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Nonlinear Adaptive H_∞ Control of Constrained Robotic Manipulators with Input Nonlinearity

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***Abstract*— The problem of constructing nonlinear adaptive H_∞ control of constrained robotic manipulators with uncertain input nonlinearities such as dead-zone or backlash, is considered in this paper. In the proposed control scheme, adaptive inverse models are introduced to compensate effects of input nonlinearities, and the trajectory converges to the desired constrained trajectory, and the constraint force also follows the desired constraint one. The resulting control strategy is derived as a solution of certain H_∞ control problem, where estimation errors of tuning parameters, errors of constraint forces and residual terms of the inverse models, are regarded as external disturbances to the process.**

I. INTRODUCTION

Motion control problems of mechanical systems are divided into two categories, that is, free motion control and constrained motion control. Free motion control problems of mechanical systems are seen in the situations where there is no contact between controlled processes and environments, and have been studied extensively as basic control problems of mechanical systems [1], [2]. On the contrary, motion control problems of constrained mechanical systems are seen in the situations where there exists a contact between controlled processes and environments, and contact forces between end-effectors of mechanical systems and environments are generated. Compared with free motion control, constrained motion control has been a difficult problem, where not only constrained trajectory control but also simultaneous constraint force control should be considered [4], [5], [6], [7] [8], and the adaptive control version of that problem for mechanical systems with parametric uncertainties, is a difficult but important problem from the practical point of view.

For that control problem, in our previous study, we provided design methods of nonlinear adaptive H_∞ control of constrained robotic manipulators based on the notion of inverse optimality [9]. In those approaches, estimation errors of tuning parameters in the adaptation mechanism and errors of constraint forces are regarded as external disturbances to the process, and the resulting control strategy is derived as a solution of corresponding H_∞ control problems [10], [11], [12]. Asymptotic stability of tracking errors of constrained trajectories and the variables concerned with errors of constraint forces, are assured. Two approaches are deduced based on that policy, and it is shown that $\mathcal{L}^2$ gains from those disturbances (errors of tuning parameters and constraint forces) to generalized outputs are prescribed by several design parameters, explicitly. The proposed control strategy contains a kind of nonlinear damping metohdology, and thus, attains good convergence and transient property with less control efforts.

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In the present work, we consider a more practical situation and present a design scheme of nonlinear adaptive H_∞ control of constrained robotic manipulators with uncertain input nonlinearities such as dead-zone or backlash. Those actuator nonlinearities are often seen in mechanical connections, electric servo motors, hydraulic servo valves and other mechanical actuators. For the input nonlinearities, adaptive inverse approaches or high-gain feedback schemes have been proposed to compensate the effect of such nonlinearities in the related previous works [13], [14], [15], [16]. In the present manuscript, the adaptive inverse approaches including smooth approximations of nonlinearities, are employed, and the resulting control strategy is derived as a solution of certain H_∞ control problem, where estimation errors of tuning parameters, errors of constraint forces and residual terms of the inverse models, are regarded as external disturbances to the process.

II. PROBLEM STATEMENT

Consider a robotic manipulator with n degrees of freedom and with rotational joints described by

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau + f, \tag{1}$$

$$\tau = N(u), \tag{2}$$

where $\theta \in \mathbf{R}^n$ is a vector of joint angles, $M(\theta) \in \mathbf{R}^{n\times n}$ is a matrix of inertia, $C(\theta,\dot{\theta}) \in \mathbf{R}^{n\times n}$ is a matrix of Coriolis and centrifugal forces, $G(\theta) \in \mathbf{R}^n$ is a vector of gravitational torques, and $\tau \in \mathbf{R}^n$ is a vector of an input torque. $N(u)$ represents actuator characteristics such as dead-zone or backlash nonlinearities. It is assumed that the system parameters in $M(\theta)$, $C(\theta,\dot{\theta})$, $G(\theta)$ and the nonlinear characteristics $N(u)$ are unknown, and τ is not an actual control signal, and is unknown. Only, θ, $\dot{\theta}$ and the actual input signal $u \in \mathbf{R}^n$ are assumed to be available for measurement. The trajectory θ of the robotic manipulator is subject to a constraint represented by a set of m geometric equations (holonomic constraint and frictionless, $m < n$) such that

$$\Psi(\theta) = 0, \quad \frac{d}{dt}\Psi(\theta) = 0, \quad (\Psi \in \mathbf{R}^m), \tag{3}$$

and f is a constraint force which is expressed as

$$f = J(\theta)^T\lambda, \qquad (\lambda \in \mathbf{R}^m), \tag{4}$$

$$J(\theta) = \frac{\partial \Psi}{\partial \theta}, \qquad (J(\theta) \in \mathbf{R}^{m\times n}), \tag{5}$$

where λ is a Lagrangian multiplier. It is assumed that the constraint force is measured by a force sensor mounted at the end-effector of the system. The control objective is to synthesize a proper input signal u such that the constrained trajectory θ and constraint force f follow the desired constrained trajectory $\theta_d(t)$ (differentiable on $t \in [0, \infty)$ and $\Psi(\theta_d) = 0$) and the desired constraint force f_d, respectively, for unknown system parameters in $M(\theta)$, $C(\theta, \dot{\theta})$, $G(\theta)$, unknown nonlinear characteristics $N(u)$ and unknown control torques τ.

$$\theta \to \theta_d, \quad (\Psi(\theta) = \Psi(\theta_d) \equiv 0), \tag{6}$$

$$f \to f_d. \tag{7}$$

Typical examples of that control problem are grinding, polishing, inserting, deburring, and scribing, etc [3], where the end-effector of the mechanical system exerts a desired force to the environment as the controlled process moves along a prescribed constrained trajectory.

