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Resonance seeking control in an event-triggered discrete-time domain

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Abstract— **Resonance tracking control of oscillatory plants whose natural frequency is unknown is investigated from a Lyapunov stability perspective. In particular, an event-triggered discrete-time system is investigated for this purpose. The proposed resonance tuner is time-synchronized with periodic sampling of the harmonic plant's output to ensure that an analytical relationship exists between the period of the driving squarewave and the tracking error. This relation defines a class of discrete-time nonlinear systems whose origin, is shown to be asymptotically stable.**

I. INTRODUCTION

Many scientific instruments and engineering applications operate with maximal performance when driven harmonically to a state of resonance. These applications include, but are not limited to, ultrasonic motors [7], piezoelectric transducers [5], [18], [3], [2], [19], micromachined gyroscopes [6], cyclotrons [4], plasma processing systems [16], microwave and induction heating systems [21], [14], [15], and wireless communication systems [20]. Typically, the lock between the driving frequency and the resonant frequency of a load is achieved by an automatic control system that is designed to update the drive frequency in the direction that reduces the system phase angle [9], or equivalently, increases the system admittance function [2], until the phase becomes zero and the power factor is maximized.

However, the stability and performance of this resonance lock is influenced by several factors including the initial frequency, the feedback gain, the low-pass filter bandwidth, the signal-to-noise ratio, the time-lag between the sensor sample and the control update, as well as the drift rate of the resonant frequency which increases with internal power-dissipation [9], [2]. In the literature[17], [8], [10], [11], the stability analysis of resonance tuning control systems has generally been initiated after non-trivial simplifications have been applied to the system model. For instance, in [17], [9] and others, conventional phase detectors are investigated according to a model that assumes the actuator current is always at steady-state. In this paper, this assumption is avoided. Towards this end, a new resonance tuning approach based on an event-triggered discrete-time system is introduced. An advantage of this approach is that the stability of the closed-loop system can be determined as a function of the feedback and filter gains without making the assumption that the gain-dependent transients of the plant are negligible.

The rest of the paper is organized as follows. In Section 2, some notation is introduced and the resonance seeking control problem is defined. In Section 3, the closed-loop dynamics are derived, and in Section 4, we prove a closed-loop stability result for this system. A simple example of a harmonic oscillator is given in Section 5, followed by conclusions in Section 6.

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II. PRELIMINARIES AND PROBLEM DEFINITION

Regarding notation, we assume the following: $\mathbb{N}$ denotes the set of positive integers. $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^+$ denotes the set of positive real numbers. I_m is the $m \times m$ identity matrix. $\|\cdot\|$ denotes the 2-norm for vectors and matrices. Lastly, for a matrix T, $\sigma_{min}(T)$ and $\sigma_{max}(T)$ denote its minimum and maximum singular values, respectively.

To demonstrate the resonance seeking control problem, we consider linear time-invariant (oscillatory) systems given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c^T x(t) \end{aligned} \tag{1}$$

where $b, c \in \mathbb{R}^{n\times 1}$ and $A \in \mathbb{R}^{n\times n}$ has the following properties,

(i) $A = EDE^{-1}$, D is diagonal (i.e. distinct eigenvalues)
(ii) $\Re\{D\} < 0$, (i.e. A is Hurwitz)
(iii) D includes a complex-conjugate pair.

To achieve resonance, we are interested in driving (1) by the following variable-period square-wave input,

$$\begin{aligned} u(t) &= \begin{cases} v_o, & t \in [t_k,\ t_k + \tau_k/2) \\ -v_o, & t \in [t_k + \tau_k/2,\ t_k + \tau_k) \end{cases} \\ t_{k+1} &= t_k + \tau_k, \quad k = 1, 2, \ldots \end{aligned} \tag{2}$$

where $v_o > 0$ is the amplitude of the square-wave (a constant), $t_1 = 0$ is the initial time, and $\tau_1 \in \mathbb{R}^+$ is the initial period of the square-waveform chosen by the user. For this system, we assert that there exists an unknown resonant period denoted by τ^* with the following property,

$$\{\tau_k = \tau^* : k = 1, 2, \ldots\} \quad \Rightarrow \quad \lim_{k\to\infty} y(t_k) = 0 \tag{3}$$

for some choice of $c \in \mathbb{R}^{n\times 1}$ in (1). This assertion follows from the assumed properties of A, and will be proven later. Intuitively, (3) implies that the input is in phase with the output, a more familiar notion associated with resonance.

