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Stabilizability of Networked Control Systems via Packet-loss Dependent Output Feedback Controllers

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Abstract— **We consider the output feedback stabilizability of networked control systems with bounded packet loss. A packet-loss dependent Lyapunov function is adopted to design packet-loss dependent stabilizing output feedback controllers by resolving some linear matrix inequalities. Moreover, two types of packet-loss processes are discussed: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. A numerical example and some simulations are worked out to demonstrate the effectiveness of the proposed design technique.**

I. INTRODUCTION

Networked control systems have many industrial applications, and typical examples are computer integrated manufacturing systems, large-scale distributed industrial processes, tele-operation and tele-control, fieldbus systems, intelligent traffic systems, satellite clusters and group maneuvers, etc. NCSs have received increasing attentions in recent years [1]-[4]. Compared with the traditional point-to-point wiring, the use of the communication channels can reduce the costs of cables and power, simplify the installation and maintenance of the whole system, and increase the reliability.

However, the insertion of communication network in the feedback control loop complicates the application of standard results in analysis and design of a networked control system (NCS) because many ideal assumptions made in the traditional control theory can not be applied to NCSs directly. In a NCS, communication capacity depends not only on the protocol, but also on the topology of the network. One of the issues raised in NCSs is the unreliable transmission paths because of limited bandwidth and large amount of data packet transmitted over one line, which may result in transmission delays, data packet dropout. The last is a potential source of instability and poor performance of NCSs. Therefore construction of a feedback controller using the most fresh information to stabilize a NCS with packet dropout is very essential to the real industrial applications. To our best knowledge, two effectively approaches have been adopted to deal with the bounded packet dropout: the first is delayed system approach [5][8], and the other is switched system approach [6][7]. In addition, two types of packet loss processes have been considered in existing results: the first regards the data packet dropout as a arbitrary process, and sufficient conditions for stabilizability have been presented [6] [7]; the other is that the data packet dropout is taken as a Markov process [7][8], and sufficient conditions for the stabilizability also have been derived. Moreover, [9] modeled NCSs with data packet dropout as asynchronous dynamic systems, but the stability condition derived in it is in bilinear matrix inequalities, which are difficult to solve.

The advantage of the switched system approach is that the controllers can make full use of the previous information to stabilize NCSs when the current state/out measurements are not available from the network. [6] introduced switched system approach earlier, and considered the stabilizability of NCSs with bounded packet losses via both state feedback and output feedback. Based on bounded packet losses, NCSs were modeled as a class of switched systems, and switched systems theories [10]-[13] can be used to design state/out feedback controllers constructed by using the feasible solutions of some linear matrix inequalities(LMIs) in [6]. Recently, [7] generalized the results in [6], and considered the state feedback stabilizability by introducing a packet-loss dependent Lyapunov function. State feedback controllers were designed by using the packet-loss dependent Lyapunov function, while it did not consider the output feedback stabilizability, and the feedback gain is constant.

In this paper, different from existing results, we try to present the design for time-varying output feedback controllers, in details, the time-varying output feedback controllers to be designed are packet-loss dependent. The advantage of such controllers is that we can regulate the gains depending on the number of the packet losses. Here, the switched system approach is adopted to discuss the output feedback stabilizability of NCSs which consists of discrete-time plants and design the packet-loss dependent stabilizing output feedback controllers. Two types of packet-loss processes are considered: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. For both cases, sufficient conditions for stabilizability are derived, and packet-loss dependent output feedback controllers are designed by solving some LMIs.

The paper is organized as follows: Section II gives the description of our systems, and some lemmas are also presented in this section. Section III deals with the stabilizability for

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NCSs with the arbitrary packet-loss process, and presents the design for stabilizing output feedback controllers constructed by using the feasible solutions of some LMIs. Section IV solves the stabilizability for NCSs with Markovian packet-loss process, and also stabilizing output feedback controllers are constructed by solving some LMIs. A numerical example and some simulations demonstrating the effectiveness of the proposed design technique are given in Section V. Finally, the conclusions are provided in Section VI.

