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On Communication in Decentralized Pole-Placement Formation Control and Parallel Estimation

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***Abstract*— We consider a class of decentralized formation flying control algorithms that stem from assigning the closed loop poles for the Multi-Input-Multi-Output formation system, and examine the role communication plays. First we revisit Smith and Hadaegh [2007], where as many parallel estimators as there are spacecraft are used. We show an intuitive way of demonstrating the existing results, and by a re-interpretation of the quantities communicated, extend the results to the case where observability of the full formation state at each spacecraft is not available. Next we show that the pole-placement formation control can be carried out with only one estimator and no communication, for the double-integrator system used to model deep space formation flying. We treat the task of having one estimator on each spacecraft as separate from the task of control, and show how it can be accomplished with communication.**

I. INTRODUCTION

There have been many formation flying control algorithms in literature; see Scharf et al. [2004] for a survey. In Smith and Hadaegh [2007], a decentralized control algorithm was proposed, where each spacecraft computes an estimate of the observable formation state, and then determines its control action in order to "implement" a specific Multi-Input-Multi-Output control law. The control law is such that, if the observable formation state x were available, a centrally computed control input $u = Kx$ would result in a closed loop system matrix of $(A + BK)$ with desired eigenvalues. Hence, we will refer to this control law as "pole-placement formation control algorithm," to distinguish it from leader-follower and other approaches. In Smith and Hadaegh [2007], spacecraft need to transmit and receive (linear transformations of) their local estimates under certain communication topology, in order to avoid instability of disagreement dynamics, i.e., instability caused by the fact that estimation is carried out in parallel at each spacecraft and can thus differ.

Our interest in this paper is to understand the role communication plays in a pole-placement formation control algorithm. Our findings are threefold. First, we obtain an intuitive understanding of the results presented in Smith and Hadaegh [2007]. Second, by re-interpreting the quantities transmitted and received, we obtain a *prescriptive procedure* for the case when none of the spacecraft can reconstruct the state estimate with its own set of measurements. Such a case was dealt with not in Smith and Hadaegh [2007], but in later papers such as Subbotin and Smith [2007], with an iterative procedure that involves solving Linear Matrix Inequalities (LMIs) iteratively. Third, we remove the implicit assumption that *every spacecraft has to compute its control input based on its local state estimate.* When control and estimation are stated as separate goals, we find that pole-placement control can be achieved *without communication*, while parallel estimators can be constructed with $(N - 1)$ communication links and communicated data of length $c(N - 1)$ where c represents the length of data of the control input to a single spacecraft. (In Smith and Hadaegh [2007], communicated data length is quadratic in N.)

The practical implications of our theoretical findings for formation flying in general and NASA's Terrestrial Planet Finder Interferometer (TPFI) mission in particular, are being investigated.

This paper is organized as follows. Section II defines the problem in a slightly more general way than Smith and Hadaegh [2007]. Section III first describes the specific result from Smith and Hadaegh [2007] that we focus on in this paper, then shows how that result can be intuitively understood and, through a new interpretation, extended to handle the case where observability is not available. In Section IV we examine the problem in a more general way, and show how pole-placement formation control can be achieved even without communication. Having each spacecraft compute locally a formation state estimate is treated as a separate task, and this is achieved through communication, as is shown in Section V. Conclusions are drawn in Section VI with a discussion of future work.

II. PROBLEM STATEMENT

In this paper we focus only on stability issues and therefore omit noise terms in the equations that follow.

Let the translational dynamics of each of the N spacecraft be described by

$$x_i(k+1) = A_i x_i(k) + b_i u_i(k),\ i = 1, \ldots, N, \tag{1}$$

where u_i is the control input and x_i is the state of interest in a deep-space formation flying problem, or more specifically, double integrators according to Scharf et al. [2002]. It is naturally assumed that (A_i, b_i) is controllable.

Since in formation flying we are interested in relative positions and velocities, let x denote the observable formation state, which qualitatively speaking is the concatenation of x_i modulo the centroid state. Let the dynamics be described by

$$x(k+1) = Ax(k) + Bu(k), \tag{2}$$

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where u is the concatenation of u_i, and A and B are suitably defined from A_i and b_i.

