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# Actuator Faults Diagnosis for a Class of Nonlinear Systems Applied to Electric Motors Drives

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***Abstract*— In this paper, the diagnosis of actuator faults is addressed for a class of nonlinear systems applied to electric motors drives. The residuals are synthesized using a model-based strategy by applying a differential geometry perspective. According to the dynamic structure of the systems considered in this work, it is proved that the observability codistribution built for this class of systems can be independent of the state under certain conditions, so that the resulting coordinates transformations in the state and outputs spaces are linear mappings. Lastly, it is deduced that the dynamical properties of this class of systems are preserved by different electrical machines. Simulation results are illustrated for a three-phase induction motor application, where it is shown that in this case, the necessary conditions of geometric nature, results in necessary and sufficient conditions to isolate the studied faults. Moreover, in order to detect the actuator faults, it is suggested a directional residual evaluation to properly identify the fault present in the system.**

## I. INTRODUCTION

In recent years, the fault detection and isolation (FDI) area has received a lot of attention by the scientific community, due to critical applications that require high levels of safety and reliability. So far in the literature, different FDI techniques have been proposed, which could be classified as: model-based techniques, data-driven and knowledge-based [6]. In this work, a model-based technique is used where the fundamental problem of residual generation (FPRG) [3] is studied for a class of nonlinear systems using differential geometry tools. In fact, the structure of the nonlinear systems can describe several electric motors. It is well-known that the differential geometry philosophy provides necessary conditions in order to check the capability of a system to isolate faults.

In previous works of nonlinear FDI [8], [15], [17], the studied dynamical systems have a similar structure to the one considered in this work, (see Eq. (1)). In [8], parametric faults were considered. Thus, a nonlinear fault distribution function was used in order to model the fault modes, which depend not only on the inputs and outputs of the system but also on unmeasured states. However, it is assumed that the nonlinear term depends only on the inputs and outputs of the system. A robust FDI scheme for sensor faults is suggested in [17], where model uncertainties in the state and output equations have been considered. Again, the nonlinear term depends only on the inputs and outputs of the system. Meanwhile, in [15], an active fault tolerant control for a ship propulsion system considering sensor faults has been studied, and a sufficient condition for the existence of the observer is given. Now, in this paper, the problem of actuator faults diagnosis is studied for a class of nonlinear systems, where in order to detect and isolate the fault scenarios, an observers bank is used, so that simultaneously faults can be considered. The nonlinearity in the system depends on the state but it satisfies the Lipschitz condition.

Electric Motors (EM) are cornerstone of many industrial processes. For this reason, their control and monitoring have received the attention of the scientific community throughout the years. In this paper, only Induction Motors (IM) have been addressed, but the application can be extended to DC and synchronous motors as well. Normally, IM are fed either directly from a three-phase supply, voltage-source inverters, matrix converters, multi-level structures, etc. However, these voltage supplies are sensitive to different kinds of faults [10], and several works have been developed in that direction [2],[14]. The goal of this work is to solve the fundamental problem of residual generation FPRG for a class of nonlinear systems considering actuator faults. Also, to highlight that the dynamical structure of the systems studied is preserved by several electric motors such that a post-fault operation strategy could be suggested for electric motor drives. Moreover, in these cases, it is possible to find that the necessary conditions of geometric nature, now are necessary and sufficient conditions.

The paper is organized as follows. First, in Section II, the studied class of nonlinear systems is described. The fundamental problem of residual generation for nonlinear systems is briefly recalled in Section III. In Section IV, the problem of actuator fault diagnosis is detailed. Finally, Section V presents the application for induction motor power supplies, and in Section VI the paper concludes with final remarks.

## II. MODEL DESCRIPTION

In this work, the following class of nonlinear systems is considered:

$$\Sigma_o : \begin{cases} \dot{x} &= Ax + g(x) + Bu + D\upsilon \\ y &= Cx \end{cases} \tag{1}$$

where $x \in \mathscr{X} \subset \mathbb{R}^n$ represents the state vector, $u \in \mathscr{U} \subset \mathbb{R}^m$ corresponds to the inputs vector, $\upsilon \in \mathscr{V} \subset \mathbb{R}^q$ to the disturbances vector, $y \in \mathscr{Y} \subset \mathbb{R}^p$ to the vector of measurements,