Notations. Throughout this paper, the following notations are used. $\mathbf{Z}^+$ denotes the set of all nonnegative integer; For any two positive integers m and n satisfying $n \geq m$, $[m,n] = \{m, m+1, \cdots, n\}$. Furthermore, denote $[m,n] \times [k,l] = \{(i,j) : i \in [m,n], j \in [k,l]\}$.

II. Problem Formulation

Consider the network control system consists of a discrete plant and a time-varying discrete controller

$$\begin{array}{l} x(t+1) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \\ u(t) = F(t)\bar{y}(t), \end{array} \tag{1}$$

where $t \in \mathbf{Z}^+$, $x(t) \in \mathbf{R}^n$ is the plant state vector, $u(t) \in \mathbf{R}^m$ is the plant input vector, and $y(t) \in \mathbf{R}^p$ is the output of the plant. A, B and C are known real constant matrices with proper dimensions. The piecewise continuous function $F(t)$ is the output feedback gain matrix to be designed. $\bar{y}(t) \in \mathbf{R}^p$ is the state measurement that is successfully transmitted over the network. We suppose that a sensor data containing the state information will be put into a single register and substitute the old data when it is successfully sent to the controller through the communication link. The controller reads out the content of the register $\bar{y}(t)$ and utilizes the data to compute the new control input, which will be applied to the plant.

We consider the NCS setup with a clock-driven sensor, and both the controller and the actuator are combined into one event-driven node, that is, network communication only occurs between the sensor and the controller through a communication channel with finite bandwidth. Further, we suppose that the update instants of $\bar{y}(t)$ is numerable and the set of successive update instants $\{t_0 = 0, t_1, \cdots, t_k, \cdots\}$ is the subset of $\mathbf{Z}^+$.

The switched system approach is used to setup a switched system in [6]. Here, we consider the NCS (1) by using the approach. For the case that there are no transmission delays between the sensor and the combined node, the switched system approach is described as follows.

Without loss of generality, we assume that the packet containing $y(0)$ is transmitted to the controller successfully, that is $\bar{y}(0) = y(0)$, then

$$x(1) = (A + BF(0)C)x(0).$$

In the next step, if the data packet containing $y(1)$ is transmitted to the controller successfully, then

$$x(2) = (A + BF(1)C)x(1),$$

otherwise,

$$\begin{array}{l} x(2) = Ax(1) + BF(1)Cx(0) \\ = (A(A + BF(0)C) + BF(1)C)x(0). \end{array}$$

We refer to the time interval between t_k and t_{k+1} as one transmission interval. In this pattern of transmission, the states of the NCS (1) at the update steps can be described as follows.

$$x(t_{k+1}) = (A^{t_{k+1}-t_k} + \sum_{l=0}^{t_{k+1}-t_k-1} A^l BF(t)C)x(t_k), \ k \in \mathbf{Z}^+.$$

Define

$$z(0) = x(0), \ z(1) = x(t_1), \cdots, \ z(k) = x(t_k), \ \cdots,$$

and

$$A(k) = A^{t_{k+1}-t_k} + \sum_{l=0}^{t_{k+1}-t_k-1} A^l BF(t)C,$$

it follows that

$$z(j) = A(j)z(j-1). \tag{2}$$

We assume that the maximum transmission period is d, therefore the upper bound of the dropped data packets is $d-1$. Further, we have

$$A(j) \in \Omega, \ \Omega = \{A_1, A_2, \cdots, A_d\},$$

where

$$A_i = A^i + \sum_{l=0}^{i-1} A^l BF(t)C.$$

We assume that there is a counter which notes the number in the last transmission interval $[t_k, t_{k-1})$. Given feedback gain set $\{F_1, F_2, \cdots, F_d\}$, for any t_k, we take the packet-loss dependent feedback gain as $F_{t_k - t_{k-1}}$.