The control objective is to make adjustments to the period of the square-wave input τ_k for $k = 1, 2, \ldots$, until it is tuned to the unknown resonant period τ^* of the plant (1). That is,

$$\lim_{k\to\infty} \tau_k = \tau^* \qquad \text{(control objective)}$$

Clearly, τ^* cannot be used in feedback since it is unknown! Instead, we assume that the period of the square-waveform

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is updated according to the following control law,

$$p_{k+1} = (1-\varepsilon)p_k + \varepsilon y(t_k)$$
$$\tau_{k+1} = \tau_k - hp_k \tag{4}$$

where $h > 0$ is chosen to set the feedback control bandwidth, and $\varepsilon \in (0,1)$ is chosen to set the bandwidth of the low-pass filtered measurements. We assume that the initial state of the filter $p_1 \in \mathbb{R}$ is arbitrary. It is also assumed that $\{y(t_k)|k = 1,2,\ldots\}$ are available discrete measurements of continuous signal $\{y(t)|t \geq 0\}$ that are triggered by the rising-edge of the square-wave input as defined by (2). As we shall see, (2) defines an event-triggered discrete-time domain that can be both implemented in hardware [1] and analyzed within a Lyapunov stability framework.

III. Closed-loop dynamics

A strong motivation for introducing the resonance seeking control problem in an event-triggered discrete-time domain, is that this approach yields closed-form analytic solutions for the closed-loop plant trajectories. As such, the stability of this class of systems can be analyzed using Lyapunov's second method. In prior work on resonance seeking control [8], [9], [11], [12], the closed-loop stability analysis assumes that the plant (i.e. harmonic oscillator) is always at steady-state (or quasi-steady-state). Relaxing this assumption is nontrivial since conventional phase detector models generally assume steady-state. (i.e. only the steady-state amplitude and phase of the actuator current is modelled). Although this assumption is justified when the feedback gain is at a low level, it increasingly becomes less valid as the gain is increased since the transient response of the harmonic oscillator becomes more apparent in this case. Using the proposed approach taken here, the transient response need not be neglected.

The goal of the following lemma is as follows: Express the closed-loop trajectories analytically in terms of discrete-time difference equations that are amenable to Lyapunov stability analysis.

Lemma 1: Given $n \in \{2,3,4,\ldots\}$, $m = n+2$, $h, \varepsilon, v_o > 0$, vectors $b, c \in \mathbb{R}^{n\times 1}$ and a Hurwitz matrix $A \in \mathbb{R}^{n\times n}$ such that $A = EDE^{-1}$ holds for a diagonal D and nonsingular E. Suppose that $x_{ss} : [0,\infty) \to \mathbb{R}^n$ and $v_{ss} : [0,\infty) \to \mathbb{R}^n$ given by

$$x_{ss}(\tau) = ED^{-1}(I - e^{D(\tau/2)})E^{-1}bv_o \tag{5}$$
$$v_{ss}(\tau) = (-1/2)Ee^{D(\tau/2)}E^{-1}bv_o$$

satisfy the following conditions

$$c^T x_{ss}(\tau^*) = 0 \tag{6}$$
$$c^T v_{ss}(\tau^*) > 0$$

for some finite $\tau^* > 0$. Then, the closed-loop trajectories of system (1) and (2) can be expressed in discrete-time as

$$x_k = \tilde{x}_k + x_{ss}(\tau_k)$$
$$p_k = \tilde{p}_k + c^T x_{ss}(\tau_k)$$
$$\tau_k = \tilde{\tau}_k + \tau^* \tag{7}$$

where $z_k = \mathrm{col}(\tilde{x}_k, \tilde{p}_k, \tilde{\tau}_k) \in \mathbb{R}^m$ $(m = n+2)$ is the state of the following discrete-time system,

$$\begin{aligned} z_{k+1} &= M(v_k)z_k + d(\ell_k), \qquad d(0) = 0 \\ v_k &= \begin{bmatrix} 0 & 1 \end{bmatrix} \ell_k \\ \ell_k &= \begin{bmatrix} 0_{2\times n} & I_2 \end{bmatrix} z_k \end{aligned} \tag{8}$$