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$g(x)$ is a smooth nonlinear vector field. Subsets $\mathscr{X}, \mathscr{U}, \mathscr{V}$ and $\mathscr{Y}$ are closed and bounded sets in their respective domains. $A$, $B$, $D$ and $C$ are real matrices of appropriate dimensions. Furthermore, $B$ matrix could be expressed by using column vectors such that $B = [b_1, \ldots, b_m]$ and $b_i \in \mathbb{R}^n$ $i = 1, \ldots, m$. It is also assumed that $B$ is full column rank, i.e., redundant actuators are not considered in this study. The following assumptions are made about the mathematical model (1):

- A.1 The linear part of $\Sigma_o$ is stable, i.e. $\Re\{\lambda_i(A)\} < 0$,
- A.2 The origin is an equilibrium point of the autonomous system $\Sigma_o$, i.e. $g(0) = 0$,
- A.3 The nonlinearity $g(x)$ holds the Lipschitz condition $\forall x \in \mathscr{X}$,
- A.4 A set of parameters $\theta$ for $\Sigma_o$ are known a priori by experimental identification.

## III. FPRG for Input-Affine Nonlinear Systems

In this section, the fundamental problem of residual generation (FPRG) for nonlinear systems is revised briefly [3]. This is problem is studied assuming an input-affine representation of the system:

$$\begin{aligned} \dot{\xi} &= f(\xi) + \sum_{k=1}^{m^*} g_k(\xi) u_k + \sum_{i=1}^{s^*} l_i(\xi) f_i + \sum_{j=1}^{d^*} n_j(\xi) w_j \\ \rho &= h(\xi) \end{aligned} \quad (2)$$

where $\xi \in \mathbb{R}^{n^*}$ represents the state vector, $u_k$ the known control inputs, $k = 1, ..., m^*$, $f_i$, $i = 1, ..., s^*$ the fault modes, $w_j$, $j = 1, ..., d^*$ the disturbances and, $\rho \in \mathbb{R}^{q^*}$ the output vector. Moreover, $f, g_1, ..., g_{m^*}$, $l_1, ..., l_{s^*}$, $n_1, ..., n_{d^*}$, and $h$ are smooth vector fields. As a result, it is assumed that $\kappa = \{1, ..., s^*\}$ fault events could be present in the system. The objective is to design an observers bank that generates $\kappa$ residuals, $r_i$, such that these signals are only affected by the $ith$ failure mode $f_i$. Necessary conditions for the solvability of the FPRG for input-affine nonlinear systems ($l$NLFPRG) have been provided in [3], and they are recalled here. Let $D_\kappa$ be the distribution generated by the vector fields $l_i$, $i \neq \kappa$, and $n_j$, $j = 1, ..., d^*$, i.e.,

$$D_\kappa = span\{l_1, ..., l_{\kappa-1}, l_{\kappa+1}, ..., l_{s^*}, n_1, ... n_{d^*}\} \quad (3)$$

Following the differential geometric approach in [3], a solution to the FPRG for input-affine nonlinear systems exists only if:

$$l_\kappa \notin (\Omega_{\mathscr{O}})^\perp \quad (4)$$

where $\Omega_{\mathscr{O}}$ denotes the largest observability codistribution contained in $(D_\kappa)^\perp$, and consequently, $(\Omega_{\mathscr{O}})^\perp$ represents an unobservability distribution. This codistribution $\Omega_{\mathscr{O}}$ is computed through the following iterative algorithm:

$$\begin{aligned} Q_0 &= \Theta \cap \text{span}\{dh\} \\ Q_{i+1} &= \Theta \cap \left( \sum_{k=0}^{m^*} L_{g_k} Q_i + \text{span}\{dh\} \right). \end{aligned} \quad (5)$$

Assuming that all codistributions of this sequence are nonsingular, then there is a integer $i^\star \leq n^* - 1$, such that $Q_i = Q_i^\star$ for all $i > i^\star$ and set $\Omega_{\mathscr{O}} = Q_i^\star$, [3]. Also, the above algorithm must be initialized in $\Theta = \left(\sum_*^{D_\kappa}\right)^\perp$ such that $\Omega_{\mathscr{O}}$ is by construction an observabilty codistribution contained in $D_\kappa$, and $\sum_*^{D_\kappa}$ is an involutive conditioned invariant distribution that contains $D_\kappa$, and is computed by the following algorithm:

$$\begin{aligned} S_0 &= \bar{D}_\kappa \\ S_{i+1} &= \bar{S}_i + \sum_{k=0}^{m^*} \left[ g_k, \bar{S}_i \cap \ker\{dh\} \right]. \end{aligned} \quad (6)$$

where $\bar{S}$ denotes the involutive closured of the distribution $S$. According to [4], [7], assuming that exist an integer $i^\star$ such that $S_{i^\star+1} = \bar{S}_{i^\star}$ then it is satisfied $\sum_*^{D_\kappa} = \bar{S}_{i^\star}$.

