2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Stability of Stochastic Systems with Nonlinear Uncertainties and Time-Varying Delays

Yun Chen, An-Ke Xue, Jian-Zhong Wang, Ming Ge and Jun-Hong Wang, Student Member of IEEE

***Abstract*— This paper considers stability of stochastic systems with nonlinear uncertainties and time-varying delays. By introducing some slack matrix matrices, a delay-dependent stability criterion is obtained based on Lyapunov-Krasovskii theory. The proposed condition, which is formulated in terms of linear matrix inequalities (LMIs), is constructed without using the model transformation and cross term bounding techniques. Some term which was ignored in previous methods is considered in our result. Two numerical examples are provided to demonstrate the less conservatism of the method.**

I. INTRODUCTION

Since there are many practical systems can be modeled as stochastic differential equations with time delays, increasing efforts have been devoted to the study of stochastic time-delay systems in the past years [1-12]. Based on Lyapunov-Krasovskii method, many stability conditions for stochastic time-delay systems have been provided by means of LMIs, for examples [2], [3], [4], [5], [6], [7], [8], and the references therein. [2] is delay-independent, and the others are delay-dependent. Delay-independent controller and filter synthesis problems for uncertain stochastic delay systems have been addressed in [9], [10], [11].

Most recently, input-output method has been applied to develop delay-dependent H_∞ controller and filter for uncertain time-delay systems with state-multiplicative noises [12]. By this approach, the system is transformed into a uncertain deterministic one without delay. The result of [12] shows less conservative than those of [7], [8], [9]. However, for time-varying delays, the approaches of [9], [10], [11], [12] require that the upper bounds of delay-derivative should be strictly small than one. It is well known that this requirement is very unrealistic and will restrict the applications. This motivates us to find new criterion without this restriction.

The results of [4], [5], [7] are obtained by model transformation and cross term estimation techniques, which may lead to conservatism. While [6], [8] are derived by introducing some slack matrices. For deterministic systems with time-varying delays, the slack matrix (free-weighting matrix) method of [15] is less conservative than the existing ones (for instances [13], [14]), because they consider some terms which were ignored previously. We hope to extend the method of [15] to stochastic delay system such that the conservatism of stability conditions could be reduced. This is the second motivation of this paper.

In this paper, the attention is focused on stability analysis for a class of uncertain stochastic systems with nonlinear uncertainties and time-varying delays. A new delay-dependent condition is obtained by employing Lyapunov-Krasovskii functional approach together with introducing some slack matrices. We avoid the use of any model transformations and bounding techniques for cross terms. The presented condition is formulated in the forms of LMIs. Similar to [15], the term $-\int_{t-h}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha$ is partitioned into $-\int_{t-h(t)}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha - \int_{t-h}^{t-h(t)} y^T(\alpha)Z_1 y(\alpha)d\alpha$. This helps us to retain the term $-\int_{t-h}^{t-h(t)} y^T(\alpha)Z_1 y(\alpha)d\alpha$ to reduce the conservatism. Finally, the advantage of our approach is verified by two illustrative examples.

Notations: Throughout this paper, the notations are standard. $\|\cdot\|$ denotes the Euclidean norm; $\mathbb{R}^n$ is n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ stands for the set of all $n\times m$ real matrices; $P>0$ $(P<0)$ means that the matrix P is positive (negative) definite and symmetric; $\mathscr{E}\{\cdot\}$ represents the expectation operator. trace$\{\cdot\}$ is the trace of a matrix. $(\Omega,\mathscr{F},\mathscr{P})$ is a probability space, where Ω is the sample space, and $\mathscr{F}$ is a σ-algebra of subsets of Ω. The symmetric term in a symmetric matrix is denoted as $*$.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following uncertain stochastic delay system:

$$\begin{cases} dx(t) = [(A+\Delta A)x(t) + (A_1+\Delta A_1)x(t-h(t))]dt \\ \qquad\qquad + g(t,x(t),x(t-h(t)))dw(t) \\ x(\theta) = \psi(\theta), \quad \forall\theta\in[-h,0], \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state; $h(t)$ is a time-varying scalar indicating the delay, which satisfies $0\le h(t)\le h, \dot{h}(t)\le \mu<\infty$; $\psi(\cdot)$ is the initial condition for all $t\in[-h,0]$; $g(t,x(t),x(t-h(t)))\in\mathbb{R}^{n\times m}$ is a nonlinear function satisfying

