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Using Orientation Agreement to Achieve Planar Rigid Formation

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Abstract— **We study a motion coordination problem where the objective is to steer group agents to move as a rigid body. We treat it as a joint orientation and formation control problem. We propose desired velocity and design decentralized feedback laws for each agent such that the rigid formation is guaranteed. Our design only employs relative information with respect to neighboring agents, and thus, can be implemented in a decentralized fashion.**

I. I

Cooperative control has been intensively studied during the past few years. The major focus in cooperative control is to design decentralized feedback control laws to achieve prescribed group motions, such as flocking, consensus, rendezvous, *etc.* [1]–[6].

The flocking algorithms [1], [2], [4], guarantee that all group agents reach to a common velocity as well as maintain constant desired relative distances in some inertial frame, which means that the group exhibits a translational motion. However, in certain situations, for example, deep-space interferometry missions, the group formation is desirable to move as a *virtual* rigid body [7]. The rigid body formation exhibits both rotational and translational motions, thereby requiring group agents to have different linear velocities and to maintain time-varying relative positions with respect to neighbors in the inertial frame. Such a rigid formation problem has been addressed by a number of studies. Reference [8] developed a centralized method to generate trajectories for group agents such that the rigid formation is preserved. In [9], a virtual structure approach was introduced, where each robot is considered as a particle embedded in a rigid body. The virtual structure approach was extended in [7], where a unified coordination scheme for formation control was proposed. Reference [10] considered single integrator agent dynamics and incorporated consensus scheme into the coordination architecture in [7] to estimate the group information. Recent research in [11] used receding horizon control to stabilize a rigid formation in a cooperative way.

In this paper, we address the 2D rigid formation problem by exploiting the relationship between the group rigid formation and the rigid body structure, and propose a decentralized leader-follower design that guarantees the rigid formation. When a rigid body is moving in 2D space, with a body frame attached, the particles on the rigid body have the same orientation and angular velocity but different linear velocities, and their relative positions, although time-varying in the inertial frame, remain constant in the body frame. By virtue of this observation, we assign each agent a local frame (heading), treat the formation of the agents as a virtual rigid body and the agents as particles on this rigid body. The orientation of this virtual rigid body is denoted by the heading of the leader, which is further chosen to be along the direction of reference velocity. When the headings of all the agents are aligned and rotate synchronously, each agent then possesses the information of the orientation of the group formation, and achieving a desired rigid formation is now equivalent to steering the relative positions between agents to some prescribed constant values in their local frames.

Since there is growing literature addressing the heading (orientation) agreement problem, for example, [13], [14], we restrict our attention to the formation control part. We draw on earlier results in [4] [12] and design desired velocity and decentralized feedback laws for each agent from its local measurements and information. The feedback laws, derived from potential function method, together with the proposed desired velocities, guarantee the convergence to the desired rigid formation. Unlike existing schemes [7], [10], where the inertial frame information is available to each agent, our design only requires the leader to have the inertial frame information and the other agents to implement the controls in their local frames.

The subsequent sections are organized as follows: Section II starts with an introduction of the notation and definitions used in the paper. We formulate our problem in Section III. The formation control laws are proposed in Section IV-A and a special case of quadratic potential function is discussed in Section IV-B. Design examples are presented in Section V.

II. P

If ω is a scalar, $\widehat{\omega}$ denotes the skew-symmetric matrix

$$\widehat{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}. \tag{1}$$

Given a vector $v \in \mathbb{R}^2$, we denote by a pair (r_v, θ_v) the polar coordinates of v. Then the following relationships are satisfied:

$$v = [r_v \cos\theta_v \;\; r_v \sin\theta_v]^T. \tag{2}$$

For the coordinate frame representation of a vector, the leading superscript indicates the reference frame while the subscript i denotes the agent i. The superscript d means

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the desired value. As an illustration, ${}^{i}v_i^d$ means the desired velocity of the ith agent in the ith frame. I_p and 0_p denote the $p\times p$ identity and zero matrices, respectively. Likewise, 1_N denotes the N-vector of ones.

