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Reliable H_∞ Flight Tracking Control via State Feedback

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Abstract— **This paper studies the problem of reliable H_∞ state feedback controller design for linear continuous-time systems. Actuator partial failures and actuator lock-in-place failures are simultaneously considered, and the signals in the case of certain actuators being locked in place are treated as zero-frequency disturbances, which gives an exact description for the lock-in-place failures. Based on the generalized KYP lemma in finite frequency domain, the controller design is developed in the framework of linear matrix inequality (LMI) approach, which can guarantee the asymptotic stability and H_∞ performances of the resulting closed-loop systems when all control components are operational as well as when some actuators fail. Finally, the design procedure and the effectiveness of the proposed method in comparison with the entire frequency approach by using bounded real lemma are illustrated via a numerical example of flight tracking control of an F-16 aircraft.**

I. INTRODUCTION

Since control components failures often occur in practical applications, it is always necessary to design controllers that achieve desired performance requirements, not only when the system is operating properly, but also in the case of any admissible failures. This motivated the study of the so-called reliable control. In the past years, a number of theoretical results as well as application examples has now been described in the literature (see, e.g., [1]-[15]). [1] presented a new methodology for the design of reliable centralized and decentralized control systems by using the algebraic Riccati equation approach. [2] and [3] proposed design approaches of the reliable H_∞ controller for linear systems with actuator failures and sensor failures, respectively. [4] extended the results given in [1] to the case of sensor and actuator partial failures. [7] solved the problem of flight tracking control in the presence of stuck-actuator faults by modeling a stuck fault as a bounded input. [11] basically followed the design guideline of the adaptive two-stage LQ reliable control, and considered the faults are not confined to a preselect set of actuators.

In the above-mentioned results for reliable control in the presence of actuators being locked in place, the inputs being locked in place were modeled as the entire frequency disturbances, which is not an exact description of the faults. The main objective of this paper is to develop an exact method of reliable H_∞ control problem for the case of actuator partial failures and actuator lock-in-place failures occurring simultaneously. The inputs being locked in place are modeled as zero-frequency disturbances and are treated by using the generalized KYP lemma in finite frequency domain [19]. The new development gives a multi-model mixed frequency H_∞ design method. The rest of this paper is arranged as follows. Section 2 presents the problem statement and some preliminaries. Section 3 proposes a new method for designing stabilizing reliable H_∞ state feedback controllers. In Section 4, the entire frequency approach based on bounded real lemma is described. In Section 5, an example of flight tracking control of an F-16 aircraft is provided to illustrate the design procedure and their effectiveness. Some concluding remarks are given in Section 6.

Notation: For a matrix A, A^* denotes its complex conjugate transpose. The Hermitian part of a square matrix A is denoted by $\mathbf{He}(A) := A + A^*$. The symbol $\mathbf{H}_n$ stands for the set of $n \times n$ Hermitian matrices. The symbol $*$ within a matrix represents the the symmetric entries. I denotes the identity matrix with an appropriate dimension. For a transfer function matrix G, its H_∞ norm is defined by

$$\|G(j\omega)\|_\infty := \sup \overline{\sigma}\{G(j\omega)\}$$

where $\overline{\sigma}(G) = \{\lambda_{\max}(G^*G)\}^{\frac{1}{2}}$ represents the maximum singular value of G, $\lambda_{\max}$ represents maximum eigenvalue.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider a linear time-invariant plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_1\omega(t) \\ z(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

with a state feedback controller of the following form:

$$u(t) = Kx(t) \tag{2}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the control input, $\omega(t) \in \mathbf{R}^r$ is the disturbance input and $z(t) \in \mathbf{R}^q$ is the regulated output, respectively. A, B, B_1, C and D are known constant matrices of appropriate dimensions.