Robotic manipulators with rotational joints have the following properties [17].

Properties of Robotic Manipulators [17]

1) $M(\theta)$ is a bounded, positive definite, and symmetric matrix.
2) $\dot{M}(\theta) - 2C(\theta, \dot{\theta})$ is a skew symmetric matrix.
3) The left-hand side of (1) can be written into the following form,

$$M(\theta)a + C(\theta, \dot{\theta})b + G(\theta) = \Omega_1(\theta, \dot{\theta}, a, b)^T \Phi_1, \tag{8}$$

where $\Omega(\theta, \dot{\theta}, a, b)$ is a known function of θ, $\dot{\theta}$, a, b, and Φ_1 is an unknown system parameter.

III. TRACKING CONTROL UNDER CONSTRAINT

First, we introduce the conventional adaptive control for constrained manipulators [7], where the control torque τ is assumed to be an actual input signal.

A. System Description Including Constraint

System descriptions of controlled processes which includes constraints implicitly, are to be obtained in the present section. The development of such descriptions is mainly owing to the previous study [4].

According to the dimension m of the geometric constraint, the output θ is divided into θ^1 and θ^2, where

$$\theta = \begin{bmatrix} \theta^1 \\ \theta^2 \end{bmatrix}, \quad \theta^1 \in \mathbf{R}^{n-m}, \ \ \theta^2 \in \mathbf{R}^m. \tag{9}$$

Then, $J(\theta)$ is also described in the following decomposed form.

$$J(\theta) = \left[\frac{\partial \Psi}{\partial \theta^1}, \ \frac{\partial \Psi}{\partial \theta^2}\right] = [J_1(\theta), \ J_2(\theta)], \tag{10}$$

$$J_1(\theta) \in \mathbf{R}^{m\times(n-m)}, \ J_2(\theta) \in \mathbf{R}^{m\times m}. \tag{11}$$

There is a proper partition such that $\det J_2(\theta) \neq 0$. Since the next relation holds,

$$0 = \frac{d}{dt}\Psi(\theta) = J(\theta)\dot{\theta} = J_1(\theta)\dot{\theta}^1 + J_2(\theta)\dot{\theta}^2, \tag{12}$$

$\dot{\theta}^2$ is represented by $\dot{\theta}^1$ such as

$$\dot{\theta}^2 = -J_2(\theta)^{-1}J_1(\theta)\dot{\theta}^1, \tag{13}$$

and it follows that $\dot{\theta}$ is represented by utilizing $\dot{\theta}^1$.

$$\dot{\theta} = L(\theta)\dot{\theta}^1, \tag{14}$$

$$L(\theta) = \begin{bmatrix} I_{n-m} \\ -J_2(\theta)^{-1}J_1(\theta) \end{bmatrix}. \tag{15}$$

For $L(\theta)$, it is easily shown that the next relation holds.

$$L(\theta)^T J(\theta)^T = J_1(\theta)^T - J_1(\theta)^T = 0. \tag{16}$$

By utilizing the property of $L(\theta)$, the system description which includes constraint implicitly, is deduced. The substitution of (14) and the next relation

$$\ddot{\theta} = L(\theta)\ddot{\theta}^1 + \dot{L}(\theta, \dot{\theta})\dot{\theta}^1, \tag{17}$$

into (1) yields

$$M(\theta)L(\theta)\ddot{\theta}^1 + M(\theta)\dot{L}(\theta, \dot{\theta}))\dot{\theta}^1 + C(\theta, \dot{\theta})L(\theta)\dot{\theta}^1 + G(\theta) = \tau + f. \tag{18}$$

By multiplying $L(\theta)^T$ to above equation, the following representation is derived.

$$M_1(\theta)\ddot{\theta}^1 + C_1(\theta, \dot{\theta}_1)\dot{\theta}_1 + G_1(\theta) = L(\theta)^T\tau, \tag{19}$$

$$M_1(\theta) = L(\theta)^T M(\theta)L(\theta), \tag{20}$$

$$C_1(\theta, \dot{\theta}) = L(\theta)^T(M(\theta)\dot{L}(\theta, \dot{\theta}) + C(\theta, \dot{\theta})L(\theta)), \tag{21}$$

$$G_1(\theta) = L(\theta)^T G(\theta). \tag{22}$$

The system description (19) does not contain constraint force nor geometric constraint, explicitly. Then, for given τ, constrained trajectories $\ddot{\theta}^1$, $\dot{\theta}^1$ and θ^1 are computed from (19), and $\ddot{\theta}^2$, $\dot{\theta}^2$ and θ^2 are also derived by considering (3), (14), (17). Finally, the constraint force f is computed from the relation $f = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) - \tau$.