with matrix function $M : \mathbb{R} \to \mathbb{R}^{m\times m}$ and vector function $d : \mathbb{R}^2 \to \mathbb{R}^m$ defined as follows:

$$M = \left[\begin{array}{c|cc} Ee^{Dr_k}E^{-1} & 0 & 0 \\ \hline \varepsilon c^T & (1-\varepsilon) & 0 \\ 0 & -h & g_k \end{array}\right] =: \left[\begin{array}{c|c} F_k & 0 \\ \hline H & G_k \end{array}\right]$$
$$d = \begin{bmatrix} e_k^T & c^T e_k & 0 \end{bmatrix}^T$$
$$g_k = 1 - h\alpha_k \qquad e_k = x_{ss}(r_k) - x_{ss}(s_k) \tag{9}$$
$$r_k = \tau^* + \tilde{\tau}_k \qquad s_k = r_k - h(\tilde{p}_k + c^T x_{ss}(r_k))$$
$$\alpha_k = \begin{cases} (1/\tilde{\tau}_k)c^T x_{ss}(r_k) & ,\tilde{\tau}_k \neq 0 \\ c^T v_{ss}(\tau^*) & ,\tilde{\tau}_k = 0 \end{cases}$$

where $\alpha_k : \mathbb{R} \to \mathbb{R}$ is a continuous function that satisfies

$$0 < k_1 \leq \alpha_k(q) \leq k_2 \quad \forall q \in [-d,d] \tag{10}$$

for some positive scalars k_1, k_2, d. Furthermore, the closed-loop trajectories given by (7–9) are valid for all timesteps $k = 1,2,\ldots,\infty$, provided $\tau_k > 0$ for all $k \in \mathbb{N}$.

Proof: Substituting (2) into (1) followed by time-integration yields the equivalent discrete-time system,

$$x_{k+1} = \Phi(\tau_k)x_k + f(\tau_k) \tag{11}$$
$$y_k = c^T x_k$$

where x_k denotes $x(t_k)$ and

$$\begin{aligned} \Phi(\tau) &= e^{A\tau} = Ee^{D\tau}E^{-1} \\ f(\tau) &= (e^{A\tau} - I)\left[\int_0^{\tau/2} e^{A\sigma}d\sigma\right]bv_o \\ &= E(I - e^{D\tau})D^{-1}(I - e^{D\tau/2})E^{-1}bv_o \end{aligned} \tag{12}$$

Next, we show that for fixed $\tau = \tau_k = \tilde{\tau}_k + \tau^*$, (11) has a unique attractive fixed-point denoted by x_{ss} that satisfies

$$x_{ss} = \Phi(\tau)x_{ss} + f(\tau) \tag{13}$$

and is given by (5). That is, rearrangement of (13) yields $x_{ss} = E(I - e^{D\tau})^{-1}E^{-1}f(\tau)$, provided inversion of $I - e^{D\tau}$ is permitted. Indeed, inversion of $I - e^{D\tau}$ is permitted for all $\tau > 0$, since D is Hurwitz and diagonal (i.e. $\forall \tau > 0$, $\|e^{D\tau}\| < 1$ and $I - e^{D\tau}$ is nonsingular). Finally, substitute (12) to get $x_{ss} = ED^{-1}(I - e^{D\tau/2})E^{-1}bv_o$ as in (5). Furthermore, one can multiply (13) by E^{-1} to get $E^{-1}x_{ss} = e^{D\tau}E^{-1}x_{ss} + E^{-1}f(\tau)$, which clearly defines a contraction map from $E^{-1}x_{ss}$ into itself due to the fact that $\|e^{D\tau_k}\| < 1$.

To show (10), note that (6) and the smoothness of (5) imply the sector condition,

$$q^2 k_1 \leq q c^T x_{ss}(\tau^* + q) \leq q^2 k_2, \quad \forall q \in [-d,d] \tag{14}$$

for some positive scalars k_1, k_2 and d. Note that α_k is the unique continuous function that permits the factorization,

$$c^T x_{ss}(\tau^* + q) = \alpha_k q, \quad \forall q \in (-\tau^*, \infty) \tag{15}$$

Indeed, $\lim_{q\to 0}[(1/q)c^T x_{ss}(\tau^*+q)] = c^T v_{ss}(\tau^*) > 0$ shows that $c^T v_{ss} = dc^T x_{ss}/d\tau$ at τ^*, and that α_k is continuous and locally positive in a neighborhood of τ^*. Hence, (10) holds.

