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Nonlinear Active Noise Control Using NARX Model Structure Selection

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***Abstract*—In this work a novel Nonlinear Active Noise Control (NANC) scheme is proposed to deal with reference and error microphone saturation issues. Polynomial NARX models are used in the adaptive controller for enhanced model flexibility. A suitable model selection approach is used off-line to find an accurate and compact model structure for the adaptive controller. The controller parameters are successively updated with an appropriate adaptive algorithm based on the error gradient and on the residual noise. Some simulation experiments are provided to show the effectiveness of the proposed approach compared to existing alternatives.**

I. Introduction

RECENT algorithmic and technologic advances in digital signal processing have fostered the research in the area of Active Noise Control (ANC), resulting in several successful applications and theoretical developments [1]. Briefly, ANC exploits the principle of destructive interference to counteract an offending noise traveling on a primary acoustic path using a secondary acoustic source suitably governed by a controller. Both broad- and narrow-band, single- and multi-channel control approaches are documented in the literature, and various model identification and adaptive algorithms have been proposed in this context using different type of filters [2].

Most of the studies presented in the literature are concerned with linear ANC, although there are several sources of nonlinearity that can affect the system. For example, the noise affecting the system may be a nonlinear deterministic, possibly chaotic, process [3]. The involved acoustic paths may also display a nonlinear behavior. For example, a common source of nonlinearity is due to high sound pressure causing distortion and saturation in some of the devices that make up the control equipment [4]. To effectively deal with these effects, the models and algorithms employed in ANC must be properly extended and rearranged. The term NANC (nonlinear ANC) collectively refers to adaptive nonlinear control schemes that address these issues [5].

Various types of nonlinear filters have been studied in the NANC literature, such as truncated Volterra expansions [6], [7], radial basis functions [6], multi-layer artificial neural networks (MLANN) [4], [8], functional link artificial neural networks (FLANN) using trigonometric functional expansions [9] or piecewise linear functional expansions [10], adaptive bilinear filters [11], and general function expansion nonlinear filters [5]. The first issue with these filters is the development of suitable adaptation mechanisms able to deal with nonlinear models, since it is easy to show that classical methods such as the Filtered-X Least Mean Squares (FXLMS) approach [2] are not applicable to the nonlinear case without proper modifications. An even more critical issue is the model size which is typically high in the nonlinear case. This greatly affects both the computational load and the memory requirements of the adaptive scheme. Also, if large over-parameterized models are employed, in order to guarantee sufficient model flexibility, several known unwanted problems may arise, such as overfitting, parameter fluctuation and even model instability [12], [13].

This paper explores the use of a different nonlinear model class, namely the polynomial Nonlinear AutoRegressive models with eXogenous variables (NARX), which is well known for its flexibility and representation capabilities [14], [15]. Also, the linear-in-the-parameters structure allows the use of simple identification algorithms of the Least Squares (LS) family. What matters most for the NANC application is that several algorithms have been developed for the identification of NARX models which are greatly concerned with the selection of a suitable and compact model structure (see, *e.g.*, [16], [17], [13]). A NANC scheme is here proposed based on the off-line structure identification of a NARX model and the on-line adaptive tuning of its parameters. The case of saturation nonlinearities acting both at the reference and the error microphone (see, *e.g.*, [11], [18]), is here considered, as represented in Figure 1.

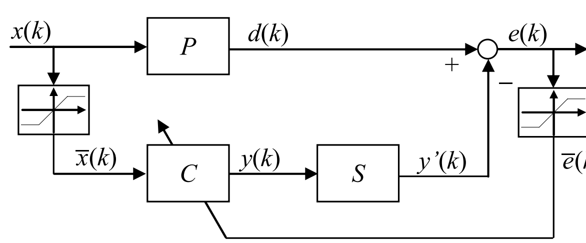

Fig. 1. Block diagram of a NANC system in the presence of saturated reference and error signals. *P* and *S* denote the primary and secondary path, respectively, and *C* the controller.

