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Composite Adaptive Control for Systems with Additive Disturbances

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***Abstract*— A novel control design is presented for the adaptive control of a general MIMO system with a gradient-based composite adaptive update law. The composite update law is driven by tracking and prediction errors with a fixed adaptation gain. An innovative scheme is developed in a swapping procedure that makes use of the recently developed Robust Integral of the Sign of the Error (RISE) technique to generate the prediction error even in the presence of nonlinear-in-the-parameter uncertainties. A Lyapunov-based stability analysis is used to derive sufficient gain conditions under which the proposed controller yields semi-global asymptotic stability for the tracking errors.**

I. INTRODUCTION

Adaptive, robust adaptive, and function approximation methods typically use tracking error feedback to update the adaptive estimates. In general, the use of the tracking error is motivated by the need for the adaptive update law to cancel cross-terms in the closed-loop tracking error system within a Lyapunov-based analysis. As the tracking error converges, the rate of the update law also converges, but drawing conclusions about the convergent value (if any) of the parameter update law is problematic. This problem led to the development of adaptive update laws that are driven, in part, by a prediction error.

The prediction error is defined as the difference between the predicted parameter estimate value and the actual system uncertainty. Including feedback of the estimation error in the adaptive update law enables improved parameter estimation. For example, some classic results [1]–[3] have proven the parameter estimation error is square integrable and may converge to the actual uncertain parameters. Since the prediction error depends on the unmeasurable system uncertainty, the swapping lemma [1], [4]–[7] is central to the prediction error formulation. The swapping technique (also described as input or torque filtering in some literature) transforms a dynamic parametric model into a static form where standard parameter estimation techniques can be applied. In [2] and [3], a nonlinear extension of the swapping lemma was derived, which was used to develop the modular z-swapping and x-swapping identifiers via an input-to-state stable (ISS) controller for systems in parametric strict feedback form. The advantages provided by prediction error based adaptive update laws led to several results that use either the prediction error or a composite of the prediction error and the tracking error (cf. the recent results in [8]–[14] and the references within).

Although prediction error based adaptive update laws have existed for approximately two decades, no stability result has been developed for systems with non-LP disturbances. In general, the inclusion of non-LP disturbances reduces the steady-state performance of continuous controllers to a uniformly ultimately bounded (UUB) result. In addition to a UUB result, the inclusion of non-LP disturbances may cause unbounded growth of the parameter estimates [15] for tracking error-based adaptive update laws without the use of projection algorithms or other update law modifications such as σ-modification [16]. The problem of non-LP disturbances is magnified for control methods based on prediction error based update laws, because the formulation of the prediction error requires the swapping (or control filtering) method. Applying the swapping approach to dynamics with non-LP disturbances is problematic because the unknown disturbance terms also get filtered and included in the filtered control input. This problem motivates the question of how can a prediction error based adaptive update law be developed for systems with additive non-LP disturbances.

To address this motivating question, a general Euler-Lagrange-like MIMO system is considered with structured and unstructured (non-LP) uncertainties, and a gradient-based composite adaptive update law is developed that is driven by both the tracking error and the prediction error. The control development is based on the recent continuous Robust Integral of the Sign of the Error (RISE) [17] technique that was originally developed in [18] and [19]. The RISE architecture is adopted since this method can accommodate for C^2 disturbances and yield asymptotic stability. For example, the RISE technique was used in [20] to develop a tracking controller for nonlinear systems in the presence of additive disturbances and parametric uncertainties. Since the swapping method will result in non-LP disturbances in the prediction error (the main obstacle that has previously limited this development), an innovative use of the RISE structure is also employed in the prediction error update (i.e., the filtered control input estimate). Sufficient gain conditions are developed under which this unique double RISE controller guarantees semi-global asymptotic tracking.

II. DYNAMIC SYSTEM AND PROPERTIES

Consider a class of MIMO nonlinear systems of the following form:

$$x^{(m)} = f(x, \dot{x}, ..., x^{(m-1)}) + G(x, \dot{x}, ..., x^{(m-2)})u + h\,(t) \tag{1}$$

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where $(\cdot)^{(i)}(t)$ denotes the i^{th} derivative with respect to time, $x^{(i)}(t) \in \mathbb{R}^n$, $i = 0, ..., m-1$ are the system states, $u(t) \in \mathbb{R}^n$ is the control input, $f\left(x, \dot{x}, ..., x^{(m-1)}\right) \in \mathbb{R}^n$ and $G\left(x, \dot{x}, ..., x^{(m-2)}\right) \in \mathbb{R}^{n\times n}$ are unknown nonlinear $\mathcal{C}^2$ functions, and $h(t) \in \mathbb{R}^n$ denotes a general nonlinear disturbance (e.g., unmodeled effects). The outputs of the system are the system states. Throughout the paper, $|\cdot|$ denotes the absolute value of the scalar argument, $\|\cdot\|$ denotes the standard Euclidean norm for a vector or the induced infinity norm for a matrix.

