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# Adaptive Neural Control of SISO Non-Affine Nonlinear Time-Delay Systems with Unknown Hysteresis Input

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*Abstract*— **In this paper, adaptive neural control is investigated for a class of SISO unknown non-affine nonlinear systems with state time-varying delays and unknown hysteresis input. The non-affine problem is solved by adopting mean value theorem and implicit function theorem. The unknown time-varying delay uncertainties are compensated for using appropriate Lyapunov-Krasovskii functionals in the design. The effect of the unknown hysteresis with the Prandtl-Ishlinskii model is also mitigated through the proposed adaptive control. By utilizing the Lyapunov synthesis, the closed-loop control system is proved to be semi-globally uniformly ultimately bounded (SGUUB).**

## I. Introduction

Control of nonlinear systems preceded by unknown hysteresis nonlinearities has been an active topic, since the hysteresis nonlinearities are common in many industrial processes. It is difficult to control a system with hysteresis nonlinearities, because they are non-differentiable nonlinearities and severely limit system performance such as giving rise to undesirable inaccuracy or oscillations, even leading to instability [1]. Due to the nonsmooth characteristics of hysteresis nonlinearities, traditional control methods are insufficient in dealing with the effects of unknown hysteresis. Therefore, the advanced control techniques to mitigate the effects of hysteresis has been called upon and has been studied for decades.

The most common approach is to construct an inverse operator to cancel the effects of the hysteresis in [1] and [2]. However, it is a challenging work to construct the inverse operator for the hysteresis, due to the complexity and uncertainty of hysteresis. As an alternative, approaches combining the hysteresis model with the control technique without constructing an inverse model have also been developed in [3], [4], [5] and [6] . However, in the above works, all the systems are affine in control inputs and the nonlinear functions are assumed to be known, which limit the applications of those proposed control. In our previous work [7], adaptive variable structure neural control was investigated for a class of unknown nonlinear systems in a Brunovsky form with state time-varying delays and unknown hysteresis inputs. To deal with the presence of function uncertainties, approximation based techniques using neural networks was used, since the neural networks has the universal approximation capabilities, learning and adaptation, parallel distributed structures [8], [9], and [10]. The unknown time-varying delay uncertainties were compensated for using appropriate Lyapunov-Krasovskii functionals in the design.

In this paper, we extend the results of [7] to the class of unknown non-affine nonlinear systems with state time-varying delays and unknown hysteresis inputs. To deal with the non-affine problem in the control variable and virtual ones, mean value theorem and implicit function theorem are adopted to transform the system to affine form, motivated by the works [11] and [12]. The control directions problem is well dealt with by using Nussbaum functions [13].

## II. Problem Formulation and Preliminaries

### A. Problem Formulation

Consider a class of SISO non-affine nonlinear time-delay systems preceded by unknown hysteresis in the following

$$\begin{aligned}\dot{x}_j &= f_j(\bar{x}_j, x_{j+1}) + h_j(\bar{x}_{\tau_j}), \quad 1 \le j \le n-1 \\ \dot{x}_n &= f_n(x,u) + h_n(x_\tau) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, ..., x_n]^T \in R^n$ is the the vector of delay-free states; $\bar{x}_j = [x_1, ..., x_j]^T \in R^j$ is the vector of delay-free states of the first $j$ differential equations; $x_{\tau_j} = x_j(t - \tau_j(t))$ denotes the delayed states, and $\tau_j(t)$ is the unknown time-varying state delays; $\bar{x}_{\tau_j} = [x_1(t-\tau_1(t)), ..., x_j(t-\tau_j(t))]^T$ is the vector of delayed states for the first $j$ differential equations; $x_\tau = [x_{\tau_1}, ..., x_{\tau_n}]^T$ includes all the delayed states; $f_j(\cdot)$ and $h_j(\cdot)$ are unknown smooth functions; $y \in R$ is the output of the system; and $u \in R$ is the input of the system and the output of the hysteresis nonlinearity, which is represented by the Prandtl-Ishlinskii model as follows

$$u(t) = p_0 v(t) - d[v](t) \tag{2}$$

$$d[v](t) = \int_0^R p(r) F_r[v](t) dr \tag{3}$$

$$\begin{aligned} F_r[v](0) &= f_r(v(0), 0) \\ F_r[v](t) &= f_r(v(t), F_r[v](t_i)), \text{ for } t_i < t \le t_{i+1}, \\ & \qquad 0 \le i \le N-1 \\ f_r(v,w) &= \max(v - r, \min(v + r), w)) \end{aligned}$$

with $p_0 = \int_0^R p(r)dr$, $p(r)$ is a density function, satisfying $p(r) \ge 0$ with $\int_0^\infty rp(r)dr < \infty$, and $F_r$ is called as the play operator. Since $p(r)$ vanishes for large values of $r$, the choice of $R = \infty$ as the upper limit of integration in the literature is just a matter of convenience. In addition, the

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function $v$ is monotone on each of the subintervals $(t_i, t_{i+1}]$, $0 \leq i \leq N-1$, $N$ is a positive integer. See [5] and [6] for the details.