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II. Nonlinear Active Noise Control

The use of a nonlinear controller has been proved to outperform linear ANC solutions if either the secondary path is non-minimum phase or the primary path is nonlinear [6], [7]. The cited works employ a 2nd order truncated Volterra series for the controller structure:

$$y(k) = \sum_{m_1=0}^{L-1} w_1(m_1; k)x(k-m_1) + \sum_{m_1=0}^{L-1} \sum_{m_2=m_1}^{L-1} w_2(m_1, m_2; k)\, x(k-m_1)\, x(k-m_2) \quad (1)$$

where $y(k)$ represents the output of the Volterra filter, and $w_1(m_1; k)$ and $w_2(m_1, m_2; k)$ denote the (time-varying) coefficients of the linear and quadratic kernel, respectively.

Notice that, since model (1) has a nonlinear Finite Impulse Response (FIR) structure, it admits only one fixed-point, so that it is not suitable for modeling nonlinear systems with a more complex static behavior. Also, due to model size explosion, practical applications are limited to models of the 2nd order such as (1). Even so, since FIR models generally require a long window L of past terms to provide sufficient accuracy, this model typically has a large number of parameters to estimate (actually, it requires $L^2 + 3L + 1$ terms). This greatly affects both the parameter estimation confidence and the robustness of the model, in terms of its generalization capabilities. Finally, as already mentioned, over-parameterization can be responsible for several unwanted dynamic effects [12], [13].

On the positive side, model (1) is a linear-in-the-parameters model, so that classical adaptation algorithms used for FIR filters can still be applied with minor modifications. The so called Volterra FXLMS (VFXLMS) algorithm is used for model (1). This algorithm is based on a multi-channel structure for feedforward active noise control. The reference signal $x(k)$ is separated in its linear and quadratic components, which are then fed to distinct controller blocks whose outputs are added to compose the overall control signal $y(k)$. Assuming that the secondary path is linear the FXLMS can be applied correctly as long as each component of $x(k)$ is independently filtered.

In a similar framework, [9] proposed the use of FLANN filters using trigonometric bases, which can be updated using a Filtered-S Least Mean Squares (FSLMS) algorithm which exploits the particular model structure. An efficient filter-bank adaptation scheme in the frequency domain was also proposed for these nonlinear filters by the same research group [19]. Other types of elementary functions have been used with FLANN filters, such as piecewise-linear expansions [10]. A filtered-x affine projection algorithm is also introduced in [10] for both the adaptive Volterra filter and the FLANN structures.

Infinite-impulse response (IIR) type filters such as the output-error bilinear filter

$$y(k) = \sum_{i=0}^{L} a_i(k)x(k-i) + \sum_{j=1}^{L} b_j(k)y(k-j) + \sum_{i=0}^{L}\sum_{j=1}^{L} c_{i,j}(k)x(k-i)y(k-j) \quad (2)$$

have been suggested [11] as an efficient alternative to Volterra models. A type of FXLMS approach is adopted also in this case, which assumes a linear secondary path and filters the gradient of the controller's output with respect to its weights. Although in terms of L models (1) and (2) have the same number of parameters, it is shown in [11] that the bilinear model approach can achieve performances comparable to Volterra filters by using a shorter filter length L, *i.e.* less parameters.

Few solutions have been proposed in the literature so far for the more general NANC problem with nonlinear secondary path. In [4], [8], MLANN are used for this purpose. More recently, [5] developed an adaptive control algorithm for general function expansion nonlinear filters that works for both linear and nonlinear secondary paths. This algorithm is based on the so called virtual secondary path filter, which is related to the gradient of the secondary path. Notice, however, that the model class employed in [5] has still a substantially FIR structure, since the controller's output is actually constructed as a linear combination of nonlinear functions of the input reference signal. As such, it still suffers from the model size problem.

III. NARX MODELS

Besides the different representation capabilities of the models used in the approaches described in Section II, the main limitation of those methods lies in the fact that they do not provide any form of structure optimization, so that largely over-parameterized models are invariantly used, with all the problems that this can cause. In this section, we introduce a different class of nonlinear models, namely the polynomial NARX models, which include both Volterra and bilinear models, and for which it is possible to devise a model selection scheme that generally yields sufficiently compact models to be efficiently used in a NANC system.