B. Adaptive Control Under Constraint

We introduce the conventional adaptive control for constrained manipulators [7]. Define the following signals.

$$\tilde{\theta}^1 = \theta^1 - \theta_d^1 \ (\in \mathbf{R}^{n-m}), \tag{23}$$

$$\tilde{\theta}^2 = \theta^2 - \theta_d^2 \ (\in \mathbf{R}^{m}), \tag{24}$$

$$\dot{\theta}_r^1 = \dot{\theta}_d^1 - \Lambda\tilde{\theta}^1 \ (\in \mathbf{R}^{n-m}), \tag{25}$$

$$\dot{\theta}_r = L(\theta)\dot{\theta}_r^1 \ (\in \mathbf{R}^{n}), \tag{26}$$

$$s = \dot{\theta}^1 - \dot{\theta}_r^1 = \dot{\tilde{\theta}}^1 + \Lambda\tilde{\theta}^1 \ (\in \mathbf{R}^{n-m}), \tag{27}$$

$$\tilde{f} = f - f_d \ (\in \mathbf{R}^{n}), \tag{28}$$

$$(\Lambda \in \mathbf{R}^{(n-m)\times(n-m)}; \ \Lambda = \Lambda^T > 0),$$

where θ_d^1 is a subset of elements in θ_d which corresponds to θ^1. μ is a variable to handle the force control part, and is synthesized from $\tilde{f}$ such as

$$\dot{\mu} = -\kappa\mu - \kappa\tilde{f}, \qquad (\mu \in \mathbf{R}^n), \qquad (\kappa > 0). \tag{29}$$

Also σ and ν are introduced as follows:

$$\sigma \equiv Ls + \mu \ (= \dot{\theta} - \nu) \ (\in \mathbf{R}^n), \tag{30}$$

$$\nu \equiv \dot{\theta}_r - \mu \ (\in \mathbf{R}^n). \tag{31}$$

For σ and ν, we obtain the following relations.

$$\dot{\sigma} = L\dot{s} + \dot{L}s - \kappa(\mu + \tilde{f}), \tag{32}$$

$$\dot{\nu} = L\ddot{\theta}_r^1 + \dot{L}\dot{\theta}_r + \kappa(\mu + \tilde{f}). \tag{33}$$

The substitution of above relations into (1) yields

$$\begin{aligned} &M(\theta)\dot{\sigma} + C(\theta,\dot{\theta})\sigma + M(\theta)\dot{\nu} + C(\theta,\dot{\theta})\nu + G(\theta) \\ &= M(\theta)\dot{\sigma} + C(\theta,\dot{\theta})\sigma + \Omega_1(\theta,\dot{\theta},\dot{\nu},\nu)^T\Phi_1 = \tau + f. \end{aligned} \quad (34)$$

This corresponds to the error equation of the traditional adaptive control [18]. For that error system, the control input is synthesized such as

$$\tau = -K\sigma - f_d + \alpha\tilde{f} + \Omega_1^T\hat{\Phi}_1, \quad (35)$$

$$\Omega_1 \equiv \Omega_1(\theta,\dot{\theta},\dot{\nu},\nu), \quad (36)$$

$$(K \in \mathbf{R}^{n\times n}:\ K = K^T > 0, \quad \alpha > 0),$$

where $\hat{\Phi}_1$ is a current estimate of Φ_1, and is tuned by the following adaptive law.

$$\dot{\hat{\Phi}}_1 = -\Gamma_1\Omega_1\sigma, \quad (\Gamma_1 = \Gamma_1^T > 0). \quad (37)$$

Then, the error equation becomes

$$M(\theta)\dot{\sigma} + C(\theta,\dot{\theta})\sigma = -K\sigma + (1+\alpha)\tilde{f} + \Omega_1^T\tilde{\Phi}_1, \quad (38)$$

$$\tilde{\Phi}_1 = \hat{\Phi}_1 - \Phi_1. \quad (39)$$

Here we define positive functions V_0, V_1

$$V_0 = \frac{1}{2}\sigma^T M(\theta)\sigma + \left(\frac{1+\alpha}{2\kappa}\right)\|\mu\|^2, \quad (40)$$

$$V_1 = V_0 + \frac{1}{2}\tilde{\Phi}_1^T\Gamma_1^{-1}\tilde{\Phi}_1, \quad (41)$$

and take the time derivative of V_1 along the trajectory of the manipulator.

$$\dot{V}_1 = -\sigma^T K\sigma - (1+\alpha)\|\mu\|^2 \le 0, \quad (42)$$

where (16) is considered. Then it follows that $\sigma, \mu \in \mathcal{L}^2 \cap \mathcal{L}^\infty$ and that $\hat{\Phi}_1 \in \mathcal{L}^\infty$. By considering the following relation

$$s = (L^TL)^{-1}L^T(\sigma - \mu), \quad (43)$$

it is shown that $s \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, if $(L^TL)^{-1}L^T \in \mathcal{L}^\infty$. Furthermore, by considering the next relation

$$s = \dot{\tilde{\theta}}^1 + \Lambda\tilde{\theta}^1 \in \mathcal{L}^2 \cap \mathcal{L}^\infty, \quad (44)$$

we obtain $\tilde{\theta}^1, \dot{\tilde{\theta}}^1 \in \mathcal{L}^\infty$ and $\tilde{\theta}^1 \to 0$. Also, by seeing $\dot{\theta} = L\dot{\theta}^1$ and $\Psi(\theta) = 0$, it follows that $\tilde{\theta} \equiv \theta - \theta_d$, $\dot{\tilde{\theta}} \in \mathcal{L}^\infty$ and $\tilde{\theta} \to 0$, if $\theta^1 \in \mathcal{L}^\infty$ implies $L \in \mathcal{L}^\infty$. Furthermore, $\dot{\theta}_r^1 = \dot{\theta}_d^1 - \Lambda\tilde{\theta}^1$ suggests that $\dot{\theta}_r^1 \in \mathcal{L}^\infty$. Hence, it is shown that $\dot{\theta}_r = L\dot{\theta}_r^1 \in \mathcal{L}^\infty$, if $\theta^1 \in \mathcal{L}^\infty$ implies $L \in \mathcal{L}^\infty$. Additionally, $\nu = \dot{\theta}_r - \mu \in \mathcal{L}^\infty$. Next, we consider constraint force. Since it holds that $\tilde{\lambda} = \lambda - \lambda_d \in \mathcal{L}^\infty$ when $(1+\alpha)I + \kappa\hat{M}$ is non-singular ($\hat{M}$ is a current estimate of M composed of the corresponding elements in $\hat{\Phi}_1$) [7], it follows that $\tilde{f} = J^T\tilde{\lambda} \in \mathcal{L}^\infty$, and that $f = J^T\lambda \in \mathcal{L}^\infty$. Then it is shown that $\tau \in \mathcal{L}^\infty$, and that $\dot{\sigma}, \dot{\mu} \in \mathcal{L}^\infty$. It suggests that $\sigma, \mu \to 0$.

