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Simultaneous Identification of Tire Cornering Stiffnesses and Vehicle Center of Gravity

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Abstract—**INS, GPS measurements are used to simultaneously estimate the location of the center of gravity of a vehicle and the tire cornering stiffnesses. The developed method uses kinematic as well as dynamic equations of a lateral vehicle model to eliminate the bias in the yaw rate and lateral acceleration measurements. An approximation of the moment of inertia is used to combine the dynamic equations of a bicycle model and thereby estimate the tire cornering stiffnesses. The chief advantage of this method is its determinate formulation which eliminates the constraint on persistency of excitation during vehicle testing. It is shown using simulations that the accuracy of the proposed method is affected by measurement noise.**

I. Introduction

Knowledge of vehicle parameters such as tire cornering stiffnesses, location of the center of gravity(CG), mass, moment of inertia etc. are required for a variety of purposes. Applications such as lane-following, steer-by-wire, friction estimation and tire monitoring critically depend on accurate knowledge of vehicle parameters [1,2]. While vehicular mass is simple to measure, parameters such as moment of inertia, cornering stiffnesses and location of CG are more difficult. These parameters are seldom provided by manufacturers too [1, 2]. Indeed, some of these parameters change over time.

The center of gravity location is typically estimated by measuring tire loads on a special tilting platform [3], Tire cornering stiffnesses have been estimated using INS measurements complemented by GPS velocity readout [1]. However, the estimation procedure required prior knowledge of the location of the center of gravity and straight lane driving for a fixed period to correct the bias in the yaw rate gyroscope measurement. A recent attempt to estimate tire cornering stiffnesses [2] concluded that the estimation problem is under-determinate.

The motivation behind this work is to develop a method to estimate vehicle CG-location and tire cornering stiffnesses through INS and GPS measurements using simple vehicle maneuvers. It is shown that the proposed method can simultaneously estimate CG-location and tire cornering stiffnesses with the use of INS and GPS measurements. Though batch least-squares has been used to solve the estimation problem, the method can also be extended to real-time estimation using recursive least square estimation.

To estimate unknown vehicle parameters, it is required to choose an appropriate vehicle dynamics model and measure vehicular motion using available sensors. The unknown parameters can then be predicted using the model and the measurements. In the proposed method, the lateral bicycle model of a vehicle is used to estimate the vehicle parameters. The bicycle model relates the lateral forces acting at the front and rear tires to the lateral and yaw acceleration experienced by the vehicle. Though this model neglects roll dynamics, it has become the standard for predicting lateral vehicle behavior. The bicycle model of a vehicle is described by equations (1), (2) which represent the effect of the tire forces on the lateral and yaw acceleration of the vehicle [4].

$$\underbrace{m(\ddot{y}+V_x\dot{\psi})}_{\substack{\text{Net lateral force}\\ \text{on the vehicle}}} = \underbrace{2C_f\left(\delta-\frac{\dot{y}+l_f\dot{\psi}}{V_x}\right)}_{\text{Lateral force from front tires}} + \underbrace{2C_r\left(-\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right)}_{\text{Lateral force from rear tires}} \quad (1)$$

$$\underbrace{I_z\ddot{\psi}}_{\substack{\text{Net torque}\\ \text{at CG}}} = \underbrace{2C_f\left(\delta-\frac{\dot{y}+l_f\dot{\psi}}{V_x}\right)l_f}_{\text{Torque at CG from front tires}} - \underbrace{2C_r\left(-\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right)l_r}_{\text{Torque at CG from rear tires}} \quad (2)$$

where $\dot{y}$, $\ddot{y}$ are the lateral velocity, acceleration at the CG relative to the vehicle's body frame, $\dot{\psi}$, $\ddot{\psi}$ are the yaw rate, yaw acceleration relative to the inertial frame, m is the mass of the vehicle, I_z is the moment of inertia of the vehicle about the vertical axis, l_f, l_r are the longitudinal distances of the front and rear wheels respectively from the CG of the vehicle, V_x is the longitudinal velocity of the vehicle, δ is the steering angle and C_f, C_r are the tire cornering stiffness of the front and rear wheels respectively. The right-hand side of equation (1) represents the sum of lateral tire forces at the front and rear wheels. Similarly, the right-hand side of equation (2) represents the yaw moment generated due to the forces at the front and rear wheels.