Recursive input-output models are widely used in black-box nonlinear model identification for their flexibility and representation capabilities (see, *e.g.*, [20] for a comprehensive review). In particular, NARX models [14], [15] have been extensively employed in applications. The deterministic NARX model is an input-output recursive model where the current output is given by:

$$y(k) = f(y(k-1), ..., y(k-n_y), x(k-1), ..., x(k-n_x)), \quad (3)$$

where $f(\cdot)$ is a generic nonlinear function, $x(\cdot)$ and $y(\cdot)$ denote the input and output signals, respectively, and n_x and n_y are the maximum input and output lags. Several types of functional expansions can be used to describe $f(\cdot)$, one of the most common being the polynomial functional expansion

$$y(k) = \sum_{m=0}^{l}\sum_{p=0}^{m}\sum_{n_1=1}^{n_y}\dots\sum_{n_m=1}^{n_u} c_{p,m-p}(n_1,...,n_m)\prod_{i=1}^{p} y(k-n_i)\prod_{h=p+1}^{m} x(k-n_h), \quad (4)$$

where l is the maximum degree of the polynomial expansion. Expression (4) is linear with respect to the parameters to be estimated, so that Least Squares (LS) type algorithms can be used for model identification.

Parameter identification is, however, the easiest part of the problem, model selection being generally the main concern, since the number of terms in the full expansion (4) grows rapidly with l, n_x and n_y, and comparably few terms are generally needed for successful modeling. The model size should be kept small, for model robustness and parameter estimation confidence, and to avoid the unwanted dynamical consequences of model redundancy [12], [13].

One of the most popular identification algorithms for NARX models is the Forward Regression Orthogonal Estimator (FROE) [16], which is based on the prediction error minimization approach and uses Orthogonal LS (OLS) to estimate the parameters, exploiting the orthogonalization to decouple the regressors. Briefly, at each iteration of the FROE the regressor that enhances most the prediction performance of the current model is included in it.

An alternative to the OLS approach is given by the Fast Recursive Algorithm (FRA) developed in [17], which is also based on the PEM approach, but solves the LS problem recursively over the model order without requiring matrix decomposition and transformation. This algorithm is notable for the reduced complexity and the improved numerical stability it achieves with respect to the FROE. This makes it a good candidate for use in a NANC scheme.

Several important drawbacks of the PEM approach for NARX identification have been discussed in the literature (see, *e.g.*, [12], [13]), and the simulation error minimization (SEM) approach has been put forward for improved structure selection and model robustness. On this line, [13] introduces the SEMP (SEM with Pruning) algorithm, which operates iteratively as the FROE, but rates candidate models based on their simulation performance. The iterative procedure includes a pruning procedure to avoid model redundancy. Notice that, since model simulation can only be computed iteratively, this approach is computationally intensive, although parameter identification is still performed with a LS approach for simplicity. Computational complexity can be somewhat reduced using cluster selection [21].

In view of the NANC application where the controller's parameters are continually adjusted, allowing for a partial compensation of an imprecise initial model estimation, it is conjectured that the PEM-based NARX identification approaches could suffice for this type of application. Here, the SEMP method has been employed to provide an accurate reference model for comparison purposes.

Remark that the envisaged model selection techniques are batch algorithms, so that to perform this task, a suitable set of data must be collected for the model selection procedure to process off-line.

IV. Methodology Description

The use of optimized NARX models can greatly enhance NANC applications. The main idea is to employ model selection in a first off-line stage to find a compact model structure for the NARX controller and then adapt its parameters on-line. With respect to other NANC approaches, this draws most of the computational load away from the adaptation task, which is critical since it has to work on-line, and transfers it to the model selection task, which can be performed off-line. The reduced complexity of the on-line adaptation is obviously beneficial in terms of hardware requirements and overall system performance, but it also allows a more flexible use of the model's degrees of freedom that is generally expected to enhance its accuracy and robustness.

A. Model selection

To estimate the NARX model of the controller with any of the model selection procedures explained in the previous section, we need to formulate the controller's model identification problem as a linear regression. This would be trivial if the desired controller output was available, since then we could use directly the linear regression (4). Unfortunately, the error is directly measurable at the output of the secondary path and not at the controller's output. To estimate the desired controller's output, a model inversion technique may be used. Notice that while the use of an inverse model of the secondary path is generally avoided in ANC systems to prevent the insertion of additional delays in the control chain, here the role of the inverse model is only functional to the off-line model structure selection of the controller and is not meant to interfere with the on-line execution of the adaptive ANC algorithm. The model selection task is based on the scheme depicted in Figure 2, which uses an inverse model of the secondary path to estimate $y(k)$ from $y'(k)$.