Property 1: $G(\cdot)$ is symmetric positive definite, and satisfies the following inequality $\forall y(t) \in \mathbb{R}^n$:

$$\underline{g}\,\|y\|^2 \leq y^T G^{-1} y \leq \bar{g}(x, \dot{x}, ..., x^{(m-2)})\,\|y\|^2 \tag{2}$$

where $\underline{g} \in \mathbb{R}$ is a known positive constant, and $\bar{g}(x, \dot{x}, ..., x^{(m-2)}) \in \mathbb{R}$ is a known positive function.

Property 2: The functions $G^{-1}(\cdot)$ and $f(\cdot)$ are second order differentiable such that G^{-1}, $\dot{G}^{-1}$, $\ddot{G}^{-1}$, f, $\dot{f}$, $\ddot{f} \in \mathcal{L}_\infty$ if $x^{(i)}(t) \in \mathcal{L}_\infty$, $i = 0, 1, ..., m+1$.

Property 3: The nonlinear disturbance term and its first two time derivatives are bounded by known constants.

Property 4: The unknown nonlinearities $G^{-1}(\cdot)$ and $f(\cdot)$ are linear in terms of unknown constant system parameters.

Property 5: The desired trajectory $x_d(t) \in \mathbb{R}^n$ is assumed to be designed such that $x_d^{(i)}(t) \in \mathcal{L}_\infty$, $i = 0, 1, ..., m+2$.

III. Control Objective

The objective is to design a continuous composite adaptive controller which ensures that the system state $x(t)$ tracks a desired time-varying trajectory $x_d(t)$ despite uncertainties and bounded disturbances in the dynamic model. To quantify this objective, a tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as

$$e_1 \triangleq x_d - x. \tag{3}$$

To facilitate a compact presentation of the subsequent control development and stability analysis, auxiliary error signals denoted by $e_i(t) \in \mathbb{R}^n$, $i = 2, 3, ..., m$ are defined as

$$\begin{aligned} e_2 &\triangleq \dot{e}_1 + \alpha_1 e_1 \\ e_3 &\triangleq \dot{e}_2 + \alpha_2 e_2 + e_1 \\ &\vdots \\ e_m &\triangleq \dot{e}_{m-1} + \alpha_{m-1} e_{m-1} + e_{m-2} \end{aligned} \tag{4}$$

where $\alpha_i \in \mathbb{R}$, $i = 1, 2, ..., m-1$ denote constant positive control gains. The error signals $e_i(t)$, $i = 2, 3, ..., m$ can be expressed in terms of $e_1(t)$ and its time derivatives as

$$e_i = \sum_{j=0}^{i-1} b_{i,j} e_1^{(j)}, \qquad b_{i,i-1} = 1 \tag{5}$$

where the constant coefficients $b_{i,j} \in \mathbb{R}$ can be evaluated by substituting (5) in (4), and comparing coefficients. A filtered tracking error [15], denoted by $r(t) \in \mathbb{R}^n$, is also defined as

$$r \triangleq \dot{e}_m + \alpha_m e_m \tag{6}$$

where $\alpha_m \in \mathbb{R}$ is a positive, constant control gain. The filtered tracking error $r(t)$ is not measurable since the expression in (6) depends on $x^{(m)}$.

IV. Control Development

The open-loop tracking error system is developed by premultiplying (6) by $G^{-1}(\cdot)$ and utilizing the expressions in (1), (4), (5) as

$$G^{-1} r = Y_d \theta + S_1 - G_d^{-1} h - u. \tag{7}$$

In (7), $Y_d\theta \in \mathbb{R}^n$ is defined as

$$Y_d \theta \triangleq G_d^{-1} x_d^{(m)} - G_d^{-1} f_d \tag{8}$$

where $Y_d(x_d, \dot{x}_d, ..., x_d^{(m)}) \in \mathbb{R}^{n\times p}$ is a desired regression matrix, and $\theta \in \mathbb{R}^p$ contains the constant unknown system parameters. In (8), the functions $G_d^{-1}(x_d, \dot{x}_d, ..., x_d^{(m-2)}) \in \mathbb{R}^{n\times n}$, and $f_d(x_d, \dot{x}_d, ..., x_d^{(m-1)}) \in \mathbb{R}^n$ are defined as

$$\begin{aligned} G_d^{-1} &\triangleq G^{-1}(x_d, \dot{x}_d, ..., x_d^{(m-2)}) \\ f_d &\triangleq f(x_d, \dot{x}_d, ..., x_d^{(m-1)}). \end{aligned} \tag{9}$$