The control objective is to design an adaptive neural controller $v(t)$ for system (1) (2) such that all signals in the closed-loop system are bounded, while the output $y$ follows the specified desired trajectory $y_d$.

For convenience of analysis, we define $g_j(\bar{x}_j, x_{j+1}) = \partial f_j(\bar{x}_j, x_{j+1})/\partial x_{j+1}$ and $g_n(x,u) = \partial f_n(x,u)/\partial u$, $j = 1, ..., n-1$, which are also unknown nonlinear functions.

To facilitate control design later in Section III, we need make the following assumptions and useful lemmas.

*Assumption 1:* The desired trajectory $y_d$, and their time derivatives up to the $n$th order, are continuous and bounded.

*Assumption 2:* There exist constants $\underline{g}_j$ and $\bar{g}_j$ such that $0 < \underline{g}_j \leq |g_j(\cdot)| \leq \bar{g}_j < \infty$, for $j = 1, ..., n$. The constants $\underline{g}_j$ and $\bar{g}_j$ are used to handle the stability analysis only. In addition, the signs of $g_j(\cdot)$, for $j = 1, ..., n-1$ are unknown, and the sign of $g_n(\cdot)$ is known. Without loss of generality, we shall assume that the sign of $g_n(\cdot)$ is positive.

*Assumption 3:* The unknown continuous function $h_j(\bar{x}_{\tau_j})$ satisfy the inequality

$$|h_j(\bar{x}_{\tau_j})| \leq \varrho_j(\bar{x}_{\tau_j}) \tag{4}$$

with $\varrho_j(\cdot)$ being known positive continuous functions.

*Assumption 4:* The unknown state time-varying state delays $\tau_j(t)$ satisfy the inequality

$$0 \leq \tau_j(t) \leq \tau_{max}, \ \dot{\tau}_j(t) \leq \bar{\tau}_{max} < 1, \quad 1 \leq j \leq n$$

with the known constants $\tau_{max}$ and $\bar{\tau}_{max}$.

*Assumption 5:* The hysteresis output, $u$, is not available.

*Assumption 6:* There exist known constants $p_{0min}$ and $p_{max}$, such that $p_0 > p_{0min}$, and $p(r) \leq p_{max}$ for all $r \in [0, R]$.

*Lemma 1:* (Implicit Function Theorem) For a continuously differentiable function $f(x,u) : R^n \times R \to R$, if there exists a positive constant $\delta$ such that $|\partial f(x,u)/\partial u| > \delta > 0$, $\forall (x,u) \in R^n \times R$. Then there exists a continuous (smooth) function $u^* = u(x)$ such that $f(x, u^*) = 0$ [11].

*Lemma 2:* (Mean Value Theorem) Assume that $f(x,y) : R^n \times R \to R$ has a derivative (finite or infinite) at each point of an open set $R^n \times (a,b)$, and assume also that it is continuous at both endpoints $y = a$ and $y = b$. Then there is a point $\xi \in (a,b)$ such that $f(x,b) - f(x,a) = f'(x,\xi)(b-a)$ [14].

### B. Nussbaum Function Properties

A function $N(\zeta)$ is called a Nussbaum-type function if it has the following properties:

$$\text{(i)} \lim_{s \to +\infty} \sup \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty$$

$$\text{(ii)} \lim_{s \to +\infty} \inf \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty$$

For clarity, the even Nussbaum function, $N(\zeta) = \exp(\zeta^2)\cos((\pi/2)\zeta)$ is used throughout this paper.

*Lemma 3:* [13] Let $V(\cdot)$, $\zeta(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0$, $\forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$\begin{aligned} V(t) \quad &\leq \quad c_0 + e^{-c_1 t} \int_0^t g(x(\tau)) N(\zeta) \dot{\zeta} e^{c_1 \tau} d\tau \\ &\quad + e^{-c_1 t} \int_0^t \dot{\zeta} e^{c_1 \tau} d\tau, \quad \forall t \in [0, t_f) \end{aligned}$$

where $c_0$ represents some suitable constant, $c_1$ is a positive constant, and $g(x(\tau))$ is a time-varying parameter which takes values in the unknown closed intervals $I = [l^-, l^+]$, with $0 \notin I$, then $V(t)$, $\zeta(t)$, $\int_0^t g(x(\tau)) N(\zeta) \dot{\zeta} d\tau$ must be bounded on $[0, t_f)$.

According to Proposition 2 [15], if the solution of the resulting closed-loop system is bounded, then $t_f = \infty$.

Throughout this paper, $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$, $\|\cdot\|$ denotes the 2-norm, $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the smallest and largest eigenvalues of a square matrix $(\cdot)$, respectively.

## III. Control Design and Stability Analysis

In this section, we will investigate the adaptive neural control for the system (1) (2) using backstepping method combined with mean value theorem, implicit function theorem and neural networks. For clarity, we define $z_j = x_j - \alpha_{j-1}$, $j = 1, ..., n$, where $\alpha_j$ is an intermediate control, $\alpha_0 = y_d$.