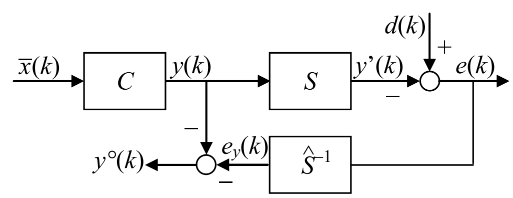

Fig. 2. Reconstruction of signal $y(k)$ for use in the model selection phase. S denotes the secondary path, C the controller, $\hat{S}^{-1}$ the estimated inverse of the secondary path, and $y°(k)$ the 'reconstructed' target controller output.

A general inverse identification scheme (see Figure 3) can be applied, where the inverse model is fed with the error signal, while the secondary path is excited with a suitable artificially generated signal $y(k)$, uncorrelated with $d(k)$. Details on the identification algorithm depend on the particular model structure chosen for the inverse. The FROE or FRA algorithms might be used in the NARX case. Using this inverse model to obtain the 'reconstructed' target controller output $y°(k)$, a dataset of $\{\bar{x}(\cdot),\ y°(k)\}$ samples may be used for the controller's model selection purpose, directly referring to the linear regression (4).

Notice that model inversion is not always applicable, so that the suggested model identification scheme could turn out to be a critical point of the overall method. Recall, however, that accurate inversion is only required in the disturbance frequency range, which can be quite circumscribed, especially in narrowband control applications. Observe also that, as will be seen in the simulation experiments section, the on-line adaptive algorithm that tunes the controller weights can partially compensate errors due to approximate inversion.

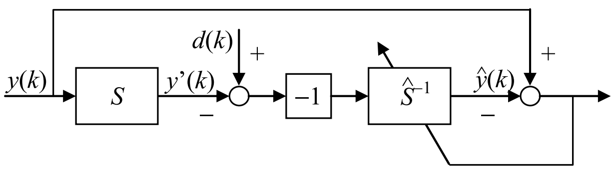

Fig. 3. Model identification scheme for the estimation of the inverse secondary path model. S denotes the secondary path, $\hat{S}^{-1}$ the estimated inverse of the secondary path.

B. On-line adaptation of the controller's parameters

At the end of the model selection phase, the controller structure is fixed and an initial parameterization is available. At this stage an adaptive algorithm must be setup for continuous updating of the model parameters, to deal with unmodeled dynamics and time-variant phenomena. The algorithm discussed in the following is an adaptation to the NARX case of that employed in [5]. The objective of the adaptive algorithm is to minimize the instantaneous squared error using the steepest descent algorithm:

$$\vartheta(k+1) = \vartheta(k) - \frac{\mu}{2}\left(\left.\frac{\partial(e(k)^2)}{\partial\vartheta}\right|_{\vartheta(k)}\right)^T$$

$$= \vartheta(k) + \mu\left(\left.\frac{\partial y'(k)}{\partial\vartheta}\right|_{\vartheta(k)}\right)^T e(k). \tag{5}$$

Assuming a deterministic NARX structure for both the controller (see equations (3-4)) and the secondary path:

$$y'(k) = f(y'(k-1), ..., y'(k-n_{y'}), y(k-1), ..., y(k-m_y))$$

$$= \sum_{m=0}^{l}\sum_{p=0}^{m}\sum_{n_1=1}^{n_{y'}}\dots\sum_{n_m=1}^{m_y} c'_{p,m-p}(n_1,...,n_m)\prod_{i=1}^{p} y'(k-n_i)\prod_{h=p+1}^{m} y(k-n_h),$$

the gradient term $\partial y'(k)/\partial\vartheta$ can be iteratively computed by means of the recursive nonlinear filter:

$$y_\vartheta'(k) = \sum_{j=1}^{n_{y'}}\frac{\partial y'(k)}{\partial y'(k-j)}y_\vartheta'(k-j) + \sum_{j=1}^{m_y}\frac{\partial y'(k)}{\partial y(k-j)}y_\vartheta(k-j). \tag{6}$$