Then, we obtain the next theorem.

Theorem 1 The adaptive control system is uniformly bounded, if the following conditions 1) $\sim$ 3) are satisfied.

1) $(L^TL)^{-1}L^T \in \mathcal{L}^\infty$.
2) $\theta^1 \in \mathcal{L}^\infty$ implies $L \in \mathcal{L}^\infty$.
3) $(1+\alpha)I + \kappa\hat{M}$ is non-singular.

Furthermore, $\tilde{\theta}$, σ, μ converge to zero asymptotically.

$$\lim_{t\to\infty}\tilde{\theta}(t) = 0, \quad \lim_{t\to\infty}\sigma(t) = 0, \quad \lim_{t\to\infty}\mu(t) = 0. \quad (45)$$

Remark Many practical constraints satisfy the conditions 1) and 2) directly.

IV. INPUT NONLINEARITY AND INVERSE CHARACTERISTIC

Next, we consider the case where the manipulator is preceded by input nonlinearities such as dead-zone or backlash [13], [14], [15], [16].

$$\begin{aligned} \tau &= [\tau_1, \cdots, \tau_n]^T \\ &= N(u) = [N_1(u_1), \cdots, N_n(u_n)]^T. \end{aligned} \quad (46)$$

(Dead-zone)

$$\begin{aligned} \tau_i &= N_i(u_i) \\ &= DZ_i(u_i) \equiv \begin{cases} m_{ri}(u_i - b_{ri}) & (u_i \ge b_{ri}) \\ 0 & (b_{li} \le u_i \le b_{ri}) \\ m_{li}(u_i - b_{li}) & (u_i \le b_{li}), \end{cases} \\ &\quad (b_{li} < 0 < b_{ri}, \ \ m_{ri}, m_{li} > 0). \end{aligned} \quad (47)$$

An inverse characteristic of dead-zone is written as follows:

$$\begin{aligned} u_i &= DZ_i^{-1}(\tau_i) \\ &= \frac{\tau_i + m_{ri}b_{ri}}{m_{ri}}\sigma_r(\tau_i) + \frac{\tau_i + m_{li}b_{li}}{m_{li}}\sigma_l(\tau_i), \end{aligned} \quad (48)$$

$$\sigma_r(\tau) = \begin{cases} 1 & (\tau > 0) \\ 0 & (\tau \le 0), \end{cases} \qquad \sigma_l(\tau) = \begin{cases} 1 & (\tau < 0) \\ 0 & (\tau \ge 0). \end{cases} \quad (49)$$

Also, the following representation is given for an input torque τ_i.

$$\tau_i = m_{ri}(u_i - b_{ri})\sigma_r(\tau_i) + m_{li}(u_i - b_{li})\sigma_l(\tau_i). \quad (50)$$

(Backlash)

$$\begin{aligned} \tau_i &= N_i(u_i) = BL_i(u_i) \\ &\equiv \begin{cases} m_i(u_i - b_{ri}) & (\dot{u}_i > 0 \ \ \& \ \ \tau_i = m_i(u_i - b_{ri})) \\ m_i(u_i - b_{li}) & (\dot{u}_i < 0 \ \ \& \ \ \tau_i = m_i(u_i - b_{li})) \\ \tau_i(t_-) & (\text{otherwise}), \end{cases} \\ &\quad (b_{li} < 0 < b_{ri}, \ m_i > 0). \end{aligned} \quad (51)$$

An inverse characteristic of backlash is written by

$$\begin{aligned} u_i &= BL_i^{-1}(\tau_i) \\ &= \begin{cases} \frac{\tau_i + m_ib_{ri}}{m_i} & \left(\dot{\tau}_i > 0 \ \ \& \ \ u_i = \frac{\tau_i + m_ib_{ri}}{m_i}\right) \\ \frac{\tau_i + m_ib_{li}}{m_i} & \left(\dot{\tau}_i < 0 \ \ \& \ \ u_i = \frac{\tau_i + m_ib_{li}}{m_i}\right) \\ u_i(t_-) + (b_{ri} - b_{li}) & \left(\dot{\tau}_i > 0 \ \ \& \ \ u_i = \frac{\tau_i + m_ib_{li}}{m_i}\right) \\ u_i(t_-) - (b_{ri} - b_{li}) & \left(\dot{\tau}_i < 0 \ \ \& \ \ u_i = \frac{\tau_i + m_ib_{ri}}{m_i}\right) \\ u_i(t_-) & (\dot{\tau}_i = 0). \end{cases} \end{aligned} \quad (52)$$

Next, we construct estimation schemes for inverse characteristics of the input nonlinearities. The rigorous inverse models of input nonlinearities include non-smooth functions such as $\sigma_r(\tau_i)$, $\sigma_l(\tau_i)$, $\sigma_r(\dot{\tau}_i)$, $\sigma_l(\dot{\tau}_i)$. However, those may not be adequate for controller design. Hence, approximate inverse models which include smooth functions [14], [15] are employed in the estimation schemes.