Also in (7), the auxiliary function $S_1\left(x, \dot{x}, ..., x^{(m-1)}, t\right) \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S_1 \triangleq\ & G^{-1}\Big(\sum_{j=0}^{m-2} b_{m,j} e_1^{(j+1)} + \alpha_m e_m\Big) + G^{-1} x_d^{(m)} \\ & - G_d^{-1} x_d^{(m)} - G^{-1} f + G_d^{-1} f_d - G^{-1} h + G_d^{-1} h \end{aligned} \tag{10}$$

where the fact that $b_{m,m-1} = 1$ was used. Based on the open-loop error system in (7), the control torque input is composed of an adaptive feedforward term plus the RISE feedback term as

$$u \triangleq Y_d \hat{\theta} + \mu_1 \tag{11}$$

where $\hat{\theta}(t) \in \mathbb{R}^p$ denotes a parameter estimate vector generated by the following gradient-based composite adaptive update law [21]–[23]:

$$\dot{\hat{\theta}} = \Gamma \dot{Y}_d^T r + \Gamma \dot{Y}_{df}^T \varepsilon \tag{12}$$

where $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite, symmetric, constant gain matrix. In (12), $Y_{df}(x_d, \dot{x}_d, ..., x_d^{(m)}) \in \mathbb{R}^{n\times p}$ is a subsequently designed filtered regression matrix, and $\varepsilon(t) \in \mathbb{R}^n$ denotes a measurable prediction error. In (11), $\mu_1(t) \in \mathbb{R}^n$ denotes the RISE feedback term defined as

$$\begin{aligned} \mu_1(t) \triangleq\ & (k_1+1) e_m(t) - (k_1+1) e_m(0) \\ & + \int_0^t \{(k_1+1)\alpha_m e_m(\sigma) + \beta_1 sgn(e_m(\sigma))\} d\sigma \end{aligned} \tag{13}$$

where $k_1, \beta_1 \in \mathbb{R}$ are positive constant control gains, and $\alpha_m \in \mathbb{R}$ was introduced in (6).

Remark 1: The parameter estimate update law in (12) depends on the unmeasurable signal $r(t)$, but the parameter estimates are independent of $r(t)$ as can be shown by directly solving (12) as

$$\hat{\theta}(t) = \hat{\theta}(0) + \Gamma\dot{Y}_d^T(\sigma)e_m(\sigma)\Big|_0^t + \int_0^t \Gamma\dot{Y}_{df}^T(\sigma)\varepsilon(\sigma)\,d\sigma - \int_0^t \left\{\Gamma\ddot{Y}_d^T(\sigma)e_m(\sigma) - \alpha_m\Gamma\dot{Y}_d^T(\sigma)e_m(\sigma)\right\}d\sigma.$$

The closed-loop tracking error system can be developed by substituting (11) into (7) as

$$G^{-1}r = Y_d\tilde{\theta} + S_1 - G_d^{-1}h - \mu_1 \tag{14}$$

where $\tilde{\theta}(t) \in \mathbb{R}^p$ denotes the parameter estimate mismatch defined as

$$\tilde{\theta} \triangleq \theta - \hat{\theta}. \tag{15}$$

To facilitate the subsequent composite adaptive control development and stability analysis, the time derivative of (14) is expressed as

$$G^{-1}\dot{r} = -\frac{1}{2}\dot{G}^{-1}r + \dot{Y}_d\tilde{\theta} - Y_d\Gamma\dot{Y}_{df}^T\varepsilon + \tilde{N}_1 + N_{1B} - (k_1+1)r - \beta_1 sgn(e_m) - e_m \tag{16}$$

where (12) and the fact that the time derivative of (13) is given as

$$\dot{\mu}_1 = (k_1+1)r + \beta_1 sgn(e_m) \tag{17}$$

was utilized. In (16), the unmeasurable/unknown auxiliary terms $\tilde{N}_1(e_1, e_2, ..., e_m, r, t)$ and $N_{1B}(t) \in \mathbb{R}^n$ are defined as

$$\tilde{N}_1 \triangleq -\frac{1}{2}\dot{G}^{-1}r + \dot{S}_1 + e_m - Y_d\Gamma\dot{Y}_d^T r \tag{18}$$

where (12) was used, and

$$N_{1B} \triangleq -\dot{G}_d^{-1}h - G_d^{-1}\dot{h}. \tag{19}$$

The structure of (16) and the introduction of the auxiliary terms in (18) and (19) is motivated by the desire to segregate terms that can be upper bounded by state-dependent terms and terms that can be upper bounded by constants. In a similar manner as in [19], the Mean Value Theorem can be used to develop the following upper bound for the expression in (18):

$$\left\|\tilde{N}_1(t)\right\| \le \rho_1(\|z\|)\|z\| \tag{20}$$

where the bounding function $\rho_1(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function, and $z(t) \in \mathbb{R}^{n(m+1)}$ is defined as

$$z(t) \triangleq \begin{bmatrix} e_1^T & e_2^T & \ldots & e_m^T & r^T \end{bmatrix}^T. \tag{21}$$

Using Properties 2 and 3, the following inequalities can be developed based on the expression in (19) and its time derivative:

$$\|N_{1B}(t)\| \le \zeta_1, \qquad \left\|\dot{N}_{1B}(t)\right\| \le \zeta_2 \tag{22}$$

where $\zeta_i \in \mathbb{R}$, $i = 1, 2$ are known positive constants.