In a centralized scheme, when x is available, a control law can be chosen as

$$u(k) = Kx(k) \tag{3}$$

so that the closed loop dynamics is given by

$$x(k+1) = (A + BK)x(k), \tag{4}$$

where K is computed from a desired set of closed loop eigenvalues. The formation control methods discussed in this paper can be regarded as various ways of "implementing" such a control law in a decentralized fashion, when x is not readily available and can only be estimated through measurements. Using the terminology of Scharf et al. [2004], this approach falls under the category of MIMO (Multi-Input-Multi-Output), and we will refer to it as pole-placement formation control, to distinguish it from leader-follower and other approaches.

Each spacecraft has local measurements described by

$$y_i(k+1) = C_i x(k), i = 1, \ldots, N. \tag{5}$$

We choose the time index to be $(k+1)$ on the left hand side so that we can use Kalman filter type of update equations with results similar in form to those in Smith and Hadaegh [2007]; interested readers are referred to Chapter 1 in Whittle [1990] and Chapter 5 in Bertsekas [2005] for details.

For the problem to be centrally solvable, we define

$$C \triangleq \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_N \end{bmatrix} \tag{6}$$

and assume that (C, A) is observable. However, we will consider two cases for decentralized control:

- **Case 1:** Each spacecraft has observability, i.e., (C_i, A) is observable, $i = 1, \ldots, N$.
- **Case 2:** Some or all spacecraft fail to have observability.

A general approach to the pole-placement formation control problem is to construct some estimators whose aggregated state $z(k)$ evolves as

$$z(k+1) = \Phi z(k) + \Psi y(k+1) \tag{7}$$

where y is the concatenation of y_i. This would correspond to the prediction-update equation in a Kalman filter. Let control be locally computed according to

$$u_i(k) = \Gamma_i z(k) + \Lambda_i y_i(k). \tag{8}$$

Note that in the above description we deliberately omit the specification of how many estimators there are and where they are located. We want the computation in (7) and (8) to be carried out locally as much as possible. If communication is called for, our interest is in understanding, in theory, how much communication would suffice.

III. A NEW STABILIZATION SCHEME IN THE N-ESTIMATOR-FEEDBACK BASED APPROACH

We first describe some results from Smith and Hadaegh [2007] that we will focus on in this paper, then present a new scheme for stabilizing the disagreement dynamics.

A. *A result from Smith and Hadaegh [2007]*

The strategy adopted in Smith and Hadaegh [2007] is to have N parallel estimators, one on each spacecraft with formation state estimate $\hat{x}^i$, $i = 1, \ldots, N$. Here we use a superscript to emphasize the estimate of x carried out at the i-th spacecraft. The estimator dynamics will be specified later.

Then tentatively the control action can be determined by

$$u_i(k) = Q_i K \hat{x}^i(k), \ i = 1, \ldots, N, \tag{9}$$

where Q_i is a "read-out" matrix such that the matrices $\Pi_i \triangleq Q_i^T Q_i$ form a partition

$$\sum_{j=1}^{N} \Pi_j = I. \tag{10}$$

It follows that the entire control vector for the formation is given by

$$u(k) = \sum_{j=1}^{N} Q_j^T u_j(k). \tag{11}$$

In this section we assume that every spacecraft has observability, i.e., (C_i, A) is observable. To further simplify, we assume that $C_1 = C_2 = \cdots = C_N$. If $u(k)$ is available to every spacecraft, then estimators can be constructed as

$$\hat{x}^i(k+1) = A\hat{x}^i(k) + Bu(k) + L(C\hat{x}^i(k) - y_i(k+1)). \tag{12}$$

This motivates the following tentative estimator update law where each spacecraft "pretends" that all others have the same estimates as itself does:

$$\begin{aligned} \hat{x}^i(k+1) &= A\hat{x}^i(k) + B\sum_{j=1}^{N} Q_j^T Q_j K \hat{x}^i(k) + \\ &\quad L(C_i\hat{x}^i(k) - y_i(k+1)) \qquad (13) \\ &= A\hat{x}^i(k) + BK\hat{x}^i(k) + \\ &\quad L(C\hat{x}^i(k) - y_i(k+1)). \qquad (14) \end{aligned}$$