## IV. FDI for a Class of Nonlinear Systems

In this study, only actuator faults are addressed. Consequently, $m$ faults could be acting on the system. The set of actuator faults is represented by $\mathscr{F}_a = \{f_1^a, \ldots, f_m^a\}$. It is only considered that $f_i^a \in \mathscr{L}_\infty[0, \infty)$ with $i = 1, \ldots, m$, i.e. each possible fault belongs to the set of essentially bounded signals. According to the differential geometry approach for residual generation [3], at most $m - q$ faults could be isolated simultaneously. As a result, the problem of isolation and identification of all the studied faults might have a solution if $p \geq m + q$. Otherwise, only subsets of faults could be isolated. In order to study the problem of FDI for system $\Sigma_o$ in (1) under actuator faults, it is considered that the faults have an additive structure, i.e.

$$\begin{aligned} \dot{x} &= Ax + g(x) + Bu + \textstyle\sum_{i=1}^m b_i f_i^a + D\upsilon \\ y &= Cx \end{aligned} \quad (7)$$

Now, for the purpose of detecting, as well as, isolating multiple concurrent faults $f_i^a$, it is defined

$$\begin{aligned} D_\kappa^i &= [\, b_1, ..., b_{i-1}, b_{i+1}, ..., b_m, D \,] \in \mathbb{R}^{n \times (m-1)+q} \\ \upsilon_\kappa^i &= col(f_1^a, ..., f_{i-1}^a, f_{i+1}^a, ..., f_m^a, \upsilon) \in \mathbb{R}^{(m-1)+q} \end{aligned} \quad (8)$$

and rewrite system (7) as:

$$\Sigma : \begin{cases} \dot{x} = Ax + g(x) + Bu + b_i f_i^a + D_\kappa^i \upsilon_\kappa^i \\ y = Cx \end{cases} \quad (9)$$

Throughout the paper, due to space limitation, proofs are not given in this version but are available from the authors.

*Proposition 1:* The observability codistribution built for (9) is independent from the state, if the following condition is satisfied:

$$\text{span}\{D_\kappa^i\} \cap \ker\{C\} = \{\emptyset\} \quad (10)$$

A relaxed condition for (10) can be expressed by:

$$A(S_0 \cap \ker\{C\}) \subseteq S_0 \quad (11)$$

$$\frac{\partial g(x)}{\partial x}(S_0 \cap \ker\{C\}) \subseteq S_0 \quad (12)$$

where $S_0 = \text{span}\{D_\kappa^i\}$. If condition (10) or (11)-(12) hold then $\Sigma_*^{D_\kappa^i}$ is given by $\Sigma_*^{D_\kappa^i} = \text{span}\{D_\kappa^i\}$.

■

From the results in [3], assume that $\Omega$ is an observability codistribution (independent of state) then there exist linear transformation matrices $\Phi$ and $\Psi$ in the state and output spaces such that

$$\tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \Psi y, \quad z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \Phi x. \tag{13}$$

In the new coordinates defined by (13), system (9) is then described by equations of the form:

$$\begin{aligned} \dot{z}_1 &= A_{11}z_1 + A_{12}z_2 + g_1(z_1,z_2) + B_1 u + b_i^1 f_i^a \\ \dot{z}_2 &= A_{21}z_1 + A_{22}z_2 + A_{23}z_3 + g_2(z_1,z_2,z_3) \\ &\quad + B_2 u + b_i^2 f_i^a + D_{\kappa 2}^i \upsilon_\kappa^i \\ \dot{z}_3 &= A_{31}z_1 + A_{32}z_2 + A_{33}z_3 + g_3(z_1,z_2,z_3) \\ &\quad + B_3 u + b_i^3 f_i^a + D_{\kappa 3}^i \upsilon_\kappa^i \\ \tilde{y}_1 &= C_1 z_1 \\ \tilde{y}_2 &= z_2 \end{aligned} \tag{14}$$

