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A Fundamental Conflict between Performance and Passivity in Haptic Rendering

Paul G. Griffiths and R. Brent Gillespie

Abstract— **This paper identifies a fundamental limitation on haptic rendering performance induced by passivity requirements. This limitation arises when the target closed-loop response requires feedback compensation of hardware dynamics. The intrinsic human-in-the-loop nature of haptic rendering further requires passivity of the closed-loop response. The conflict between performance and passivity that we discuss is a consequence of feedback bandwidth constraints, not sampling or quantization. Key to our analysis is an interpretation of a Bode gain-phase integral relationship that relates magnitude at low frequencies to phase at high frequencies.**

I. Introduction

FEEDBACK control in haptic rendering shapes the dynamic response of a motorized user interface, called a haptic device, to match a target dynamic response. Depending on the intrinsic dynamics of the haptic device and the desired dynamic response, compensation for the haptic device dynamics may be required for the actual closed-loop response to closely match the desired dynamic response. In general, the mechanical properties of the device such as damping and inertia are minimized during hardware design to avert the need for compensation in the feedback design. However, minimizing these hardware dynamics is in conflict with other design considerations such as structural strength or component cost. Feedback control offers the ability to compensate for hardware dynamics; however the designer must be simultaneously aware of inherent costs and limitations.

The human-in-the-loop aspect of haptic rendering raises special stability concerns. Typical frequency-domain techniques for assessing stability such as phase margin and gain margin are not appropriate for haptic rendering since the mechanical interaction with the human operator introduces significant nonlinear, time-varying dynamics. This particular stability problem is termed *coupled stability* [1], and a common solution is to design the controller such that the closed-loop response presented to the user through the haptic device has a passive dynamic response. Coupled stability is assured if the human operator behaves passively. Given that passivity is a requirement for stability, it is important to determine the class of dynamic responses that can be rendered by the haptic device while maintaining the passivity condition for coupled stability.

Previous analysis of haptic rendering identifies limitations on the feedback design due to sampling effects and quantization error. When rendering a *virtual wall*, passivity imposes limitations on the gains of the controller parameters [2]–[7]. Extensions to the work on the virtual wall problem predict limits for more general nonlinear virtual environments [8], [9]. Prior literature on haptic rendering has not analyzed the challenges of accurately rendering passive linear time-invariant dynamics without regard to sampling or quantization issues. In this respect, our analysis is similar to analysis of limitations in end-point impedance control for robots [1]. However, unlike typical impedance control which uses force sensing, we consider haptic rendering using position sensing as the feedback signal.

In this paper we show that certain linear time-invariant passive dynamic systems cannot be approximated passively over a given finite bandwidth. This fundamental conflict between performance and passivity holds for all linear time-invariant controllers. In contrast to previous work, the limitation we reveal is not mitigated by faster sampling or finer sensor quantization. A necessary condition is determined for the existence of a feasible feedback design, and interpretation of this condition reveals certain compensation of hardware dynamics cannot be achieved while maintaining a passive closed-loop response.

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P. G. Griffiths and R. B. Gillespie are with the Department of Mechanical Engineering at the University of Michigan, Ann Arbor, MI 48109-2125 USA (e-mail: paulgrif@umich.edu; brentg@umich.ed).

II. Background

A. Haptic rendering using position feedback

In a standard configuration of haptic rendering, a human operator grasps and applies forces to a motorized, computer-controlled manipulator. Figure 1 depicts a direct-drive, single-axis, impedance-type haptic device. We assume that the user's torque f applied to the handwheel and the motor torque u affect the handwheel position y through the same dynamics. In general, the haptic device may be linear or rotary; however, without loss of generality, we refer to f and u as forces rather than torques.