For the control input u, let u^F denote the signal vector in the

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case of some actuator failures. The actuator partial failure model is as follows:

$$u^F = \alpha u, \tag{3}$$

where

$$\alpha := diag\begin{bmatrix}\alpha_1 & \alpha_2 & \dots & \alpha_m\end{bmatrix} \tag{4}$$

with α_i satisfies

$$0 \le \underline{\alpha}_i \le \alpha_i \le \overline{\alpha}_i, \qquad (i = 1,2,\dots,m). \tag{5}$$

Denote

$$\underline{\alpha} = diag\begin{bmatrix}\underline{\alpha}_1 & \underline{\alpha}_2 & \dots & \underline{\alpha}_m\end{bmatrix}, \tag{6}$$
$$\overline{\alpha} = diag\begin{bmatrix}\overline{\alpha}_1 & \overline{\alpha}_2 & \dots & \overline{\alpha}_m\end{bmatrix}. \tag{7}$$

While the actuator lock-in-place failure model is:

$$u^F = F_j u + (I - F_j)\beta_j, \qquad (j = 1,2,\dots,L) \tag{8}$$

where

$$F_j = diag\begin{bmatrix}f_{j1} & f_{j2} & \dots & f_{jm}\end{bmatrix}, \tag{9}$$
$$\beta_j = \begin{bmatrix}\beta_{j1} & \beta_{j2} & \dots & \beta_{jm}\end{bmatrix}^T \tag{10}$$

with

$$f_{ji} = \begin{cases} 1 & \text{the } i\text{th actuator is operational} \\ 0 & \text{the } i\text{th actuator is locked in place} \end{cases} \quad i = 1,\dots,m. \tag{11}$$

Here, the index j denotes the jth failure mode and L is the total failure modes. And β_{ji} is an unknown constant. Actually, the actuator failure mode adopted in this paper is:

$$u^F = F_j \alpha u + (I - F_j)\beta_j, \qquad (j = 1,2,\dots,L) \tag{12}$$

Remark 1: It considers the case of actuator partial failure and actuator locking in place simultaneously. When $f_{ji} = 1$ and $\underline{\alpha}_i = \overline{\alpha}_i = 0$, it covers the case of outage of the ith actuator u_i in the jth failure mode. When $f_{ji} = 1$ and $\underline{\alpha}_i > 0$, it corresponds to the case of partial failure of the ith actuator u_i in the jth failure mode. When $f_{ji} = 0$, the ith actuator u_i is locked in place in the jth failure mode. Without loss of generality, we assume that $F_0 = I$. Note that, when $F_0 = I$ and $\underline{\alpha} = \overline{\alpha} = I$, it corresponds to the normal control input vector $u^F(t) = u(t)$.

Then the resulting closed-loop system in the event of the actuator failures described by (12) is

$$\begin{aligned} \dot{x}(t) &= (A + BF_j\alpha K)x(t) + B_1\omega(t) + B(I - F_j)\beta_j \\ z(t) &= (C + DF_j\alpha K)x(t). \end{aligned} \tag{13}$$

Remark 2: Here, z is defined as the regulated output. But the inputs being locked in place are uncontrollable. So we do not consider the lock-in-place inputs in z.

In the closed-loop system, if β_j in the failed actuator is regarded as a zero-frequency disturbance, then the transfer function matrices $G_i(s)(i = 1,2)$ from ω and β_j to z are respectively denoted by

$$G_i(s) = \mathbf{C}(sI - \mathbf{A})^{-1}\mathbf{B}_i + \mathbf{D}_i, \tag{14}$$

where state space realizations $(\mathbf{A}, \mathbf{B}_i, \mathbf{C}, \mathbf{D}_i)$ of $G_i(s)$ are correspondingly given by

$$\left[\begin{array}{c|cc} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \hline \mathbf{C} & \mathbf{D}_1 & \mathbf{D}_2 \end{array}\right] = \left[\begin{array}{c|cc} A + BF_j\alpha K & B_1 & B(I - F_j) \\ \hline C + DF_j\alpha K & 0 & 0 \end{array}\right]. \tag{15}$$

The control synthesis problem under consideration is to find a state feedback controller (2) such that the resulting closed-loop system is asymptotically stable and the following H_∞-norm bound constraints

$$\left\|G_1(j\omega)\right\|_\infty < \gamma_1, \quad \omega \in R \cup \{\infty\} \tag{16}$$

$$\left\|G_2(j\omega)\right\|_\infty < \gamma_2, \quad \omega = 0 \tag{17}$$

hold not only when all control components are operational, but also in the case of some actuator failures by (12).

