \quad \bar{H}_{75} = -\bar{N}_5 \\ \bar{H}_{74} &= -\bar{N}_4, \quad \bar{H}_{77} = -\bar{N}_7^T - \bar{N}_7 \\ \bar{H}_{81} &= -\bar{N}_1 - \theta_8 A\bar{X} - \theta_1 \bar{Y}^T B^T, \quad \bar{H}_{82} = \bar{N}_8^T - \bar{N}_2 \\ &\quad -\theta_2 \bar{Y}^T B^T \quad \bar{H}_{83} = -\bar{N}_3 + \theta_8 \bar{X} - \theta_3 \bar{Y}^T B^T, \\ \bar{H}_{84} &= -\bar{N}_4 - \theta_4 \bar{Y}^T B^T, \quad \bar{H}_{85} = -\bar{N}_5 - \theta_5 \bar{Y}^T B^T \\ \bar{H}_{86} &= -\bar{N}_6 - \theta_6 \bar{Y}^T B^T, \\ \bar{H}_{87} &= -\bar{N}_8^T - \bar{N}_7 - \theta_7 \bar{Y}^T B^T \\ \bar{H}_{88} &= -\bar{S} - \bar{N}_8^T - \bar{N}_8 - \theta_8 B\bar{Y} - \theta_8 \bar{Y}^T B^T \end{aligned}$$

and (34) is satisfied. Then under the static controller with gain obtained by

$$K = \bar{Y} \bar{X}^{-1}. \tag{36}$$

then the system is stochastically stable.

Proof: In order to transform the nonconvex LMI in (32) into a solvable LMI, assume that $M_i = \theta_i M_0$ where θ_i is known and given. Define $\bar{X} = M_0^{-1}$,

$$\hat{W} = diag(\bar{X}, \bar{X}, \bar{X}, \bar{X}, \bar{X}, \bar{X}, \bar{X}, \bar{X})$$

and $\bar{Y} = K\bar{X}$. Then by pre-multiplying the inequality in (32) by $\hat{W}^T$ and post-multiplying by $\hat{W}$, we can obtain the inequality (35). ■

IV. Experimental Studies

In order to study the network property in the real environment, a real-time network induced delay and packet loss measurement system has been developed based on MATLAB applications.

A. Experimental Setup

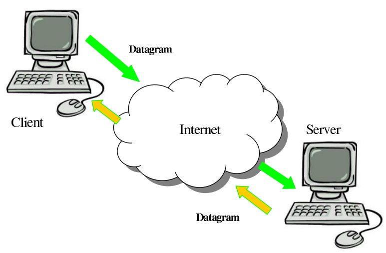

Fig. 3. Real-time network measurement system

With Instrument Control Toolbox in MATLAB, two computers located in different places can communicate with each other via networks following UDP protocols directly from MATLAB. The delay and packet loss information could be recorded and saved in MATLAB for further analysis.

B. Network Induced Delays and Packet Losses

During one measurement, 6,000 data packets were transmitted between the client and the server once per hour the average delay of each measurement is very close, Fig.4. When

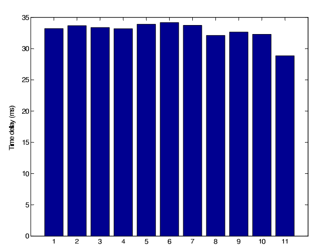

Fig. 4. Histogram of average time delay vs hour index

the information of delays recorded in one measurement has been drawn in Fig.5, the minimum time delay value is 10 ms and up to 93% of packets can be received in 50 ms.

The packet loss rates in four different cases were recorded and analyzed as shown in the pie chart Fig.6. In order to vary the case of the network, FlashGet has been used to download and upload data files from an internet web service. The measurement in Case 1 was processed without running FlashGet. In Cases 2, 3 and 4 the FlashGet was used to

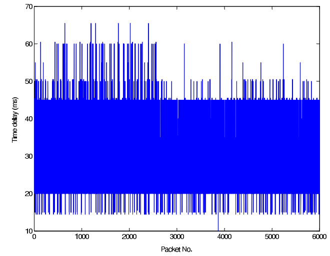

Fig. 5. Time delay vs numbers of data packet

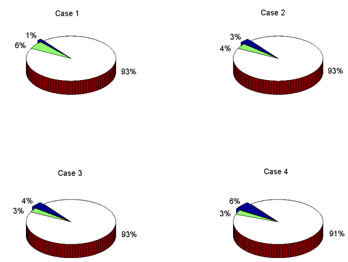

Fig. 6. Pie chart of packet loss rate

download date files with the rate of 60 KB/s, 100 KB/s and 400KB/s respectively.

In the pie chart, the slices which present the packet loss rates in four cases were pulled out. It is clear that the packet loss rate increases as the download rate increases. For the rest two slices, the small one presents the rate of the packets which have been received but exceeded the bounded delay value, and the big one presents the rate of the packets received in a shorter time than the bound delay value.