V. SWAPPING

A measurable form of the prediction error $\varepsilon(t) \in \mathbb{R}^n$, used in the composite adaptive update law in (12), is defined as the difference between the filtered control input $u_f(t) \in \mathbb{R}^n$ and the estimated filtered control input $\hat{u}_f(t) \in \mathbb{R}^n$ as

$$\varepsilon \triangleq u_f - \hat{u}_f, \tag{23}$$

where the filtered control input $u_f(t) \in \mathbb{R}^n$ is generated by

$$\dot{u}_f + \omega u_f = \omega u \qquad u_f(0) = 0, \tag{24}$$

where $\omega \in \mathbb{R}$ is a known positive constant, and $\hat{u}_f(t) \in \mathbb{R}^n$ is subsequently designed. The differential equation in (24) can be directly solved to yield

$$u_f = v * u, \tag{25}$$

where $*$ is used to denote the standard convolution operation, and the scalar function $v(t)$ is defined as

$$v \triangleq \omega e^{-\omega t}. \tag{26}$$

Using (1), the expression in (25) can be rewritten as

$$u_f = v * \left(G^{-1}x^{(m)} - G^{-1}f - G^{-1}h\right). \tag{27}$$

Since the system dynamics in (1) include non-LP bounded disturbances $h(t)$, they also get filtered and included in the filtered control input in (27). To compensate for the effects of these disturbances, the typical prediction error formulation is modified to include a RISE-like structure in the design of the estimated filtered control input. With this motivation, the open-loop prediction error system is engineered to facilitate the RISE-based design of the estimated filtered control input.

Adding and subtracting the term $G_d^{-1}x_d^{(m)} + G_d^{-1}f_d + G_d^{-1}h$ to the expression in (27) yields

$$u_f = v * (G_d^{-1}x_d^{(m)} + G_d^{-1}f_d + G^{-1}x^{(m)} - G_d^{-1}x_d^{(m)} - G^{-1}f - G_d^{-1}f_d - G^{-1}h + G_d^{-1}h - G_d^{-1}h). \tag{28}$$

Using (8), the expression in (28) is simplified as

$$u_f = v * \left(Y_d\theta + S - S_d - G_d^{-1}h\right) \tag{29}$$

where $S(x, \dot{x}, ..., x^{(m)})$, $S_d(x_d, \dot{x}_d, ..., x_d^{(m)}) \in \mathbb{R}^n$ are defined as

$$S \triangleq G^{-1}x^{(m)} - G^{-1}f - G^{-1}h \tag{30}$$

$$S_d \triangleq G_d^{-1}x_d^{(m)} - G_d^{-1}f_d - G_d^{-1}h. \tag{31}$$

The expression in (29) is further simplified as

$$u_f = Y_{df}\theta + v * S - v * S_d + h_f \tag{32}$$

where the filtered regressor matrix $Y_{df}(x_d, \dot{x}_d, ..., x_d^{(m)}) \in \mathbb{R}^{n\times p}$ is defined as

$$Y_{df} \triangleq v * Y_d \tag{33}$$

and the disturbance $h_f(t) \in \mathbb{R}^n$ is defined as

$$h_f \triangleq -v * G_d^{-1}h.$$

The term $v * S(x, \dot{x}, ..., x^{(m)}) \in \mathbb{R}^n$ in (32) depends on $x^{(m)}$. Using the following property of convolution [15]:

$$g_1 * \dot{g}_2 = \dot{g}_1 * g_2 + g_1(0)\, g_2 - g_1 g_2(0) \tag{34}$$

an expression independent of $x^{(m)}$ can be obtained. Consider

$$v * S = v * \left(G^{-1} x^{(m)} - G^{-1} f - G^{-1} h\right)$$

which can be rewritten as

$$\begin{aligned} v * S \;=\; & v * \Big(\frac{d}{dt}(G^{-1} x^{(m-1)}) - \dot{G}^{-1} x^{(m-1)} \\ & -G^{-1} f - G^{-1} h\Big). \end{aligned} \tag{35}$$