For the later development, the following preliminaries are required. First, the below Lemma 1 and Lemma 2 respectively follow from Theorem 1 and Theorem 2 in [17].

Lemma 1: Let $(\mathbf{A}, \mathbf{B}_2, \mathbf{C}, \mathbf{D}_2)$ in (15) be given. The following statements are equivalent:

(i)

$$J^* \begin{bmatrix} \mathbf{B}_2^* & \mathbf{D}_2^* \\ 0 & I \end{bmatrix}^* \Pi_2 \begin{bmatrix} \mathbf{B}_2^* & \mathbf{D}_2^* \\ 0 & I \end{bmatrix} J < 0, \tag{18}$$

$$J := \begin{bmatrix} (j\omega I - \mathbf{A}^*)^{-1}\mathbf{C}^* \\ I \end{bmatrix}, \quad \Pi_2 := \begin{bmatrix} I & 0 \\ 0 & -\gamma_2^2 I \end{bmatrix} \tag{19}$$

holds for $\omega = 0$.

(ii) There exist $P = P^*$ and $Q = Q^* > 0$ such that

$$\begin{bmatrix} M^* \\ I \end{bmatrix}^* X \begin{bmatrix} M^* \\ I \end{bmatrix} < 0, \tag{20}$$

where

$$M := \begin{bmatrix} \mathbf{A} & \mathbf{B}_2 \\ \mathbf{C} & \mathbf{D}_2 \end{bmatrix}, \quad X := \begin{bmatrix} -Q & 0 & P & 0 \\ 0 & I & 0 & 0 \\ P & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_2^2 I \end{bmatrix}. \tag{21}$$

Proof. Lemma 1 is a special condition of Theorem 1 in [17].

Lemma 2: Let $P, Q \in \mathbf{H}_n$ and $R = \begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}$ be given. The following statements are equivalent:

(i) The inequality (20) holds.

(ii) There exist $\mathscr{W}$ such that

$$X < \mathbf{He}\left(\begin{bmatrix} I \\ -M \end{bmatrix} \mathscr{W}\right) \tag{22}$$

where X is defined by (21).

Proof. The result basically follows from Theorem 2 of [17] by changing $\begin{bmatrix} -I \\ M \end{bmatrix}$ into $\begin{bmatrix} I \\ -M \end{bmatrix}$ which is also the range space of $\begin{bmatrix} M & I \end{bmatrix}^*$ for a special case.

Lemma 3: Let $(\mathbf{A}, \mathbf{B}_1, \mathbf{C}, \mathbf{D}_1)$ in (15) be given. The following statements are equivalent:

(i)

$$J^* \begin{bmatrix} \mathbf{B}_1^* & \mathbf{D}_1^* \\ 0 & I \end{bmatrix}^* \Pi_1 \begin{bmatrix} \mathbf{B}_1^* & \mathbf{D}_1^* \\ 0 & I \end{bmatrix} J < 0, \tag{23}$$

$$\Pi_1 := \begin{bmatrix} I & 0 \\ 0 & -\gamma_1^2 I \end{bmatrix} \tag{24}$$

holds for $\omega \in R \cup \{\infty\}$ with J defined by (19).
(ii) There exists a matrix $P = P^*$ such that

$$\begin{bmatrix} \mathbf{A} & I \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A} & I \\ \mathbf{C} & 0 \end{bmatrix}^* + \begin{bmatrix} \mathbf{B}_1 & 0 \\ \mathbf{D}_1 & I \end{bmatrix} \Pi_1 \begin{bmatrix} \mathbf{B}_1 & 0 \\ \mathbf{D}_1 & I \end{bmatrix}^* < 0. \tag{25}$$

Proof. The result basically follows by dualizing Theorem 1 of [16].

III. Reliable H_∞ control via state feedback

In this section, we would like to give the design method of reliable H_∞ controller for the linear time-invariant system (1) via state feedback. The following theorem presents a sufficient condition for the solvability of this problem.