**Step j** ($1 \leq j \leq n-1$)**:** The derivative of $z_j$ is given by

$$\dot{z}_j = f_j(\bar{x}_j, x_{j+1}) + h_j(\bar{x}_{\tau_j}) - \dot{\alpha}_{j-1} \tag{5}$$

Using Lemma 1 and Lemma 2, there exists a constant $\lambda_j$ $(0 < \lambda_j < 1)$ such that

$$f_j(\bar{x}_j, x_{j+1}) \quad = \quad f_j(\bar{x}_j, x^*_{1,i_1+1}) + g_{\lambda_j}(x_{j+1} - x^*_{j+1}) \tag{6}$$

where $g_{\lambda_j} = g_{\lambda_j}(\bar{x}_j, \xi_{j+1}) = \frac{\partial f_j(\cdot)}{\partial x_{j+1}}\Big|_{x_{j+1} = \xi_{j+1}}$, and $\xi_{j+1} = \lambda_j x_{j+1} + (1 - \lambda_j) x^*_{j+1}$. Substituting (6) into (5), we obtain the tracking error dynamic as follows

$$\begin{aligned} \dot{z}_j \quad &= \quad g_{\lambda_j}(z_{j+1} + \alpha_j) + f_j(\bar{x}_j, x^*_{j+1}) - g_{\lambda_j} x^*_{j+1} \\ &\quad + h_j(\bar{x}_{\tau_j}) - \dot{\alpha}_{j-1} \end{aligned} \tag{7}$$

where

$$\dot{\alpha}_{j-1} \quad = \quad \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_k} \Big[ f_k(\bar{x}_k, x_{k+1}) + h_k(\bar{x}_{\tau_k}) \Big] + \omega_{j-1}$$

$$\begin{aligned} \omega_{j-1} \quad &= \quad \frac{\partial \alpha_{j-1}}{\partial \zeta_{j-1}} \dot{\zeta}_{j-1} + \sum_{k=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial y_d^{(k)}} \dot{y}_d^{(k+1)} \\ &\quad + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \hat{W}_{1,k}} \dot{\hat{W}}_{1,k} \end{aligned}$$

Define $V_{z_j} = \frac{1}{2}z_j^2$, its time derivative along (7) is

$$\begin{aligned} \dot{V}_{z_j} &= z_j\Big[g_{\lambda_j}(z_{j+1}+\alpha_j) + f_j(\bar{x}_j, x_{j+1}^*) - g_{\lambda_j}x_{j+1}^* \\ &\quad -\omega_{j-1} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_k}f_k(\bar{x}_k, x_{k+1})\Big] \\ &\quad +z_j\Big[-\sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_k}h_k(\bar{x}_{\tau_k}) + h_j(\bar{x}_{\tau_k})\Big] \end{aligned} \tag{8}$$

Applying Assumption 3 and using Young's Inequality, we have

$$\begin{aligned} \dot{V}_{z_j} &\leq z_j\Big[g_{\lambda_j}(z_{j+1}+\alpha_j) + f_j(\bar{x}_j, x_{j+1}^*) - g_{\lambda_j}x_{j+1}^* \\ &\quad -\omega_{j-1} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_k}f_k(\bar{x}_k, x_{k+1})\Big] \\ &\quad +\frac{z_j^2}{2}\sum_{k=1}^{j-1}\Big(\frac{\partial\alpha_{j-1}}{\partial x_k}\Big)^2 + \frac{z_j^2}{2} + \frac{1}{2}\sum_{k=1}^{j}\varrho_k^2(\bar{x}_{\tau_k}) \end{aligned} \tag{9}$$

Consider the following Lyapunov-Krasovskii functional

$$V_{U_j}(t) = \frac{1}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{j}\int_{t-\tau_k(t)}^{t}\varrho_k^2(\bar{x}_k(\tau))d\tau \tag{10}$$

Its time derivative is

$$\begin{aligned} \dot{V}_{U_j}(t) &= \frac{1}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{j}\varrho_k^2(\bar{x}_k(t)) - \\ &\quad \frac{1}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{j}\varrho_k^2(\bar{x}_{\tau_k}(t))(1-\dot{\tau}_k(t)) \end{aligned} \tag{11}$$

Combining (9) and (11), we have

$$\begin{aligned} &\dot{V}_{z_j} + \dot{V}_{U_j}(t) \\ &\leq z_j\Big[g_{\lambda_j}(z_{j+1}+\alpha_j) + Q_j(Z_j)\Big] \\ &\quad +\frac{1}{2(1-\bar{\tau}_{max})}\Big[1-\frac{z_j^2}{c_{z_j}^2}\Big]\sum_{k=1}^{j}\varrho_k^2(\bar{x}_j(t)) \end{aligned} \tag{12}$$

where

$$\begin{aligned} &Q_j(Z_j) \\ =\;& f_j(\bar{x}_j, x_{j+1}^*) - g_{\lambda_j}x_{j+1}^* - \omega_{j-1} \\ &-\sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_k}f_k(\bar{x}_k, x_{k+1}) + \frac{z_j}{2}\sum_{k=1}^{j-1}\Big(\frac{\partial\alpha_{j-1}}{\partial x_k}\Big)^2 \\ &+\frac{z_j}{2} + \frac{z_j}{2(1-\bar{\tau}_{max})c_{z_j}^2}\sum_{k=1}^{j}\varrho_k^2(\bar{x}_k(t)) \end{aligned} \tag{13}$$