The left hand sides of equations (1), (2) are a function of the lateral acceleration $(\ddot{y}+V_x\dot{\psi})$ and yaw rate $(\dot{\psi})$ measurements in the inertial frame. The other measurements that are possible in this model include the steering angle δ, and the longitudinal velocity V_x. The parameters that need to be identified are $l_f, l_r, \frac{C_f}{m}, \frac{C_r}{m}, \frac{C_f}{I_Z}$ and $\frac{C_r}{I_Z}$.

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Apart from the vehicular parameters, two biases, which are unknown constant errors in the lateral accelerometer and yaw gyroscope measurements, need to be estimated as well. Otherwise, it would not be possible to calculate $\dot{y}, \psi$ accurately by integration of $\ddot{y}, \dot{\psi}$ respectively.

II. Procedure for Parameter Estimation

i. *GPS global velocity measurement for bias correction* Carrier-phase GPS velocity measurements are used to estimate the bias in the acceleration and yaw rate measurements. Kinematic equations of velocity angles are used to achieve this instead of the dynamic equations from the bicycle model. This subsequently allows accurate computation of lateral velocity and yaw angle in the bicycle model by integrating the corrected lateral acceleration and yaw rate respectively.
ii. *GPS global velocity measurement to estimate l_f, l_r* The GPS velocity measurement depends on the longitudinal distance of the GPS antenna from the CG of the vehicle. Thus, the global velocity angle measurement from the GPS is also used to estimate the position of the GPS antenna w.r.t the CG of the vehicle and subsequently, l_f and l_r.
iii. *Approximating $I_z = ml_f l_r$ to combine equations (1), (2)* Upon obtaining the bias values and the estimates for l_f, l_r; the only unknowns in equations (1), (2) are $\frac{C_f}{m}, \frac{C_r}{m}$. To avoid constraints on the persistency of excitation while estimating two parameters simultaneously, one of the parameters is eliminated by substituting equation (2) appropriately in equation (1). To achieve this, an approximate expression for I_z in terms of m, l_f, l_r is used. The remaining parameter is then obtained simply by least squares estimation.

A. Estimation of accelerometer bias

The global velocity angle is related to the global velocity measurements $\dot{X}, \dot{Y}$ (from the GPS antenna) as follows:

$$\theta_{GPS} = \tan^{-1}\left(\frac{\dot{Y}}{\dot{X}}\right) \tag{7}$$

where θ_{GPS} is the global velocity angle i.e. the angle made between the velocity vector at the GPS antenna and the global x-axis.

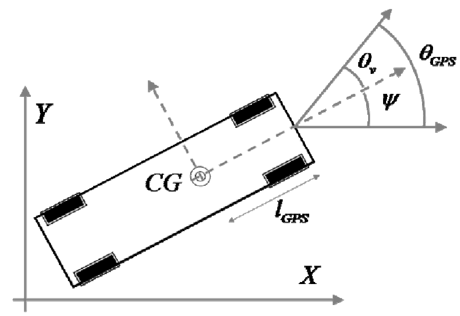

Fig. 1. Definition of vehicle, global co-ordinates

The global velocity angle may be written as a sum of the orientation of the vehicle ψ and the velocity angle at the GPS antenna relative to the longitudinal axis of the vehicle, $\theta_v = \tan^{-1}\left(\frac{\dot{y} + l_{GPS}\dot{\psi}}{V_x}\right)$, as shown in Fig.1, where l_{GPS} is the longitudinal distance between the GPS antenna and the CG.