It was shown in Smith and Hadaegh [2007] that the above would lead to closed loop eigenvalues given by

$$\sigma(A + BK) \bigcup \sigma(A + LC) \bigcup_{j=1}^{N-1} \sigma(A + BK + LC) \tag{15}$$

where $\sigma(\cdot)$ denotes the set of eigenvalues for the given matrix. Since $(A + BK + LC)$ may not be stable, it was then proposed that some spacecraft j should send $v_l(k) \triangleq H_l \hat{x}^j(k)$ to some other spacecraft i, and that the communicated quantity should be incorporated into the estimator update equation as

$$\begin{aligned} \hat{x}^i(k+1) &= A\hat{x}^i(k) + BK\hat{x}^i(k) + L(C\hat{x}^i(k) \\ &\quad -y_i(k+1)) + F_l(v_l(k) - H_l\hat{x}^i(k)) \qquad (16) \end{aligned}$$

In general a communication topology is used to specify how many such v_l's are communicated and between which spacecraft. In Smith and Hadaegh [2007], several classes of communication topologies are analyzed, and closed loop eigenvalues are given in expressions that involve the eigenvalues of the *Laplacian* of the graph describing the communication topology. For the purpose of this paper we will not go into the details of all the results, but instead only focus on the result for a special class of communication topologies, two of which are illustrated in Figure 1.

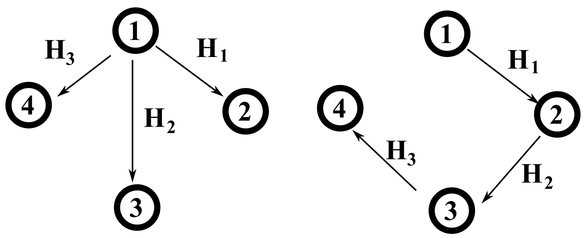

Fig. 1. Two examples of communication topologies in a certain class.

With such communication topologies, the closed loop eigenvalues are given by

$$\sigma(A+BK) \bigcup \sigma(A+LC) \bigcup_{l=1}^{N-1} \sigma(A+BK+LC-F_l H_l). \tag{17}$$

It follows that $F_l H_l$ can be chosen appropriately to make $A+BK+LC-F_l H_l$ stable.

B. Re-interpreting the result

We can see from (17) that the choice of $F_l = L$ and $H_l = C$ would make the closed loop system asymptotically stable. In such a case the LC term could be seen as a "nuisance" that has to be removed by communication. This motivates us to re-examine the estimator equation (14). We start with

$$\begin{aligned} \hat{x}^i(k+1) &= A\hat{x}^i(k) + BK\hat{x}^i(k) + \text{(TBD)}, \\ & \quad i = 1, \ldots, N, \end{aligned} \tag{18}$$

where the term (TBD) is To Be Determined later. Define

$$e_i \triangleq \hat{x}^i - x. \tag{19}$$

Then the closed loop system is described by

$$x(k+1) = (A+BK)x(k) + B\sum_{j=1}^{N} \Pi_j K e_j(k), \tag{20}$$

$$\begin{aligned} e_i(k+1) &= (A+BK)e_i(k) - B\sum_{j=1}^{N} \Pi_j K e_j(k) + \text{(TBD)}, \\ & \quad i = 1, \ldots, N. \end{aligned} \tag{21}$$

For simpler notations, in the following we will omit time (k) and use a superscripted "+" to denote time $(k+1)$. We define

$$\eta_N \triangleq e_N, \tag{22}$$

$$\eta_i \triangleq e_i - e_N, \; i = 1, \ldots, N-1. \tag{23}$$

It follows that

$$\eta_i^+ = (A+BK)\eta_i, \; i = 1, \ldots, N-1, \tag{24}$$

$$\eta_N^+ = A\eta_N - B\sum_{j=1}^{N-1} \Pi_j K \eta_j + \text{(TBD)}. \tag{25}$$

Now it is clear that if we choose the term (TBD) such that η_N is stabilized, then the closed loop system will be asymptotically stable. A natural choice is

$$\text{(TBD)} \triangleq L_N(C_N \hat{x}^N - y_N^+) = L_N C_N \eta_N, \tag{26}$$

which leads to

$$\eta_N^+ = (A + L_N C_N)\eta_N - B\sum_{j=1}^{N-1} \Pi_j K \eta_j. \tag{27}$$

Thus the estimators in (18) take the form

$$\hat{x}^{N\,+} = (A+BK)\hat{x}^N + L_N(C_N\hat{x}^N - y_N^+), \tag{28}$$

$$\begin{aligned} \hat{x}^{i\,+} &= (A+BK)\hat{x}^i + L_N(C_N\hat{x}^N - y_N^+), \\ & \quad i = 1, \ldots, N-1. \end{aligned} \tag{29}$$