Theorem 1: Consider the linear time-invariant system (1). For all $j = 1,2,\ldots,L$, if there exist scalars $\tau > 0$, $\delta > 0$ and matrices $P = P^*$, $Q = Q^* > 0$, $W = W^* > 0$, $V = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}$, $\mathscr{K}$ satisfying the following inequalities

$$\begin{bmatrix} AW + WA^* + BF_j a\mathscr{K} + \mathscr{K}^* aF_j B^* + \tau B_1 B_1^* & WC^* + \mathscr{K}^* aF_j D^* \\ * & -\tau\gamma_1^2 I \end{bmatrix} < 0, \quad a \in \{\underline{\alpha}, \overline{\alpha}\} \tag{26}$$

$$\begin{bmatrix} -Q & -V_1^* & P - W + V_1^* \overline{F}_j B^* & 0 \\ * & \delta I - \mathbf{He}(V_2) & -V_3 + V_2^* \overline{F}_j B^* & -V_4 \\ * & * & \mathbf{He}\left[AW + B\overline{F}_j V_3 + BF_j a\mathscr{K}\right] & B\overline{F}_j V_4 + WC^* \mathscr{K}^* aF_j D^* \\ * & * & * & -\delta\gamma_2^2 I \end{bmatrix} < 0, \quad a \in \{\underline{\alpha}, \overline{\alpha}\} \tag{27}$$

where $\overline{F}_j := I - F_j$, then the resulting closed-loop system (13) is asymptotically stable and satisfies H_∞-norm bound constraints (16) and (17) not only when all control components are operational, but also in the case of some actuator failures by (12). In this case, the gain of the controller (2) is given by

$$K := \mathscr{K} W^{-1}. \tag{28}$$

Proof. The H_∞-norm bound constraints (16) and (17) are respectively equivalent to the followings

$$J^* \begin{bmatrix} \mathbf{B}_1^* & \mathbf{D}_1^* \\ 0 & I \end{bmatrix}^* \tau\Pi_1 \begin{bmatrix} \mathbf{B}_1^* & \mathbf{D}_1^* \\ 0 & I \end{bmatrix} J < 0, \quad \omega \in R \cup \{\infty\}, \tag{29}$$

$$J^* \begin{bmatrix} \mathbf{B}_2^* & \mathbf{D}_2^* \\ 0 & I \end{bmatrix}^* \delta\Pi_2 \begin{bmatrix} \mathbf{B}_2^* & \mathbf{D}_2^* \\ 0 & I \end{bmatrix} J < 0, \quad \omega = 0 \tag{30}$$

where J and Π_2 are defined by (19), and Π_1 is defined by (24). Then we can get the following from (30) by using Lemma 1

$$\begin{bmatrix} M^* \\ I \end{bmatrix}^* \delta X \begin{bmatrix} M^* \\ I \end{bmatrix} < 0. \tag{31}$$

where X is defined by (21). In view of Lemma 2, (31) is equivalent

$$X < \mathbf{He}\left(\begin{bmatrix} I \\ -M \end{bmatrix} \mathscr{W} \right). \tag{32}$$

To make the problem tractable, the multiplier $\mathscr{W}$ is given by

$$\mathscr{W} := \begin{bmatrix} I \\ 0 \end{bmatrix} WR + \begin{bmatrix} 0 \\ I \end{bmatrix} V \tag{33}$$

where $W = W^* > 0$, $V = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}$. Then (32) is equivalent to

$$X < \mathbf{He}\left(\begin{bmatrix} I & 0 \\ 0 & I \\ -\mathbf{A} & -\mathbf{B}_2 \\ -\mathbf{C} & -\mathbf{D}_2 \end{bmatrix} \begin{bmatrix} 0 & 0 & W & 0 \\ V_1 & V_2 & V_3 & V_4 \end{bmatrix} \right). \tag{34}$$

By defining $\mathscr{K} := KW$ and $\overline{F}_j := I - F_j$, it is equivalent to

$$\begin{bmatrix} -Q & -V_1^* & P - W + V_1^* \overline{F}_j B^* & 0 \\ * & \delta I - \mathbf{He}(V_2) & -V_3 + V_2^* \overline{F}_j B^* & -V_4 \\ * & * & \mathbf{He}\left[AW + B\overline{F}_j V_3 + BF_j \alpha\mathscr{K}\right] & B\overline{F}_j V_4 + WC^* \mathscr{K}^* \alpha F_j D^* \\ * & * & * & -\delta\gamma_2^2 I \end{bmatrix} < 0. \tag{35}$$

We can see that (35) is convex. So if (27) hold, then (35) holds accordingly.