We model the dynamics of the haptic device and the controller as transfer functions of the Laplace variable s. Let P denote the haptic device dynamics such that

$$y = P(f+u), \tag{1}$$

and let C describe the controller such that

$$u = -Cy. \tag{2}$$

The primary goal of the controller is to shape the dynamic behavior of the haptic device from f and y to match desired dynamics, termed the *virtual environment*. We denote linear time-invariant virtual environments by the transfer function

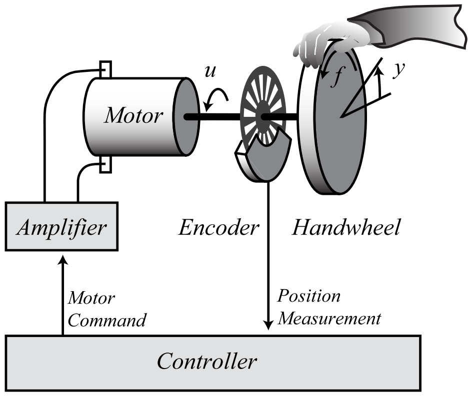

Fig. 1: Schematic of a rotary haptic device and controller.

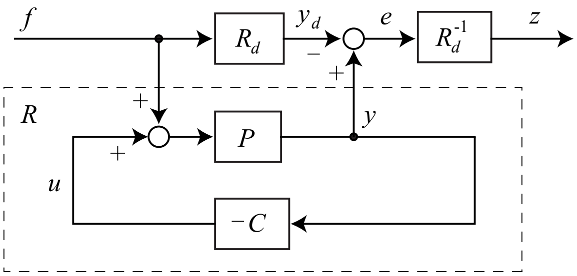

Fig. 2: General control configuration for haptic rendering with position feedback. The closed-loop response from f to z is given by distortion Θ.

R_d. We denote the actual closed-loop dynamics between f and y rendered to the user by R. These dynamics are defined by the feedback interconnection of P with C:

$$R \triangleq \frac{P}{1+PC}. \tag{3}$$

As in [10], we measure the accuracy of haptic rendering by *distortion*, defined as the relative error between the actual and desired closed-loop dynamics:

$$\Theta \triangleq \frac{R-R_d}{R_d}. \tag{4}$$

The dynamic response presented to the user closely matches the desired response if Θ is small along the entire $j\omega$-axis. Due to bandwidth limitations imposed by the hardware, the loop-gain PC must roll-off at high frequencies. It follows from (3) that the rendered dynamics R approach the open-loop device dynamics P at high frequencies. As a practical performance specification, let

$$|\Theta(j\omega)| \leq M_\Theta, \quad \text{for} \quad 0 \leq \omega \leq \omega_c. \tag{5}$$

The block diagram in Figure 2 depicts the model-matching feedback design problem in the general control configuration. The signal y_d is the desired position of the haptic device given by $y_d \triangleq R_d f$, and the error between the actual and desired position is given by $e \triangleq y - y_d$. The performance variable z is given by the normalized error signal $z \triangleq R_d^{-1} e$ such that distortion Θ describes the closed-loop response from f to z.

In contrast to the block diagram shown in Fig. 2, it is common in the literature on haptic rendering to use the virtual environment as the controller [3], [4], [6]–[8], or to partition the controller into a virtual coupler and virtual environment [9], [11]. In the former case, the virtual environment must be expressed as a dynamic response from position to force rather than from force to position as we have defined it here, and no separate compensation for the hardware dynamics is included. When the virtual environment represents the dynamics of some physical system, it is preferable to express the virtual environment in forward dynamics from force to motion. By partitioning the controller into a virtual coupler and virtual environment, the virtual environment can then be expressed in forward dynamics. The virtual coupler can also provide compensation for hardware dynamics as described in [10]. For our present discussion we do not partition the controller; however this does not imply that the controller is synonymous with the virtual environment. We assume that the controller accounts for the virtual environment and any compensation of hardware dynamics.