with $Z_j = [\bar{x}_j, \alpha_{j-1}, \omega_{j-1}, \frac{\partial\alpha_{j-1}}{\partial x_1}, \frac{\partial\alpha_{j-1}}{\partial x_2}, ..., \frac{\partial\alpha_{j-1}}{\partial x_{j-1}}] \in \Omega_{Z_j} \subset R^{2j+1}$ and $c_{z_j}$ is a positive design constant that can be chosen arbitrarily small.

Let $\hat{W}_j^T S(Z_j)$ be the approximation of the function $Q_j(Z_j)$ (13) on the compact set $\Omega_{Z_j}$, then we have

$$Q_j(Z_j) = \hat{W}_j^T S(Z_j) - \tilde{W}_j^T S(Z_j) + \epsilon_j(Z_j) \tag{14}$$

where the approximation error $\epsilon_j(Z_j)$ satisfies $|\epsilon_j(Z_j)| \leq \epsilon_j^*$ with positive constant $\epsilon_j^*$.

Consider the following fictitious control law $\alpha_j$:

$$\alpha_j = N(\zeta_j)\Big[k_j(t)z_j + \hat{W}_j^T S(Z_j)\Big] \tag{15}$$

$$\dot{\zeta}_j = k_j(t)z_j^2 + \hat{W}_j^T S(Z_j)z_j \tag{16}$$

$$k_j(t) = k_{ja} + k_{jb}(t)$$

where $N(\zeta_j) = e^{\zeta_j^2}\cos((\pi/2)\zeta_j)$, $k_{ja}$ is a positive constant and $k_{jb}(t)$ is chosen as

$$k_{jb}(t) = \frac{k_{jc}}{2(1-\bar{\tau}_{max})z_j^2}\sum_{k=1}^{j}\int_{t-\bar{\tau}_{\max}}^{t}\varrho_k^2(\bar{x}_k(t))d\tau \tag{17}$$

with $k_{jc}$ a positive constant specified by the designer.

The adaptive tuning law is defined as

$$\dot{\hat{W}}_j = \Gamma_{wj}[S(Z_j)z_j - \sigma_{wj}\hat{W}_j] \tag{18}$$

where $\Gamma_{wj} = \Gamma_{wj}^T > 0$, $\sigma_{wj}$ is a positive design constant.

Consider the following Lyapunov function candidate

$$V_j = V_{z_j} + V_{U_j} + \frac{1}{2}\tilde{W}_j^T\Gamma_{wj}^{-1}\tilde{W}_j \tag{19}$$

Its derivative with respect to time $t$ is

$$\dot{V}_j = \dot{V}_{z_j} + \dot{V}_{U_j} + \tilde{W}_j^T\Gamma_{wj}^{-1}\dot{\hat{W}}_j \tag{20}$$

Substituting (12) into (20), noting (14) and using control laws (15) and (16), it follows that

$$\begin{aligned} \dot{V}_j &\leq g_{\lambda_j}z_jz_{j+1} + g_{\lambda_j}N(\zeta_j)\dot{\zeta}_j + \tilde{W}_j^T\Gamma_{wj}^{-1}\dot{\hat{W}}_j \\ &\quad +z_j[\hat{W}_j^T S(Z_j) - \tilde{W}_j^T S(Z_j) + \epsilon_j(Z_j)] \\ &\quad +\frac{1}{2(1-\bar{\tau}_{max})}\Big[1-\frac{z_j^2}{c_{z_j}^2}\Big]\sum_{k=1}^{j}\varrho_k^2(\bar{x}_k(t)) \end{aligned} \tag{21}$$

Adding and subtracting $\dot{\zeta}_j$ on the right hand side of (21) and using adaption law (18), we have

$$\begin{aligned} \dot{V}_j &\leq g_{\lambda_j}z_jz_{j+1} + g_{\lambda_j}N(\zeta_j)\dot{\zeta}_j + \dot{\zeta}_j \\ &\quad -k_j(t)z_j^2 + |z_j|\epsilon_j^* - \sigma_{wj}\tilde{W}_j^T\hat{W}_j \\ &\quad +\frac{1}{2(1-\bar{\tau}_{max})}\Big[1-\frac{z_j^2}{c_{z_j}^2}\Big]\sum_{k=1}^{j}\varrho_k^2(\bar{x}_k(t)) \end{aligned} \tag{22}$$

By completion of squares and using Young's inequality, the following inequalities hold:

$$-\sigma_{wj}\tilde{W}_j^T\hat{W}_j \leq -\frac{\sigma_{wj}\|\tilde{W}_j\|^2}{2} + \frac{\sigma_{wj}\|W_j^*\|^2}{2} \tag{23}$$

$$|z_j|\epsilon_j^* \leq \frac{k_{ja}z_j^2}{2} + \frac{\epsilon_j^{*2}}{2k_{ja}} \tag{24}$$

$$g_{\lambda_j}z_jz_{j+1} \leq \frac{k_{ja}z_j^2}{4} + \frac{\bar{g}_j z_{j+1}^2}{k_{ja}} \tag{25}$$