(Inverse model of dead-zone)

An estimation scheme for the inverse characteristic of dead-zone is given by

$$u_i = \hat{N}_i^{-1}(\tau_{di}) = \hat{DZ}_i^{-1}(\tau_{di}) \equiv \frac{\tau_{di} + (\widehat{m_{ri}b_{ri}})}{\hat{m}_{ri}}\chi_{ri}(\tau_{di}) + \frac{\tau_{di} + (\widehat{m_{li}b_{li}})}{\hat{m}_{li}}\chi_{li}(\tau_{di}), \quad (53)$$

$$\begin{cases} \chi_{ri}(\tau_{di}) \equiv \frac{1}{2}\left\{1 + \tanh\left(\frac{\tau_{di}}{e_{0i}}\right)\right\} \\ \chi_{li}(\tau_{di}) \equiv \frac{1}{2}\left\{1 - \tanh\left(\frac{\tau_{di}}{e_{0i}}\right)\right\}, \end{cases} \quad (e_{0i} > 0), \quad (54)$$

where τ_{di} is an ideal input torque. Then, the following relation is deduced for τ_{di}.

$$\tau_{di} = \hat{\phi}_{2i}^T \omega_{2i} + \epsilon_i, \quad (55)$$

$$\hat{\phi}_{2i} = \left[\hat{m}_{ri},\, -(\widehat{m_{ri}b_{ri}}),\, \hat{m}_{li},\, -(\widehat{m_{li}b_{li}})\right]^T, \quad (56)$$

$$\omega_{2i} = \left[u_i\chi_{ri}(\tau_{di}),\, \chi_{ri}(\tau_{di}),\, u_i\chi_{li}(\tau_{di}),\, \chi_{li}(\tau_{di})\right]^T, \quad (57)$$

where ϵ_i is a residual term. It is shown that for $\hat{\phi}_{2i}$ satisfying

$$0 < \hat{m}_{ri},\, \hat{m}_{li} < \infty, \quad |(\widehat{m_{ri}b_{ri}})|,\, |(\widehat{m_{li}b_{li}})| < \infty,$$

the next inequality holds [14], [15].

$$|\epsilon_i| < \infty. \quad (58)$$

(Inverse model of backlash)

An estimation scheme for the inverse characteristic of backlash is given by

$$u_i = \hat{N}_i^{-1}(\tau_{di}) = \hat{BL}_i^{-1}(\tau_{di}) \equiv \frac{1}{\hat{m}_i}\left\{\tau_{di} + (\widehat{m_ib_{ri}})\chi_{ri}(\dot{\tau}_{di}) + (\widehat{m_ib_{li}})\chi_{li}(\dot{\tau}_{di})\right\}, \quad (59)$$

where τ_{di} is an ideal input torque. Then, the following relation is deduced for τ_{di}.

$$\tau_{di} = \hat{\phi}_{2i}^T \omega_{2i}, \quad (60)$$

$$\hat{\phi}_{2i} = \left[\hat{m}_i,\, -(\widehat{m_ib_{ri}}),\, -(\widehat{m_ib_{li}})\right]^T, \quad (61)$$

$$\omega_{2i} = \left[u_i,\, \chi_{ri}(\dot{\tau}_{di}),\, \chi_{li}(\dot{\tau}_{di})\right]^T. \quad (62)$$

(Error of estimation scheme)

The difference between $\tau_i = N_i(u_i)$ and $\tau_{di} = \hat{N}_i(u_i)$ (τ_{di} is deduced from $u_i = \hat{N}_i^{-1}(\tau_{di})$) is evaluated by the following relation.

$$\tau_i - \tau_{di} = (\phi_{2i} - \hat{\phi}_{2i})^T \omega_{2i} + d_i, \quad (63)$$

$$d_i = N_i(u_i) - \phi_{2i}^T \omega_{2i} - \epsilon_i, \quad (64)$$

where ϕ_{2i} is a true value of $\hat{\phi}_{2i}$, and $\epsilon_i = 0$ for $\hat{N}_i^{-1} = \hat{BL}_i^{-1}$. Then, for $\hat{\phi}_{2i}$ satisfying

$$0 < \hat{m}_i,\, \hat{m}_{ri},\, \hat{m}_{li} < \infty, \quad |(\widehat{m_{ri}b_{ri}})|,\, |(\widehat{m_{li}b_{li}})| < \infty, \quad (65)$$

the total residual term d_i satisfies the next inequality [14], [15].

$$|d_i| < \infty. \quad (66)$$

V. ADAPTIVE CONTROL WITH INPUT NONLINEARITY

First, we construct a conventional adaptive control with input nonlinearities. Ω_2 and Φ_2 are defined by

$$\Omega_2 = \begin{bmatrix} \omega_{21} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \omega_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \omega_{2n} \end{bmatrix}, \quad (67)$$

$$\Phi_2 = \left[\ \Phi_{21}^T,\ \ \Phi_{22}^T,\ \ \cdots,\ \ \Phi_{2n}^T\ \right]^T. \quad (68)$$