Now, letting $t_{k+1} - t_k = i, t_k - t_{k-1} = j$, we have

$$\bar{A}_{ij} = A^i + \sum_{l=0}^{i-1} A^l BF_j C, \tag{3}$$

then it is obvious that the evolution of NCS (1) at the transmission instants can be described by the following switched system

$$z(t+1) = \bar{A}_{\eta(t)} z(t), \ t \in \mathbf{Z}^+ \tag{4}$$

for arbitrary switching, where

$$\begin{array}{c} \bar{A}_{\eta(t)} = A^{r(t)} + \sum_{l=0}^{r(t)-1} A^l BF_{r(t-1)}C \in \bar{\Omega} = \\ \{\bar{A}_{11}, \bar{A}_{12}, \cdots, \bar{A}_{1d}, \cdots, \bar{A}_{d1}, \bar{A}_{d2}, \cdots, \bar{A}_{dd}\}. \end{array}$$

$\eta(t) = (r(t), r(t-1)) \in [1,d] \times [1,d]$, Here, $\eta(1) = (r(1), 1), \forall i \in [1,d]$, that is, $y(0)$ is transmitted to the controller successfully.

Now, we present the following definitions and technical lemmas for later use.

Definition 1: [7] A packet-loss process $\{r(t_k) \in \mathbf{N} : r(t_k) = t_{k+1} - t_k\}$ is said to be arbitrary if it takes values in $[1,d]$ arbitrarily.

Definition 2: A packet-loss process $\{r(t_k) \in \mathbf{N} : r(t_k) = t_{k+1} - t_k\}$ is said to be Markovian if it is a Markov chain, and the transition probability matrix $P = [p_{ij}] \in \mathbf{R}^{d\times d}$, where $p_{ij} = \Pr(r(t_{k+1}) = i | r(t_k) = j) \geq 0$ for any $(i,j) \in [1,d] \times [1,d]$, and $\sum_{i=1}^{d} p_{ij} = 1$.

Lemma 1: [14] Given the symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}, \tag{5}$$

where S_{11} is $r \times r$, then the three states are equivalent as followed:

a) $S < 0$;

b) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

c) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2: [13] For a given $C \in \mathbf{R}^{p\times n}$ with $rank(C) = p$, assume that the singular value decomposition of C as

$$C = M \begin{bmatrix} C_0 & 0 \end{bmatrix} N^T \tag{6}$$

where $M \in \mathbf{R}^{p\times p}$ and $N \in \mathbf{R}^{n\times n}$ are unitary matrices and $C_0 \in \mathbf{R}^{p\times p}$ is a diagonal matrix with positive diagonal elements in decreasing order. $Q \in \mathbf{R}^{n\times n}$ is a symmetric matrix, then there exists a matrix $Z \in \mathbf{R}^{p\times p}$ such that $CQ = ZC$ if and only if

$$Q = N \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} N^T \tag{7}$$

where $Q_1 \in R^{p\times p},\ Q_2 \in R^{(n-p)\times(n-p)}$.

III. Stabilizability of NCSs with arbitrary packet-loss process

In this section, we suppose that the packet loss of the NCS (1) is arbitrary. Sufficient conditions for the stabilizability via output feedback is derived by a switched system approach and stabilizing controllers are designed by resolving some LMIs.

Definition 3: A function $\phi : \mathbf{R}^n \to \mathbf{R}_+$ is of class K if it is continuous, strictly increasing, and $\phi(0) = 0$.

Without loss of generality, we assume that 0 is an equilibrium of NCS (1), and the state response starts at $t_0 = 0$ with the initial condition $x(0)$. The following result will ensure the asymptotic stability of NCS (1). It is a generality of Lemma 1 in [6].

Lemma 3: If there exists a piecewise continuous function $V : \mathbf{R}^n \to \mathbf{R}_+$ belonging to the function set $\Omega = \{V_1, V_2, \cdots, V_q\}$ with $q \geq 1$, where $V_l : \mathbf{R}^n \to \mathbf{R}_+$ is a continuous differentiable, locally positive definite function for any $l \in [1,q]$, and functions α, β, γ of class K such that for all $x \in B_r = \{x : \|x\| \leq r\}$,

$$\alpha(\|x\|) \leq V_l(x) \leq \beta(\|x\|),\ \forall l \in [1,q], \tag{8}$$

and

$$\Delta V^k(x(t_k)) = V(x(t_{k+1})) - V(x(t_k)) \leq -\gamma(\|x(t_k)\|), \tag{9}$$

then NCS (1) is uniformly asymptotically stable.