For the last term in (22), if $|z_j| > c_{z_j}$, then it is less than zero; if $|z_j| \leq c_{z_j}$, then it is bounded. Therefore, we have

$$\frac{1-\frac{z_j^2}{c_{z_j}^2}}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{j}\varrho_k^2(\bar{x}_k(t)) \leq \frac{\varrho_{j\max}}{2(1-\bar{\tau}_{max})} \tag{26}$$

where $\varrho_{j\max} = \max \sum_{k=1}^{j} \varrho_k^2(\bar{x}_k(t))$ as $|z_j| \le c_{z_j}$.

Substituting (23)-(26) to (22), we have

$$\dot{V}_j \le -c_j V_j + g_{\lambda_j} N(\zeta_j)\dot{\zeta}_j + \dot{\zeta}_j + \mu_j + \frac{\bar{g}_j z_{j+1}^2}{k_{ja}} \quad (27)$$

where

$$c_j = \min\left\{\frac{k_{ja}}{2}, k_{jc}, \frac{\sigma_{wj}}{\lambda_{\max}(\Gamma_{wj}^{-1})}\right\}$$
$$\mu_j = \frac{\sigma_{wj}\|W_j^*\|^2}{2} + \frac{\epsilon_j^{*2}}{2k_{ja}} + \frac{\varrho_{j\max}}{2(1-\bar{\tau}_{max})} \quad (28)$$

Multiplying (27) by $e^{c_j t}$ and integrating it over $[0,t]$, we have

$$0 \le V_j \le C_j + e^{-c_j t}\int_0^t [g_{\lambda_j} N(\zeta_j) + 1]\dot{\zeta}_j e^{c_j\tau} d\tau$$
$$+ e^{-c_j t}\int_0^t \frac{\bar{g}_j z_{j+1}^2}{k_{ja}} e^{c_j\tau} d\tau \quad (29)$$

with $C_j = V_j(0) + \mu_j/c_j$.

By carefully examining the extra term $e^{-c_j t}\int_0^t \frac{\bar{g}_j z_{j+1}^2}{k_{ja}} e^{c_j\tau} d\tau$ in (29), we have the following inequality

$$e^{-c_j t}\int_0^t \frac{\bar{g}_j z_{j+1}^2}{k_{ja}} e^{c_j\tau} d\tau \le \frac{\bar{g}_j}{k_{ja} c_j} \sup_{\tau\in[0,t]} [z_{j+1}^2(t)] \quad (30)$$

From (30), if $z_{j+1}$ can be regulated as bounded, then the extra term $e^{-c_j t}\int_0^t \frac{\bar{g}_j z_{j+1}^2}{k_{ja}} e^{c_j\tau} d\tau$ in (29) is also bounded. Then, from Lemma 3, the boundedness of $z_j$ and $\hat{W}_j$ can be guaranteed. The boundedness of $z_{j+1}$ will be proved in the following steps.

**Step n:** This is the final step. In this step, we will design the control input $v(t)$. Since $z_n = x_n - \alpha_{n-1}$, its derivative is given by

$$\dot{z}_n = f_n(x,u) + h_n(x_\tau) - \dot{\alpha}_{n-1} \quad (31)$$

Similarly, by implicit function theorem in Lemma 1 and mean value theorem in Lemma 2, and using (2) lead to

$$\dot{z}_n = g_{\lambda_n} p_0 v(t) - g_{\lambda_n} d[v](t) + f_n(x,u^*)$$
$$- g_{\lambda_n} u^* + h_n(x_\tau) - \dot{\alpha}_{n-1} \quad (32)$$

where

$$g_{\lambda_n} = g_{\lambda_n}(x,\xi_{n+1}) = \frac{\partial f_n(\cdot)}{\partial u}\Big|_{u=\xi_{n+1}}$$
$$\xi_{n+1} = \lambda_n u + (1-\lambda_n)u^*, \quad 0 < \lambda_n < 1$$
$$\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1} \frac{\partial\alpha_{n-1}}{\partial x_k}\dot{x}_k + \omega_{n-1}$$
$$= \sum_{k=1}^{n-1} \frac{\partial\alpha_{n-1}}{\partial x_k}\Big[f_k(\bar{x}_k, x_{k+1}) + h_k(\bar{x}_{\tau_k})\Big] + \omega_{n-1}$$
$$\omega_{n-1} = \frac{\partial\alpha_{n-1}}{\partial\zeta_{n-1}}\dot{\zeta}_{n-1} + \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}} y_d^{(k+1)} + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{W}_k}\dot{\hat{W}}_k$$