Hence, the measured global velocity angle is given by

$$\theta_{GPS} = \tan^{-1}\left(\frac{\dot{y} + l_{GPS}\dot{\psi}}{V_x}\right) + \psi \tag{8}$$

Since the lateral velocity is much smaller compared to the longitudinal velocity,

$$\theta_{GPS} \approx \frac{\dot{y} + l_{GPS}\dot{\psi}}{V_x} + \psi = \frac{(\dot{y} + V_x\psi) + l_{GPS}\dot{\psi}}{V_x} \tag{9}$$

Assuming constant longitudinal velocity V_x, the bias in the lateral accelerometer is estimated by substituting for $\dot{y} + V_x\psi$ as

$$\dot{y} + V_x\psi = \int(\ddot{y} + V_x\dot{\psi}) = \int y_{acc} + b_1 t \tag{10}$$

where y_{acc} is the measured lateral acceleration in the inertial frame and $-b_1$ is the bias in the lateral accelerometer. Substituting equation (10) in equation (9),

$$\theta_{GPS} = \frac{1}{V_x}\int y_{acc} + \frac{b_1 t}{V_x} + \frac{l_{GPS}\dot{\psi}}{V_x} \tag{11}$$

Rewriting the above equation with the unknowns on the right hand side,

$$\left[V_x\theta_{GPS} - \int y_{acc}\right] = l_{GPS}\dot{\psi} + b_1 t = \begin{bmatrix}\dot{\psi} & t\end{bmatrix}\begin{bmatrix}l_{GPS} \\ b_1\end{bmatrix} \tag{12}$$

Equation (12) may be solved in a least squares fashion to estimate the unknowns – b_1, l_{GPS}. In the above formulation, persistency of excitation is not expected to be a problem since the regressors $\dot{\psi}$ and t have very different behavior. While the yaw rate varies with the road profile, the time increases linearly. Thus, by measuring the global velocity angle of the GPS, lateral acceleration of the vehicle; it is possible to estimate the bias in the lateral accelerometer and the distance between the GPS antenna and the CG.

In the simulations, a batch-total-least squares approach is used to estimate the parameters. Simple least-squares is not used since the regressor matrix is noisy for this estimation problem. Total least squares aims to get a best fit of the parameters so that the sum of the orthogonal distances between the predicted and observed measurements is minimized. In this method, the regressor matrix is first augmented by the measurement matrix. Subsequent singular value decomposition of the augmented matrix results in the last column of the matrix 'V' becoming the scaled optimal estimate for the parameters where the scaling factor is given by the last element of the matrix 'V'.

B. *Estimation of l_f, l_r*

Once the value of l_{GPS} is known, it is possible to find l_f by measuring the distance between the GPS antenna and the front axle as shown in Fig. 2. If this distance be l_{GPS-fw}, then l_f is given by

$$l_f = l_{GPS} + l_{GPS-fw} \tag{13}$$

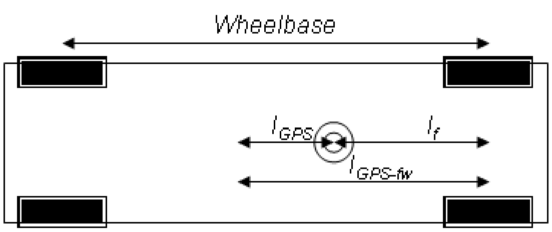

Fig. 2. Top view of vehicle

Assuming that the wheel base is known,

$$l_r = Wheelbase - l_f$$

C. *Estimation of tire cornering stiffnesses*

The procedure to estimate the cornering stiffnesses starts with estimation of $\frac{C_f}{m}$ using equations (1),(2). Clearly, equations (1), (2) cannot be combined to obtain a single equation with $\frac{C_f}{m}$ as the only unknown. In this work, this problem is circumvented by approximating $I_z = m l_f l_r$ [4]. This approximation comes from the lumped-mass model of the vehicle. If the vehicle was modeled as a sum of two masses at the front and rear at distances l_f, l_r from the CG respectively as shown in Fig. 3, the moment of inertia may be written as

$$I_z = m l_f l_r$$

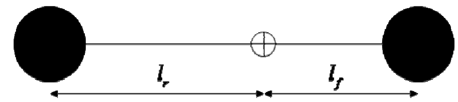

Fig. 3. Lumped mass model of vehicle

Substituting in (2),

$$\Rightarrow m l_f l_r \ddot{\psi} = 2C_f \left(\delta - \frac{\dot{y} + l_f \dot{\psi}}{V_x} \right) l_f - 2C_r \left(-\frac{\dot{y} - l_r \dot{\psi}}{V_x} \right) l_r \tag{14}$$