Consider a rigid body moving in 2D space. Assume that there is an inertial frame E fixed in the space and that a body-fixed frame B is attached to the point O on the rigid body, as shown in Figure 1. Suppose that O moves along a curve with the linear velocity $v(t)\in\mathbb{R}^2$ and angular velocity $\omega(t)\in\mathbb{R}$, both represented in the local frame B. Given an arbitrary point P in the rigid body with the position vector r from O in the local frame B, the linear velocity of P, v_p, propagated from O is given by

$$v_p = v + \hat{\omega} r \tag{3}$$

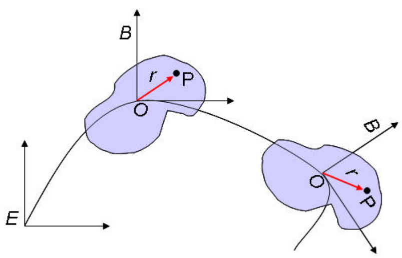


Fig. 1. Rigid body moving along a curve in the 2D space

III. P F

A. Agent Dynamics and Information flows

Consider N fully actuated agents moving in the plane with the inertial frame E, where each agent $i=1,\cdots,N$ is represented by a vector $x_i\in\mathbb{R}^2$. The dynamics of each agent is modeled as

$$\ddot{x}_i = f_i \tag{4}$$

where f_i is the input force of the ith agent. In order for the group to achieve some task, we choose one agent, say agent 1, to be the *group leader*, who has the inertial frame information. The desired velocity of agent 1 is predesigned as $v^d(t)=[v_x^d(t)\ v_y^d(t)]^T\in\mathbb{R}^2$ in E.

The information flows among agents are modeled as graphs. Throughout the paper, we consider the following two graphs: position graph and leader graph.

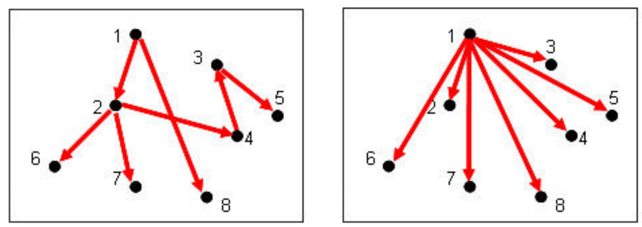
1) Position Graph: If the ith and jth agents have access to the relative information x_i-x_j, then the nodes i and j in the position graph G are connected by a link. To simplify our notation, we assign an orientation to the graph G by denoting one of the nodes of each link to be the positive end. The choice of orientation does not change the results because the position graph is assumed to be bidirectional and time-invariant. We further assume that G is connected. Suppose that M is the total number of links, and recall that the $N\times M$ *incidence matrix* D is defined as

$$d_{ik} := \begin{cases} +1 & \text{if the } i\text{th node is the positive end of the } k\text{th link} \\ -1 & \text{if the } i\text{th node is the negative end of the } k\text{th link} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Then z_k, the difference variable of link k, is defined as

$$z_k := \sum_{l=1}^{N} d_{lk} x_l = \begin{cases} x_i - x_j & \text{if the } i\text{th node is the positive end} \\ x_j - x_i & \text{if the } j\text{th node is the positive end.} \end{cases} \tag{6}$$

2) Leader Graph: If the jth agent can receive information from the ith agent, then the ith agent is considered as the *local* leader of the jth agent and the nodes i and j in the leader graph G_ℓ are connected by a directional link from i to j. Hence, the leader graph G_ℓ is directed. As we show later, the information received from the local leader of agent j is used to design the desired velocity of agent j. Therefore, each agent $i=2,\cdots,N$ is restricted to have only one local leader. We further assume that there is a path from agent 1 to any other agent and that no cycle exists in G_ℓ, as shown in Figure 2.



(a) General leader graph: The *local* leaders of the agents are not the same.

(b) Special leader graph: The group leader is the *local* leader of all the agents.

Fig. 2. Two types of leader graphs: agent 1 is the group leader and there exists a unique leader for each of the other agents.

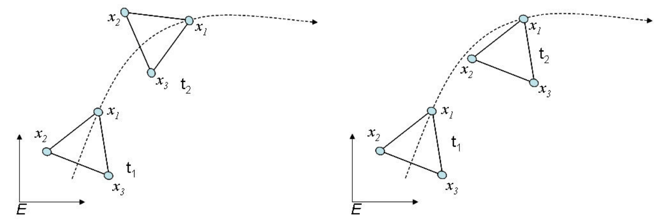
B. Virtual Rigid Body Formation

To facilitate our definition of *virtual* rigid body formation, we denote the direction of $v^d(t)$ by a unit vector $T(t)$. The direction of $T(t)$, denoted by $\theta_T(t)$, can be considered as the *virtual* heading (orientation) of the group formation. Recall that in a rigid body, the relative positions between any two points are fixed in the body frame, which implies that if the agents achieve a rigid formation, the relative positions between any two agents are invariant in the frame of θ_T. Thus, a group of agents is said to converge to a virtual rigid body formation as in Fig. 3(a) if and only if the following two conditions are satisfied:

A1) If ith and jth agents are connected by link k, then the difference variable z_k in (6) converges to a prescribed compact set $\mathcal{B}_k=\{z_k|\ \ |z_k|=d_k\}\subset\mathbb{R}^2$, $k=1,\cdots,M$.