Define $V_{z_n} = \frac{1}{2p_0} z_n^2$, its time derivative along (32) is

$$\dot{V}_{z_n} = z_n g_{\lambda_n} v(t) - \frac{1}{p_0} z_n g_{\lambda_n} d[v](t)$$
$$+ \frac{1}{p_0} z_n \Big[f_n(x,u^*) - g_{\lambda_n} u^* - \omega_{n-1} - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k} f_k(\bar{x}_k, x_{k+1})\Big]$$
$$+ \frac{1}{p_0} z_n \Big[-\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k} h_k(\bar{x}_{\tau_k}) + h_n(x_\tau)\Big]$$

Applying Assumption 3 and using Young's Inequality, we have

$$\dot{V}_{z_n} \le z_n g_{\lambda_n} v(t) - \frac{1}{p_0} z_n g_{\lambda_n} d[v](t) + \frac{1}{p_0} z_n \Big[f_n(x,u^*)$$
$$- g_{\lambda_n} u^* - \omega_{n-1} - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k} f_k(\bar{x}_k, x_{k+1})\Big]$$
$$+ \frac{z_n^2}{2p_0^2}\sum_{k=1}^{n-1}\Big(\frac{\partial\alpha_{n-1}}{\partial x_k}\Big)^2 + \frac{z_n^2}{2p_0^2} + \frac{1}{2}\sum_{k=1}^{n}\varrho_k^2(\bar{x}_{\tau_k}) \quad (33)$$

Similarly, we consider the following Lyapunov-Krasovskii functional

$$V_{U_n}(t) = \frac{1}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{n}\int_{t-\tau_k(t)}^{t} \varrho_k^2(\bar{x}_k(\tau)) d\tau$$

Its time derivative is

$$\dot{V}_{U_n}(t) = \frac{1}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{n}\varrho_k^2(\bar{x}_k(t)) - \frac{1}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{n}\varrho_k^2(\bar{x}_{\tau_k}(t))(1-\dot{\tau}_k(t)) \quad (34)$$

Combining (33) and (34), we have

$$\dot{V}_{z_n} + \dot{V}_{U_n}(t) \le z_n\Big[g_{\lambda_n} v(t) - \frac{1}{p_0} g_{\lambda_n} d[v](t) + Q_n(Z_n)\Big]$$
$$+ \frac{1}{2(1-\bar{\tau}_{max})}\Big[1 - \frac{z_n^2}{c_{z_n}^2}\Big]\sum_{k=1}^{n}\varrho_k^2(\bar{x}_k(t)) \quad (35)$$

where

$$Q_n(Z_n) = \frac{1}{p_0}\Big[f_n(x,u^*) - g_{\lambda_n} u^* - \omega_{n-1} - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k} f_k(\bar{x}_k, x_{k+1})\Big] + \frac{z_n}{2p_0^2}\sum_{k=1}^{n-1}\Big(\frac{\partial\alpha_{n-1}}{\partial x_k}\Big)^2$$
$$+ \frac{z_n}{2p_0^2} + \frac{z_n}{2(1-\bar{\tau}_{max})c_{z_n}^2}\sum_{k=1}^{n}\varrho_k^2(\bar{x}_k(t)) \quad (36)$$

with $Z_n = [x, \alpha_{n-1}, \omega_{n-1}, \frac{\partial\alpha_{n-1}}{\partial x_1}, \frac{\partial\alpha_{n-1}}{\partial x_2}, ..., \frac{\partial\alpha_{n-1}}{\partial x_{n-1}}] \in \Omega_{Z_n} \subset R^{2n+1}$ and $c_{z_n}$ is a positive design constant that can be chosen arbitrarily small.

Let $\hat{W}_n^T S(Z_n)$ be the approximation of the function $Q_n(Z_n)$ (36) on the compact set $\Omega_{Z_n}$, then we have

$$Q_n(Z_n) = \hat{W}_n^T S(Z_n) - \tilde{W}_n^T S(Z_n) + \epsilon_n(Z_n) \quad (37)$$

where the approximation error $\epsilon_n(Z_n)$ satisfies $|\epsilon_n(Z_n)| \leq \epsilon_n^*$ with positive constant $\epsilon_n^*$.

Choose the following Lyapunov function candidate

$$V_n = V_{z_n} + V_{U_n} + \frac{1}{2}\tilde{W}_n^T \Gamma_{wn}^{-1} \tilde{W}_n + \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr$$

Its derivative along (35) is

$$\begin{aligned}
\dot{V}_n &= \dot{V}_{z_n} + \dot{V}_{U_n} + \tilde{W}_n^T \Gamma_{wn}^{-1} \dot{\hat{W}}_n \\
&\quad + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\
&\leq z_n\Big[g_{\lambda_n} v(t) - \frac{1}{p_0} g_{\lambda_n} d[v](t) + Q_n(Z_n)\Big] \\
&\quad + \frac{1}{2(1-\bar{\tau}_{max})}\Big[1 - \frac{z_n^2}{c_{z_n}^2}\Big]\sum_{k=1}^n \varrho_k^2(\bar{x}_k(t)) \\
&\quad + \tilde{W}_n^T \Gamma_{wn}^{-1}\dot{\hat{W}}_n + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr
\end{aligned} \quad (38)$$