Let us introduce one additional closed-loop transfer function that plays a key role in our analysis. The Bode sensitivity function for the feedback loop between P and C is given by

$$S \triangleq \frac{1}{1+PC}. \tag{6}$$

The value of the Bode sensitivity function along the $j\omega$-axis describes several important feedback properties including stability robustness, attenuation of exogenous inputs entering the plant input, and the sensitivity of the four closed-loop transfer functions associated with the feedback interconnection of P and C to variations in the hardware dynamics [12]. There are two facts, however, about the Bode sensitivity function which are relevant to our present discussion. First, comparing (6) and (3), we note that S and the rendered virtual environment R are related algebraically by

$$R = PS. \tag{7}$$

Through this relationship we can translate specifications on the rendered virtual environment into specifications on the Bode sensitivity function. The second fact is that, for practical systems, the loop-gain PC will be strictly proper. It follows that

$$\lim_{s\to\infty} S(j\omega) = 1. \tag{8}$$

B. Passivity for coupled stability

While the controller C should be designed to stabilize the haptic device P, stability of this feedback interaction is not sufficient to guarantee well-behaved interaction between the human operator and the controlled device. When in physical contact with the device, the human operator forms an additional feedback path between the haptic device position y and the force f. The dynamics in this path are variable and depend on many factors such as grasp, posture, muscle co-contraction, and volitional responses. As a result, the coupled user and powered haptic device may give rise to undesired oscillations.

A practical method to assure *coupled stability* of the human operator and haptic interface system is to design the dynamic response of the haptic interface system to be passive [1]. A necessary and sufficient condition for a linear time-invariant transfer function between a pair of power variables—such force and velocity—to be passive is that its poles lie in the closed left-half plane and its Nyquist plot lies in the closed-right half plane [13]. Thus, for a stable feedback design, the user is presented with a passive dynamic response if

$$\mathrm{Real}[j\omega R(j\omega)] \geq 0 \quad \forall \omega. \tag{9}$$

In other words, the Nyquist plot of sR must lie entirely in the right-half plane, or equivalently, the positive-ω locus of R must remain below the real-axis.

C. Bode gain-phase relationship

Analyticity of transfer functions imposes relationships between the magnitude and phase that may conflict with feedback design goals. The Bode gain-phase integral predicts that the phase of a proper, stable, minimum phase transfer function along the $j\omega$-axis is completely determined by the magnitude of the transfer function along the $j\omega$-axis [14], [15]. We use a related integral expression to relate performance and passivity requirements in haptic rendering. We state the result in terms of S; however we note that the result is not particular to the Bode sensitivity function.

Let p_i for $i = 1,\ldots,n_p$ be the poles of S and z_i for $i = 1,\ldots,n_z$ be the zeros of S such that

$$S(s) = \frac{k\prod_{i=1}^{n_z}(s-z_i)}{\prod_{i=1}^{n_p}(s-p_i)}. \tag{10}$$

We assume that S is normalized such that k is positive. Let $\arg(s)$ assume values from $-\pi$ to π and define

$$\arg S(s) = \sum_{i=1}^{n_z}\arg(s-z_i) - \sum_{i=1}^{n_p}\arg(s-p_i). \tag{11}$$

Then $\log S(s)$ is given by $\log|S(s)| + j\arg S(s)$.

Bode gain-phase relationships follow from Cauchy's integral theorem applied to contour integrals that enclose the right-half plane. We consider the contour integral of $\log S/\sqrt{1+s^2/\omega_0^2}$. Assume that all p_i's lie in the open left-half plane and that all z_i's lie in the closed left-half plane. Then the integrand $\log S/\sqrt{1+s^2/\omega_0^2}$ is analytic on and inside a contour defined by a large semicircle around the right-half plane and the $j\omega$-axis. Small semi-circle indentations at +/-$j\omega_0$ and at any zeros of S on the imaginary axis are added to avoid singularities. We take $\sqrt{1-\omega^2/\omega_0^2}$ to be positive for $-\omega_0 < \omega < \omega_0$, positive imaginary for $\omega > \omega_0$, and negative imaginary for $\omega < \omega_0$. By the hypotheses on P and C, it follows that $S(j\omega) \to 1$ as $\omega \to \infty$. Then S satisfies (cf. [14, Eqn. 13-36])

$$\int_0^{\omega_0} \frac{\log|S(j\omega)|}{\sqrt{1-\omega^2/\omega_0^2}}d\omega = -\int_{\omega_0}^{\infty} \frac{\arg S(j\omega)d\omega}{\sqrt{\omega^2/\omega_0^2-1}}. \tag{12}$$