Based on Lemma 3, for NCS (1) we can get the following sufficient condition for the stabilizability via output feedback.

Theorem 1: If there exist d symmetric positive definite matrices $X_1, X_2, \cdots, X_d$ and d matrices $Y_1, Y_2, \cdots, Y_d$, $Z_1, Z_2, \cdots, Z_d$ such that

$$CX_i = Z_i C \tag{10}$$

$$\begin{bmatrix} X_j & (A^i X_j + \sum_{l=0}^{i-1} A^l B Y_j C)^T \\ A^i X_j + \sum_{l=0}^{i-1} A^l B Y_j C & X_i \end{bmatrix} > 0, \quad \forall (i,j) \in [1,d] \times [1,d], \tag{11}$$

then NCS (1) is stabilizable via the output feedback controller

$$u(t) = Y_{r(t_{k-1})} Z_{r(t_{k-1})}^{-1} \bar{y}(t),\ t \in [t_k, t_{k+1}).$$

Proof: Based on Lemma 3, we only need to prove that there exists a feedback gain set $\{F_1, F_2, \cdots, F_d\}$ guaranteeing the stability of switched system (4) for any switching.

It is obvious that the switched system (4) can be represented equivalently by

$$z(t+1) = \sum_{j=1}^{d} \sum_{i=1}^{d} \xi_{ij}(t) \bar{A}_{ij} z(t), t \in \mathbf{Z}^+, \tag{12}$$

where

$$\bar{A}_{ij} = A^i + \sum_{l=0}^{i-1} A^l B F_j C.$$

Based on (12), we know that every number pair $(i,j) \in [1,d] \times [1,d]$ denotes only one subsystem, and $\sum_{j=1}^{d} \sum_{i=1}^{d} \xi_{ij}(t) = 1$ if the (i,j) subsystem is active and 0 otherwise.

Now, for switched system (12), we adopt the following switched Lyapunov function

$$V(t, z(t)) = z^T(t) \sum_{i=1}^{d} \xi_i(t) P_i z(t), \tag{13}$$

where $P_i = X_i^{-1}$.

From Lemma 3, we only need to show that the Lyapunov function (13) proves the stability of the system (12).

In fact, the difference of (13) along the trajectory of (12) is defined as

$$\begin{aligned} \Delta V(z(t)) &= V(z(t+1)) - V(z(t)) \\ &= z^T(t+1) \sum_{i=1}^{d} \xi_i(t+1) P_i z(t+1) \\ &\quad - z^T(t) \sum_{i=1}^{d} \xi_i(t) P_i z(t) \end{aligned}$$

$$= z^T \sum_{j=1}^{d}\sum_{i=1}^{d}\xi_{ij}(t)(A^i + \sum_{l=0}^{i-1} A^l BF_jC)^T$$
$$\sum_{i=1}^{d}\xi_i(t+1)P_i \sum_{j=1}^{d}\sum_{i=1}^{d}\xi_{ij}(t)$$
$$\times(A^i \sum_{l=0}^{i-1} A^l BF_jC)z(t)$$
$$-z^T(t)\sum_{i=1}^{d}\xi_i(t)P_i z(t).$$

Then system (12) is stability proved by the Lyapunov function (13) if and only if

$$(A^i + \sum_{l=0}^{i-1} A^l BF_jC)^T P_i(A^i \sum_{l=0}^{i-1} A^l BF_jC) - P_j < 0. \quad (14)$$

From Lemma 1, we know that the inequality above holds if and only if

$$\begin{bmatrix} P_j & (A^i + \sum_{l=0}^{i-1} A^l BF_jC)^T P_i \\ P_i(A^i + \sum_{l=0}^{i-1} A^l BF_jC) & P_i \end{bmatrix} > 0. \quad (15)$$

By post- and pre-multiplying the both sides of the inequality (15) using the following matrix $diag\{P_j^{-1}, P_i^{-1}\}$, and letting $X_i = P_i^{-1}$, $Y_i = F_iX_i$, we can get (11). Thus, this completes the proof. ■

Remark 1: Based on the analysis in Section II, we can get the state evolvement equation of the close-loop system as follows:

$$x(t+1) = Ax(t) + BF_{r(t_{k-1})}x(t_k),$$
$$t \in [t_k, t_{k+1}),\ k \in \mathbf{N}.$$

For the system above, we adopt the following packet-loss dependent Lyapunov function

$$V(t) = x^T(t)P_{r(t_k)}x(t), t \in [t_k, t_{k+1}). \quad (16)$$

Let $r(t_k) = t_{k+1} - t_k = i$, then (16) is nothing but (13).