The following control laws are proposed:

$$\begin{aligned}
v &= N(\zeta_n)\Big[k_n(t) z_n + \hat{W}_n^T S(Z_n)\Big] \\
&\quad - \mathrm{sign}(z_n)\int_0^R \frac{\hat{p}(t,r)}{p_{0\,\min}}|F_r[v](t)|dr \quad (39) \\
\dot{\zeta}_n &= k_n(t) z_n^2 + \hat{W}_n^T S(Z_n) z_n \quad (40) \\
k_n(t) &= k_{na} + k_{nb}(t)
\end{aligned}$$

where $N(\zeta_n) = e^{\zeta_n^2}\cos((\pi/2)\zeta_n)$, $k_{na}$ is a positive constant and $k_{nb}(t)$ is chosen as

$$k_{nb}(t) = \frac{k_{nc}}{2(1-\bar{\tau}_{max}) z_n^2}\sum_{k=1}^n \int_{t-\bar{\tau}_{\max}}^t \varrho_k^2(\bar{x}_k(t))d\tau \quad (41)$$

with $k_{nc}$ a positive constant specified by the designer.

The adaptive tuning laws are defined as

$$\dot{\hat{W}}_n = \Gamma_{wn}[S(Z_n) z_n - \sigma_{wn}\hat{W}_n] \quad (42)$$

$$\frac{\partial}{\partial t}\hat{p}(t,r) = \begin{cases} 0, & if\ \hat{p}(t,r) = p_{max} \\ \frac{\eta|z_n| g_n}{p_{0\,\min}}|F_r[v](t)|, & if\ 0 \leq \hat{p}(t,r) < p_{max} \end{cases} \quad (43)$$

with $\Gamma_{wn} = \Gamma_{wn}^T > 0$, $\sigma_{wn}$ and $\eta$ are positive design constants.

Substituting (37), (39) and (42) into (38) , and add and subtracting $\dot{\zeta}_n$ on the right hand side of (38) lead to

$$\begin{aligned}
\dot{V}_n &\leq g_{\lambda_n} N(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n - k_n(t) z_n^2 + |z_n|\epsilon_n^* \\
&\quad - \sigma_{wn}\tilde{W}_n^T\hat{W}_n - \frac{g_{\lambda_n}|z_n|}{p_{0\,\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|dr \\
&\quad - \frac{z_n}{p_0} g_{\lambda_n} d[v](t) + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\
&\quad + \frac{1}{2(1-\bar{\tau}_{max})}\Big[1 - \frac{z_n^2}{c_{z_n}^2}\Big]\sum_{k=1}^n \varrho_k^2(\bar{x}_k(t))
\end{aligned} \quad (44)$$

By completion of squares and using Young's inequality, the following inequalities hold:

$$\begin{aligned}
-\sigma_{wn}\tilde{W}_n^T\hat{W}_n &\leq -\frac{\sigma_{wn}\|\tilde{W}_n\|^2}{2} + \frac{\sigma_{wn}\|W_n^*\|^2}{2} \quad (45) \\
|z_n|\epsilon_n^* &\leq \frac{k_{na} z_n^2}{2} + \frac{\epsilon_n^{*2}}{2k_{na}} \quad (46)
\end{aligned}$$

Notice

$$\begin{aligned}
&-\frac{g_{\lambda_n}|z_n|}{p_{0\,\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|dr - \frac{z_n}{p_0} g_{\lambda_n} d[v](t) \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\
\leq\ &-\frac{g_{\lambda_n}|z_n|}{p_{0\,\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|dr \\
&+\frac{|z_n|}{p_{0\,\min}} g_{\lambda_n}\int_0^R p(r)|F_r[v](t)|dr \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\
\leq\ &-\frac{g_{\lambda_n}|z_n|}{p_{0\,\min}}\int_0^R \tilde{p}(t,r)|F_r[v](t)|dr \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr
\end{aligned} \quad (47)$$

Case(i): When $r \in R_{max} = \{r : \hat{p}(t,r) = p_{max}\}$, and $R_{max} \subset [0,R]$, according to (43), we have

$$\tilde{p}(t,r) \geq 0, \quad \frac{\partial}{\partial t}\hat{p}(t,r) = 0 \quad (48)$$

Substituting (48) into (47), we obtain

$$\begin{aligned}
&-\frac{g_{\lambda_n}|z_n|}{p_{0\,\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|dr - \frac{z_n}{p_0} g_{\lambda_n} d[v](t) \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \leq 0
\end{aligned} \quad (49)$$

Case (ii): When $r \in R_{max}^c$, which is the complement set of $R_{max}$ in $[0,R]$, i.e., $0 \leq \hat{p}(t,r) < p_{max}$, according to (43), we have

$$\frac{\partial}{\partial t}\hat{p}(t,r) = \frac{\eta|z_n| g_n}{p_{0\,\min}}|F_r[v](t)| \quad (50)$$

Substituting (50) into (47), we obtain

$$\begin{aligned}
&-\frac{g_{\lambda_n}|z_n|}{p_{0\,\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|dr - \frac{z_n}{p_0} g_{\lambda_n} d[v](t) \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \leq 0
\end{aligned}$$