This integral equality implies that amplification of $S(j\omega)$ below ω_0 must be balanced with negative phase area above ω_0. If we change the variable of integration to $v \triangleq \log\omega/\omega_0$, the weighting factors $1/\sqrt{1-\omega^2/\omega_0^2}$ and $1/\sqrt{\omega^2/\omega_0^2-1}$ become $e^v/\sqrt{1-e^{2v}}$ and $e^v/\sqrt{e^{2v}-1}$. Inspection of these weighting factors (shown in Fig. 3) reveals that the left-hand side of (12) is strongly influenced by $\log|S(j\omega)|$ within a decade below ω_0. The contribution of the phase of $S(j\omega)$ to the right-hand side of (12) is more evenly distributed with bias towards values near ω_0.

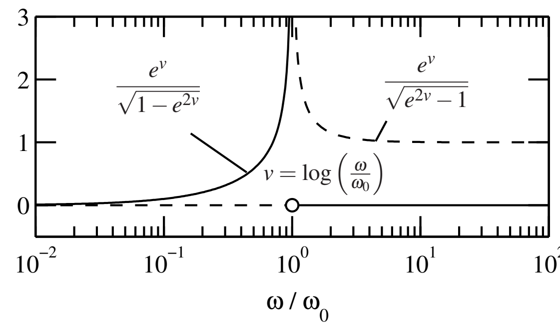

Fig. 3: Weighting factors from (12) shown on a logarithmic frequency scale.

III. A LIMITATION ON PARTIAL CANCELLATION OF DEVICE DYNAMICS

The performance specification (5) and passivity requirement (9) generate constraints on the magnitude and phase of closed-loop transfer functions Θ and R. The Bode gain-phase relationship (12) provides a means to relate magnitude and phase requirements if they are expressed in terms of a single closed-loop transfer function. To do this, we express performance and passivity requirements in terms of the Bode sensitivity function S. From (7), the magnitude and phase of S are given by

$$|S| = |R|/|P| \tag{13}$$

$$\arg S = \arg R - \arg P. \tag{14}$$

From (4), distortion may be written as

$$R = (1+\Theta)R_d. \tag{15}$$

It follows from (13) and (15) that

$$\log|S| = \log\left|\frac{R_d}{P}\right| + \log|1+\Theta|. \tag{16}$$

Assume that the performance specification (5) is satisfied with $M_\Theta < 1$. Then for $0 \leq \omega \leq \omega_c$

$$\log|S(j\omega)| \geq \log\left|\frac{R_d(j\omega)}{P(j\omega)}\right| + \log|1-M_\Theta|. \tag{17}$$

This inequality provides a lower bound on the magnitude of $S(j\omega)$ that depends only on the hardware P, the desired rendered dynamics R_d and the performance specification (5). Notably, the lower bound on the magnitude of $S(j\omega)$ does not depend on the feedback design.

The constraint on the phase of $R(j\omega)$ given by (9) can be expressed as a constraint on the phase of $S(j\omega)$ using (14). For $0 \leq \omega \leq \omega_0$ we require that

$$-\pi - \arg P(j\omega) \leq \arg S(j\omega) \leq -\arg P(j\omega). \qquad (18)$$

For our present discussion, we focus on the implications of the lower bound in (18) when the haptic device dynamics are dominated by inertial effects at high frequencies. For such devices, the phase of $P(j\omega)$ approaches $-\pi$ at high frequencies. It follows that any phase lag in $S(j\omega)$ must be small at high frequencies so as to not violate the lower bound in (18).