Applying the property in (34) to the first term of (35) yields

$$v * S = S_f + W \tag{36}$$

where the state-dependent terms are included in the auxiliary function $S_f(x, \dot{x}, ..., x^{(m-1)}) \in \mathbb{R}^n$, defined as

$$\begin{aligned} S_f \;\triangleq\; & \dot{v} * \left(G^{-1} x^{(m-1)}\right) + v(0)\, G^{-1} x^{(m-1)} \\ & -v * \dot{G}^{-1} x^{(m-1)} - v * G^{-1} f - v * G^{-1} h \end{aligned} \tag{37}$$

and the terms that depend on the initial states are included in $W(t) \in \mathbb{R}^n$, defined as

$$W \triangleq -vG^{-1}\left(x(0), \dot{x}(0), ..., x^{(m-2)}(0)\right) x^{(m-1)}(0). \tag{38}$$

Similarly, following the procedure in (35)-(38), the expression $v * S_d$ in (32) is evaluated as

$$v * S_d = S_{df} + W_d \tag{39}$$

where $S_{df}(x_d, \dot{x}_d, ..., x_d^{(m-1)}) \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S_{df} \;\triangleq\; & \dot{v} * (G_d^{-1} x_d^{(m-1)}) + v(0)\, G_d^{-1} x_d^{(m-1)} \\ & -v * \dot{G}_d^{-1} x_d^{(m-1)} - v * G_d^{-1} f_d - v * G_d^{-1} h \end{aligned} \tag{40}$$

and $W_d(t) \in \mathbb{R}^n$ is defined as

$$W_d \triangleq -vG_d^{-1}(x_d(0), \dot{x}_d(0), ..., x_d^{(m-2)}(0)) x_d^{(m-1)}(0). \tag{41}$$

Substituting (36)-(41) into (32), and then substituting the resulting expression into (23) yields

$$\varepsilon = Y_{df}\theta + S_f - S_{df} + W - W_d + h_f - \hat{u}_f. \tag{42}$$

Based on (43) and the subsequent analysis, the filtered control input estimate is designed as

$$\hat{u}_f = Y_{df}\hat{\theta} + \mu_2, \tag{43}$$

where $\mu_2(t) \in \mathbb{R}^n$ is a RISE-like term defined as

$$\mu_2(t) \triangleq \int_0^t [k_2 \varepsilon(\sigma) + \beta_2 sgn(\varepsilon(\sigma))] d\sigma, \tag{44}$$

where $k_2, \beta_2 \in \mathbb{R}$ denote constant positive control gains. In a typical prediction error formulation, the estimated filtered control input is designed to include just the first term $Y_{df}\hat{\theta}$ in (43). But as discussed earlier, due to the presence of non-LP disturbances in the system model, the unmeasurable form of the prediction error in (42) also includes the filtered disturbances. Hence, the estimated filtered control input is augmented with an additional RISE-like term $\mu_2(t)$ to cancel the effects of disturbances in the prediction error measurement as illustrated in the subsequent design and stability analysis.

Substituting (43) into (42) yields the following closed-loop prediction error system:

$$\varepsilon = Y_{df}\tilde{\theta} + S_f - S_{df} + W - W_d + h_f - \mu_2. \tag{45}$$

To facilitate the subsequent composite adaptive control development and stability analysis, the time derivative of (45) is expressed as

$$\dot{\varepsilon} = \dot{Y}_{df}\tilde{\theta} - Y_{df}\Gamma\dot{Y}_{df}^T\varepsilon + \tilde{N}_2 + N_{2B} - k_2\varepsilon - \beta_2 sgn(\varepsilon), \tag{46}$$

where (12) and the fact that

$$\dot{\mu}_2 = k_2\varepsilon + \beta_2 sgn(\varepsilon) \tag{47}$$

were utilized. In (46), the unmeasurable/unknown auxiliary term $\tilde{N}_2(e_1, e_2, ..., e_m, r, t) \in \mathbb{R}^n$ is defined as

$$\tilde{N}_2 \triangleq \dot{S}_f - \dot{S}_{df} - Y_{df}\Gamma\dot{Y}_d^T r, \tag{48}$$

where the update law in (12) was utilized, and the term $N_{2B}(t) \in \mathbb{R}^n$ is defined as

$$N_{2B} \triangleq \dot{W} - \dot{W}_d + \dot{h}_f. \tag{49}$$

In a similar fashion as in (20), the following upper bound can be developed for the expression in (48):

$$\left\|\tilde{N}_2(t)\right\| \leq \rho_2(\|z\|)\|z\|, \tag{50}$$

where the bounding function $\rho_2(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function, and $z(t) \in \mathbb{R}^{n(m+1)}$ was defined in (21). Using Property 3, and the fact that $v(t)$ is a linear, strictly proper, exponentially stable transfer function, the following inequality can be developed based on the expression in (49) with a similar approach as in Lemma 2 of [7]:

$$\|N_{2B}(t)\| \leq \xi, \tag{51}$$

where $\xi \in \mathbb{R}$ is a known positive constant.