A2) The relative angle between $T(t)$ and $z_k(t)$ achieves a constant value γ_k in the limit; that is, $\lim_{t\to\infty}\theta_T-\theta_{z_k}=\gamma_k$.

Our objective is to design decentralized control laws such that the group formation moves as a virtual rigid body, that is, the conditions A1 and A2 are satisfied. The virtual rigid body formation distinguishes from the flocking formation, where the relative positions between neighbors remain time-invariant in the inertial frame, shown in Fig. 3(b). In the virtual rigid body formation, however, the N agents maintain a time-varying geometric relationship in the inertial frame E.



(a) Rigid formation: All the agents have different velocities and the group formation moves as a rigid body.

(b) Flocking formation: All the agents have the same velocity and the group formation translates in the plane.

Fig. 3. Comparison of rigid body formation and flocking formation: The formation of three agents are sampled at time constants t_1 and t_2, $t_1 < t_2$.

IV. F C O A

To achieve A1 and A2, we assign each agent a local frame R_i, represented by the heading $\theta_i \in S^1$, $i = 1, \cdots, N$, where

$$R_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix}. \tag{7}$$

If the agents are considered as point robots, the heading assignment can be arbitrary. We further let the group leader's heading θ_1 be the same as the *virtue* group heading $\theta_T(t)$. It then follows that

$$\dot\theta_1 = \omega(t) \tag{8}$$

where

$$\omega(t) = \frac{v_x^d(t)\dot v_y^d(t) - v_y^d(t)\dot v_x^d(t)}{\|v^d\|^2}. \tag{9}$$

Assumption 1: $\omega(t)$, $\dot\omega(t)$ are continuous and bounded. □

We note that when each agent achieves the same heading as $\theta_1(t)$, it keeps a copy of $\theta_T(t)$ information, thereby simplifying the objective A2 to

$$\lim_{t\to\infty} \theta_i - \theta_{z_k} = \gamma_k, \tag{10}$$

if the ith agent is the positive end of link k. Since the agents can obtain the $\omega(t)$ information and its local leader's heading through G_ℓ, the following simple scheme guarantees the agreement and the synchronous rotation of the headings θ_j's, $j = 1, \cdots, N$,

$$\dot\theta_i = \omega - (\theta_i - \theta_{L(i)}), \quad i = 2, \cdots, N \tag{11}$$

where $L(i)$ is the *local* leader of agent i in G_ℓ. Due to significant results in the heading (orientation) agreement problem, we refer to [5], [13], [14] and references therein for more robust designs. In particular, when $\omega(t)$ and $\dot\omega(t)$ is available only to agent 1, [14], [15] presented adaptive decentralized designs to reconstruct the $\omega(t)$ information. Therefore, to focus on the formation control part, we make the following assumption:

Assumption 2: $\omega(t)$ and $\dot\omega(t)$ are available to each agent. The headings of all the agents achieve agreement and rotate synchronously at the angular velocity $\omega(t)$. □

Since the headings of all the agents are synchronized with $\theta_T(t)$, the objective A1, together with (10), implies that in the ith frame, the ith agent needs to maintain a desired relative distance d_k and a desired relative bearing γ_k with respect to its neighbor of link k. To design f_i's that guarantee these objectives, we rewrite the agent dynamics (4) in the ith frame as

$${}^i\ddot x_i = {}^i f_i \tag{12}$$

where ${}^i\ddot x_i = R_i^T \ddot x_i$ and ${}^i f_i$ is the applied force to ith agent in the ith frame. An internal feedback

$${}^i f_i = -K_i({}^i\dot x_i(t) - {}^i v_i^d(t)) + {}^i\dot v_i^d(t) + {}^i u_i + \hat\omega\, {}^i\dot x_i, \quad K_i > 0 \tag{13}$$

where

$${}^i\dot x_i = R_i^T \dot x_i \tag{14}$$

and a change of variable ${}^i\xi_i = {}^i\dot x_i(t) - {}^i v_i^d(t)$ bring the agent dynamics to be of the form [4, Example 1]

$${}^i\dot x_i = {}^i\xi_i + {}^i v_i^d(t) \tag{15}$$

$${}^i\dot\xi_i = -K_i {}^i\xi_i + {}^i u_i \tag{16}$$

where ${}^i v_i^d(t)$ is the desired velocity of the ith agent and ${}^i u_i$ is the external feedback from the neighbors of agent i, both of which are represented in the ith frame. We note that the design of ${}^i f_i$ now becomes the designs of the desired velocity ${}^i v_i^d(t)$ and the external feedback ${}^i u_i$, which are proposed in the following sections.