To show (8), we assert that

$$z_{k+1} = \lambda_k - d(\ell_k)$$
$$\lambda_k := \begin{bmatrix} \Phi(\tau_k)x_k + f(\tau_k) \\ (1-\varepsilon)p_k + \varepsilon c^T x_k \\ \tau_k - hp_k - \tau^* \end{bmatrix}$$

holds, and then substitute (7), (13) and (15) into λ_k, to get

$$\lambda_k = \begin{bmatrix} \Phi(\tau_k)(\tilde{x}_k + x_{ss}(\tau_k)) + f(\tau_k) \\ (1-\varepsilon)(\tilde{p}_k + c^T x_{ss}(\tau_k)) + \varepsilon c^T(\tilde{x}_k + x_{ss}(\tau_k)) \\ (\tilde{\tau}_k + \tau^*) - h(\tilde{p}_k + c^T x_{ss}(\tau_k)) - \tau^* \end{bmatrix}$$
$$= \begin{bmatrix} \Phi(\tau_k)\tilde{x}_k + x_{ss}(\tau_k) \\ (1-\varepsilon)\tilde{p}_k + \varepsilon c^T \tilde{x}_k + c^T x_{ss}(\tau_k) \\ -h\tilde{p}_k + (1 - h\alpha(\tilde{\tau}_k))\tilde{\tau}_k \end{bmatrix} = M(v_k)z_k$$

which is equivalent to (8). ■

The above result assumes that its possible to satisfy (6). In the following lemma, we show that this is indeed the case.

Lemma 2: Let $\mathscr{C}$ denote the set of all unit-norm n-vectors that have exactly one non-zero element, i.e.,

$$\mathscr{C} := \left\{ [0_{k-1} \; e \; 0_{n-k}]^T \in \mathbb{R}^{n\times 1} \;\middle|\; \begin{matrix} k \in \{1,2,\ldots,n\} \\ e \in \{-1,1\} \end{matrix} \right\}$$

where $0_m \in \mathbb{R}^{1\times m}$ is the zeros-vector. Then, for the linear system (1) driven by a square-waveform of constant period $\tau > 0$ according to (2), it follows that there exists $c \in \mathscr{C}$ and a $\tau^* > 0$ such that the resonance condition (3) holds.

Proof: It suffices to show that $c \in \mathscr{C}$ exists to satisfy $c^T x_{ss}(\tau^*) = 0$ and $c^T v_{ss}(\tau^*) > 0$ for some $\tau^* > 0$, where x_{ss} and v_{ss} are given in (5). Toward this end, we introduce $\{x_{unit}(t) : t \geq 0\}$ to denote the unit-step response of (1) with initial condition $x(0) = 0$, and derive the following identity,

$$x_{ss}(2\tau) + x_{unit}(\tau)v_o = 0, \quad \forall \tau \geq 0 \tag{16}$$

Clearly, $x_{unit} : [0,\infty) \to \mathbb{R}^n$ includes two oscillatory trajectories due to the complex-conjugate pair of poles present in (1). Without loss of generality, we let these trajectories be denoted by $x_i(t)$ for $i = 1,2$ such that $\dot{x}_1 = x_2$. Hence, there exists a finite time t^* for which $x_1(t^*)$ reaches maximum overshoot and $x_2(t^*) = 0$. By choosing $e \in \mathscr{C}$ such that $e^T x_{unit} = x_2$, it follows from (16) that $c = e$ and $c = -e$ satisfies $c^T x_{ss}(\tau^*) = 0$ for $\tau^* = t^*/2$. This zero crossing of $c^T x_{ss}(\tau)$ at τ^* is isolated (i.e. unique on some finite interval $[-d,d]$) because the state-trajectories are oscillatory and smooth. ■

IV. Closed-loop stability

Recall that the goal of the feedback controller (4) is to drive the actual system state given by (x_k, p_k, τ_k) to the state of resonance given by $(x_{ss}(\tau^*), 0, \tau^*)$ in an efficient and stable manner. When this occurs, resonance is achieved. Inspection of equation (7), reveals that this occurs if the error variables in $z_k = \text{col}(\tilde{x}_k, \tilde{p}_k, \tilde{\tau}_k)$ are driven to zero. That is,

$$\begin{bmatrix} \tilde{x}_k \\ \tilde{p}_k \\ \tilde{\tau}_k \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{implies} \quad \begin{bmatrix} x_k \\ p_k \\ \tau_k \end{bmatrix} \to \begin{bmatrix} x_{ss}(\tau^*) \\ 0 \\ \tau^* \end{bmatrix}$$