$$\begin{aligned} &\text{trace}\{g^T(t,x(t),x(t-h(t)))g(t,x(t),x(t-h(t)))\} \\ &\quad \le \| G_1 x(t)\|^2 + \| G_2 x(t-h(t))\|^2, \end{aligned} \tag{2}$$

where $G_1,\ G_2\in\mathbb{R}^{n\times n}$ are matrix functions; $A, A_1\in\mathbb{R}^{n\times n}$ are known real constant matrices, ΔA, ΔA_1 are time-varying parametric uncertainties, which can be described by

$$\begin{bmatrix} \Delta A & \Delta A_1 \end{bmatrix} = LF(t)\begin{bmatrix} E_1 & E_2 \end{bmatrix}, \tag{3}$$

where L, E_1, E_2 are constant matrices with compatible dimensions, and $F(t)$ is an unknown matrix function with Lebesgue

This work was supported by the National Natural Science Foundation of China under Grant 60434020, the Research Foundation of Education Bureau of Zhejiang Province under Grant Y200701897.

The authors are with the Institute of Information and Control, Hangzhou Dianzi University, Hangzhou 310018, P.R.China

Y. Chen is also with the National Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, P.R.China cloudscy@hdu.edu.cn

978-1-4244-2079-7/08/$25.00 ©2008 AACC.

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & A^T X & hN_1 & hS_1 & hM_1 & PL \\ * & \Gamma_{22} & \Gamma_{23} & A_1^T X & hN_2 & hS_2 & hM_2 & 0 \\ * & * & \Gamma_{33} & 0 & hN_3 & hS_3 & hM_3 & 0 \\ * & * & * & \Gamma_{44} & 0 & 0 & 0 & X^T L \\ * & * & * & * & -hZ_1 & 0 & 0 & 0 \\ * & * & * & * & * & -hZ_1 & 0 & 0 \\ * & * & * & * & * & * & -hZ_2 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{5}$$

measurable elements and such that $\| F(t) \| \leq 1$; $w(t)$ is an m-dimensional Wiener process defined on the probability space $(\Omega, \mathscr{F}, \mathscr{P})$ satisfying $\mathscr{E}\{dw(t)\} = 0, \mathscr{E}\{dw^2(t)\} = dt$.

Definition 1: The uncertain stochastic system (1) is said to be robustly stable in mean-square sense for all admissible uncertainties (2) and (3), if for any scalar $\varepsilon > 0$ there exists a scalar $\sigma(\varepsilon) > 0$ such that

$$\mathscr{E}\{| x(t) |^2\} < \varepsilon, \forall t > 0$$

when

$$\sup_{-h \leq s \leq 0} \mathscr{E}\{|\psi(s)|^2\} < \sigma(\varepsilon).$$

Additionally, system (1) is said to be robustly asymptotically stable in mean-square sense, if for all admissible uncertainties (2) and (3)

$$\lim_{t \to \infty} \mathscr{E}\{\| x(t) \|^2\} = 0$$

holds for any initial conditions.

III. MAIN RESULTS

In this section, the standard LMI approach is employed to address the delay-dependent stability of system (1).

Theorem 1: System (1) is robustly asymptotically stable in mean-square sense, if there exist positive definite symmetric matrices P, Q, $R \in \mathbb{R}^{n \times n}$, matrices X, M_i, N_i, $S_i (i = 1,2,3) \in \mathbb{R}^{n \times n}$ and positive scalars ε, δ such that

$$P < \delta I \tag{4}$$

and LMI (5) (shown at the top of this page), where

$$\begin{aligned} \Gamma_{11} =& PA + A^T P + Q + R + M_1 + M_1^T \\ & + N_1 + N_1^T + \delta G_1^T G_1 + \varepsilon E_1^T E_1, \\ \Gamma_{12} =& PA_1 + M_2^T + N_2^T - N_1 + S_1 + \varepsilon E_1^T E_2, \\ \Gamma_{22} =& -(1-\mu)Q - N_2 - N_2^T + S_2 + S_2^T \\ & + \delta G_2^T G_2 + \varepsilon E_2^T E_2. \\ \Gamma_{13} =& -M_1 - S_1 + M_3^T + N_3^T, \\ \Gamma_{23} =& -M_2 - S_2 - N_3^T + S_3^T, \\ \Gamma_{33} =& -R - M_3 - M_3^T - S_3 - S_3^T, \\ \Gamma_{44} =& h(Z_1 + Z_2) - X - X^T. \end{aligned}$$