Now, we suppose that C is of full row rank. We present the singular value decomposition of C as

$$C = M \begin{bmatrix} C_0 & 0 \end{bmatrix} N^T, \quad (17)$$

where $M \in \mathbf{R}^{p\times p}$ and $N \in \mathbf{R}^{n\times n}$ are unitary matrices and $C_0 \in \mathbf{R}^{p\times p}$ is a diagonal matrix with positive diagonal elements in decreasing order. For any $i \in [1, q]$, assume that X_i satisfies

$$X_i = N \begin{bmatrix} X_{1i} & 0 \\ 0 & X_{2i} \end{bmatrix} N^T, \quad (18)$$

where $X_{1i} \in R^{p\times p},\ X_{2i} \in R^{(n-p)\times(n-p)}$.

Based on Lemma 2, we can get the following result:

Theorem 2: If there exist symmetric positive definite matrices $X_i = N \begin{bmatrix} X_{1i} & 0 \\ 0 & X_{2i} \end{bmatrix} N^T$ with $i \in [1, d]$ and matrices $Y_1, Y_2, \cdots, Y_d$ such that

$$\begin{bmatrix} X_j & (A^iX_j + \sum_{l=0}^{i-1} A^l BY_jC)^T \\ A^iX_j + \sum_{l=0}^{i-1} A^l BY_jC & X_i \end{bmatrix} > 0,$$
$$\forall (i, j) \in [1, d] \times [1, d], \quad (19)$$

then the NCS (1) is stabilizable via the output feedback controller

$$u(t) = Y_{r(t_{k-1})}\bar{Z}_{r(t_{k-1})}\bar{y}(t),\ t \in [t_k, t_{k+1}),$$

where

$$\bar{Z}_i = MC_0X_{1i}^{-1}C_0^{-1}M^T,$$

and $C = M \begin{bmatrix} C_0 & 0 \end{bmatrix} N^T$ is the singular value decomposition of C.

IV. Stabilizability of NCSs with Markovian packet-loss process

In this section, we suppose that the packet loss of the NCS (1) abide by the Markovian process. Sufficient condition for the stabilizability via output feedback is derived by the switched systems approach, and the feedback controllers are designed by resolving some LMIs.

Definition 4: The NCS (1) with Markovian packet-loss process is said to be mean square stable(MS) if $\lim_{t\to\infty} E[\|x(t)\|^2] = 0$ for any initial state (x_0, r_0).

The following result is a generalization of Theorem 9 in [7].

Lemma 4: The NCS (1) with Markovian packet-loss process is to be mean square stable(MS) if there exist positive definite matrices P_i, $i \in [1, d]$, such that

$$\sum_{i=1}^{s}[p_{ij}(A^i + \sum_{l=0}^{i-1} A^l BF_jC)P_i(A^i + \sum_{l=0}^{i-1} A^l BF_jC)] - P_j < 0.$$

Here, we omit its proof since it is similar with that of Theorem 9 in [7].

the states of NCS (1) with Markovian packet-losses process at the update steps can be described as follows:

$$z(t+1) = \bar{A}_{\eta(t)}z(t), \quad (20)$$

where $\bar{A}_{\eta(t)} \in \bar{\Omega}$.

Using Lyapunov function (13) for (20), it is easy to get the sufficient condition for the stabilizability of the NCS (1) via output feedback.