A. Formation Control

To specify the leader's desired velocity ${}^1 v_1^d(t)$ in its own frame, recall that the leader's desired velocity in the inertial frame is $v^d(t)$ and that its heading is always along $T(t)$, the direction of $v^d(t)$. It then follows that

$${}^1 v_1^d(t) = \begin{pmatrix} \|v^d\| \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{(v_x^d)^2 + (v_y^d)^2} \\ 0 \end{pmatrix}. \tag{17}$$

For the other agents, $i = 2, \cdots, N$, inspired by the velocity propagation law of rigid body in (3), we propose ${}^i v_i^d(t)$ to be of the following form

$${}^i v_i^d(t) = {}^{L(i)} v_{L(i)}^d(t) + \widehat{\omega}\ {}^i z_{i,L(i)} \tag{18}$$

where

$${}^i z_{i,j} = {}^i(x_i - x_j) \tag{19}$$

is the relative position between the ith and the jth agents represented in the ith frame. The ${}^{L(i)} v_{L(i)}^d(t)$ and ${}^i z_{i,L(i)}$ information in (18) are obtained through G_ℓ and G, respectively. Therefore, the design (18)-(19) requires that G_ℓ be a subgraph of G.

We now design the external feedback ${}^i u_i$. To simplify our analysis, since all the frames R_i's are aligned, for each link k, we let agent k^+, k^- be the positive and negative end, and denote by ${}^{k^+} z_k$ the relative distance z_k measured in the k^+th frame. We note that the objectives A1 and A2 in (10) are equivalent to regulating the relative distance ${}^{k^+} z_k$ such that

$$\lim_{t\to\infty} {}^{k^+} z_k = {}^{k^+} z_k^d \tag{20}$$

where ${}^{k^+}z_k^d$ is the desired value of ${}^{k^+}z_k$ in the k^+th frame. Indeed, the vector ${}^{k^+}z_k^d$ is available from d_k in A1 and γ_k in (10), and satisfies

$$ {}^{k^+}z_k^d = \begin{pmatrix} d_k \cos\gamma_k \\ d_k \sin\gamma_k \end{pmatrix}. \tag{21} $$

Denoting the desired target set of ${}^{k+}z_k$ by

$$ \mathcal{A}_k = \{{}^{k^+}z_k | {}^{k^+}z_k = {}^{k^+}z_k^d\}, \quad k = 1, \cdots, M, \tag{22} $$

we then propose ${}^iu_i$ to be of the following form:

$$ {}^iu_i = -\sum_{k=1}^{M} d_{ik}\psi_k({}^{k^+}z_k, {}^{k^+}z_k^d), \tag{23} $$

where the nonlinearities $\psi_k({}^{k^+}z_k, {}^{k^+}z_k^d)$ are of the form

$$ \psi_k({}^{k^+}z_k, {}^{k^+}z_k^d) = \nabla P_k({}^{k^+}z_k, {}^{k^+}z_k^d) \tag{24} $$

in which $P_k({}^{k^+}z_k, {}^{k^+}z_k^d)$ is a nonnegative C^2 potential function such that

$$ P_k({}^{k^+}z_k, {}^{k^+}z_k^d) \to \infty \text{ as } z_k \to \infty \tag{25} $$

$$ P_k({}^{k^+}z_k, {}^{k^+}z_k^d) = 0 \Leftrightarrow z_k \in \mathcal{A}_k \tag{26} $$

$$ \nabla P_k({}^{k^+}z_k, {}^{k^+}z_k^d) = 0 \Leftrightarrow z_k \in \mathcal{A}_k. \tag{27} $$

To state our main result, we introduce the concatenated vectors

$$ \Psi = [\psi_1({}^{1^+}z_1, {}^{1^+}z_1^d)^T, \cdots, \psi_M({}^{M^+}z_M, {}^{M^+}z_M^d)^T]^T $$

$$ x_R = [{}^1x_1^T, \cdots, {}^Nx_N^T]^T \quad v_R^d = [({}^1v_1^d)^T, \cdots, ({}^Nv_N^d)^T]^T $$

$$ z_R = [({}^{1^+}z_1)^T, \cdots, ({}^{M^+}z_M)^T]^T \quad z_R^d = [({}^{1^+}z_1^d)^T, \cdots, ({}^{M^+}z_M^d)^T]^T $$

$$ \xi_R = [({}^1\xi_1)^T, \cdots, ({}^N\xi_N)^T]^T \quad u_R = [({}^1u_1)^T, \cdots, ({}^Nu_N)^T]^T, $$

and note from (6) and (23) that

$$ z_R = (D^T \otimes I_2)x_R \tag{28} $$

and

$$ u_R = -(D \otimes I_2)\Psi \tag{29} $$

where $\otimes$ represents the Kronecker product. For the objective (20) to be feasible, the target sets $\mathcal{A}_k$ in (22) must be such that

$$ \{\mathcal{A}_1 \times \cdots \times \mathcal{A}_M\} \cap \mathcal{R}(D^T \otimes I_2) \neq \emptyset \tag{30} $$

since, from (28), z_R is restricted to be in the range space $\mathcal{R}(D^T \otimes I_2)$.