Proof: First, if denote

$$y(t)dt = dx(t), \tag{6}$$

then system (1) becomes to

$$0 = [-y(t) + A(t)x(t) + A_1(t)x(t-h(t))]dt + g(t)dw(t). \tag{7}$$

where $g(t) = g(t, x(t), x(t-h(t)))$, $A(t) = A + \Delta A$, $A_1(t) = A_1 + \Delta A_1$.
According to Newton-Leibniz formula, we have

$$\begin{aligned} 0 =& x(t) - x(t-h) - \int_{t-h}^{t} y(\alpha) d\alpha, \\ 0 =& x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} y(\alpha) d\alpha, \\ 0 =& x(t-h(t)) - x(t-h) - \int_{t-h}^{t-h(t)} y(\alpha) d\alpha. \end{aligned} \tag{8}$$

Choose the Lyapunov-Krasovskii functional as

$$V(t, x_t) = \sum_{i=1}^{4} V_i(t, x_t) \tag{9}$$

where

$$\begin{aligned} V_1(t, x_t) &= x^T(t) P x(t), \\ V_2(t, x_t) &= \int_{t-h(t)}^{t} x^T(\alpha) Q x(\alpha) \mathrm{d}\alpha, \\ V_3(t, x_t) &= \int_{t-h}^{t} x^T(\alpha) R x(\alpha) \mathrm{d}\alpha, \\ V_4(t, x_t) &= \int_{-h}^{0} \int_{t+\beta}^{t} y^T(\alpha)(Z_1 + Z_2) y(\alpha) \mathrm{d}\alpha d\beta, \end{aligned}$$

with $P > 0,\ Q > 0,\ R > 0,\ Z_1 > 0,\ Z_2 > 0$.
By Itô differential formula [1], the stochastic differential $dV(t, x_t)$ along the trajectories of system (1) is

$$dV(t, x_t) = \mathscr{L}V(t, x_t)dt + 2x^T(t)Pg(t)dw(t). \tag{10}$$

$$\begin{aligned} \mathscr{L}V(t, x_t) =& 2x^T(t)P\left[\ A(t)x(t) + A_1(t)x(t-h(t))\ \right] \\ & + \mathrm{tr}\{g^T(t)Pg(t)\} + \sum_{i=2}^{4} \mathscr{L}V \end{aligned} \tag{11}$$

with

$$\begin{aligned} \mathscr{L}V_2 =& x^T(t)Qx(t) - (1 - \dot{h}(t))x^T(t-h(t))Qx(t-h(t)) \\ & \leq x^T(t)Qx(t) - (1-\mu)x^T(t-h(t))Qx(t-h(t)), \\ \mathscr{L}V_3 =& x^T(t)Rx(t) - x^T(t-h)Rx(t-h), \\ \mathscr{L}V_4 =& hy^T(t)(Z_1 + Z_2)y(t) - \int_{t-h}^{t} y^T(\alpha)(Z_1 + Z_2)y(\alpha)\mathrm{d}\alpha \\ =& hy^T(t)(Z_1 + Z_2)y(t) - \int_{t-h}^{t} y^T(\alpha) Z_2 y(\alpha)\mathrm{d}\alpha \\ & - \int_{t-h(t)}^{t} y^T(\alpha) Z_1 y(\alpha)\mathrm{d}\alpha - \int_{t-h}^{t-h(t)} y^T(\alpha) Z_1 y(\alpha)\mathrm{d}\alpha. \end{aligned} \tag{12}$$

Thus, there arrives at

$$dV(t,x_t) = \mathscr{L}V(t,x_t)dt + 2x^T(t)Pg(t)dw(t) + \sum_{i=1}^{4} v_i(t) \quad (13)$$

where $0 = v_i (i = 1,2,3,4)$,

$$\begin{aligned} v_1(t) =& 2\xi^T(t)M[x(t)-x(t-h)-\int_{t-h}^{t} y(\alpha)d\alpha], \\ v_2(t) =& 2\xi^T(t)N[x(t)-x(t-h(t))-\int_{t-h(t)}^{t} y(\alpha)d\alpha], \\ v_3(t) =& 2\xi^T(t)S[x(t-h(t))-x(t-h)-\int_{t-h}^{t-h(t)} y(\alpha)d\alpha], \\ v_4(t) =& 2y^T(t)X^T\{g(t)dw(t) \\ &+[A(t)x(t)+A_1(t)x(t-h(t))-y(t)]dt\}, \end{aligned} \quad (14)$$

and $\xi^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-h)]$,

$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}. \quad (15)$$

From (4), the following is true

$$\begin{aligned} \mathrm{tr}\{g^T(t)Pg(t)\} <& \delta x^T(t)G_1^T G_1 x(t) \\ &+\delta x^T(t-h(t))G_2^T G_2 x(t-h(t)). \end{aligned} \quad (16)$$