Furthermore, we can get the following from (29) by using Lemma 3

$$\begin{bmatrix} \mathbf{A} & I \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} 0 & P_1 \\ P_1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A} & I \\ \mathbf{C} & 0 \end{bmatrix}^* + \begin{bmatrix} \mathbf{B}_1 & 0 \\ \mathbf{D}_1 & I \end{bmatrix} \tau\Pi_1 \begin{bmatrix} \mathbf{B}_1 & 0 \\ \mathbf{D}_1 & I \end{bmatrix}^* < 0, \tag{36}$$

where $P_1 = P_1^* > 0$. It is equivalent to

$$\begin{bmatrix} P_1\mathbf{A}^* + \mathbf{A}P_1 + \tau\mathbf{B}_1\mathbf{B}_1^* & P_1\mathbf{C}^* + \tau\mathbf{B}_1\mathbf{D}_1^* \\ \mathbf{C}P_1 + \tau\mathbf{D}_1\mathbf{B}_1^* & \tau\mathbf{D}_1\mathbf{D}_1^* - \tau\gamma_1^2 I \end{bmatrix} < 0. \tag{37}$$

Using Schur complement Lemma and multiplying by $diag\begin{bmatrix} I & I & \tau I \end{bmatrix}$ from the left side and the right side, respectively, we can get

$$\begin{bmatrix} P_1\mathbf{A}^* + \mathbf{A}P_1 & P_1\mathbf{C}^* & \tau\mathbf{B}_1 \\ \mathbf{C}P_1 & -\tau\gamma_1^2 I & \tau\mathbf{D}_1 \\ \tau\mathbf{B}_1^* & \tau\mathbf{D}_1^* & -\tau I \end{bmatrix} < 0. \tag{38}$$

If (38) holds, it follows that

$$P_1\mathbf{A}^* + \mathbf{A}P_1 < 0, \tag{39}$$

that is to say, the closed-loop system (13) is asymptotically stable.

Let $P_1 := W$, (37) is equivalent to

$$\begin{bmatrix} AW + WA^* + BF_j\alpha KW + WK^*\alpha F_j B^* + \tau B_1 B_1^* & WC^* + WK^*\alpha F_j D^* \\ * & -\tau\gamma_1^2 I \end{bmatrix} < 0. \tag{40}$$

It is easy to see that (40) is also convex for all $j = 1, 2, \ldots, L$. Similarly, if (26) hold, then (40) holds accordingly. Thus, the proof is complete.

Remark 2: As a matter of fact, Theorem 1 is the generalized eigenvalue problem (GEVP). The GEVP is a quasi-convex optimization problem since the constraint is convex and the objective is quasi-convex. It can be solved by using Toolbox of MATLAB.

Then, based on Theorem 1, the following algorithm is presented for the design of a reliable H_∞ state feedback controller:

Algorithm 1:

Step 1. Let $\gamma_2 > 0$ be given. Then conditions (27) are changed to be convex.

Step 2. Define a scalar $\xi > 0$. Replace $\tau\gamma_1^2$ with ξ in (26). Then minimize γ_1^2 subject to the LMI constraints (26) and (27) for all $j = 1, 2, \ldots, L$ and $\delta > 0$, $\xi > 0$, $Q > 0$, $W > 0$, $\tau > 0$, $\xi < \tau\gamma_1^2$.

Step 3. The optimal value of γ_1 and the corresponding feedback gain K are obtained.

IV. Entire frequency approach

In this section, we give the entire frequency approach that the inputs being locked in place are treated as the entire frequency disturbances by using bounded real lemma in the H_∞ framework.