VI. Stability Analysis

Theorem 1: The controller given in (11) and (13) in conjunction with the composite adaptive update law in (12), where the prediction error is generated from (23), (24), (43), and (44), ensures that all system signals are bounded under closed-loop operation and that the position tracking error and the prediction error are regulated in the sense that

$$\|e_1(t)\| \to 0 \text{ and } \|\varepsilon(t)\| \to 0 \qquad \text{as} \qquad t \to \infty$$

provided the control gains k_1 and k_2 introduced in (13) and (44) are selected sufficiently large (see the subsequent proof), and the following conditions are satisfied:

$$\alpha_{m-1} > \frac{1}{2}, \qquad \alpha_m > \frac{1}{2}, \tag{52}$$

$$\beta_1 > \zeta_1 + \frac{1}{\alpha_m}\zeta_2 \ , \qquad \beta_2 > \xi, \tag{53}$$

where the gains α_{m-1} and α_m were introduced in (4), β_1 was introduced in (13), β_2 was introduced in (44), ζ_1 and ζ_2 were introduced in (22), and ξ was introduced in (51).

Proof: Let $\mathcal{D} \subset \mathbb{R}^{n(m+2)+p+2}$ be a domain containing $y(t) = 0$, where $y(t) \in \mathbb{R}^{n(m+2)+p+2}$ is defined as

$$y \triangleq [z^T \quad \varepsilon^T \quad \sqrt{P_1} \quad \sqrt{P_2} \quad \tilde{\theta}^T]^T. \tag{54}$$

In (54), the auxiliary function $P_1(t) \in \mathbb{R}$ is defined as

$$P_1(t) \triangleq \beta_1 \sum_{i=1}^{n} |e_{mi}(0)| - e_m(0)^T N_{1B}(0) - \int_0^t L_1(\tau)d\tau, \tag{55}$$

where $e_{mi}(0) \in \mathbb{R}$ denotes the ith element of the vector $e_m(0)$, and the auxiliary function $L_1(t) \in \mathbb{R}$ is defined as

$$L_1 \triangleq r^T(N_{1B} - \beta_1 sgn(e_m)), \tag{56}$$

where $\beta_1 \in \mathbb{R}$ is a positive constant chosen according to the sufficient condition in (53). Provided the sufficient condition introduced in (53) is satisfied, the following inequality is obtained [19]:

$$\int_0^t L_1(\tau)d\tau \leq \beta_1 \sum_{i=1}^{n} |e_{mi}(0)| - e_m(0)^T N_{1B}(0). \tag{57}$$

Hence, (57) can be used to conclude that $P_1(t) \geq 0$. Also in (54), the auxiliary function $P_2(t) \in \mathbb{R}$ is defined as

$$P_2(t) \triangleq -\int_0^t L_2(\tau)d\tau, \tag{58}$$

where the auxiliary function $L_2(t) \in \mathbb{R}$ is defined as

$$L_2 \triangleq \varepsilon^T(N_{2B} - \beta_2 sgn(\varepsilon)), \tag{59}$$

where $\beta_2 \in \mathbb{R}$ is a positive constant chosen according to the sufficient condition in (53). Provided the sufficient condition introduced in (53) is satisfied, then $P_2(t) \geq 0$.

Let $V_L(y,t) : \mathcal{D} \times [0,\infty) \to \mathbb{R}$ be a continuously differentiable, positive definite function defined as

$$\begin{aligned} V_L \ &\triangleq\ \frac{1}{2}\sum_{i=1}^{m} e_i^T e_i + \frac{1}{2} r^T G^{-1} r + \frac{1}{2}\varepsilon^T\varepsilon \\ &\quad + P_1 + P_2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} \end{aligned} \tag{60}$$

which satisfies the inequalities

$$U_1(y) \leq V_L(y,t) \leq U_2(y) \tag{61}$$

provided the sufficient conditions introduced in (53) are satisfied. In (61), the continuous positive definite functions $U_1(y)$, $U_2(y) \in \mathbb{R}$ are defined as $U_1(y) \triangleq \lambda_1 \|y\|^2$ and $U_2(y) \triangleq \lambda_2(x, \dot{x}, ..., x^{(m-2)}) \|y\|^2$, where λ_1, $\lambda_2(x, \dot{x}, ..., x^{(m-2)}) \in \mathbb{R}$ are defined as