Or

$$m \ddot{\psi} = \frac{2C_f}{l_r} \left(\delta - \frac{\dot{y} + l_f \dot{\psi}}{V_x} \right) - \frac{2C_r}{l_f} \left(-\frac{\dot{y} - l_r \dot{\psi}}{V_x} \right) \tag{15}$$

Multiplying the above equation by l_f and adding with equation (1),

$$m(\ddot{y} + V_x \dot{\psi}) + m l_f \ddot{\psi} = 2C_f \frac{l_f}{l_r} \left(\delta - \frac{\dot{y} + l_f \dot{\psi}}{V_x} \right) + 2C_f \left(\delta - \frac{\dot{y} + l_f \dot{\psi}}{V_x} \right) \tag{16}$$

$$\Rightarrow (\ddot{y} + V_x \dot{\psi}) + l_f \ddot{\psi} = \frac{2C_f}{m} \left(\delta - \frac{\dot{y} + l_f \dot{\psi}}{V_x} \right) \left(1 + \frac{l_f}{l_r} \right) \tag{17}$$

$$\Rightarrow l_r . \frac{(\ddot{y} + V_x \dot{\psi}) + l_f \ddot{\psi}}{l_f + l_r} = \frac{2C_f}{m} \left(\delta - \frac{\dot{y} + l_f \dot{\psi}}{V_x} \right) \tag{18}$$

In the above equation, all the quantities in the left hand side of the equation are known. l_f, l_r are assumed to be known after estimating it using the GPS measurement as shown in the sections II A,B. On the right hand side of the equation, $\frac{2C_f}{m}$ is the unknown parameter to be estimated. However, a measurement of $\frac{\dot{y} + l_f \dot{\psi}}{V_x}$ is not directly available since the global velocity measurement at the GPS antenna is given by $\frac{\dot{y} + l_{GPS} \dot{\psi}}{V_x} + \psi$. Since $l_f - l_{GPS}, V_x, \dot{\psi}$ are known it is possible to obtain $\frac{\dot{y} + l_f \dot{\psi}}{V_x} + \psi$ as

$$\frac{\dot{y}+l_f\dot{\psi}}{V_x}+\psi=\underbrace{\frac{\dot{y}+l_{GPS}\dot{\psi}}{V_x}+\psi}_{\theta_{GPS}}+\underbrace{(l_f-l_{GPS})}_{front-axle-GPS\,distance}\frac{\dot{\psi}}{V_x} \tag{19}$$

However to obtain $\frac{\dot{y}+l_f\dot{\psi}}{V_x}$, the value of ψ needs to be subtracted from the above equation. ψ can be obtained by integrating the yaw rate measurement. But, in doing so, the bias in the yaw rate measurement causes a drift as described earlier in the case of the accelerometer integration. Defining the yaw rate bias to be $-b_2$, the value of ψ is given by

$$\psi=\int\dot{\psi}_{measured}+b_2t \tag{20}$$

Substituting in equation (22),

$$l_r\frac{(\ddot{y}+V_x\dot{\psi})+l_f\ddot{\psi}}{l_f+l_r}=\frac{2C_f}{m}\left(\delta-(\theta_{GPS}+(l_f-l_{GPS})\frac{\dot{\psi}}{V_x}-\int\dot{\psi}_{measured}-b_2t)\right)$$

$$\Rightarrow l_r.\frac{(\ddot{y}+V_x\dot{\psi})+l_f\ddot{\psi}}{(l_f+l_r)}=\frac{2C_f}{m}\left(\delta-\theta_{GPS}-(l_f-l_{GPS})\frac{\dot{\psi}}{V_x}+\int\dot{\psi}_{measured}\right)+\frac{2C_fb_2}{m}t$$

Or,

$$\boxed{\left[l_r.\frac{(\ddot{y}+V_x\dot{\psi})+l_f\ddot{\psi}}{(l_f+l_r)}\right]=\left[\left(\delta-\theta_{GPS}-(l_f-l_{GPS})\frac{\dot{\psi}}{V_x}+\int\dot{\psi}_{measured}\right)\;t\right]\begin{bmatrix}\frac{2C_f}{m}\\ \frac{2C_fb_2}{m}\end{bmatrix}} \tag{21}$$

Thus, by measuring the lateral acceleration, yaw rate, steering angle, longitudinal velocity of the vehicle, global velocity angle of the GPS antenna; it is possible to estimate the cornering stiffness of the front tires normalized by the mass of the vehicle. In the above formulation, persistency of excitation is not expected to be a problem since the regressors $\left(\delta-\theta_{GPS}-(l_f-l_{GPS})\frac{\dot{\psi}}{V_x}+\int\dot{\psi}_{measured}\right),t$ have very different behavior. While the first regressor varies with the road profile, the time increases linearly.