Lemma 4: For all $j = 1, 2, \ldots, L$, if there exist matrices $W = W^* > 0$ and $\mathscr{K}$ such that the following inequalities

$$\begin{bmatrix} \mathbf{He}(AW + BF_j a\mathscr{K}) & WC^* + \mathscr{K}^* aF_j D^* & B_1 \\ CW + DF_j a\mathscr{K} & -\gamma_1 I & 0 \\ B_1^* & 0 & -\gamma_1 I \end{bmatrix} < 0, \tag{41}$$

$$\begin{bmatrix} \mathbf{He}(AW + BF_j a\mathscr{K}) & WC^* + \mathscr{K}^* aF_j D^* & B(I - F_j) \\ CW + DF_j a\mathscr{K} & -\gamma_2 I & 0 \\ (I - F_j)B^* & 0 & -\gamma_2 I \end{bmatrix} < 0, \tag{42}$$

hold with $a \in \{\underline{\alpha}, \overline{\alpha}\}$, then the resulting closed-loop system (13) is asymptotically stable and meets H_∞-norm bound constraints $\|G_1\|_\infty < \gamma_1$ and $\|G_2\|_\infty < \gamma_2$ not only when all control components are operational, but also in the case of some actuator failures by (12). The parameter K of the controller (2) can be obtained by solving (28).

Remark 3: Lemma 4 is a corollary of Theorem 1 in [2]. Here, the inputs being locked in place are modeled as the entire frequency disturbances and dealt with in the robust H_∞ framework by using bounded real lemma. The result is not exact enough. The conditions (41) and (42) are all convex. Lemma 4 is the eigenvalue problem (EVP). This is a convex optimization problem.

Based on Lemma 4, we summarize:

Algorithm 2:

Step 1. Fix the value of the positive scalar γ_2.

Step 2. Minimize γ_1 subject to the LMI constraints (41) and (42) for all $j = 1, 2, \ldots, L$ and $W > 0$.

Step 3. The values of the parameters γ_1 and K are found.

V. Example

In this section, the proposed design method, of a reliable H_∞ state feedback controller, that control input signals of the actuators being locked in place are treated as zero-frequency disturbances, and the advantage of the proposed method in comparison with the existing one by using bounded real lemma in the H_∞ framework are illustrated via a numerical example of flight tracking control of an F-16 aircraft in [5]. We consider actuator lock-in-place failures only.

The trimmed values of the F-16 aircraft equations and the aircraft model are given by [5].

Our objective is to design a state feedback controller such that

- During normal operation, the closed-loop system is stable, and the output $Sy(t)$ tracks the reference signal $r(t)$ without steady-state error, that is
$$\lim_{t\to\infty} e(t) = 0, \qquad e(t) = r(t) - Sy(t)$$
and with good closed-loop disturbance attenuation performance, where S is a known constant matrix used to form the output required to track the reference signals.
- In the event of actuator lock-in-place failures, the closed-loop system is still stable, and the output $Sy(t)$ tracks the reference signal $r(t)$ without steady-state error and with an acceptable level of performance.

Then we have the following augmented system in the event of the actuator lock-in-place failures

$$\begin{aligned} \dot{\xi}(t) &= A_a\xi(t) + B_aF_ju(t) + G_a\omega_a(t) + B_a(I - F_j)\beta_j, \\ z_a(t) &= C_a\xi(t) + D_aF_ju(t) \end{aligned}$$

where $\xi(t) = [(\int_0^t e(\tau)d\tau)^*, x^T(t)]^*$, $\omega_a(t) = [r^*(t), \omega^*(t)]^*$, and

$$A_a = \begin{bmatrix} 0 & -SC \\ 0 & A \end{bmatrix}, \quad B_a = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad G_a = \begin{bmatrix} I & 0 \\ 0 & G \end{bmatrix},$$

$$C_a = \begin{bmatrix} Q^{\frac{1}{2}} \\ 0 \end{bmatrix}, \quad D_a = \begin{bmatrix} 0 \\ R^{\frac{1}{2}} \end{bmatrix}$$

with $Q = Q^* \geq 0$ and $R = R^* > 0$. Actually, the design problem can be reduced to the following: find a state feedback controller $u(t) = K\xi(t)$ such that the resulting closed-loop system is asymptotically stable and the H_∞-norm bound constraints (16) and (17) hold not only when all control components are operational, but also in the case of some actuators being locked in place, where

$$\begin{aligned} G_1(j\omega) &:= (C_a + D_aF_jK)(j\omega I - A_a - B_aF_jK)^{-1}G_a, \\ G_2(j\omega) &:= (C_a + D_aF_jK)(j\omega I - A_a - B_aF_jK)^{-1}B_a(I - F_j). \end{aligned}$$

We set the parameter value

$$\gamma_2 = 1,$$

and S, Q and R are also given by [5]. Subsequently, minimize γ_1 subject to the above constraints.