Theorem 1 below, proves that the set in (30) is globally asymptotically stable if the following property holds:

Property 1: $(D \otimes I_2)\Psi = 0$ and $z_R \in \mathcal{R}(D^T \otimes I_2)$ imply $z_R \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M \cap \mathcal{R}(D^T \otimes I_2)$. □

Theorem 1: Suppose that G_ℓ is a subgraph of G and consider the agent dynamics (15)-(16), where the desired velocity ${}^iv_i^d(t)$ and the feedback law ${}^iu_i$ are defined in (17)-(18) and (23)-(27). Then the trajectories $(z_R(t), \xi_R(t))$ are bounded and converge to the equilibria set

$$ \mathcal{E} = \left\{(z_R, \xi_R) |\ \xi_R = 0,\ (D \otimes I_2)\Psi(z_R) = 0 \text{ and } z_R \in \mathcal{R}(D^T \otimes I_2)\right\}. \tag{31} $$

Furthermore, if Property 1 holds, the set

$$ \mathcal{A} = \left\{(z_R, \xi_R) | \xi_R = 0, z_R \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M \cap \mathcal{R}(D^T \otimes I_2)\right\} \tag{32} $$

is globally asymptotically stable.

Proof: We take the Lyapunov function V as

$$ V = \frac{1}{2}\xi_R{}^T\xi_R + \sum_{k=1}^{M} P_k. \tag{33} $$

and its derivative is

$$ \dot{V} = \xi_R^T\dot{\xi}_R + \Psi^T\dot{z}_R. \tag{34} $$

Since in Assumption 2 we assume that all the frames R_i are aligned and rotate at the angular velocity $\omega(t)$, we obtain

$$ \frac{d({}^{k^+}z_k)}{dt} = \frac{dR_{k^+}^Tz_k}{dt} \tag{35} $$

$$ = -\widehat{\omega}({}^{k^+}z_k) + R_{k^+}^T\dot{z}_k. \tag{36} $$

or in a compact form

$$ \dot{z}_R = -(I_M \otimes \widehat{\omega})z_R + (D^T \otimes I_2)\dot{x}_R, \tag{37} $$

which is rewritten from (15) as

$$ \dot{z}_R = -(I_M \otimes \widehat{\omega})z_R + (D^T \otimes I_2)(\xi_R + v_R^d). \tag{38} $$

Because there exists a path from agent 1 to all the other agents in G_ℓ and because all the agents have the same heading, v_R^d can be further written from (18) as

$$ v_R^d = 1_N \otimes {}^1v_1^d + (I_N \otimes \widehat{\omega})z_{R1} \tag{39} $$

where

$$ z_{R1} = [({}^1z_{1,1})^T, \cdots, ({}^Nz_{N,1})^T]^T \tag{40} $$

and ${}^iz_{i,1}$ is defined in (19). Noting that

$$ z_R = (D^T \otimes I_2)z_{R1} \tag{41} $$

and substituting (39), (41) into (38), we obtain

$$ \dot{z}_R = (D^T \otimes I_2)\xi_R \tag{42} $$

because 1_N spans the null space of D^T and because

$$ (D^T \otimes I_2)(I_N \otimes \widehat{\omega}) = (I_M \otimes \widehat{\omega})(D^T \otimes I_2) = D^T \otimes \widehat{\omega}. \tag{43} $$

Thus, it follows from (29) and (42) that

$$ \Psi^T\dot{z}_R = -u_R^T\xi_R, \tag{44} $$

which, together with (16) and (34), leads to

$$ \dot{V} = -\sum_i K_i {}^i\xi_i{}^2 \leq 0. \tag{45} $$

Since the dynamics of ξ_R and z_R in (16) and (42) are time-invariant, we apply LaSalle Invariance Principle to analyze the largest invariant set where $\dot{V} = 0$. We then conclude form (16) that $\xi_i = 0$ implies ${}^iu_i = 0$, which proves the convergence to the set of equilibria $\mathcal{E}$ in (31). Moreover, when Property 1 is satisfied, the set $\mathcal{A}$ in (32) is globally asymptotically stable. ■

Convergence to $\mathcal{A}$ in (32) means that (20) is achieved. It also guarantees that $\xi_R \to 0$ and thus from (42) that $\dot{z}_R \to 0$, which implies that the desired rigid formation is maintained.