If there exist matrices R_i such that

$$R_i C_i = C_N, \; i = 1, \ldots, N-1, \tag{30}$$

then an alternative to Equation (29) is

$$\hat{x}^{i\,+} = (A+BK)\hat{x}^i + L_N(C_N\hat{x}^N - R_i y_i^+), \; i = 1, \ldots, N-1. \tag{31}$$

If we choose to communicate $C_N\hat{x}^N$ in the above equation, then this would correspond to the choice of $F_l = L_N$ and $H_l = C_N$ in Smith and Hadaegh [2007]. However, we can also choose to communicate $(C_N\hat{x}^N - y_N^+)$ in Equation (29), which means that (part of) the error correction term used by the N-th spacecraft is communicated to all other spacecraft. With this interpretation, ignoring communication delays, we can see that the two communication topologies shown in Figure 1 achieve the same goal: In the first scheme, the error correction term is passed directly, while in the second scheme, it is passed with "relays." This gives us an intuitive understanding why such topologies work.

Before we extend this new interpretation to the case where observability is not available, we will show another, symmetric form of estimators, just to bring out the essence of disagreement dynamics.

Symmetric form: We consider the case when $C_i = C$ and (C, A) is observable. Starting from Equations (20) and (21), we define

$$\bar{e} \triangleq \frac{1}{N}\sum_{j=1}^{N} e_j, \tag{32}$$

$$\eta_i \triangleq e_i - \bar{e}, \; i = 1, \ldots, N-1, \tag{33}$$

$$\eta_N \triangleq \bar{e}, \tag{34}$$

and obtain the following estimators that have asymptotically stable disagreement dynamics:

$$\hat{x}^{i\,+} = (A+BK)\hat{x}^i + \frac{1}{N}\sum_{j=1}^{N} L(C\hat{x}^j - y_j), \; i = 1, \ldots, N. \tag{35}$$

Compare the above with

$$\hat{x}^{i\ +} = (A + BK)\hat{x}^i + L(C\hat{x}^i - y_i),\ i = 1, \dots, N, \quad (36)$$

and we can see that stability is guaranteed when spacecraft "coordinate" their error correction terms, and not guaranteed if each one "acts alone."

To implement such a symmetric scheme, one spacecraft has to be the "averager": It collects the error correction terms $(C\hat{x}^j - y_j)$ from all other spacecraft, and sends back the average $\frac{1}{N}\sum_{j=1}^{N}(C\hat{x}^j - y_j)$.

C. Extending to the missing-observability-case

We make the simplifying assumption that communication delay is small compared to the estimation cycle, and therefore can be ignored in the following. As a consequence we do not make a distinction between information sent through one hop or multiple hops. The objective is to see how, in this extreme case, disagreement dynamics can be eliminated and what information needs to be communicated.

If there exists an i-th spacecraft with (C_i, A) observable, then it can play the role that the N-th spacecraft played in the last section, i.e., it stabilizes its own estimator, and sends the error correction term to others.

When none of (C_i, A), $i = 1, \dots, N$, is observable, we can adopt the following procedure:

1) Pick a spacecraft to be the "aggregator," e.g., the spacecraft with the "richest" set of measurements. Say this is the N-th spacecraft. Call a set of measurements "supplementary" if it can augment y_N to become y_{total} such that (C_{total}, A) is observable. Since we assume that the spacecraft fleet is centrally observable, such a set always exists.
2) Ask those spacecraft that can provide measurements in a chosen supplementary set to send their measurements to the aggregator.
3) The aggregator uses an error correction term $L_N v_N \triangleq L_N(C_{total}\hat{x}^N - y_{total})$ in its estimator, and sends v_N to all other spacecraft.
4) The other spacecraft incorporate the same error correction term in their estimation equations.

Such a procedure would be more prescriptive and transparent than the trial-and-error procedure described in Subbotin and Smith [2007]. Its applicability to NASA formation flight missions such as TPFI is being investigated.