Then, (13) can be rewritten as

$$\begin{aligned} dV(t,x_t) =& \mathscr{L}\tilde{V}(t,x_t)dt + 2x^T(t)Pg(t)dw(t) \\ &+ 2y^T(t)Xg(t)dw(t), \end{aligned} \quad (17)$$

where

$$\begin{aligned} &\mathscr{L}\tilde{V}(t,x_t) \\ =& \mathscr{L}V(t,x_t)+v_1(t)+v_2(t)+v_3(t) \\ &+2y^T(t)X^T[A(t)x(t)+A_1(t)x(t-h(t))-y(t)] \\ <& 2x^T(t)P\left[\ A(t)x(t)+A_1(t)x(t-h(t))\ \right] \\ &+\delta x^T(t)G_1^T G_1 x(t)+\delta x^T(t-h(t))G_2^T G_2 x(t-h(t)) \\ &+x^T(t)Qx(t)-(1-\mu)x^T(t-h(t))Qx(t-h(t)) \\ &+x^T(t)Rx(t)-x^T(t-h)Rx(t-h) \\ &+v_1(t)+v_2(t)+v_3(t) \\ &+2y^T(t)X^T[A(t)x(t)+A_1(t)x(t-h(t))-y(t)] \\ &+h\xi^T(t)MZ_2^{-1}M^T\xi(t) \\ &+h\xi^T(t)NZ_1^{-1}N^T\xi(t)+h\xi^T(t)SZ_1^{-1}S^T\xi(t) \\ &-\int_{t-h}^{t}[\xi^T(t)M+y^T(s)Z_2]Z_2^{-1}[M^T\xi(t)+Z_2 y(s)]ds \\ &-\int_{t-h(t)}^{t}[\xi^T(t)N+y^T(s)Z_1]Z_1^{-1}[N^T\xi(t)+Z_1 y(s)]ds \\ &-\int_{t-h}^{t-h(t)}[\xi^T(t)S+y^T(s)Z_1]Z_1^{-1}[S^T\xi(t)+Z_1 y(s)]ds. \end{aligned} \quad (18)$$

Obviously, the last three terms of the above inequality are negative definite, so $\mathscr{L}\tilde{V}(t,x_t) < 0$ is guaranteed by $[\xi^T(t) \quad y^T(t)]\Phi[\xi^T(t) \quad y^T(t)]^T < 0$, where $\Phi < 0$ is equivalent to $\Gamma < 0$ by following the routine techniques to handle the norm-bounded uncertainties (see Ref. [11]).

Therefore, if $\Gamma < 0$, which implies $\mathscr{L}\tilde{V}(t,x_t) < 0$, then the stochastic system (1) is robustly asymptotically stable in mean-square sense by Definition 1 and the stochastic stability theory in [1]. This completes the proof. ■

Remark 1: Model transformation and cross term bounding techniques are both avoided in Theorem 1. Moreover, in some existing reports, the term $-\int_{t-h}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha$ is bounded as $-\int_{t-h(t)}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha$, and the negative definite one $-\int_{t-h}^{t-h(t)} y^T(\alpha)Z_1 y(\alpha)d\alpha$ is ignored, which may result in some conservatism [15]. Inspired by [15], in order to reduce the conservatism, $-\int_{t-h}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha$ in Theorem 1 is decomposed as

$$\begin{aligned} &-\int_{t-h}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha = \\ &-\int_{t-h(t)}^{t} y^T(\alpha)Z_1 y(\alpha)d\alpha - \int_{t-h}^{t-h(t)} y^T(\alpha)Z_1 y(\alpha)d\alpha, \end{aligned}$$

such that $-\int_{t-h}^{t-h(t)} y^T(\alpha)Z_1 y(\alpha)d\alpha$ can be retained by introducing some slack matrices.