C. Bounded Delays and Bounded Packet Losses

The time delay distribution in one measurement can help us to establish the upper bound of delays. In Fig.7 (a), it

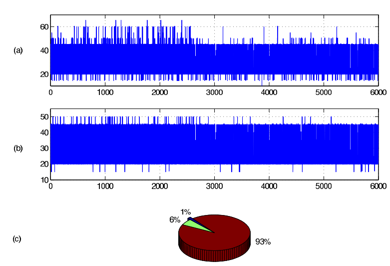

Fig. 7. Bounded delay and packet lost

shows the delay distribution in one measurement. Since the minimum delay is 10 ms and over 90% packets arrived in 50 ms, the upper and lower bound of delays were chosen as 50 ms and 10 ms respectively. The packets exceed the bounded delay were dropped and the new delay distribution under bounded delay was shown in Fig.7 (b). In the pie chart Fig.7 (c), the slice marked with 1% presents the original packet loss rate, the small slice 6% in the rest two presents the rate of packets which exceed the bounded delay and were dropped. The big slice shows us the data rate received in the bounded delay.

D. Markov Chains

As we discussed in Section II, in this work the packet loss process is governed by a Markov Chain. The order of the MC is determined due to the upper bound of the packet loss amount between two successful transmissions. From the results obtained from experiments (Fig.6), the upper bounds of the packet loss in Case 1 and Case 4 are both three packets. For each case, there are two different MC transition matrixes, they are calculated before and after dropping the packets which exceed the bounded delay. Only the transition matrices in Case 4 (Fig.6) are listed here as an example and those transition matrices are applied in the simulation example in subsection IV-E.

Case 4, before dropping the packets exceed the bounded delay:

$$\prod = \begin{bmatrix} 0.9426 & 0.0553 & 0.0021 \\ 0.9159 & 0.0779 & 0.0062 \\ 0.9231 & 0.0769 & 0 \end{bmatrix}. \qquad (37)$$

Case 4, after dropping the packets:

$$\prod = \begin{bmatrix} 0.9128 & 0.0788 & 0.0084 \\ 0.8984 & 0.0831 & 0.0185 \\ 0.92 & 0.08 & 0 \end{bmatrix}. \qquad (38)$$

E. Simulations

Consider the following nominal continuous-time system which is controlled through networks with packet losses and time varying delays recorded in the experiment:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -3 & -0.001 \\ -1 & 0.001 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}(t - d_2(t)), \quad (39)$$

with $\mathbf{x}(0) = [0.5, -0.5]^T$. The continuous system is

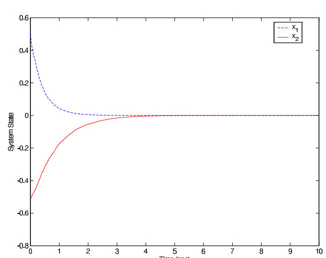

Fig. 8. The state response with bounded delays and packet losses

open loop unstable with eigenvalues of A as -3.0013 and 0.0013. The plant is sampled with a sampling period $T_s = 0.01\ seconds$. The packet-loss upper bound is $S = 3$. The transition probability matrix (38) was used to represent the packet losses in this example. The time varying delay is set to be $m_J T_s,\ m_J \in \{1, 2, 3, 4, 5\}$. The sampled-data controller is designed as in (30), applying Theorem 3 with $\beta = 3$, we obtain a networked controller gain

$$K = YX^{-1} = \begin{bmatrix} 0.1078 & -1.7060 \end{bmatrix}.$$

Fig.8 shows the state response of the system. Since the control gain was well designed, the system can be stochastically stabilized in around 5 seconds. In next subsection, a real-time networked control system based on MATLAB applications will be introduced.

F. Real-Time NCSs Experiment

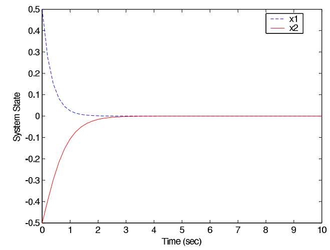

Fig. 9. The state response of real-time networked control system

We simulate the example in IV-E through real-time network channel based on MATLAB application. The experiment setup is as same as shown in Fig.3. Fig.9 shows that the system can be stochastically stabilized in 3 second due to the proper design of the controller.

V. CONCLUSIONS

This paper mainly dealt with the stabilization problem of NCSs with bounded time varying delays and Markovian packet loss via sampled-data control approach. The system can be stochastically stabilized according to the proper design of the static feedback controller. Lyapunov method and LMI techniques were applied to ensure the stochastic stability of the networked control systems. A real-time network induced delay and packet loss measurement system has been built and experiments were shown to study the real network characters.

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