It is important to point out that since the nonlinear term $g(x)$ in (9) is Lipschitz, then after the linear coordinates transformation $z = \Phi x$, this property is preserved by the nonlinear terms $g_1(\cdot), g_2(\cdot)$ and $g_3(\cdot)$ in (14). Now, consider the $z_1$-*subsystem* of (14), where $z_2 = \tilde{y}_2$ and this term can be viewed as an independent input, namely the resulting subsystem:

$$\begin{aligned} \dot{z}_1 &= A_{11}z_1 + A_{12}\tilde{y}_2 + g_1(z_1,\tilde{y}_2) + B_1 u + b_i^1 f_i^a \\ \tilde{y}_1 &= C_1 z_1 \end{aligned} \tag{15}$$

where it has been shown in [3], that this subsystem is *locally weakly observable* [5], also that this condition is not sufficient for the existence of an asymptotic observer of $z_1$. Nevertheless under some additional assumptions an asymptotic observer can be designed, [1], [9], [13], [16]. One important property is that if conditions in Proposition 1 hold, the dynamical structure of the decoupled subsystems in (14) is preserved with respect to (9).

*Proposition 2:* The dynamical system shown in (17) is an asymptotically stable observer of the nonlinear subsystem (15) in the absence of the input $f_i^a$, which can be used as a residual generator to diagnose the fault $f_i^a$, if the two following assumptions are hold:

A3.1 The pair $(A_{11}, C_1)$ is observable (at least detectable), which implies that it is possible to find a matrix $\Lambda \in \mathbb{R}^{n_1 \times (p-n_2)}$ such that $\Re\{\lambda_i(A_{11} - \Lambda C_1)\} < 0$.

A3.2 The nonlinearity holds the Lipschitz condition

$$\|g_1(z_1,\tilde{y}_2) - g_1(\hat{z}_1,\tilde{y}_2)\| \le \gamma \|z_1 - \hat{z}_1\| \tag{16}$$

with $\gamma \le \frac{\lambda_{min}(Q)}{2\|P\|}$ where $P, Q > 0$ such that, $(A_{11} - \Lambda C_1)^T P + P(A_{11} - \Lambda C_1) = -Q$.

Then the proposed observer has the following structure:

$$\begin{aligned} \dot{\hat{z}}_1 &= A_{11}\hat{z}_1 + A_{12}\tilde{y}_2 + g_1(\hat{z}_1,\tilde{y}_2) + B_1 u + \Lambda(\tilde{y}_1 - \hat{\tilde{y}}_1) \\ \hat{\tilde{y}}_1 &= C_1 \hat{z}_1 \end{aligned} \tag{17}$$

■

## V. APPLICATION TO ELECTRIC MOTOR DRIVES

Nowadays, Induction Motors (IM) are the workhorse of the power industry [12]. These motors are fed by balanced three-phase sources. However, if a fault is present in the power source, the IM will exhibit severe torque and velocity fluctuations. Thus, an actuator fault diagnosis on this machine would be useful in order to avoid money losses or even accidents. Furthermore, according to the identified faults, post-fault operation modes can be suggested to reduce the performance degradation.

### A. Stator Reference Frame Model

In this section, the dynamic modeling of IM is briefly detailed. This is derived by using a two-phase motor representation (direct and quadrature axes). Different reference frames can be used for the IM model. In this work, a stator reference frame model is considered in order to detect the actuator faults. Hence, the resulting model is showed in (18) [12], where $p$ denotes the differential operator, the subscripts $s$ and $r$ stand for stator and rotor; $(d,q)$ represents the components of a vector with respect to a stator reference frame; $L_s$, $L_r$ are stator and rotor self-inductances, $L_m$ is the mutual inductance, and $R_s$, $R_r$ are the stator and rotor equivalent resistances.

Only under balanced conditions, there are four system equations, as given in (18). Under unbalanced conditions, two more system equations, one for the stator zero-sequence voltage and the other for the rotor zero-sequence voltage are required. They are given by [11]

$$v_{os} = (R_s + L_{ls}p)i_{os}, \quad \& \quad v_{or} = (R_r + L_{lr}p)i_{or} \tag{19}$$

where in the variables the subscript $o$ denotes the zero-sequence component and $L_{ls} = L_s - L_m$, $L_{lr} = L_r - L_m$ are the leakage inductances. The electric torque and rotor speed are related by

$$T_e = J\frac{d\omega_m}{dt} + T_L + f\omega \tag{20}$$

where $J$ is the moment of inertia, $f$ is the mechanical friction, $n_p$ is the pair of poles and $T_L$ is the load torque.