In this example, the following possible actuator failures are considered:

(1) $F_1 = diag\{0,1,1,1,1\}$-Right elevator actuator is locked in place;
(2) $F_2 = diag\{1,0,1,1,1\}$-Left elevator actuator is locked in place;
(3) $F_3 = diag\{1,1,0,1,1\}$-Right aileron actuator is locked in place;
(4) $F_4 = diag\{1,1,1,0,1\}$-Left aileron actuator is locked in place.
i) Then minimizing γ_1 by using Theorem 1, the optimal value of γ_1 and the corresponding feedback gain K are found to be:

$$\gamma_{1min} = 2.9110,$$

$$\left[\begin{matrix} -21.5845 & -37.8539 & 4.3525 & 3.5453 & 13.4723 \\ 21.5579 & -37.8419 & -4.8298 & 3.5469 & 13.4874 \\ -18.4254 & -15.6318 & 23.6533 & -18.1407 & 3.7668 \\ 18.4555 & -15.5528 & -24.2170 & -18.3710 & 3.7574 \\ 7.6028 & -0.2723 & 76.2222 & 0.3875 & 0.0908 \end{matrix}\right.$$

$$\left.\begin{matrix} 186.4367 & -1.6488 & 118.1748 & 44.1512 \\ 186.3557 & 1.9534 & -118.1374 & -45.2656 \\ 57.4487 & -9.1727 & 98.2422 & 151.6987 \\ 58.2752 & 9.3023 & -98.4415 & -152.3296 \\ -0.7919 & -27.9242 & -50.9081 & 374.5282 \end{matrix}\right].$$

ii) And then minimize γ_1 by using Lemma 4. As a result, we obtain:

$$\gamma_{1min} = 8.0696,$$

$$\left[\begin{matrix} -63.6357 & -55.4925 & 72.2317 & 34.7712 & 55.6320 \\ 63.9780 & -56.0264 & -72.6867 & 36.0239 & 56.2004 \\ -82.2185 & -28.9185 & 79.7869 & -63.1665 & 25.1098 \\ 82.4320 & -29.3711 & -80.3975 & -61.9038 & 25.5939 \\ 79.8268 & -0.1380 & 76.5266 & -0.2387 & 0.1151 \end{matrix}\right.$$

$$\left.\begin{matrix} 425.3992 & -74.6129 & 226.8712 & 370.3879 \\ 432.8249 & 75.4229 & -228.4671 & -372.0664 \\ 117.5343 & -82.8557 & 277.4170 & 360.3813 \\ 124.6874 & 83.1263 & -278.1580 & -360.9192 \\ 0.5645 & -76.5288 & -198.7086 & 575.2798 \end{matrix}\right].$$

The actual achieved γ_1 for the normal case are respectively 2.8542 by the proposed method and 3.8306 by the entire frequency approach, and the actual achieved values of γ_1 and γ_2 for some failure modes are as follows:

F_i	$diag\{1,1,0,1,1\}$		$diag\{1,1,1,0,1\}$	
Actual values	γ_1	γ_2	γ_1	γ_2
Our design	2.8920	0.3673	2.8701	0.3674
BRL	3.8384	0.3669	3.8287	0.3670

Comparisons of simulation results are given in Fig.1-Fig.8.