B. A Special Case: Quadratic Potential Function

When the potential function P_k is restricted to be of the quadratic form [11], [13], that is $P_k = |{}^{k^+}z_k - {}^{k^+}z_k^d|^2$, the design of ${}^i v_i^d(t)$ in (18) can be simplified to

$$ {}^i v_i^d(t) = {}^{L(i)} v_{L(i)}^d(t) + \widehat{\omega}\ {}^i z_{i,L(i)}^d. \quad i = 2, \cdots, N \tag{46} $$

where ${}^i z_{i,L(i)}^d$ is the desired *constant* value of $z_{i,L(i)}$ in ith frame. Note that the ith agent now does not have to know the relative position of its local leader $L(i)$ since only the constant ${}^i z_{i,L(i)}^d$ is required in (46). Therefore, the ${}^i \dot{v}_i^d(t)$ term in the control law (13) is simplified and the restriction in Theorem 1 that G_ℓ be a subgraph of G is eliminated.

Theorem 2 below proves that with the choice of quadratic potential function and the design (46), all trajectories $(z_R(t), \xi_R(t))$ converge to the equilibria set $\mathcal{A}$ in (32). Before proceeding, we point out the following lemma that guarantees that no equilibria arise outside $\mathcal{A}$:

Lemma 1: When $P_k = |{}^{k^+}z_k - {}^{k^+}z_k^d|^2$, $k = 1, \cdots, M$, Property 1 is satisfied.

Proof: We obtain from (24) that $\psi_k = {}^{k^+}z_k - {}^{k^+}z_k^d$, or in the stacked form

$$ \Psi = z_R - z_R^d. \tag{47} $$

Because z_R and z_R^d both belong to the range space of $D^T \otimes I_2$, $(D \otimes I_2)(z_R - z_R^d) = 0$ is satisfied only when $z_R = z_R^d$, which implies $z_R \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M \cap \mathcal{R}(D^T \otimes I_2)$. ■

Theorem 2: Consider the agent dynamics (15)-(16), where the desired velocity ${}^i v_i^d(t)$ and the feedback law ${}^i u_i$ are defined in (17), (46) and (23)-(27), with $P_k = |{}^{k^+}z_k - {}^{k^+}z_k^d|^2$, $k = 1, \cdots, M$, in (24). Then the trajectories $(z_R(t), \xi_R(t))$ are bounded and converge to the set $\mathcal{A}$ in (32).

Proof: We take the same Lyapunov function as in (33), whose derivative is the same as in (34). Because there exists a path from the group leader x_1 to all the other agents and because all the agents have the same heading, v_R^d can be written as

$$ v_R^d = 1_N \otimes {}^1 v_1^d + (I_N \otimes \widehat{\omega}) z_{R1}^d \tag{48} $$

where

$$ z_{R1}^d = [({}^1 z_{1,1}^d)^T, \cdots, ({}^N z_{N,1}^d)^T]^T \tag{49} $$

and ${}^i z_{i,1}^d$ is the desired relative positions from the ith agent to agent 1. Since the desired formation is considered to be rigid, ${}^i z_{i,1}^d$'s are uniquely defined. Substituting (48) into (38) yields

$$ \dot{z}_R = -(I_M \otimes \widehat{\omega}) z_R + (D^T \otimes I_2)\xi_R + (D^T \otimes I_2)(I_N \otimes \widehat{\omega}) z_{R1}^d \tag{50} $$

since 1_N belongs to the null space of D^T. Due to (43), we further obtain that

$$ \dot{z}_R = -(I_M \otimes \widehat{\omega}) z_R + (D^T \otimes I_2)\xi_R + (I_M \otimes \widehat{\omega})(D^T \otimes I_2) z_{R1}^d. \tag{51} $$

Because $(D^T \otimes I_2) z_{R1}^d = z_R^d$, it follows from (51) that

$$ \dot{z}_R = (I_M \otimes \widehat{\omega})(z_R^d - z_R) + (D \otimes I_2)^T \xi_R. \tag{52} $$

From (52), (47) and (29), the second term in (34) now becomes

$$ \Psi^T \dot{z}_R = \Psi^T (D^T \otimes I_2)(\xi_R) = -u_R^T \xi_R \tag{53} $$

since $(z_R - z_R^d)$ is perpendicular to $(I_M \otimes \widehat{\omega})(z_R^d - z_R)$.