Now, let $a = R_r/L_r$, $b = L_m/\sigma L_s L_r$, $c = (L_m^2 R_r/\sigma L_s L_r^2) + (R_s/\sigma L_s)$, $m = 3n_p L_m/2JL_r$, $d = 1/\sigma L_s$, $k = 1/J$, $e = 1/L_{ls}$, $\sigma = 1 - (L_m^2/L_s L_r)$, be a reparametrization of the induction motor model, where $a$, $b$, $c$, $m$, $d$, $k$, $e$, are known parameters, and take $(u_d, u_q, u_o)^T$ as the control vector in the system. Then, the resulting system showed in (21) is the mathematical model of the induction motor in the stator reference frame:

$$\begin{aligned} \dot{x}_1 &= -cx_1 + abx_4 + n_p b x_5 x_6 + du_d \\ \dot{x}_2 &= -cx_2 + abx_5 - n_p b x_4 x_6 + du_q \\ \dot{x}_3 &= -eR_s x_3 + eu_o \\ \dot{x}_4 &= -ax_4 + aL_m x_1 - n_p x_5 x_6 \\ \dot{x}_5 &= -ax_5 + aL_m x_2 + n_p x_4 x_6 \\ \dot{x}_6 &= -kfx_6 + m(x_2 x_4 - x_1 x_5) - kT_L \end{aligned} \tag{21}$$

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} (R_s+L_sp) & 0 & L_mp & 0 \\ 0 & (R_s+L_sp) & 0 & L_mp \\ L_mp & -n_p\omega_mL_m & (R_r+L_rp) & -n_p\omega_mL_r \\ n_p\omega_mL_m & L_mp & n_p\omega_mL_r & (R_r+L_rp) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \quad (18)$$

with the state vector given by $x = (x_1, x_2, x_3, x_4, x_5, x_6)^T = (i_{ds}, i_{qs}, i_{os}, \lambda_{dr}, \lambda_{qr}, \omega_m)^T$. In the representation (21), the $o$-component of the rotor flux is omitted since the rotor equivalent circuits are short circuited. As a result, this dynamic equation is independent of the rest of the variables. Moreover, to have a $o$-component in the stator current, the neutral point in the stator windings must be grounded. On the other hand, the available and commons measurements in a real scenario are given by stator currents $i_{dqos}$ and the mechanical velocity $\omega_m$. Thus, system (21) can be rewritten in compact form as:

$$\begin{aligned} \dot{x} &= Ax + g(x) + Bu + DT_L \\ y &= Cx \end{aligned} \quad (22)$$

where,

$$A = \begin{bmatrix} -c & 0 & 0 & ab & 0 & 0 \\ 0 & -c & 0 & 0 & ab & 0 \\ 0 & 0 & -eR_s & 0 & 0 & 0 \\ aL_m & 0 & 0 & -a & 0 & 0 \\ 0 & aL_m & 0 & 0 & -a & 0 \\ 0 & 0 & 0 & 0 & 0 & -kf \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k \end{bmatrix}$$

$$g(x) = \begin{bmatrix} n_pbx_5x_6 \\ -n_pbx_4x_6 \\ 0 \\ -n_px_5x_6 \\ n_px_4x_6 \\ m(x_2x_4 - x_1x_5) \end{bmatrix}, \quad B = \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

### *B. Actuator Fault Scenarios for Induction Motor*

Three different faults are considered as generic faults ($f_a^1 \to d$-axis, $f_a^2 \to q$-axis, $f_a^3 \to o$-axis) with an arbitrary time profile, and they are studied here with the geometric FDI analysis. Note that in (22), there are 4 measurements, 1 unknown disturbance and 3 fault modes. In order to check the possibility of isolating the three faults from each other, three cases shown in Table I must be studied. Next, the geometric FDI analysis is used, and it is obtained that $(f_a^1, f_a^2, f_a^3)$ can be considered as a strongly identifiable family, because they fulfilled the conditions given in [3] and recalled here in Sec. III.