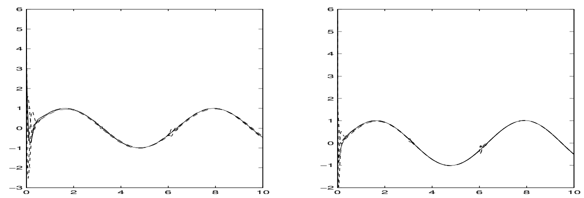

Fig. 1. Stability axis roll rate for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

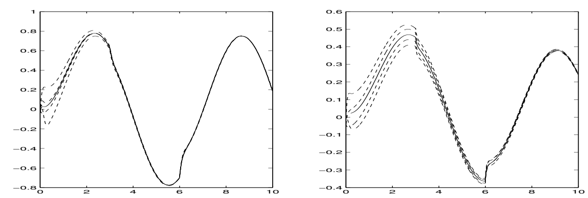

Fig. 2. Angle of attack for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

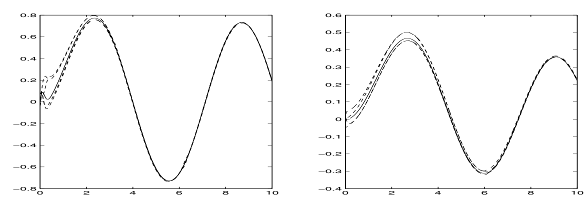

Fig. 3. Angle of sideslip for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

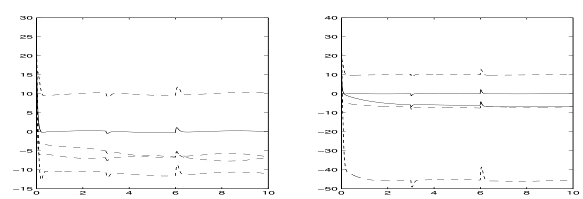

Fig. 4. Right elevator deflection for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

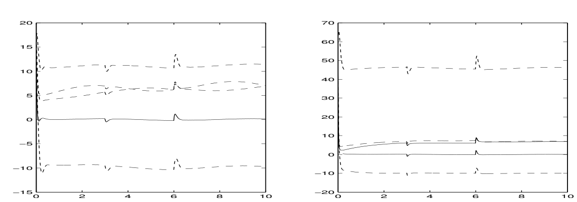

Fig. 5. Left elevator deflection for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

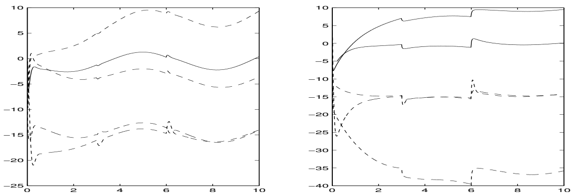

Fig. 6. Right aileron deflection for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

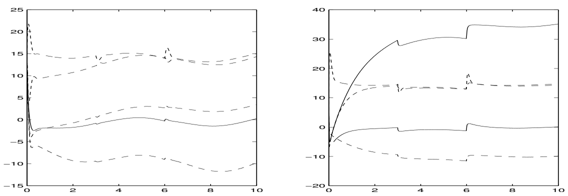

Fig. 7. Left aileron deflection for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

Based on the output responses and the actual achieved values of γ_1 and γ_2, it is easy to see that the new proposed method is less conservative than the method given by bounded real lemma in most conditions for the example, and the obtained H_∞ performance (tracking performance) by using our design procedure is better than that by using bounded real lemma. So, compared with the existing method given by bounded real lemma, the new proposed method can be a good alternative for designing reliable H_∞ state feedback controllers with actuators being locked in place.

VI. CONCLUSION

In this paper, the design problem of reliable H_∞ state feedback controllers for linear continuous-time systems has been addressed. A new method is developed to guarantee the H_∞-norm bound constraints in addition to closed-loop asymptotic stability not only when the system is operating properly, but also in the event of some actuator failures. Sufficient conditions for existence of feasible controllers are given in terms of solutions to a set of linear matrix inequalities (LMIs). A numerical example of flight tracking control of an F-16 aircraft is given to illustrate the proposed design method and demonstrate its effectiveness.

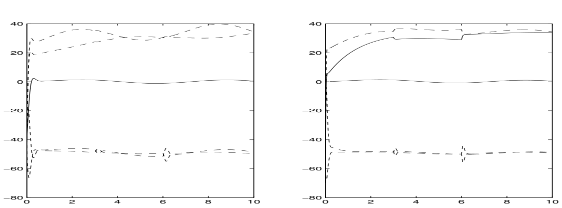

Fig. 8. Rudder deflection for normal case (solid) and faulty cases 1-4 (dashed) by using the proposed method (left) and the entire frequency method (right)

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