$\hat{\Phi}_2$ is introduced as an estimate of Φ_2. Then, τ is written as follows:

$$\tau = \tau_d + \Omega_2^T(\Phi_2 - \hat{\Phi}_2) + d, \quad (69)$$

$$\tau_d = [\tau_{d1},\, \cdots,\, \tau_{dn}]^T, \quad (70)$$

$$d = [d_1,\, \cdots,\, d_n]^T. \quad (71)$$

Here, by considering (35), (36), we determine τ_d such as

$$\tau_d = -f_d + \alpha\tilde{f} + \Omega_1^T\hat{\Phi}_1 + v, \quad (72)$$

and synthesize $u = \hat{N}^{-1}(\tau_d)$ from τ_d (72). For stability analysis, a positive function V_2 is defined by

$$V_2 = V_1 + \frac{1}{2}\tilde{\Phi}_2^T\Gamma_2^{-1}\tilde{\Phi}_2, \quad (73)$$

$$\tilde{\Phi}_2 = \hat{\Phi}_2 - \Phi_2, \quad (74)$$

$$(\Gamma_2 = \Gamma_2^T > 0).$$

The time derivative of V_2 is given as follows:

$$\dot{V}_2 = -(1+\alpha)\|\mu\|^2 + \sigma^T\Omega_1(\theta, \dot{\theta}, a, b)^T(\hat{\Phi}_1 - \Phi_1) + \sigma^T\{v + \Omega_2^T(\Phi_2 - \hat{\Phi}_2) + d\} + \sum_{i=1}^{2}(\hat{\Phi}_i - \Phi_i)^T\Gamma_i^{-1}\dot{\hat{\Phi}}_i. \quad (75)$$

The tuning law of $\hat{\Phi}_i$ and the control law of v are

$$\dot{\hat{\Phi}}_1 = \mathrm{Pr}\{-\Gamma_1\Omega_1\sigma\}, \quad (76)$$

$$\dot{\hat{\Phi}}_2 = \mathrm{Pr}\{\Gamma_2\Omega_2\sigma\}, \quad (77)$$

$$v = -K\sigma \ \ (K = K^T > 0), \quad (78)$$

where $\mathrm{Pr}\{\cdot\}$ represent projection-type adaptive laws [18] which assure the constraints (65). Then, $\dot{V}_2$ is evaluated by

$$\dot{V}_2 \le -\sigma^T K\sigma - (1+\alpha)\|\mu\|^2 + \sigma^T d \le -(1+\alpha)\|\mu\|^2 - \frac{\lambda_{\min}(K)}{2}\|\sigma\|^2 + \frac{1}{2\lambda_{\min}(K)}\|d\|^2, \quad (79)$$

where $\lambda_{\min}(K)$ is an minimal eigenvalue of K. Since d and $\hat{\Phi}_i$ are bounded, we obtain the next theorem.

Theorem 2 The adaptive control system is uniformly bounded, if the conditions 1) ∼ 3) (Theorem 1) are satisfied, and $\tilde{\theta}$ and $\tilde{\mu}$ converge to the residual regions defined by $\|(\tilde{\theta}^T,\, \tilde{\mu}^T)\| \sim \lambda_{\min}(K)^{-1}$.

VI. ADAPTIVE H_∞ CONTROL WITH INPUT NONLINEARITY I

Based on the adaptive control scheme in Section V, we construct the nonlinear adaptive H_∞ control systems, where estimation errors of tuning parameters $\tilde{\Phi}_1$, errors of constraint forces $\tilde{f}$ and residual terms of inverse models of input nonlinearities d are regarded as external disturbances to the process [12]. First, τ_d is synthesized by (72), where v

is a stabilizing signal derived from H_∞ control criterion. A positive function V_3 is defined by

$$V_3 = V_0 + \frac{1}{2}\tilde{\Phi}_2^T \Gamma_2^{-1} \tilde{\Phi}_2. \quad (80)$$

The time derivative of V_3 is evaluated as follows:

$$\dot{V}_3 \le (1+\alpha)(\sigma-\mu)^T \tilde{f} + \sigma^T \Omega_1^T \tilde{\Phi}_1 -(1+\alpha)\|\mu\|^2 + \sigma^T(d+v), \quad (81)$$

where the tuning law of $\hat{\Phi}_2(t)$ is the same as (77). By considering (81), a virtual system is introduced.

$$\frac{d}{dt}\begin{bmatrix} \sigma \\ \mu \end{bmatrix} = \begin{bmatrix} -M^{-1}C\sigma \\ -\kappa\mu \end{bmatrix} + \begin{bmatrix} M^{-1}(1+\alpha) \\ -\kappa I \end{bmatrix} \tilde{f} + \begin{bmatrix} M^{-1}\Omega_1^T \\ 0 \end{bmatrix} \tilde{\Phi}_1 + \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix} d + \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix} v. \quad (82)$$

(82) is rewritten into the next form.

$$\frac{d}{dt}x = f(x) + g_{11}\tilde{f} + g_{12}\tilde{\Phi}_1 + g_{13}d + g_2 v, \quad (83)$$

$$x \equiv [\sigma^T,\, \mu^T]^T. \quad (84)$$

We are to stabilize the above system via a control input v by utilizing H_∞ criterion, where $\tilde{f}$, $\tilde{\Phi}_1$, and d are regarded as external disturbances to the process [12]. For that purpose, we introduce the following Hamilton-Jacobi-Isaacs (HJI) equation

$$\frac{\partial}{\partial t}V + \mathcal{L}_f V + \frac{1}{4}\left\{\sum_{i=1}^{3}\frac{\|\mathcal{L}_{g_{1i}}V\|^2}{\gamma_i^2} - \mathcal{L}_{g_2}VR^{-1}\left(\mathcal{L}_{g_2}V\right)^T\right\} + q(x) \le 0, \quad (85)$$