where $y_\vartheta'(k) = \dfrac{\partial y'(k)}{\partial\vartheta(k)}$, $y_\vartheta(k) = \dfrac{\partial y(k)}{\partial\vartheta(k)}$, and the approximations $\dfrac{\partial y'(k-j)}{\partial\vartheta(k)} \approx \dfrac{\partial y'(k-j)}{\partial\vartheta(k-j)}$ and $\dfrac{\partial y(k-j)}{\partial\vartheta(k)} \approx \dfrac{\partial y(k-j)}{\partial\vartheta(k-j)}$ have been used, assuming that the step size is sufficiently small to yield slow convergence [2]. The derivatives of $y'(k)$ with respect to terms $y'(k-j)$, $j = 1, \dots, n_{y'}$, and $y(k-j)$, $j = 1, \dots, m_y$, appearing in expression (6) can be computed as follows:

$$\frac{\partial y'(k)}{\partial y'(k-\bar{n})} = \sum_{m=0}^{l}\sum_{p=0}^{m}\sum_{j=1}^{p}\sum_{n_1=1}^{n_{y'}}\dots\sum_{n_j=\bar{n}}^{\bar{n}}\dots\sum_{n_m=1}^{n_u} c'_{p,m-p}(n_1,\dots,\bar{n},\dots,n_m)$$

$$\times \prod_{\substack{i=1\\ i\neq j}}^{p} y'(k-n_i)\prod_{h=p+1}^{m} y(k-n_h),$$

$$\frac{\partial y'(k)}{\partial y(k-\bar{n})} = \sum_{m=0}^{l}\sum_{p=0}^{m}\sum_{j=p+1}^{m}\sum_{n_1=1}^{n_y}\dots\sum_{n_j=\bar{n}}^{\bar{n}}\dots\sum_{n_m=1}^{n_u} c'_{p,m-p}(n_1,\dots,\bar{n},\dots,n_m)$$

$$\times \prod_{i=1}^{p} y'(k-n_i)\prod_{\substack{h=p+1\\ h\neq j}}^{m} y(k-n_h).$$

Finally, the derivative $y_{\bar\vartheta}(k)$ of $y(k)$ with respect to a generic controller weight $\bar\vartheta(k) = c_{\bar{p},\bar{m}-\bar{p}}(\bar{n}_1,...,\bar{n}_m;\ k)$ can be computed recursively using the following nonlinear dynamic filter:

$$y_{\bar\vartheta}(k) = \prod_{i=1}^{\bar{p}} y(k-\bar{n}_i)\prod_{h=\bar{p}+1}^{\bar{m}} x(k-\bar{n}_h) + \sum_{m=0}^{l}\sum_{p=0}^{m}\sum_{n_1=1}^{n_y}\dots\sum_{n_m=1}^{n_u} c_{p,m-p}(n_1,...,n_m)$$

$$\times \sum_{j=1}^{p}\Big[y_{\bar\vartheta}(k-n_j)\prod_{\substack{i=1\\ i\neq j}}^{p} y(k-n_i)\Big]\prod_{h=p+1}^{m} x(k-n_h), \tag{7}$$

employing once again the approximation $\dfrac{\partial y(k-j)}{\partial\vartheta(k)} \approx \dfrac{\partial y(k-j)}{\partial\vartheta(k-j)}$.

Recursions (6) and (7) are particularly costly for on-line implementation, and some simplification is highly desired. A typical approximation in ANC systems is Feintuch's assumption [2], [11]. This consists in neglecting the recursion based on the old output gradients, on the grounds that all derivatives of past outputs with respect to current weights are zero. With this simplification, expression (6) simplifies to

$$y_\vartheta'(k) = \sum_{j=1}^{m_y}\frac{\partial y'(k)}{\partial y(k-j)}y_\vartheta(k-j), \tag{8}$$

and the dynamic filter (7) to

$$y_{\bar\vartheta}(k) = \prod_{i=1}^{\bar{p}} y(k-\bar{n}_i)\prod_{h=\bar{p}+1}^{\bar{m}} x(k-\bar{n}_h) \tag{9}$$

These more manageable expressions have been used in the following with satisfactory results.

The developed adaptation algorithm is denoted Nonlinear Filtered-Gradient LMS (NFGLMS), since it involves filtering the gradient of the controller output with respect to its weights, instead of the reference signal. The NFGLMS algorithm is related to the FXLMS-based NANC algorithm presented in [5], in the way it employs the gradient information in the updating process, but the different assumptions of the two methods lead to different solutions. Moreover, in [5] the idea is to keep a structure similar to the one of FXLMS, and the function expansion block is introduced to avoid the problems related to the use of the FXLMS algorithm for nonlinear systems. On the contrary, the NFGLMS concentrates all nonlinearities in the adaptive filter. The proposed algorithm can also be envisaged as an extension of the output-error bilinear filter approach [11], in terms of the model structure and the filtering operations concerning the secondary path.