Combining Case (i) with Case (ii), we have

$$-\frac{g_{\lambda_n}|z_n|}{p_{0\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|dr - \frac{z_n}{p_0}g_{\lambda_n}d[v](t) + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \le 0 \quad (51)$$

For the last term in (44), if $|z_n| > c_{z_n}$, then it is less than zero; if $|z_n| \le c_{z_n}$, then it is bounded. Therefore, we have

$$\frac{1-\frac{z_n^2}{c_{z_n}^2}}{2(1-\bar{\tau}_{max})}\sum_{k=1}^{n}\varrho_k^2(\bar{x}_k(t)) \quad \le \quad \frac{\varrho_{n\max}}{2(1-\bar{\tau}_{max})} \quad (52)$$

where $\varrho_{n\max} = \max\sum_{k=1}^n \varrho_k^2(\bar{x}_k(t))$ as $|z_n| \le c_{z_n}$.

From Assumption 6 and the adaptation law (43), we know the boundedness of $|\tilde{p}(t,r)| \le p_{max}$, which leads to the boundedness of $\frac{\sigma_p}{2\eta}\int_0^R \tilde{p}^2(t,r)dr \le \frac{\sigma_p R}{2\eta}p_{\max}^2$, where $\sigma_p$ is a positive design constant. Adding and subtracting the term $\frac{\sigma_p}{2\eta}\int_0^R \tilde{p}^2(t,r)dr$ on the right hand side of (44), and substituting (45), (46), (51) and (52) into (44), we have

$$\dot{V}_n \quad \le \quad -c_n V_n + g_{\lambda_n}N(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n + \mu_n \quad (53)$$

where

$$c_n = \min\left\{\frac{k_{na}}{2}, k_{nc}, \frac{\sigma_{wn}}{\lambda_{\max}(\Gamma_{wn}^{-1})}, \sigma_p\right\}$$

$$\mu_n = \frac{\sigma_{wn}\|W_n^*\|^2}{2} + \frac{\epsilon_n^{*2}}{2k_{na}} + \frac{\varrho_{n\max}}{2(1-\bar{\tau}_{max})} + \frac{\sigma_p R}{2\eta}p_{\max}^2$$

Multiplying (53) by $e^{c_n t}$ and integrating it over $[0,t]$, we have

$$0 \quad \le \quad V_n \le C_n + e^{-c_n t}\int_0^t [g_{\lambda_n}N(\zeta_n)+1]\dot{\zeta}_n e^{c_n\tau}d\tau$$

with $C_n = V_n(0) + \mu_n/c_n$. According to Lemma 3, we can conclude that $V_n(t), \zeta_n(t)$, hence $z_n(t), \hat{W}_n$ are SGUUB on $[0,t_f)$. From the boundedness of $z_n$, the boundedness of the extra term $\frac{\bar{g}_{n-1}}{k_{(n-1)a}c_{n-1}}\sup_{\tau\in[0,t]}[z_n^2(t)]$ at Step $(n-1)$ is readily obtained. Applying Lemma 3 backward $(n-1)$ times, we can guarantee that $V_j, z_j, \hat{W}_j$ and hence $x_j$ are SGUUB on $[0,t_f)$. Therefore, no finite time escape phenomenon may happen and $t_f = \infty$. Let $C_{0j}$ be the upper bound of $e^{-c_j t}\int_0^t [g_{\lambda_j}N(\zeta_j)+1]\dot{\zeta}_j e^{c_j\tau}d\tau + \frac{\bar{g}_j}{k_{ja}c_j}\sup_{\tau\in[0,t]}[z_{j+1}^2(t)]$, $j = 1,2,...,n-1$, and $C_{0n}$ be the upper bound of $e^{-c_n t}\int_0^t [g_{\lambda_n}N(\zeta_n)+1]\dot{\zeta}_n e^{c_n\tau}d\tau$, then there exists $T$, such that for all $t > T$, we have $|z_i| \le \sqrt{2(C_j + C_{0j})}$, $\|\tilde{W}_j\| \le \sqrt{2(C_j+C_{0j})/\lambda_{\min}(\Gamma_{wj}^{-1})}$, for $j = 1,2,...,n$.

*Theorem 1:* Consider the closed-loop system consisting of the plant (1), preceded by unknown hysteresis nonlinearities (2) and (3), and the control laws (39) (40) and adaptation laws (42) (43). Under Assumptions 1-6, for bounded initial conditions, the overall closed-loop neural control system is SGUUB in the sense that all of the signals in the closed-loop system are bounded, and the tracking error remains in a compact set.

**Proof:** The proof can be easily completed by following the above design procedures from Step 1 to Step n. ■

## IV. CONCLUSION

Adaptive neural control has been proposed for a class of unknown SISO non-affine nonlinear systems with state time-varying delays and unknown hysteresis input. The non-affine problem has been solved using mean value theorem and implicit function theorem. The unknown time-varying delay uncertainties have been compensated for using appropriate Lyapunov-Krasovskii functionals in the design. The closed-loop control system has been theoretically shown to be SGUUB using Lyapunov synthesis method.

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