Hence, the remaining task is to determine when the discrete-time trajectories of (8) will asymptotically converge to zero. Towards this end, a Lyapunov stability argument is presented in the following theorem. Therein, we list simple sufficient conditions that can be evaluated numerically, to determine if the dynamics of the harmonic oscillator and the resonance controller given by (8) converges with asymptotic stability to the desired state of resonance. In the following section, a control analysis example is illustrated based on the results of this theorem.

Theorem 1: For the closed-loop system of (8) and (9) subjected to a bounded external disturbance w_k as follows,

$$\begin{aligned} z_{k+1} &= M(v_k)z_k + d(\ell_k) + w_k \\ v_k &= \begin{bmatrix} 0 & 1 \end{bmatrix} \ell_k \\ \ell_k &= \begin{bmatrix} 0_{2\times n} & I_2 \end{bmatrix} z_k \\ \|w_k\| &\leq c_1 \qquad\qquad k = 1,2,\ldots \end{aligned} \tag{17}$$

there exists positive scalars κ_1, κ_2, c_0, c_2 and r such that

$$\begin{aligned} \|TM(v)T^{-1}\| &\leq c_0 < 1, \qquad \forall v \in B_r^1 \\ \|Td(\ell)\| &\leq c_2\|N\ell\|, \qquad \forall \ell \in B_r^2 \end{aligned} \tag{18}$$

where $B_r^m := \{x \in \mathbb{R}^m \mid \|x\| \leq r\}$ and T is given by

$$T = \begin{bmatrix} E^{-1} & 0 \\ 0 & N \end{bmatrix}, \quad N = \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix}$$

Moreover, suppose that c_0 and c_2 also satisfy:

$$c_0 + c_2 < 1 \tag{19}$$

Assuming conditions (17), (18) and (19) hold, then the following statements are valid.

(i) Equilibrium point $z_k = 0$ in (17) is stable if (18) holds with $c_1 < (1 - c_0 - c_2)r\sigma_{min}(T)/\sigma_{max}(T)$, and asymptotically stable in the absence of external disturbances (i.e. when $c_1 = 0$).

(ii) The region of attraction is approximated (conservatively) by the following compact set

$$\Omega := \left\{ z \in \mathbb{R}^n \;\middle|\; \|z\| \leq r\frac{\sigma_{min}(T)}{\sigma_{max}(T)} - r_o \right\} \tag{20}$$

where r_o is an arbitrarily small positive constant.

(iii) If $c_1 = 0$, then every trajectory starting in Ω, remains in Ω and approaches the origin as $t \to \infty$.

Proof: Let k_1 and k_2 satisfy $0 < k_1 \leq \alpha_k \leq k_2$ (sector condition). To show that (18) is satisfied with $c_0 < 1$, let $a_o = \min\{|\text{real}(D_{ii})| : i = 1,\ldots,n\}$ and choose $\kappa_2 = \kappa_1^2$ to get

$$TMT^{-1} = \left[\begin{array}{c|cc} e^{Dr_k} & 0 & 0 \\ \hline \varepsilon c^T E\kappa_1 & (1-\varepsilon) & 0 \\ 0 & -h\kappa_1 & (1-h\alpha_k) \end{array}\right]$$

Observe that as $\kappa_1 \to 0$, $\|TMT^{-1}\| \to c_0^*$ where we define $c_0^* := \max\{(1-\varepsilon), (1-hk_1), \exp(a_o(r-\tau^*))\}$. Clearly, $c_0^* < 1$ provided r, ε and h are chosen according to $r < \tau^*$, $\varepsilon < 1$, and $0 < h < 1/k_2$. Next, the existence of c_2 satisfying (18), follows from the fact that $d(\ell)$ is a vanishing perturbation, i.e. $d(0) = 0$. Statements (i), (ii) and (iii) are verified as follows. Consider the Lyapunov function candidate,