An inherent conflict emerges between performance and passivity when (17) and (18) are related through the Bode gain-phase relationship (12).

Proposition 1: Assume that P has relative degree of two and has no open right-half plane poles. Given $0 \leq M_\Theta < 1$, a necessary condition for the existence of a proper, stabilizing controller C that meets the performance specification (5) and passivity requirement (9) is

$$\int_0^{\omega_0} \frac{\log\left|\frac{R_d(j\omega)}{P(j\omega)}\right| + \log|1-M_\Theta|}{\sqrt{1-\omega^2/\omega_0^2}} d\omega \leq \int_{\omega_0}^{\infty} \frac{\pi + \arg P(j\omega)}{\sqrt{\omega^2/\omega_0^2 - 1}} d\omega. \qquad (19)$$

for all $0 < \omega_0 \leq \omega_c$.

Proof: Suppose that there exists a proper stabilizing controller C that achieves the performance specification and satisfies the passivity requirement on sR. Since open right-half plane poles of C are open right-half plane zeros of R (recalling that $R = P/(1+PC)$, it follows that C has no open right-half plane poles. Together with the hypotheses on P, it follows that S is stable and has no open right-half plane zeros. Then S satisfies the Bode gain-phase relationship (12).

For $0 \leq \omega \leq \omega_c$, the left-hand side of (12) is lower bounded by (17) which captures the performance requirement. We use the passivity condition (18) to upper bound $-\arg S(j\omega)$ by $\pi + \arg P(j\omega)$ for all positive frequencies. Thus the resulting inequality (19) holds for any controller that satisfies the hypotheses. ■

We can demonstrate the existence of passive virtual environment dynamics R_d that violate (19) by choosing any non-zero passive virtual environment and multiply it by a sufficiently large scalar. Suppose that ω_0 and M_Θ are fixed. The left-hand side of (19) grows with the magnitude of R_d, but the right-hand side depends only on the phase of P. A sufficiently large scaling of any passive virtual environment will violate (19). It follows that no proper, stabilizing controller C exists that meets the performance specification while presenting passive dynamics to the user.

Proposition 1 implies a limitation on the ability to compensate for hardware dynamics while presenting a passive response to the user. We say that the feedback design *partially cancels* hardware dynamics at frequencies where $|R(j\omega)| > |P(j\omega)|$. At these frequencies the magnitude of the closed-loop response y to the human operator's force f exceeds the open-loop response. Examining (19), we note that only virtual environments that require partial cancellation can cause the inequality to be violated. The unpowered hardware dynamics sP are passive, so $\pi + \arg P(j\omega)$ is positive. It follows that the right-hand side of (19) is always positive. Furthermore, the term $\log|1-M_\Theta|$ is always negative, so (19) will always hold if $|R_d(j\omega)| < |P(j\omega)|$ for $0 < \omega < \omega_c$.

A practical implication of Proposition 1 is that even a small amount of compensation for hardware inertia may be impossible without violating the passivity requirement on sR. Consider haptic device dynamics P with only inertia. Since $\arg P(j\omega)$ is $-\pi$, the right-hand side of (19) is exactly zero. The left-hand side of (19) must remain less than or equal to zero. This requirement implies that $|R_d(j\omega)/P(j\omega)|$ must not significantly exceed unity over any range of frequencies below ω_c.

IV. Example

We now consider the conflict between performance and passivity for a simple example problem. Let the haptic device model be a damped mass system $P = \frac{1}{s^2+s}$ and let the virtual environment dynamics be an ideal parallel spring-damper $R_d = \frac{1}{s+10}$. For the performance specification, we let $M_\Theta = 0.5$ and we consider several different values for ω_c. Figure 4 shows the left-hand and right-hand sides of (19) using four values for ω_c indicated by lines A, B, C, and D. Proposition 1 predicts that there is no proper stabilizing feedback design that achieves $|\Theta(j\omega)| \leq 0.5$ up to 10 (rad/s) and that makes sR passive. Thus the performance bandwidth indicated by A cannot be achieved passively.