*1) $FPRG_1$ Solution:* According to Sec. III, the sufficient condition shown in (10) is satisfied to $FPRG_1 - FPRG_3$. By brevity, it only is analyzed $FPRG_1$. Thus, the span $\{D_\kappa^1\}$ and $\ker\{C\}$ are given by:

TABLE I

STUDIED SETS OF ACTUATOR FAULTS

| FPRG | Fault | Disturbances |
|---|---|---|
| $FPRG_1$ | $f_a^1$ | $f_a^2$, $f_a^3$, $T_L$ |
| $FPRG_2$ | $f_a^2$ | $f_a^1$, $f_a^3$, $T_L$ |
| $FPRG_3$ | $f_a^3$ | $f_a^1$, $f_a^2$, $T_L$ |

$$\text{span}\{D_\kappa^1\} = \text{span}\left\{\begin{bmatrix} 0 & 0 & 0 \\ d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k \end{bmatrix}\right\}, \ \ker\{C\} = \text{span}\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\right\}$$

such that, condition (10) is fulfilled, which implies a linear coordinates transformation in the state and output spaces. After this, the following subsystem is obtained and expressed in original coordinates:

$$\begin{aligned} \dot{x}_1 &= -cx_1 + abx_4 + n_pbx_5y_4 + du_d + df_a^1 \\ \dot{x}_4 &= -ax_4 + aL_mx_1 - n_px_5y_4 \\ \dot{x}_5 &= -ax_5 + aL_my_2 + n_px_4y_4 \\ y_1 &= x_1 \end{aligned} \quad (23)$$

which is only sensible to fault $f_a^1$. Thus, by constructing an observer for this subsystem a residual generator is achieved. The dynamical structure of the observer for this subsystem is given by

$$\begin{aligned} \dot{\hat{x}}_1 &= -c\hat{x}_1 + ab\hat{x}_4 + n_pb\hat{x}_5y_4 + du_d + k_{11}(y_1 - \hat{y}_1) \\ \dot{\hat{x}}_4 &= -a\hat{x}_4 + aL_m\hat{x}_1 - n_p\hat{x}_5y_4 + k_{12}(y_1 - \hat{y}_1) \\ \dot{\hat{x}}_5 &= -a\hat{x}_5 + aL_my_2 + n_p\hat{x}_4y_4 + k_{13}(y_1 - \hat{y}_1) \\ \hat{y}_1 &= \hat{x}_1 \end{aligned} \quad (24)$$

and the residual signal is given by

$$r_d(t) = y_1 - \hat{y}_1 \quad (25)$$

*2) $FPRG_2$ Solution:* The resulting observer for this case is

$$\begin{aligned} \dot{\hat{x}}_2 &= -c\hat{x}_2 + ab\hat{x}_5 - n_pb\hat{x}_4y_4 + du_q + k_{21}(y_2 - \hat{y}_2) \\ \dot{\hat{x}}_4 &= -a\hat{x}_4 + aL_my_1 - n_p\hat{x}_5y_4 + k_{22}(y_2 - \hat{y}_2) \\ \dot{\hat{x}}_5 &= -a\hat{x}_5 + aL_m\hat{x}_2 + n_p\hat{x}_4y_4 + k_{23}(y_2 - \hat{y}_2) \\ \hat{y}_2 &= \hat{x}_2 \end{aligned} \quad (26)$$

and the residual signal

$$r_q(t) = y_2 - \hat{y}_2 \quad (27)$$

*3) $FPRG_3$ Solution:* Finally, the observer presents for this case a simple form:

$$\begin{aligned} \dot{\hat{x}}_3 &= -eR_s\hat{x}_3 + eu_o + k_{31}(y_3 - \hat{y}_3) \\ \hat{y}_3 &= \hat{x}_3 \end{aligned} \quad (28)$$

Once more, the residual signal is constructed by

$$r_o(t) = y_3 - \hat{y}_3 \quad (29)$$

*Remark 1:* All the previous residuals (25), (27), and (29) will give an indication of a fault present in the system related to each axis in the $dqo$-frame. However, these residuals cannot identify or quantify explicitly the fault present in the system.