where the solution V is given by $V = V_0$. $q(x)$ and R are a positive function and a positive definite matrix, respectively, and those are derived from HJI equation based on inverse optimality for the given solution V and the positive constants γ_i $(i = 1 \sim 3)$. The substitution of the solution $V = V_0$ into HJI equation (85) yields

$$-(1+\alpha)\|\mu\|^2 + \frac{1}{4\gamma_1^2}(1+\alpha)^2\|\sigma-\mu\|^2 + \frac{1}{4\gamma_2^2}\sigma^T\Omega_1^T\Omega_1\sigma + \frac{1}{4\gamma_3^2}\|\sigma\|^2 - \frac{1}{4}\sigma^T R^{-1}\sigma + q(x) \le 0. \quad (86)$$

In order to obtain $q(x)$ and R, we consider the following relation (87) which is a sufficient condition for the above inequality (86).

$$-(1+\alpha)\|\mu\|^2 + \frac{1}{2\gamma_1^2}(1+\alpha)^2(\|\sigma\|^2 + \|\mu\|^2) + \frac{1}{4\gamma_2^2}\sigma^T\Omega^T\Omega\sigma + \frac{1}{4\gamma_3^2}\|\sigma\|^2 - \frac{1}{4}\sigma^T R^{-1}\sigma + q(x) \le 0. \quad (87)$$

Then, $q(x)$ and R satisfying (87) are given as follows:

$$q(x) = \frac{1}{4}\sigma^T K_R \sigma + (1+\alpha)\left(1 - \frac{1+\alpha}{2\gamma_1^2}\right)\|\mu\|^2, \quad (88)$$

$$R = \left\{\frac{2(1+\alpha)^2}{\gamma_1^2}I + \frac{\Omega_1^T\Omega_1}{\gamma_2^2} + \frac{1}{\gamma_3^2}I + K_R\right\}^{-1}, \quad (89)$$

$$K_R = K_R^T > 0. \quad (90)$$

In order that $q(x)$ is a positive function, α and γ_1 should satisfy the next relation.

$$\gamma_1^2 > \frac{1+\alpha}{2}. \quad (91)$$

By utilizing R, v is deduced as a solution for the corresponding H_∞ control problem.

$$v = -\frac{1}{2}R^{-1}\left(\mathcal{L}_{g_2}V\right)^T = -\frac{1}{2}R^{-1}\sigma = -\frac{1}{2}\left\{\frac{2(1+\alpha)^2}{\gamma_1^2}I + \frac{\Omega_1^T\Omega_1}{\gamma_2^2} + \frac{1}{\gamma_3^2}I + K_R\right\}\sigma. \quad (92)$$

By considering HJI equation, the time derivative of V_3 is evaluated as follows:

$$\dot{V}_3 \le \left(v + \frac{1}{2}R^{-1}\sigma\right)^T R\left(v + \frac{1}{2}R^{-1}\sigma\right) - v^T R v -\gamma_1^2\left\|\tilde{f} - \left(\frac{1+\alpha}{2\gamma_1^2}\right)(\sigma-\mu)\right\|^2 + \gamma_1^2\|\tilde{f}\|^2 -\gamma_2^2\left\|\tilde{\Phi}_1 - \frac{1}{2\gamma_2^2}\Omega_1\sigma\right\|^2 + \gamma_2^2\|\tilde{\Phi}_1\|^2 -\gamma_3^2\left\|d - \frac{1}{2\gamma_3^2}\sigma\right\|^2 + \gamma_3^2\|d\|^2 - q(x). \quad (93)$$

The tuning law of $\hat{\Phi}_1$ is the same as (76). Then, the positive function V_2 satisfies the next relation.

$$\dot{V}_2 \le -\frac{1}{2}\sigma^T\left\{\frac{2(1+\alpha)^2}{\gamma_1^2}I + \frac{\Omega^T\Omega}{\gamma_2^2} + \frac{1}{2\gamma_3^2}I + K_R\right\}\sigma -(1+\alpha)\|\mu\|^2 + \gamma_3^2\|d\|^2. \quad (94)$$

From the evaluation of V_2 and V_3, we obtain the next theorem.

Theorem 3 The adaptive control system is uniformly bounded under the same conditions 1) $\sim$ 3) (Theorem 1), and $\tilde{\theta}$ and $\tilde{\mu}$ converge to residual regions defined by $\|(\tilde{\theta}^T,\, \tilde{\mu}^T)\| \sim \gamma_1^2, \gamma_3^2, \lambda_{\min}(K_R)^{-1}$. Also, v is an optimal control solution which minimizes the following cost functional.

$$J = \sup_{\tilde{\Phi}_1,\tilde{f},d\in\mathcal{L}^2}\left\{\int_0^t (q + v^T R v)d\tau + V_3(t) -\gamma_1^2\int_0^t\|\tilde{f}\|^2 d\tau - \gamma_2^2\int_0^t\|\tilde{\Phi}_1\|^2 d\tau - \gamma_3^2\int_0^t\|d\|^2 d\tau\right\}. \quad (95)$$

Additionally, the next inequality holds for any finite t.

$$\int_0^t (q + v^T R v)d\tau + V_3(t) \le \gamma_1^2\int_0^t\|\tilde{f}\|^2 d\tau + \gamma_2^2\int_0^t\|\tilde{\Phi}_1\|^2 d\tau + \gamma_3^2\int_0^t\|d\|^2 d\tau + V_3(0). \quad (96)$$

Remark It is shown that the $\mathcal{L}^2$ gains from the disturbances $\tilde{f}$, $\tilde{\Phi}_1$, d to the generalized output $\sqrt{q + v^T R v}$ are prescribed by positive constants $\gamma_1, \gamma_2, \gamma_3$. However, $\mathcal{L}^2$ gain γ_1 is restricted by the control parameters α (91).