Thus, we obtain from (34), (16) and (53) that

$$ \dot{V} = -\sum_i K_i {}^i \xi_i^{\,2}, \tag{54} $$

which implies that the signals $(\xi_R(t), z_R(t))$ are bounded. We further conclude from Barbalat's Lemma that ${}^i \xi_i \to 0$. We next show that ${}^i u_i \to 0$. To see this, we note

$$ {}^i \ddot{\xi}_i = -K_i {}^i \dot{\xi}_i + {}^i \dot{u}_i \tag{55} $$

is continuous and uniformly bounded because ${}^i \dot{u}_i$ and ${}^i \dot{\xi}_i$ are continuous functions of the bounded signals $(z_R(t), \xi_R(t), \omega(t), \dot{\omega}(t))$. We then obtain from [16, Lemma 1] that $\dot{\xi}_R \to 0$, which means that $u_R \to 0$ from (16). Using Lemma 1, we conclude the global convergence to the set $\mathcal{A}$ in (32). ■

V. D E

Consider three agents x_i, $i = 1, 2, 3$, where x_1 is the group leader and x_j is the local leader of x_{j+1}, $j = 1, 2$. In G, each agent has the other two as its neighbors. We choose

$$ \omega = 1 \quad \text{and} \quad {}^1 v_1^d = [2\ 0]^T, \tag{56} $$

which means that the leader's trajectory is a circle with radius 2. To stabilize the desired formation in Figure 4, recall that

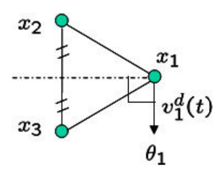


Fig. 4. The desired formation of three agents: The desired relative distances for every two agents are 1. z_{32} is always aligned with the direction of $v_1^d(t)$.

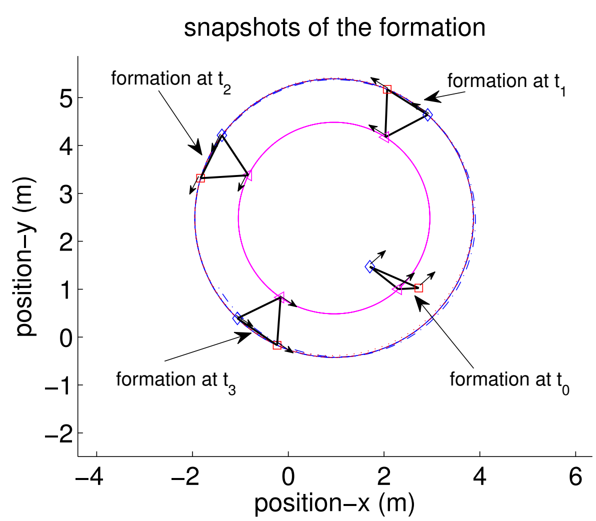


Fig. 5. The desired formation in Figure 4 is achieved: The arrow denotes the heading of each agent and initially the headings of all the agents are aligned. Magenta △, blue ⋄ and red □ represent x_1, x_2 and x_3, respectively.

the leader's heading θ_1 is always along the direction $v_1^d(t)$ and that all the agents have the same heading. We then compute the desired relative position of each link as

$$^1z^d_{12} = [\frac{1}{2}\ \ \frac{\sqrt{3}}{2}]^T \quad ^1z^d_{13} = [-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}]^T \quad ^1z^d_{23} = [-1\ \ 0]^T \qquad (57)$$

and choose the potential function P_k in (24) as

$$P_k({}^{k^+}z_k, {}^{k^+}z_k{}^d) = \|{}^{k^+}z_k - {}^{k^+}z^d_k\|^2, \qquad (58)$$

where ${}^{k^+}z_k{}^d$ is available from (57).

Figure 5 illustrates that the design in (18) and (23), which employs the information in (56)-(58), achieves the desired rigid formation.

We next consider four agents and make use of the simplified design in Section IV-B to achieve the square formation in Figure 6, where the desired relative positions in the local frames are given by

$$^1z^d_{12} = {}^1z^d_{43} = [\frac{\sqrt{2}}{2}\ \ -\frac{\sqrt{2}}{2}]^T \quad ^1z^d_{23} = {}^1z^d_{14} = [\frac{\sqrt{2}}{2}\ \ \frac{\sqrt{2}}{2}]^T. \quad (59)$$

In G_ℓ, agent 1 serves as the local leader of agent 2 while both

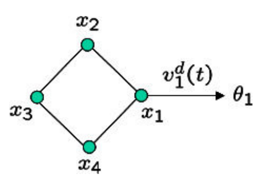


Fig. 6. The desired formation of four agents is a square with side length 1, where z_{13} is always aligned with the leader's heading θ_1.

agents 3 and 4 take agent 2 as its leader. G is a ring graph, where agent i, $i = 2,3$, is the neighbor of both agents $i-1$ and $i+1$, and agent 1 is the neighbor of agent 4. Note that G_ℓ is not a subgraph of G. Instead of assuming that the heading agreement has been achieved, we apply the methodology in [14] to achieve the agreement. Since the headings of the agents are synchronized eventually, the desired formation is guaranteed as shown in Figure 7.