### C. Residual Evaluation

In this section, the evaluation of the $dqo$-residual is made. This step is necessary because the real actuator faults appear in the $abc$-frame. For this reason, the relationship between the $dqo$ and $abc$-frames is used [12]:

$$x_{dqo} = [T_{abc}]x_{abc} \quad (30)$$

where $x_{dqo} = [x_d, x_q, x_o]^T$, $x_{abc} = [x_a, x_b, x_c]^T$ and

$$[T_{abc}] = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Let $e_a = (1,0,0)^T$, $e_b = (0,1,0)^T$ y $e_c = (0,0,1)^T$, and $v_a = \frac{[T_{abc}]e_a}{\|[T_{abc}]e_a\|}$, $v_b = \frac{[T_{abc}]e_b}{\|[T_{abc}]e_b\|}$ and $v_c = \frac{[T_{abc}]e_c}{\|[T_{abc}]e_c\|}$. Then, unitary vectors $(v_a, v_b, v_c)$ are related to the directions of the $abc$-actuators faults in the $dqo$-frame. Consequently, it is suggested to evaluate the contribution of the residual in those directions taking the vector inner-product:

$$\begin{aligned} r_a &= |\langle v_a, r_{dqo}\rangle| \\ r_b &= |\langle v_b, r_{dqo}\rangle| \\ r_c &= |\langle v_c, r_{dqo}\rangle| \end{aligned} \quad (31)$$

where $r_{dqo} = (r_d, r_q, r_o)^T$ denotes the residual vector formed by (25), (27) and (29). Thus, the residuals $(r_a, r_b, r_c)$ quantify the fault magnitudes and directionality with respect to $a, b, c$ axes.

### D. Simulation Results

In order to validate the ideas presented in this work, a simulation evaluation is carried out using MATLAB/SIMULINK©. In the simulation, two faults scenarios in the three-phase motor voltage were considered:

- **Case A** there is a complete actuator outage in phase-$a$ of the motor at $t = 3.75\,s$.
- **Case B** a soft-failure occurs at $t = 3.75\,s$, where the actuator voltage of phase-$b$ and $c$ is reduced 50 % from its nominal value.

In both cases, it is considered an initial load torque $T_L = 1\,N\cdot m$, and at $t = 3.35\,s$ there is step change to $T_L = 4.5\,N\cdot m$. The induction motor parameters are $R_s = 7.73\,\Omega$, $L_s = 0.113\,H$, $R_r = 2.79\,\Omega$, $L_r = 0.112\,H$, $L_m = 0.1\,H$, $J = 0.015\,kg - m^2$ and $f = 0.002\,N.m/(rad/sec)$. The nominal line voltage of the motor is $220\,Vrms$. The observers gains in (24), (26) and (28) were calculated using the resulting $(A, C)$ matrices and the command $lqr$ in MATLAB. In fact, the conditions of Proposition 3 were fulfilled in the three cases (24), (26) and (28). The results for Case A are illustrated in Figs. 1 and 2.

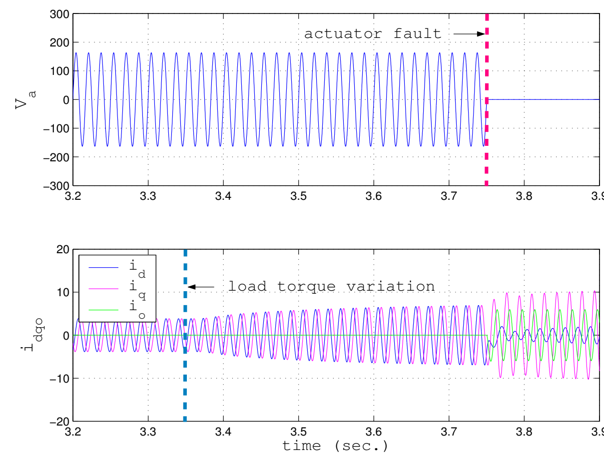


Fig. 1. Simulation Results Case A: (TOP) Phase-$a$ Voltage, (BOTTOM) Line currents in $dqo$-frame.

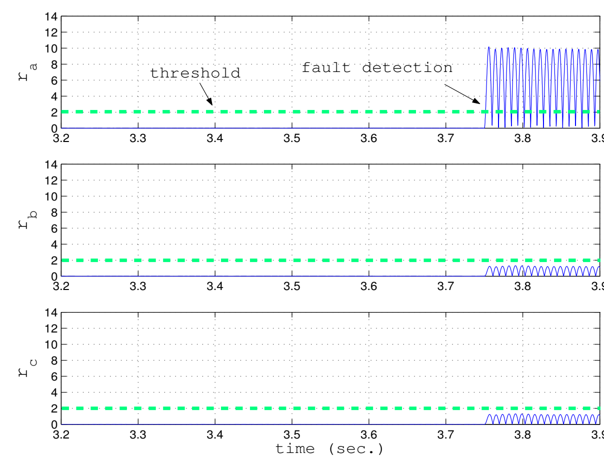


Fig. 2. Simulation Results Case A: Residuals (TOP) Phase-$a$, (MIDDLE) Phase-$b$, and (BOTTOM) Phase-$c$.