C. *Comments on the algorithm's complexity and usage*

The complexity of the the on-line adaptive phase scales with the number of model terms to be adapted, that is generally reduced to a minimum thanks to the selection process, so that the on-line adaptation and control effort is much lower than with the Volterra and bilinear model-based algorithms.

As for model selection, a full analysis of the computational complexity of the FRA is provided in [17]. Model selection gets increasingly intensive as the set of candidate terms grows larger, so that there is a trade-off between model flexibility and computational load. It is however important to note that such a stage is unavoidable for robust nonlinear identification to avoid the well-known drawbacks of over-parameterization.

Besides, the active noise controller need not be idle during model selection. On the contrary, the last computed model structure should be employed for on-line adaptation and control until a new model structure is available, and model selection should be periodically repeated. The process could be started, *e.g.*, using a simple linear FIR model and FXLMS.

V. Simulation Experiments

In the following tests a multi-harmonic reference signal $x(k)$, sum of three sine waves at the normalized frequencies of 0.02, 0.04, and 0.08, is employed, with a sampling frequency of 8000 Hz. The primary and secondary paths are modeled by two linear FIR filters (taken from [22]) with coefficients of the impulse response given by [0.0179 0.1005 0.279 0.489 0.586 0.489 0.279 0.1005 0.0179] and [0.7756 0.5171 –0.362], respectively. The nonlinearity consists in the saturation of the reference and/or the error microphone. Two levels of saturations are considered for the reference signal, namely "weak" and "strong", obtained by setting the clipping threshold at 90% and 50% of the maximum signal value, respectively [11]. The error signal is either non saturated or weakly saturated with a 50% threshold [18]. Overall, three settings have been studied:

WU) weakly saturated reference, unsaturated error,

SU) strongly saturated reference, unsaturated error,

SW) strongly saturated reference, weakly saturated error.

The step size, or convergence factor, μ is chosen in order to guarantee stability and to ensure, at the same time, a sufficiently rapid convergence. Finally, perfect knowledge of the secondary path is assumed, *i.e.* $\hat{S}(z) = S(z)$, and the adaptive coefficients of the controller are initialized to zero.

A. *Test 1: setting WU*

The first test concerns setting WU. An inverse FIR filter of length $L_{inv} = 8$ is used for the reconstruction of signal $y(k)$, that is estimated using LMS with a step size $\mu_{inv} = 0.1$, while an adaptive NARX filter of order $L = 8$ including quadratic terms is identified with the FRA algorithm for 8 iterations. Finally, the weight update is performed with NFGLMS with a convergence factor of $\mu = 0.1$.

Apparently the model selection phase (see Figure 4) is capable of achieving significant noise reduction by itself (10^{-4} error amplitude), but the NFGLMS (after a brief transient) can still improve the performance by several orders of magnitude (10^{-8} error amplitude after 10000 samples). The 8-parameter model identified by the FRA algorithm is reported in Table I.

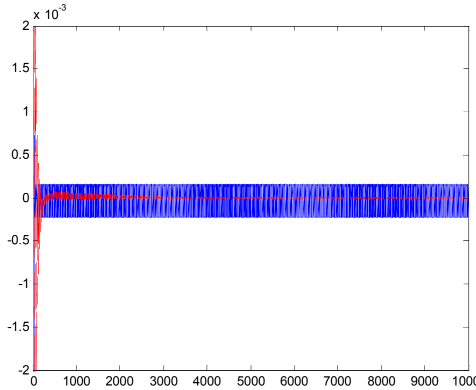

Fig. 4. Setting WU, residual noise with the proposed NANC scheme, after model selection (blue line) and during weight adaptation (red line).

B. *Test 2: setting SU*

Setting SU has been tested in the same experimental conditions. Although the error transient in the weight update phase is quite different to that of case WU (see Figure 5), convergence is finally achieved (10^{-6} error amplitude after 10000 samples). As for the controller's model structure (see Table I), some similarities may be noticed with the model obtained with setting WU. In particular, the first (and more important) selected terms are common to both models. Observe that these terms are linear, and may actually be explained as necessary for the description of the system itself (the first and secondary paths, the noise), while saturation effects are modeled by the nonlinear terms.