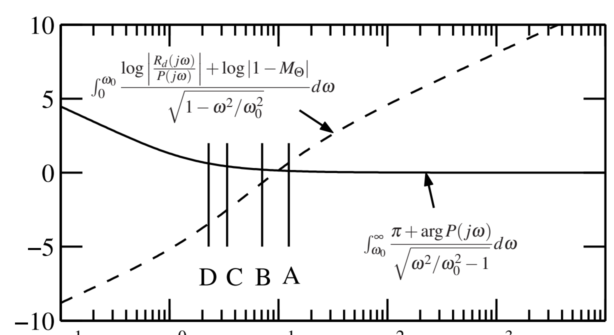

Fig. 4: Right and left sides of the inequality in Proposition 1 for $M_\Theta = 0.5$. Lines A, B, C, and D indicate four specification for ω_c.

Let us examine the conflict between performance and passivity through example controller designs. Since the rendered dynamics R are given by $P/1+PC$, we can algebraically solve for the controller that makes $R \equiv R_d$. However, the solution $C = R_d^{-1} - P^{-1}$ is not necessarily proper or stabilizing. In the present example, the exact algebraic solution for C is $-s^2 + 10$. This improper controller is not practical, and furthermore, multiplying this expression by a low-pass filter to obtain a proper controller does not necessarily result in closed-loop stability. We note that classical design techniques for loop-shaping do not provide a direct method of shaping the closed-loop dynamics, and optimal synthesis tools do not necessarily produce stable controllers, a prerequisite for sR to be passive. To generate feedback designs which approximate the desired closed-loop response using a stable, proper controller, we use controllers of the form

$$C = \left(\frac{(\tau s + 1)^2}{R_d} - \frac{1}{P} \right) \left(\frac{1}{\gamma s + 1} \right)^3. \qquad (20)$$

For the values of τ and γ given in Table IV, the controller given by (20) yields closed-loop stability and the desired performance. The frequency responses of distortion Θ are shown in Fig. 5.

Design	$(2\pi\tau)^{-1}$ Hz	$(2\pi\gamma)^{-1}$ Hz
A	8	400
B	4	100
C	1.6	15
D	0.6	2

TABLE I

PARAMETERS τ AND γ OF THE FOUR FEEDBACK DESIGNS.

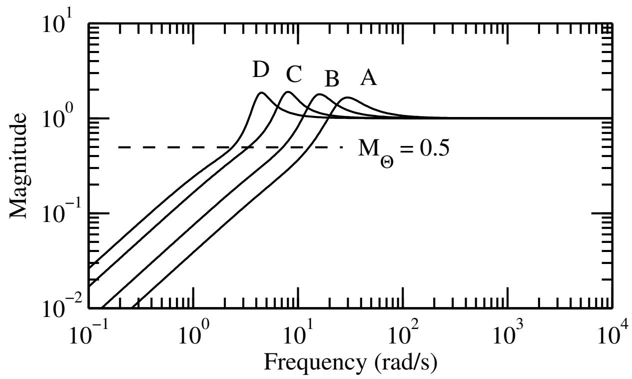

Fig. 5: Distortion Θ for designs A, B, C and D.