First, it is observed, after the load torque step at $t = 3.35\,s$, an increment in the line currents. However, the residuals were not affected by this perturbation change. Next, after the actuator outage in phase-$a$, residual $r_a$ clearly indicates the presence of this fault. A threshold of 2 was set for the residual evaluation, in order to avoid false alarms. Now, Figs. 3 and 4 present the simulation plots for Case B. In this last scenario, concurrent faults are affecting the system, but the residuals are clearly isolating the incident faults, and are insensitive to the perturbation change at $t = 3.35\,s$. As a result, the model-based FDI strategy can successfully detect and isolate the actuator faults in the induction motor.

*Remark 2:* Note that from the results in Figs. 2 and 4, the residuals are not exactly zero after the fault, although

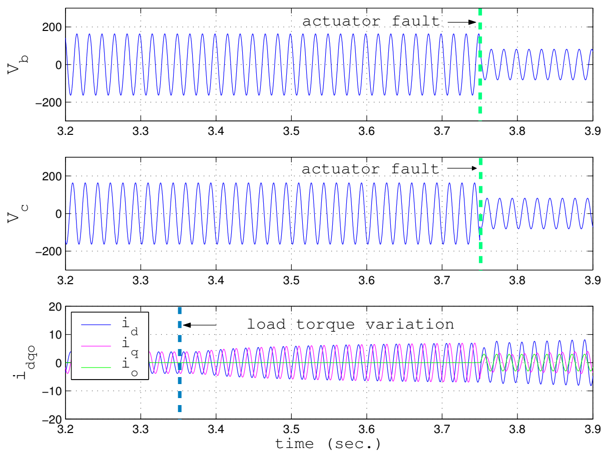


Fig. 3. Simulation Results Case B: (TOP) Phase-$b$ voltage, (MIDDLE) Phase-$c$ voltage, and (BOTTOM) Line currents in $dqo$-frame.

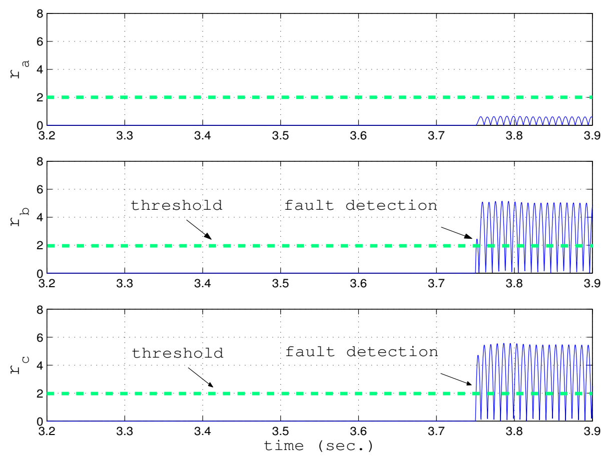


Fig. 4. Simulation Results Case B: Residuals (TOP) phase-$a$, (MIDDLE) phase-$b$, and (BOTTOM) phase-$c$.

the faulty axis is not aligned with that residual. First, the original faults are in a $abc$-frame, therefore the effect of a fault is induced in the three $dqo$-axes. Finally, the directional evaluation cannot cancel this effect since the faults are not quantified by the residuals. Nevertheless, the residual clearly show a substantial increase in the axis effected by the fault.

## VI. CONCLUSIONS AND FINAL REMARKS

In this paper, the FDI-geometric analysis was applied for a class of nonlinear systems. Necessary conditions were provided to be able isolate faults from perturbations for the studied class. In fact, the class of nonlinear systems can be directly applied to represents most electrical motors. As an application, the actuator fault diagnosis problem for the induction motor was analyzed in this work. Thus, departing from the mathematical model of the motor in the stator reference frame ($dqo$), it was concluded that the necessary conditions to construct dedicated observers to isolate all the actuator faults were provided. Finally, in order to isolate the faults in the original $abc$-frame, a directional residual evaluation was suggested. Simulation results were obtained in order to validate the proposed ideas. As a future work, it is intended to test this FDI strategy experimentally, and to extend the application to other electric motors.

## VII. ACKNOWLEDGMENTS

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