IV. FORMATION CONTROL WITHOUT COMMUNICATION

When we presented the problem definition in Section II, we did not specify how many estimators have to be used. In this section we show that, to carry out a pole-placement formation control, it is possible to use only one estimator, and no communication at all.

To follow closely relevant literature, in this section we use a continuous time set up. We consider the entire spacecraft formation dynamics of interest (i.e., positions and velocities stacked together, "minus" the centroid position and velocity, qualitatively speaking) given as follows, where we omit time t for simpler notations:

$$\dot{x} = Ax + Bu \triangleq Ax + \sum_{i=1}^{N} B_i u_i. \quad (37)$$

The measurements are given by

$$y_i = C_i x. \quad (38)$$

We assume that (A, B) is centrally controllable and that (C, A) is centrally observable, where C is C_i stacked up as defined by Equation (6).

A. The control law

Pole-placement formation control can be carried out by first "soft linking" the spacecraft using local output feedback. When the formation dynamics does not have the so called *fixed mode* (Wang and Davison [1973], Corfmat and Morse [1976]), which will be defined later, suitable local output feedback can be constructed such that the new system is both observable and controllable from *every single* spacecraft. Therefore the formation can be controlled from a chosen spacecraft by using feedback from an estimator.

More specifically, the procedure is as follows (Corfmat and Morse [1976]):

1) For each spacecraft, use a local output feedback:
$$u_i = f_i y_i + \bar{v}_i = f_i C_i x + \bar{v}_i,\ i = 1, \dots, N, \quad (39)$$
where f_i is a feedback gain suitably chosen. The formation dynamics (37) then becomes
$$\dot{x} = Ax + \sum_{i=1}^{N} B_i(f_i C_i x + \bar{v}_i) \triangleq \bar{A}x + \sum_{i=1}^{N} B_i \bar{v}_i. \quad (40)$$
2) Pick, without loss of generality, the N-th spacecraft to be the controlling spacecraft. When the system (C, A, B) does not have any fixed mode and f_i's are suitably chosen, it can be shown that $(\bar{A}, B_N)$ is controllable and $(C_N, \bar{A})$ is observable.
3) Construct a state estimator
$$\dot{\hat{x}}^N = \bar{A}\hat{x}^N + L_N(C_N\hat{x}^N - y_N) + \sum_{i=1}^{N} B_i \bar{v}_i, \quad (41)$$
so that the error equation for $e_N \triangleq \hat{x}^N - x$ is
$$\dot{e}_N = \bar{A}e_N + L_N C_N e_N \triangleq A_L e_N, \quad (42)$$
where A_L is stable by construction since $(C_N, \bar{A})$ is observable.
4) Define
$$\bar{v}_N = K\hat{x}^N + v_N^* \quad (43)$$
and
$$\bar{v}_i = v_i^*, i = 1, \dots, N-1, \quad (44)$$
so that the closed loop system is
$$\begin{aligned} \dot{x} &= (\bar{A} + B_N K)x + B_N K e_N + \sum_{i=1}^{N} B_i v_i^* \\ &\triangleq A_K x + B_N K e_N + BV^*, \end{aligned} \quad (45)$$

where A_K is stable by construction since $(\bar{A}, B_N)$ is controllable.

5) Determine (offline) the constants V^* for any desired constant formation state x^*.

It can be seen that the N-th spacecraft knows what other spacecraft are doing with their pre-determined feedback gains and offsets, so no communication is needed.

B. Fixed mode

The above procedure is possible if and only if the formation dynamics (C, A, B) does not possess any fixed-mode. There are several equivalent definitions of fixed-mode. In this section we present the definition given in Wang and Davison [1973].

Consider output feedback of a system (C, A, B), i.e.,

$$\dot{x} = Ax + Bu,\ y = Cx,\ u = Ky. \tag{46}$$

In a decentralized control scheme, the output feedback gain K is in a block diagonal form, i.e.,

$$K \in \mathcal{K} \triangleq \{K : K = \text{diag}(K_1, \ldots, K_N)\}. \tag{47}$$

The fixed modes of the system associated with $\mathcal{K}$, denoted by $\Lambda(C, A, B, \mathcal{K})$, is defined as those eigenvalues of the closed loop system that are common to all such output feedback gains, i.e.,

$$\Lambda(C, A, B, \mathcal{K}) \triangleq \bigcap_{K \in \mathcal{K}} \sigma(A + BKC). \tag{48}$$

There are a number of papers on characterization of fixed modes and how to check their existence. One way is the following. Since $0 \in \mathcal{K}$, we can exhaustively examine all the eigenvalues of A, denoted by $\lambda_1, \ldots, \lambda_N$, and see whether any of them will satisfy

$$\det(\lambda_i I - (A + BKC)) \equiv 0 \tag{49}$$

for all choices of the entries in $K \in \mathcal{K}$.