Frequency responses of the rendered virtual environment R for each design, labeled A through D, are shown in Fig. 6. Inspection of the phase plot reveals that designs A, B, and C violate the lower bound of -180 degrees and thus do not present the user with passive dynamics. Proposition 1 proves that no feasible design exists that meets the performance specification for design A and satisfies the phase criteria for passivity. Progressive relaxations of the bandwidth incur smaller excursions below -180 degrees. Only design D both meets the performance specification and the passivity requirement. For designs B and C, inequality (19) is not violated so Proposition 1 provides no determination on

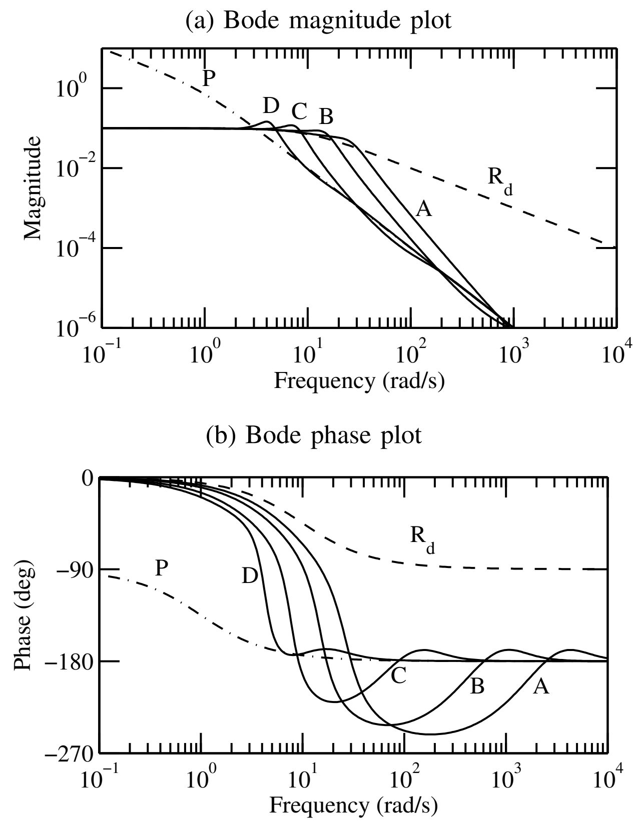

Fig. 6: Frequency response of the haptic device P, the virtual environment R_d, and four causal approximations to R_d (designs A, B, C, & D). Designs A, B, and C violate passivity since their phase drops below -180 degrees.

whether some other feedback design exists that achieves the bandwidth of C without violating passivity. There may then exist a higher-order controller than (20) which satisfies both performance and passivity requirements.

V. CONCLUSIONS

In this paper we have shown that, for given haptic device hardware and performance specification for the feedback design, a certain class of passive transfer functions cannot be approximated while presenting a passive response to the human operator. This class of transfer functions is characterized by compensation of hardware dynamics. In Proposition 1 we have provided a necessary condition for the existence of a stabilizing, proper feedback design that can passively approximate a desired closed-loop response. If the inequality (19) is violated at any frequency, we can conclude that no feasible feedback design exists that satisfies both performance and passivity requirements.

Passivity is a strong requirement which may be overly restrictive. Less conservative coupled stability criteria may be available through a robust stability analysis assuming a set of possible user dynamics. Although relaxing passivity would mitigate the conflict between performance and passivity, other tradeoffs may exist such as between performance bandwidth and the closed-loop bandwidth. As seen in the example, small increases in the performance bandwidth cause large growth in the controller bandwidth. While a high-order controller may be used to increase the rate of roll-off, such a design will also increase the amount by which the phase requirement of passivity is violated.

While our analysis and design example assume only position feedback, a force or torque sensor may be installed to measure the user's applied input f. While additional sensing offers benefits, it does not circumvent the intrinsic conflict between performance and passivity. Our Bode gain-phase analysis of S is simply an analysis of the closed-loop response from f to $f+u$. Regardless of the sensor suite, performance and passivity impose the same requirements on this closed-loop response. Furthermore, due to bandwidth limitations of the motor and amplifier, the closed-loop response from f to $f+u$ must always approach unity (as is true of S). The conflict between performance and passivity is thus intrinsic to impedance-type haptic devices where the inputs f and u affect the output y through the same dynamics.

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