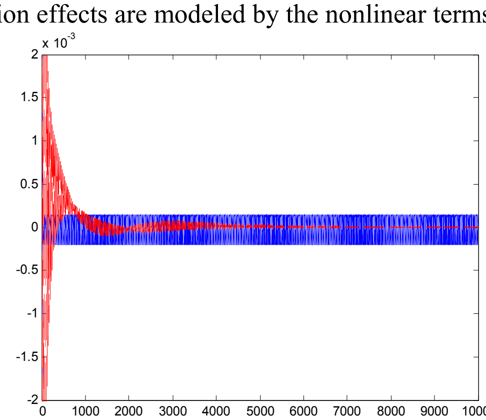

Fig. 5. Setting SU, residual noise with the proposed NANC scheme, after model selection (blue line) and during weight adaptation (red line).

C. *Test 3: setting SW*

Setting SW was tested, using LMS for the identification of the inverse with $\mu_{inv} = 0.1$, and NFGLMS with $\mu = 0.01$. The model identified by the FRA algorithm is reported in Table I. Both the model inversion and the weight update are affected by error saturation since they heavily rely on the information conveyed by the error signal.

The residual behavior (see Figure 6) shows that significant noise reduction can be achieved even in this case and that the NFGLMS can improve the results.

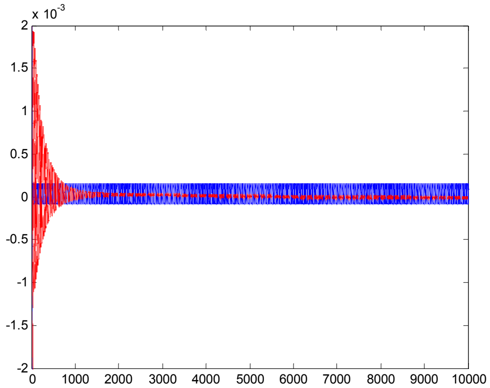

Fig. 6. Setting SW, residual noise with the proposed NANC scheme, after model selection (blue line) and during weight adaptation (red line).

TABLE I
FILTER MODELS ESTIMATED IN THE 3 TESTS (IN THE OFF-LINE PHASE)

Test 1		Test 2		Test 3	
Regressors	Par.s	Regressors	Par.s	Regressors	Par.s
$y(k-5)$	0.0278	$y(k-5)$	0.0268	$y(k-5)$	0.3088
$u(k-4)$	0.1137	$u(k-4)$	0.1812	$u(k-4)$	0.1776
$u(k-1)$	−0.0647	$u(k-1)$	−0.1038	$u(k-1)$	−0.0861
$y(k-3)u(k-8)$	−0.0323	$u(k-3)u(k-4)$	0.0578	$u(k-3)u(k-4)$	0.0680
$y(k-8)u(k-3)$	0.2379	$y(k-7)u(k-4)$	0.0045	$y(k-2)u(k-4)$	0.0324
$y(k-4)y(k-6)$	0.6724	$u(k-3)^2$	0.0045	$u(k-3)^2$	0.0280
$u(k-6)u(k-8)$	−0.0258	$y(k-6)u(k-7)$	0.1531	$y(k-1)u(k-3)$	0.1437
$y(k-8)u(k-1)$	0.0418	$y(k-5)y(k-8)$	0.2933	$y(k-6)u(k-7)$	0.0148

VI. CONCLUSIONS

The problem of the presence of nonlinearities in real ANC systems has been faced focusing on the effects of saturation of the reference and error microphones. A novel NANC scheme has been proposed that generalizes the Volterra and bilinear filter-based FXLMS methods, and overcomes some limitations of those methods. The main idea is to identify a compact NARX model of the noise canceling controller, using well known model selection algorithms, suitably readapted to the ANC context. A filter gradient–based LMS algorithm is then employed for adaptive parameter tuning. Such algorithm has also the capability to partially compensate for model structure errors left out by the previous stage.

The proposed method has been tested on some realistic scenarios simulating saturation of the reference and error microphones, and has provided interesting results that outperform previously proposed techniques. Compared to these techniques, the presented one allows some additional degrees of freedom concerning the model selection scheme itself and its integration with the parameter update algorithm, that may be tailored to the system's characteristics and expected performance.

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