$$\begin{aligned} \lambda_1 \ &\triangleq\ \frac{1}{2}\min\left\{1, \underline{g}, \lambda_{\min}\left\{\Gamma^{-1}\right\}\right\} \qquad (62) \\ \lambda_2 \ &\triangleq\ \max\{\frac{1}{2}\bar{g}(x, \dot{x}, ..., x^{(m-2)}), \frac{1}{2}\lambda_{\max}\left\{\Gamma^{-1}\right\}, 1\} \end{aligned}$$

where $\underline{g}$, $\bar{g}(x, \dot{x}, ..., x^{(m-2)})$ are introduced in (2), and $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ denote the minimum and maximum eigenvalue of the arguments, respectively. After using (4), (6), (12), (16), (46), (55), (56), (58) and (59), the time derivative of (60) can be expressed as

$$\begin{aligned} \dot{V}_L \ &=\ -\sum_{i=1}^{m}\alpha_i e_i^T e_i + e_{m-1}^T e_m - r^T r - k_1 r^T r \\ &\quad + r^T \dot{Y}_d \tilde{\theta} + r^T \tilde{N}_1 + r^T N_{1B} - r^T Y_d \Gamma \dot{Y}_{df}^T \varepsilon \\ &\quad - \beta_1 r^T sgn(e_m) + \varepsilon^T \dot{Y}_{df}\tilde{\theta} + \varepsilon^T \tilde{N}_2 + \varepsilon^T N_{2B} \\ &\quad - k_2 \varepsilon^T \varepsilon - \varepsilon^T Y_{df}\Gamma \dot{Y}_{df}^T \varepsilon - \beta_2 \varepsilon^T sgn(\varepsilon) \\ &\quad - r^T(N_{1B} - \beta_1 sgn(e_m)) - \varepsilon^T N_{2B} \\ &\quad + \varepsilon^T \beta_2 sgn(\varepsilon) - \tilde{\theta}^T \Gamma^{-1}(\Gamma \dot{Y}_d^T r + \Gamma \dot{Y}_{df}^T \varepsilon). \end{aligned} \tag{63}$$

After canceling the similar terms and using the fact that $a^T b \leq \frac{1}{2}(\|a\|^2 + \|b\|^2)$ for some $a, b \in \mathbb{R}^n$, the expression in (63) is upper bounded as

$$\begin{aligned} \dot{V}_L \ &\leq\ -\sum_{i=1}^{m}\alpha_i e_i^T e_i + \frac{1}{2}\|e_{m-1}\|^2 + \frac{1}{2}\|e_m\|^2 \\ &\quad - \|r\|^2 - k_1\|r\|^2 + r^T\tilde{N}_1 - r^T Y_d \Gamma \dot{Y}_{df}^T \varepsilon \\ &\quad + \varepsilon^T \tilde{N}_2 - k_2\varepsilon^T\varepsilon - \varepsilon^T Y_{df}\Gamma\dot{Y}_{df}^T\varepsilon. \end{aligned}$$

Using the following upper bounds:

$$\left\|Y_d \Gamma \dot{Y}_{df}^T\right\| \leq c_1, \qquad \left\|Y_{df}\Gamma\dot{Y}_{df}^T\right\| \leq c_2,$$

where $c_1, c_2 \in \mathbb{R}$ are positive constants, $\dot{V}_L(y,t)$ is upper bounded using the squares of the components of $z(t)$ as

$$\begin{aligned} \dot{V}_L \ &\leq\ -\lambda_3\|z\|^2 - k_1\|r\|^2 + \|r\|\left\|\tilde{N}_1\right\| \\ &\quad + c_1\|\varepsilon\|\|r\| + \|\varepsilon\|\left\|\tilde{N}_2\right\| - (k_2 - c_2)\|\varepsilon\|^2 \end{aligned} \tag{64}$$

where

$$\lambda_3 \triangleq \min\{\alpha_1, \alpha_2, ..., \alpha_{m-2}, \alpha_{m-1} - \frac{1}{2}, \alpha_m - \frac{1}{2}, 1\}.$$

Letting

$$k_2 = k_{2a} + k_{2b}$$

where $k_{2a}, k_{2b} \in \mathbb{R}$ are positive constants, and using the inequalities in (20) and (50), the expression in (64) is upper bounded as

$$\begin{aligned} \dot{V}_L \ &\leq\ -\lambda_3\|z\|^2 - k_{2b}\|\varepsilon\|^2 \\ &\quad - \left[k_1\|r\|^2 - \rho_1(\|z\|)\|r\|\|z\|\right] \\ &\quad - \left[(k_{2a} - c_2)\|\varepsilon\|^2 - (\rho_2(\|z\|) + c_1)\|\varepsilon\|\|z\|\right]. \end{aligned} \tag{65}$$