$$V(z_k) = \|Tz_k\| \tag{21}$$

Fix $k \in \mathbb{N}$, and choose z_k such that $V(z_k) < r\sigma_{min}(T)$. It follows that $\|z_k\| < r$, and $\ell_k \in B_r^2$, $v_k \in B_r^1$. Consequently, (21) evolves along the trajectories of (17) as follows

$$\begin{aligned}
\Delta V(z_k) &:= V(z_{k+1}) - V(z_k) \\
&= \|TM(\ell_k)z_k + Td(\ell_k) + Tw_k\| - \|Tz_k\| \\
&\leq \|TM(\ell_k)z_k\| + \|T(d(\ell_k) + w_k)\| - \|Tz_k\| \\
&\leq (\|TM(\ell_k)T^{-1}\| - 1)\|Tz_k\| + \|Td(\ell_k)\| + \|Tw_k\| \\
&\leq (c_0 - 1)\|Tz_k\| + c_2\|N\ell_k\| + c_1\|T\| \\
&\leq (c_0 - 1)\|Tz_k\| + c_2\|Tz_k\| + c_1\|T\| \\
&= -c_3 V(z_k) + c_1\sigma_{max}(T) \quad \Leftarrow \quad c_3 := 1 - c_0 - c_2
\end{aligned}$$

Hence, $\Delta V(z_k) < 0$ if $\sigma_{max}(T)c_1c_3^{-1} < V(z_k) < r\sigma_{min}(T)$. Furthermore, if $c_1 = 0$, then $\Delta V(z_k) < 0$ for all $\ell_k \in B_r^2$ (asymptotic stability). Using the following inequality,

$$\sigma_{min}(T)\|z_k\| \leq V(z_k) \leq \sigma_{max}(T)\|z_k\|$$

the region of attraction (20) can be derived [13]. ■

V. Example

In this section, we consider a simple example to illustrate how the theorem can be used to analyze the stability domain of a simple harmonic oscillator, whose plant parameters defined in (1) are as follows.

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \quad b, c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The plant is driven with a square-wave of amplitude $v_o = 1$, a feedback gain of $h = 0.05$, and a filtering constant of $\varepsilon = 0.1$. For this system, we compute the constant c_0 and c_2 that satisfy (18) as a function of the resonance tracking error $\tilde{\tau}$ and the filtering error $\tilde{p}$ and plot the contour lines in Fig. 1. Observing that $c_0 + c_2 < 1$ holds within the bold lines in Fig. 1, i.e. condition (18), we conclude that there exists an circle within these bold lines that defines the region of attraction of the closed-loop system.

VI. Conclusion

In this article, we introduce a new approach to resonant frequency tracking by means of an event-triggered feedback control. It is shown that the synchronization of the square-wave driver waveforms with periodic sampling of the plants output yields a closed-form analytical solution of the closed-loop system trajectories (i.e. on-line time-integration is not required). This feature is instrumental in proving asymptotic stability of closed loop system using a simple Lyapunov function. The region of attraction for this class of systems

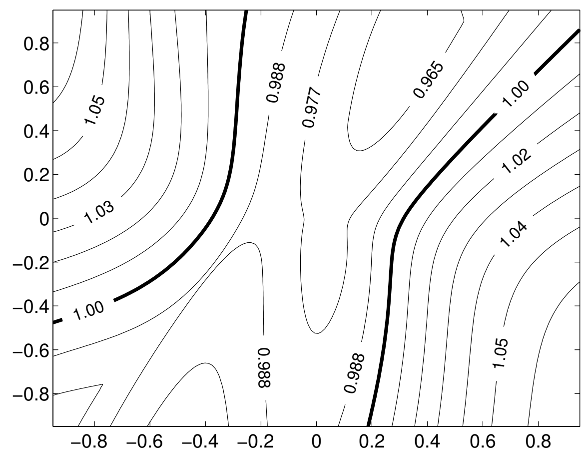

Fig. 1. Level curves of $\|TM(\ell)T^{-1}\| + \|Td(\ell)\|/\|N\ell\|$ versus tracking error $\tilde{\tau}$ (x-axis) and filtering error $\tilde{p}$ (y-axis), where $\ell = (\tilde{p}, \tilde{\tau})$.

is estimated, based on derived sufficient conditions that ensure stability of the resonance seeking control system. Simulations demonstrate the feasibility of this approach. Regarding future work, additional disturbance modelling is needed to assess the stability and performance robustness of the proposed controller in the presence of heavy external disturbances and time-varying loads.

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