The cornering stiffness for the rear tires may be estimated by proceeding with a similar derivation as above. Starting with equation (15),

$$m\ddot{\psi}=\frac{2C_f}{l_r}\left(\delta-\frac{\dot{y}+l_f\dot{\psi}}{V_x}\right)-\frac{2C_r}{l_f}\left(-\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right)$$

Multiplying the above equation by l_r and subtracting from equation (1)

$$m(\ddot{y}+V_x\dot{\psi})-ml_r\ddot{\psi}=-2C_r\left(\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right)-2C_r\frac{l_r}{l_f}\left(\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right) \tag{22}$$

$$\Rightarrow(\ddot{y}+V_x\dot{\psi})-l_r\ddot{\psi}=-\frac{2C_r}{m}\left(\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right)\left(1+\frac{l_r}{l_f}\right)$$

$$\Rightarrow l_f.\frac{(\ddot{y}+V_x\dot{\psi})-l_r\ddot{\psi}}{(l_f+l_r)}=-\frac{2C_r}{m}\left(\frac{\dot{y}-l_r\dot{\psi}}{V_x}\right) \tag{23}$$

As described above, the quantity $\frac{\dot{y}-l_r\dot{\psi}}{V_x}$ may be expressed in terms of measurements as

$$\frac{\dot{y}-l_r\dot{\psi}}{V_x}=\theta_{GPS}-(l_r+l_{GPS})\frac{\dot{\psi}}{V_x}-\int\dot{\psi}_{measured}-b_2t \tag{24}$$

Substituting in the equation (24) in (23),

$$l_f.\frac{(\ddot{y}+V_x\dot{\psi})-l_r\ddot{\psi}}{(l_f+l_r)}=-\frac{2C_r}{m}(\theta_{GPS}-(l_r+l_{GPS})\frac{\dot{\psi}}{V_x}-\int\dot{\psi}_{measured}-b_2t))$$

$$\boxed{\Rightarrow\left[l_f.\frac{(\ddot{y}+V_x\dot{\psi})-l_r\ddot{\psi}}{(l_f+l_r)}\right]=\left[\left(-\theta_{GPS}+(l_r+l_{GPS})\frac{\dot{\psi}}{V_x}+\int\dot{\psi}_{measured}\right)\;t\right]\begin{bmatrix}\frac{2C_r}{m}\\ \frac{2C_rb_2}{m}\end{bmatrix}} \tag{25}$$

The cornering stiffness of the rear tires normalized to the mass of the vehicle may be obtained by using total least squares estimation of the parameters in the above equation.

III. RESULTS AND DISCUSSION

The above estimation procedure was verified by simulation for a vehicle with the following parameters:

TABLE 1. VEHICLE PARAMETERS FOR SIMULATION

QUANTITY	*SYMBOL*	*Value*
Mass	*m*	*1573kg*
Moment of inertia	*Iz*	*2873 Nm^2*
Front wheel distance from CG	l_f	*1.1m*
Rear wheel distance from CG	l_r	*1.58m*
Front tire cornering stiffness	C_f	*80000 N/rad*
Rear tire cornering stiffness	C_r	*80000 N/rad*

The longitudinal velocity was assumed to be constant at 10 m/s for the simulations. The noise and bias in the measurements were chosen to match real experimental observations. Earlier experiments [5, 6] on a Navistar Safetruck yielded measurements with the following stochastic characteristics, which were used for the simulations.

TABLE 2. MEASUREMENT CHARACTERISTICS FROM EXPERIMENTAL DATA

Signal	*Variance*	*Bias*
Yaw rate	*6.8*10^-5^* → $6.8*10^{-5}$	*-0.005*
GPS global velocity angle	$2.4*10^{-5}$	*0*
Lateral acceleration	*0.0222*	*0.039*
Longitudinal velocity	$9*10^{-4}$	*0*
Steering angle	$3.1*10^{-5}$	*0*

A simple steering maneuver was chosen for the simulation to keep the implementation realistic. This input is shown in Fig. 4. Steering angle input for vehicle model. The simulated yaw rate, lateral acceleration are shown in Fig. 6.