VII. ADAPTIVE H_∞ CONTROL WITH INPUT NONLINEARITY II

Next, the proposed H_∞ control strategy is deduced from the simplified description, where (16) is also considered.

First, τ_d is synthesized by (72). Since (16) holds, $\dot{V}_3$ is written as follows:

$$\dot{V}_3 = \sigma^T \Omega_1^T \tilde{\Phi}_1 + \sigma^T d + \sigma^T v - (1+\alpha)\|\mu\|^2. \quad (97)$$

From the above relation, we introduce the following virtual system.

$$\frac{d}{dt}\begin{bmatrix} \sigma \\ \mu \end{bmatrix} = \begin{bmatrix} -M^{-1}C\sigma \\ -\kappa\mu \end{bmatrix} + \begin{bmatrix} M^{-1}\Omega_1^T \\ 0 \end{bmatrix}\tilde{\Phi}_1 + \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix} d + \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix} v. \quad (98)$$

The virtual process is rewritten into the next form.

$$\frac{d}{dt}x = f(x) + g_{11}\tilde{\Phi}_1 + g_{12}d + g_2 v. \quad (99)$$

We are to stabilize the virtual system via the control input v by utilizing H_∞ control criterion, where $\tilde{\Phi}_1$ and d are regarded as external disturbances to the process. Similarly to the previous section, for HJI equation

$$\frac{\partial}{\partial t}V + \mathcal{L}_f V + \frac{1}{4}\left\{\sum_{i=1}^{2}\frac{\|\mathcal{L}_{g_{1i}}V\|^2}{\gamma_i^2} - \mathcal{L}_{g_2}VR^{-1}\left(\mathcal{L}_{g_2}V\right)^T\right\} + q(x) \le 0, \quad (100)$$

together with the solution $V = V_0$, or for the following equivalent relation

$$-(1+\alpha)\|\mu\|^2 + \frac{1}{4\gamma_1^2}\sigma^T\Omega_1^T\Omega_1\sigma + \frac{1}{4\gamma_2^2}\|\sigma\|^2 - \frac{1}{4}\sigma^T R^{-1}\sigma + q(x) \le 0, \quad (101)$$

$q(x)$, R and the optimal solution v are given as follows:

$$q(x) = \frac{1}{4}\sigma^T K_R \sigma + (1+\alpha)\|\mu\|^2, \quad (102)$$

$$R = \left\{\frac{\Omega_1^T\Omega_1}{\gamma_1^2} + \frac{1}{\gamma_2^2}I + K_R\right\}^{-1}, \quad (103)$$

$$K_R = K_R^T > 0, \quad (104)$$

$$v = -\frac{1}{2}R^{-1}\sigma = -\frac{1}{2}\left\{\frac{\Omega_1^T\Omega_1}{\gamma_1^2} + \frac{1}{\gamma_2^2}I + K_R\right\}\sigma. \quad (105)$$

Theorem 4 The adaptive control system is uniformly bounded under the same conditions 1) $\sim$ 3) (Theorem 1), and $\tilde{\theta}$ and $\tilde{\mu}$ converge to residual regions defined by $\|(\tilde{\theta}^T, \tilde{\mu}^T)\| \sim \gamma_2^2,\ \lambda_{\min}(K_R)^{-1}$. Furthermore, v is an optimal control solution which minimizes the following cost functional.

$$J = \sup_{\tilde{\Phi}_1, d \in \mathcal{L}^2}\left\{\int_0^t (q + v^T R v)d\tau + V_3(t) - \gamma_1^2\int_0^t \|\tilde{\Phi}_1\|^2 d\tau - \gamma_2^2\int_0^t \|d\|^2 d\tau\right\}. \quad (106)$$

Additionally, the next inequality holds for any finite t.

$$\int_0^t (q + v^T R v)d\tau + V_3(t) \le \gamma_1^2\int_0^t \|\tilde{\Phi}_1\|^2 d\tau + \gamma_2^2\int_0^t \|d\|^2 d\tau + V_3(0). \quad (107)$$

Remark The $\mathcal{L}^2$ gains from the disturbances $\tilde{\Phi}_1$ and d to the generalized output $\sqrt{q + v^T R v}$ are prescribed by positive constants γ_1, γ_2. However, $\mathcal{L}^2$ gain from $\tilde{f}$ to the generalize output $\sqrt{q + v^T R v}$ is not prescribed in the present control scheme.

VIII. CONCLUDING REMARKS

Design methodologies of nonlinear adaptive H_∞ control for constrained robotic manipulators with uncertain input nonlinearities, are proposed, where tracking control of constrained trajectories and control of constraint forces are considered. The adaptive inverse approaches are employed to compensate the effect of the input nonlinearities. The resulting control strategy is derived as a solution of certain H_∞ control problem, where estimation errors of tuning parameters, errors of constraint forces and residual terms of the inverse models, are regarded as external disturbances to the process. Two approaches are deduced based on that policy, and it is shown that $\mathcal{L}^2$ gains from those disturbances to generalized outputs are prescribed by several design parameters, explicitly. Although the effectiveness of the proposed methodology was assured in the several simulation examples, the experimental verification is left in our future research.

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