Remark 2: The existence of the positive definite symmetric term $hy^T(t)(Z_1+Z_2)y(t)$, which is shown as $h\dot{x}^T(t)(Z_1+Z_2)\dot{x}(t)$ in the deterministic delay systems ([14], [15]), drives us to introduce the free matrix X in (14). In Theorem 1, the cross relationships among $x(t)$, $x^T(t-h(t))$, $x(t-h)$ and $y(t)$ are involved by some slack matrices.

Remark 3: Due to the slack matrices N_2, S_2 and their transposes introduced in (5), the constraint that the upper bound of delay derivative is less than one (i.e. $\mu < 1$), is removed in Theorem 1.

If the uncertainty $g(t)$ is restricted to be $g(t) = \Delta Cx(t) + \Delta Dx(t-h(t))$, with

$$\left[\ \Delta C \quad \Delta D\ \right] = LF(t)\left[\ E_3 \quad E_4\ \right], \quad (19)$$

where L, E_3, E_4 are constant matrices with compatible dimensions, and $F(t)$ is an unknown matrix function satisfying $\| F(t) \| \leq 1$, then system (1) reduces to

$$\begin{cases} dx(t) = [(A+\Delta A)x(t)+(A_1+\Delta A_1)x(t-h(t))]dt \\ \qquad\quad + [\Delta C(t)x(t)+\Delta D(t)x(t-h(t))]dw(t) \\ x(\theta) = \psi(\theta), \quad \forall\theta \in [-h,0]. \end{cases} \quad (20)$$

Following the similar lines as in the proof of Theorem 1, we can obtain the result as follows.

Corollary 1: System (20) is robustly asymptotically stable in mean-square sense, if there exist positive definite symmetric matrices P, Q, $R \in \mathbb{R}^{n\times n}$, matrices X, M_i, N_i, $S_i (i = 1,2,3) \in \mathbb{R}^{n\times n}$ and positive scalars $\varepsilon_1, \varepsilon_2$ satisfying LMI (21) (shown at the top the next page), where

$$\begin{aligned} \Pi_{11} =& PA+A^T P+Q+R+M_1+M_1^T \\ &+N_1+N_1^T+\varepsilon_1 E_1^T E_1+\varepsilon_2 E_3^T E_3, \\ \Pi_{12} =& PA_1+M_2^T+N_2^T-N_1+S_1+\varepsilon_1 E_1^T E_2+\varepsilon_2 E_3^T E_4, \\ \Pi_{22} =& -(1-\mu)Q+M_1+M_1^T+N_1+N_1^T \\ &+\varepsilon_1 E_2^T E_2+\varepsilon_2 E_4^T E_4. \end{aligned}$$

$$
\Pi = \begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Gamma_{13} & A^T X & hN_1 & hS_1 & hM_1 & PL & 0 & 0 \\
* & \Pi_{22} & \Gamma_{23} & A_1^T X & hN_2 & hS_2 & hM_2 & 0 & 0 & 0 \\
* & * & \Gamma_{33} & 0 & hN_3 & hS_3 & hM_3 & 0 & 0 & 0 \\
* & * & * & \Gamma_{44} & 0 & 0 & 0 & X^T L & 0 & 0 \\
* & * & * & * & -hZ_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -hZ_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -hZ_2 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\
* & * & * & * & * & * & * & * & -P & PL \\
* & * & * & * & * & * & * & * & * & -\varepsilon_2 I
\end{bmatrix} < 0, \quad (21)
$$

TABLE I
THE UPPER BOUNDS OF DELAY OF EXAMPLE 2 FOR DIFFEREN μ

μ	0	0.5	0.9
Yue[6]	1.1812	0.8502	0.4606
Chen [16]	2.1491	1.2956	0.8180
Theorem 1	2.1491	1.3224	0.9748

IV. ILLUSTRATIVE EXAMPLES

In this section, we will present numerical examples to illustrate the effectiveness of our method.

Example 1: Consider the uncertain stochastic time-delay system (1) with

$$
\begin{aligned}
& A = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix}, \\
& L = I, E_1 = E_2 = 0.1I, G_1 = G_2 = \sqrt{0.1}I, \\
& \mu = 0.
\end{aligned} \quad (22)
$$

The upper bounds of delay for system (22) given by [6], [7], [8], [12] are 1.1812, 1.3640, 1.5270 and 1.56, respectively. However, the maximal allowable delay by Theorem 1 is 2.1491.