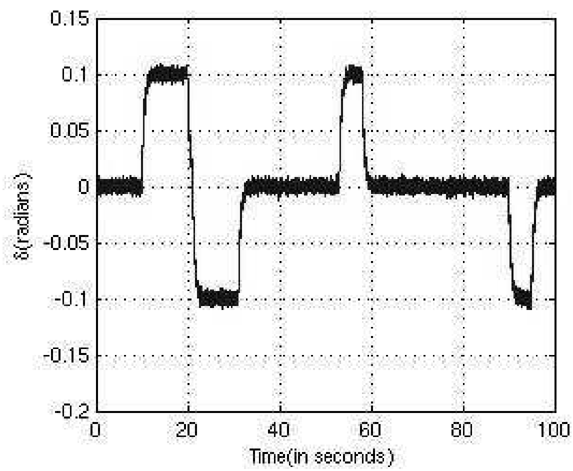

Fig. 4. Steering angle input for vehicle model

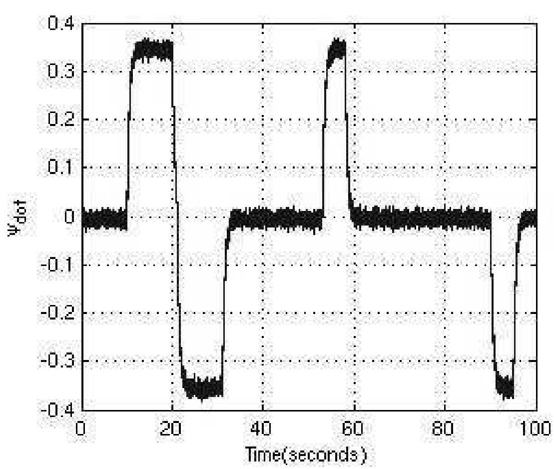

Fig. 5.Simulated yaw rate measurement

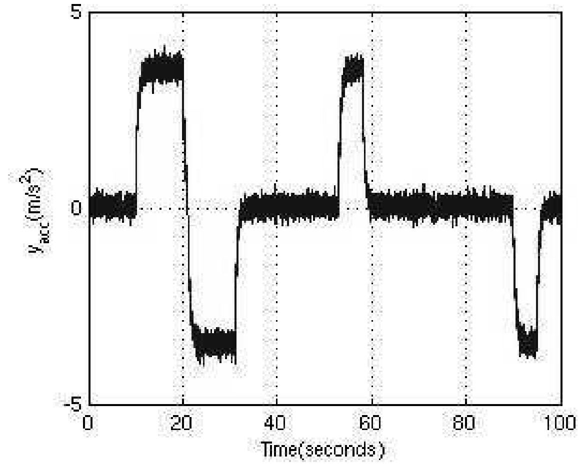

Fig. 6. Simulated lateral acceleration measurement

As opposed to using simulated measurements, correction for road bank angle would be required before utilizing experimental data. However, road bank values are expected to be available for standard vehicle test pads and facilities. Further, the effect of vehicular roll is neglected in the above model which may be acceptable for slow steering inputs. Table 3. Comparison of predicted, true parameter values compares the parameter estimates from the simulation to the true values.

TABLE 3. COMPARISON OF PREDICTED, TRUE PARAMETER VALUES

Parameter	*Estimated Value*	*True Value*
l_f *(m)*	1.16	1.1
l_r *(m)*	1.52	1.58
$\frac{C_f}{m}$ *(m/rad-s²)* → $(m/rad\text{-}s^2)$	77.19	50.86
$\frac{C_r}{m}$ $(m/rad\text{-}s^2)$	136.15	50.86
$\frac{C_f}{I_z}$ $(m^{-1}rad^{-1}s^{-2})$	43.77	27.85
$\frac{C_r}{I_z}$ $(m^{-1}rad^{-1}s^{-2})$	77.22	27.85
Accelerometer bias(m/s^2)	0.0394	0.0390
Gyro bias*(rad/s)*	0.005	0.0049