Completing the squares for the terms inside the brackets in (65) yields

$$\begin{aligned} \dot{V}_L \ &\leq\ -\lambda_3\|z\|^2 - k_{2b}\|\varepsilon\|^2 \\ &\quad + \frac{\rho_1^2(\|z\|)\|z\|^2}{4k_1} + \frac{(\rho_2(\|z\|) + c_1)^2\|z\|^2}{4(k_{2a} - c_2)} \\ &\leq\ -\lambda_3\|z\|^2 + \frac{\rho^2(\|z\|)\|z\|^2}{4k} - k_{2b}\|\varepsilon\|^2 \\ &\leq\ -U(y), \end{aligned} \tag{66}$$

where $k \in \mathbb{R}$ is defined as

$$k \triangleq \frac{k_1 (k_{2a} - c_2)}{\max\{k_1, (k_{2a} - c_2)\}}, \quad (67)$$

and $\rho(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function defined as

$$\rho^2(\|z\|) \triangleq \rho_1^2(\|z\|) + (\rho_2(\|z\|) + c_1)^2.$$

In (66), $U(y) = c \left\| \begin{bmatrix} z^T & \varepsilon^T \end{bmatrix}^T \right\|^2$, for some positive constant c, is a continuous, positive semi-definite function that is defined on the domain

$$\mathcal{D} \triangleq \left\{ y(t) \in \mathbb{R}^{n(m+2)+p+2} \mid \|y\| \leq \rho^{-1}\left(2\sqrt{\lambda_3 k}\right) \right\}.$$

The inequalities in (61) and (66) can be used to show that $V_L(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_i(t) \in \mathcal{L}_\infty$ and $\varepsilon(t), r(t), \tilde{\theta}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_i(t) \in \mathcal{L}_\infty$ and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard linear analysis methods can be used to prove that $\dot{e}_i(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (4) and (6). Since $e_i(t) \in \mathcal{L}_\infty$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 5 can be used along with (3)-(6) to conclude that $x^{(i)}(t) \in \mathcal{L}_\infty$, $i = 0, 1, ..., m$ in $\mathcal{D}$. Since $\tilde{\theta}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (15) can be used to prove that $\hat{\theta}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $x^{(i)}(t) \in \mathcal{L}_\infty$, $i = 0, 1, ..., m$ in $\mathcal{D}$, Property 2 can be used to conclude that $G^{-1}(\cdot)$ and $f(\cdot) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Thus, from (1) and Property 3, we can show that $u(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Therefore, $u_f(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, and hence, from (23), $\hat{u}_f(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (17) can be used to show that $\dot{\mu}_1(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, and since $\dot{G}^{-1}(\cdot)$ and $\dot{f}(\cdot) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (16) can be used to show that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, and (46) can be used to show that $\dot{\varepsilon}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_i(t) \in \mathcal{L}_\infty$, $\dot{r}(t)$, and $\dot{\varepsilon}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the definitions for $U(y)$ and $z(t)$ can be used to prove that $U(y)$ is uniformly continuous in $\mathcal{D}$.

Let $\mathcal{S} \subset \mathcal{D}$ denote a set defined as

$$\mathcal{S} \triangleq \left\{ y(t) \subset \mathcal{D} \mid U_2(y(t)) < \lambda_1 \left(\rho^{-1}\left(2\sqrt{\lambda_3 k}\right)\right)^2 \right\}. \quad (68)$$

The region of attraction in (68) can be made arbitrarily large to include any initial conditions by increasing the control gain k (i.e., a semi-global stability result). Theorem 8.4 of [24] can now be invoked to state that

$$c \left\| \begin{bmatrix} z^T & \varepsilon^T \end{bmatrix}^T \right\|^2 \to 0 \qquad \text{as} \qquad t \to \infty \qquad \forall y(0) \in \mathcal{S}. \quad (69)$$

Based on the definition of $z(t)$, (69) can be used to show that

$$\begin{aligned} \|e_1(t)\| &\to 0 \qquad \text{as} \qquad t \to \infty \qquad \forall y(0) \in \mathcal{S} \quad (70) \\ \|\varepsilon(t)\| &\to 0 \qquad \text{as} \qquad t \to \infty \qquad \forall y(0) \in \mathcal{S}. \end{aligned}$$

VII. Conclusion

A novel approach for the design of a gradient-based composite adaptive controller was proposed for generic MIMO systems subjected to bounded disturbances. A model-based feedforward adaptive component was used in conjunction with the RISE feedback, where the adaptive estimates were generated using a composite update law driven by both the tracking and prediction error with the motivation of using more information in the adaptive update law. To account for the effects of non-LP disturbances, the typical prediction error formulation was modified to include a second RISE-like term in the estimated filtered control input design. Using a Lyapunov stability analysis, sufficient gain conditions were derived under which the proposed controller yields semi-global asymptotic stability.

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