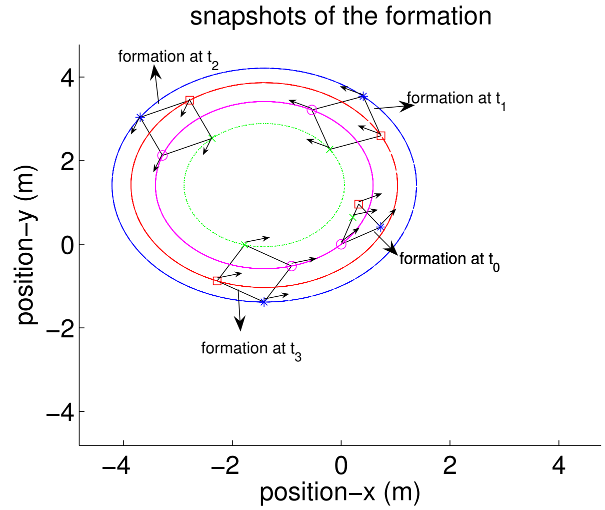


Fig. 7. The desired formation is achieved: The arrow denotes the heading of each agent and initially the headings of all the agents are different. Magenta ∘, green ×, red □ and blue ∗ represent x_1, x_2, x_3 and x_4, respectively.

VI. C F W

We study a motion coordination problem where the objective is to achieve a rigid group formation. We treat this problem as a joint orientation and formation control problem. We develop decentralized leader-follower control laws such that the rigid formation is guaranteed. The proposed design is further simplified when the potential function is quadratic. We then show by numerical examples that our designs achieve the desired rigid formations. Future directions include extensions to time-varying information topology and to more complex agent dynamics.

R

[1] N. E. Leonard and E. Fiorelli, "Virtual leader, artificial potentials and coordinated control of groups." in *Proceedings of the 40th IEEE Conference on Decision and Control*, 2001, pp. 2968–2973.

[2] H. Tanner, A. Jadbabaie, and G. Pappas, "Stable flocking of mobile agents, part i: Fixed topology." in *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, 2003, pp. 2010–2015.

[3] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 916–922, 2007.

[4] M. Arcak, "Passivity as a design tool for group coordination." *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1380–1390, 2007.

[5] W. Ren, R. W. Beard, and E. M. Atkins, "A survey of consensus problems in multi-agent coordination." in *Proceedings of 2005 American Control Conference*, 2005, pp. 1859–1864.

[6] J. R. Lawton and R. W. Beard, "Synchronized multiple spacecraft rotations," *Automatica*, vol. 38, pp. 1359–1364, 2002.

[7] R. W. Beard, J. Lawton, and F. Y. Hadaegh, "A coordination architecture for spacecraft formation control." *IEEE Transactions on Control Systems Technology*, vol. 9, no. 6, November 2001.

[8] C. Belta and V. Kumar, "Motion generation for groups of robots: a centralized, geometric approach," in *Proceedings of the ASME Design Engineering Technical Conference*, Montreal, Canada, September 29 - October 2 2002.

[9] K.-H. Tan, M. A. Lewis, and T. C. Lab, "Virtual structure for high-precision cooperative mobile robotic control," in *Proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1996.

[10] N. Sorensen and W. Ren, "A unified formation control scheme with a single or multiple leaders," in *Proceedings of the 2007 American Control Conference*, New York City, New York, July 2007.

[11] W. B. Dunbar and R. M. Murray, "Distributed receding horizon control for multi-vehicle formation stabilization," *Automatica*, vol. 42, no. 4, pp. 549–558, 2006.

[12] M. Mesbahi and F. Y. Hadaegh, "Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching," in *Proceedings of the 1999 IEEE International Conference on Control Applications*, Hawaii, August 1999.

[13] D. A. Paley, N. E. Leonard, R. Sepulchre, D. Grünbaum, and J. K. Parrish, "Oscillator models and collective motion," *IEEE Control Systems Magazine*, vol. 27, no. 4, pp. 89–105, August 2007.

[14] H. Bai, M. Arcak, and J. T. Wen, "A decentralized design for group alignment and synchronous rotation without inertial frame information," in *Proceedings of the 46th Conference on Decision and Control*, New Orleans, LA, 2007.

[15] ——, "Group coordination when the reference velocity is available only to the leader: An adaptive design." in *Proceedings of the 2007 American Control Conference*, New York City, New York, USA, July 2007.

[16] J. S.-C. Yuan and W. M. Wonham, "Probing signals for model reference identification." *IEEE Transactions on Automatic Control*, vol. 22, no. 4, August 1977.