It may be observed that the biases are estimated accurately from the table above. The estimate of the yaw rate gyroscope's bias is used to accurately predict the orientation of the vehicle. Substituting back these values in equation (1),

$$(\ddot{y}+V_x\dot{\psi})=\frac{2C_f}{m}\left(\delta-(\theta_{GPS}-\psi)\right)+\frac{2C_r}{m}\left(-(\theta_{GPS}-\psi)+\frac{(l_f+l_r)\dot{\psi}}{V_x}\right)$$

Approximating $C_f = C_r$,

$$(\ddot{y}+V_x\dot{\psi}) = \frac{2C_f}{m}\left(\delta - 2\left(\theta_{GPS}-\psi\right)+\frac{\left(l_f+l_r\right)\dot{\psi}}{V_x}\right) \tag{25}$$

where ψ is the orientation of the vehicle that is given by

$$\psi = \int \dot{\psi}_{meas} - b_{gyro} t$$

$$\boxed{\Rightarrow \frac{C_f}{m} = \frac{C_r}{m} = \frac{\left(\delta - 2\left(\theta_{GPS}-\psi\right)+\frac{\left(l_f+l_r\right)\dot{\psi}}{V_x}\right)}{(\ddot{y}+V_x\dot{\psi})}} \tag{26}$$

This approximation results in more accurate estimates of the mass-normalized cornering stiffness. The new corrected estimates are shown in Table 4 which may be seen to be closer to the true values.

TABLE 4.COMPARISON OF ORIGINAL AND CORRECTED ESTIMATES

Parameter	***Estimated Value***	***Corrected Value***	***True Value***
$\frac{C_f}{m}$ *(m/rad-s^2)*	77.19	52.58	50.86
$\frac{C_r}{m}$ *(m/rad-s^2)*	136.15	52.58	50.86
$\frac{C_f}{I_z}$ *($m^{-1}rad^{-1}s^{-2}$)*	43.77	29.80	27.85
$\frac{C_r}{I_z}$ *($m^{-1}rad^{-1}s^{-2}$)*	77.22	29.80	27.85

Analysis of the simulations shows that the accuracy of the parameter estimates strongly depends on the noise in the measurements. In particular, noisy GPS velocity measurements affect the estimation accuracy of CG location ($l_{f,r}$) in sections II A,B. Further, the division of GPS velocity measurements in equation (7) results in non-gaussian noise in the global velocity angle measurement which could result in biased estimates. The faulty estimates of $l_{f,r}$ in turn affect the estimation accuracy of the tire cornering stiffnesses in section II C. Apart from the GPS noise, the noise in the yaw rate measurement also affects the estimation accuracy. This is because the yaw rate is differented to obtain the yaw acceleration in equations (21) and (25). Since differentiation amplifies the effect of noise, the estimation accuracy is affected by noisy yaw gyroscope measurements. Indeed, estimates of $l_{f,r}$ and consequently other parameters are found to be accurate in the absence of measurement noise. However, with the experimentally measured noise level, the estimates are found to be biased as shown in Table 4.

Another factor that may cause problems during experimental implementation of the above technique is the change in longitudinal velocity of the vehicle. Indeed, it is found that the estimates of $l_{f,r}$ suffer in the presence of velocity variation.

IV. CONCLUSION

An estimation scheme using total least square estimation is proposed for the estimation of a vehicle's CG location and tire cornering stiffnesses. The proposed technique solves problems related to persistency of excitation inherent in earlier methods. Kinematic models are used to augment dynamic models of the system to estimate certain unknown parameters. Further, an approximation for the moment of the inertia of the vehicle is used to combine the dynamic equations to eliminate other unknowns in the equations. Finally, decoupled measurement equations for the mass-normalized cornering stiffnesses are derived that may be directly used for estimating the stiffnesses.

The error in estimation is found to increase with measurement noise. Simulations with practical measurement noise show that the estimates are reasonably close to the true values. This method presents an improvement over existing least square estimation techniques whose estimation is not guaranteed to converge even under ideal no-noise, constant velocity conditions. However, to be practically useful, additional measurements/low noise sensors need to be utlized to predict the parameter values accurately.

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