Example 2: Consider system (22) with time-varying delay (i.e. $\mu \neq 0$).

Table I lists the maximal admissible delays for different μ of this example. When $\mu = 0$, Theorem 1 and [16] can obtain the same result. However, if the delay is time-varying, Theorem 1 is less conservative than [16], since Theorem 1 considers the negative definite term "$-\int_{t-h}^{t-h(t)} y^T(\alpha) Z_1 y(\alpha) d\alpha$" which is ignored by [16].

Therefore, the advantage of our method is obvious by Examples 1 and 2.

V. CONCLUSIONS

A new delay-dependent stability condition for uncertain stochastic delayed systems with nonlinear uncertainties has been presented in terms of linear matrix inequalities (LMIs). The result has been derived based on Lyapunov-Krasovskii method and slack matrix technique. Some term which has been ignored in the previous reports has been retained in this paper, thus our result is less conservative than those existing ones. Two illustrative examples have been provided to show the superiority of the new method.

In (14), an auxiliary vector $y(t)$ is introduced, which plays an important role in the derivation of Theorem 1. The proposed method based on an additional vector can be extended to the controller and filter design problems for stochastic systems with time-delays.

REFERENCES

[1] E.K. Boukas and Z.K. Liu, *Deterministic and Stochastic Systems with Time-Delay*, Birkhauser, Boston, 2002.

[2] E. I. Varriest and P. Florchinger, Stability of stochastic systems with uncertain time delays, *Systems & Control Letters*, vol. 24, no. 1, 1995, pp. 41-47.

[3] X. Mao, Robustness of exponential stability of stochastic differential delay equations, *IEEE Trans. Autom. Control*, vol. 41, 1996, pp. 442-447.

[4] D. Yue, and S. Won, Delay-dependent robust stability of stochastic systems with time delay and nonlinear uncertainties, *Electronics Letters*, vol. 37, no. 15, 2001, pp 992-993.

[5] D. Yue, J. Fang and S. Won, Delay-dependent robust stability of stochastic uncertain systems with time delay and Markovian jump parameters, *Circuits, Systems & Signal Processing*, vol. 22, no. 4, 2003, pp 351-365.

[6] D. Yue and Q.-L. Han, Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching, *IEEE Trans. Automatic Control*, vol. 50, no. 2, 2005, pp 217-222.

[7] W.-H. Chen, Z.-H. Guan and X. Lu, Delay-dependent exponential stability of uncertain stochastic systems with multiple delays: an LMI approach, *Systems & Control Letters*, vol. 54, 2005, pp 547-555.

[8] S. Xu, J. Lam, X. Mao and Y. Zou, A new LMI condition for delay dependent robust stability of stochastic time-delay systems, *Asian Journal Control*, vol. 7, 2005, pp. 419-423.

[9] S. Xu and T. Chen, Robust H_∞ control for uncertain stochastic systems with state delay, *IEEE Trans. Automatic Control*, vol. 47, no. 12, 2002, pp 2089-2094.

[10] H. Gao, C. Wang and L. Zhao, Comments on "An LMI-based approach for rbust stabilization of uncertain stochastic systems with time-varying delays", *IEEE Trans. Automatic Control*, vol. 48, no. 12, 2003, pp 2073-2074.

[11] H. Gao, J. Lam and C. Wang, Robust energy-to-peak filter design for stochastic time-delay systems, *Systems & Control Letters*, vol. 55, 2006, pp 101-111.

[12] E. Gershon, U. Shaked and N. Berman. H_∞ control and estimation of retarded state-multiplicative stochastic systems, *IEEE Trans. Automatic Control*, vol. 52, no. 9, 2007, pp 1773-1779.

[13] E. Fridman and U. Shaked, An improved stabilization method for linear time-delay systems, *IEEE Trans. Automatic Control*, vol. 47, no. 11, 2002, pp 1931-1937.

[14] X. Jiang and Q.-L. Han, On H_∞ control for linear systems with interval time-varying delay, *Automatica*, vol. 41, 2005, pp 2099-2106.

[15] Y. He, Q.-G. Wang, C. Lin and M. Wu, Delay-range-dependent stability for systems with time-varying delay, *Automatica*, vol. 43, 2007, pp 371-376.

[16] Y. Chen, A. Xue, S. Zhou and R, Lu, Delay-dependent robust control for uncertain stochastic time-delay systems, *Circuits, Systems & Signal Processing*, to be published.