C. A conjecture

The following are necessary conditions for the control procedure described in the previous section to work:

- Every spacecraft has to have some measurements to do output feedback with.
- The entire formation state is centrally controllable and observable.

For the spacecraft formation flying problem that we are interested in, each spacecraft is modeled as double integrators in three dimensions, with one thruster in each dimension as the controller.

We conjecture that for such a system, the above necessary conditions guarantee that no fixed mode exists in the formation dynamics, and therefore are also sufficient for the viability of the control law given in the last section.

For example, we can prove that this is the case with the measurement topology shown in Figure 2, for N spacecraft.

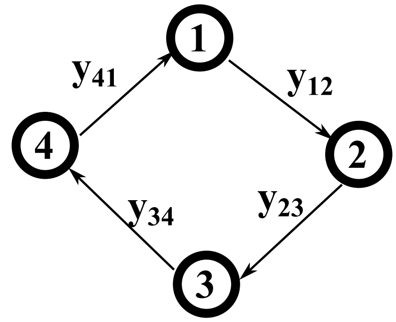

Fig. 2. A “ring” measurement topology does not have fixed mode.

V. PARALLEL ESTIMATORS WITH COMMUNICATION

Sometimes it is beneficial to have one estimator on each spacecraft, as described in Smith and Hadaegh [2007]. We treat this as a separate task that is not necessarily coupled with the pole-placement formation control, and proceed as follows.

Since the system (40) after output feedback is observable by every spacecraft, we can use $(N-1)$ communication links to send the part of the control input that is not pre-determined, i.e., $K\hat{x}^N$, from the “controlling spacecraft” to all other spacecraft. We then construct state estimators as follows:

$$\begin{aligned}\dot{\hat{x}}^i &= \bar{A}\hat{x}^i + B_N(K\hat{x}^N) + \sum_{k=1}^{N} B_k v_k^* + L_i(C_i\hat{x}^i - y_i),\\ & i = 1, \ldots, N-1. \end{aligned} \tag{50}$$

The estimation errors evolve as

$$\dot{e}_i = (\bar{A} + L_i C_i)e_i \triangleq A_{L_i} e_i,\ i = 1, \ldots, N-1. \tag{51}$$

Since $(C_i, \bar{A})$ is an observable pair, the spectrum of A_{L_i} can be arbitrarily assigned. Thus, under this scheme,

- we use $(N-1)$ communication links,
- we transmit data of size $c(N-1)$, where c represents the size of $K\hat{x}^N$, i.e., the size of the control input for a single spacecraft, and
- we achieve asymptotically converging estimates for all spacecraft.

Again, the effect of communication delays have been ignored in the above analysis. Applicability of this scheme to NASA formation flight missions such as TPFI is being investigated.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we examined the role communication plays in a type of formation control algorithms which we call “pole-placement formation control.” We presented an intuitive understanding of the stabilization of disagreement dynamics presented in Smith and Hadaegh [2007], where multiple estimators are constructed and used in feedback. We also, by a re-interpretation of the quantities communicated, extended the result to the case when observability of the entire formation state is not available at each spacecraft. With a more general statement of the problem, we separated the task of estimation and control, and showed that for pole-placement formation control, local output feedback and one estimator would suffice, with no communication. To have parallel estimators on each spacecraft, communication is needed with $(N-1)$ communication links and communicated data length of $c(N-1)$ where c is the length of a single control input.

We conjectured that the double integrators system used to model deep space formation flying does not have any fixed mode, which enabled the new control algorithm described above. Special cases of measurement topologies have been proved, and we are working on the general case.

The practical implications of these findings for NASA formation flight missions such as TPFI are being investigated.

VII. ACKNOWLEDGMENTS

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