Theorem 3: The NCS (1) is MS if there exist symmetric positive definite matrices $X_1, X_2, \cdots, X_d$, and matrices $W_1, W_2, \cdots, W_d$, $G_1, G_2, \cdots, G_d$, $Y_1, Y_2, \cdots, Y_d$ satisfying

$$CG_i = W_iC, \quad (21)$$

$$\begin{bmatrix} \Lambda & Q_i^T \\ Q_i & X_i \end{bmatrix} > 0,\ \forall i \in [1, d], \quad (22)$$

where

$$Q_i = [\sqrt{p_{1i}}(AG_i + BY_iC)^T \ \cdots \ \sqrt{p_{di}}(A^dG_i + B_dY_iC)^T],$$

$$\Lambda = diag(G_1+G_1^T-X_1, \cdots, G_d+G_d^T-X_d), \ B_j = \sum_{l=0}^{j-1} A^j B,$$

and the stabilizing controller is given by

$$u(t) = F_{r(t_{k-1})}\bar{y}(t) = Y_{r(t_{k-1})}W^{-1}_{r(t_{k-1})}\bar{y}(t), \ t \in [t_k, t_{k+1}).$$

Further, we suppose that C is of full row rank. By using Lemma 2, we can get another sufficient condition in the form of LMIs for the stabilizability of NCS (1) via output feedback.

Theorem 4: If there exist symmetric positive definite matrices X_i, symmetric matrices $G_i = N \begin{bmatrix} G_{1i} & 0 \\ 0 & G_{2i} \end{bmatrix} N^T$ and matrices Y_i with $i \in [1, d]$ such that

$$\begin{bmatrix} \Lambda & Q_i^T \\ Q_i & X_i \end{bmatrix} > 0, \ \forall i \in [1, d], \qquad (23)$$

where

$$Q_i = [\sqrt{p_{1i}}(AG_i + BY_iC)^T \ \cdots \ \sqrt{p_{di}}(A^dG_i + B_dY_iC)^T],$$

$$\Lambda = diag(2G_1 - X_1, \cdots, 2G_d - X_d), \ B_j = \sum_{l=0}^{j-1} A^j B,$$

then the NCS (1) is stabilizable via output feedback controller

$$u(t) = Y_{r(t_{k-1})}\bar{W}_{r(t_{k-1})}\bar{y}(t), \ t \in [t_k, t_{k+1}),$$

where

$$\bar{W}_i = MC_0G_{1i}^{-1}C_0^{-1}M^T,$$

and $C = M \begin{bmatrix} C_0 & 0 \end{bmatrix} N^T$ is the singular value decomposition of C.

Remark 2: For NCS (1), we can find the maximum allowable bound of data packet dropout by solving the following optimal problem

$$\begin{array}{c} \max\limits_{X_i, Y_i, G_{1i}, G_{2i}} \ d \\ subject \ to \ (23). \end{array} \qquad (24)$$

V. NUMERICAL EXAMPLES

In this section, A numerical example and some simulations are given to demonstrate the effectiveness of the proposed design technique.

Example 1: Consider the second-order NCS

$$\begin{aligned} x(t+1) \ &= \ \begin{bmatrix} 0.66 & 0.209 \\ -0.123 & -0.5 \end{bmatrix} x(t) \\ &\quad + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u(t), \\ &y(t) = [2 \ 1]x(t), \\ &u(t) = F_i\bar{y}(t), \forall i \in [1, d], \end{aligned} \qquad (25)$$

where

$$\bar{y}(t) = \begin{cases} y(t), & \text{if the packet containing y(t) is transmitted successfully;} \\ \bar{y}(t-1), & \text{otherwise.} \end{cases}$$

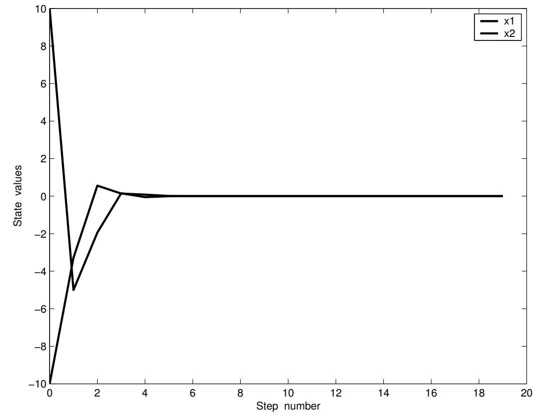

Fig. 1a. State response (arbitrary packet loss).

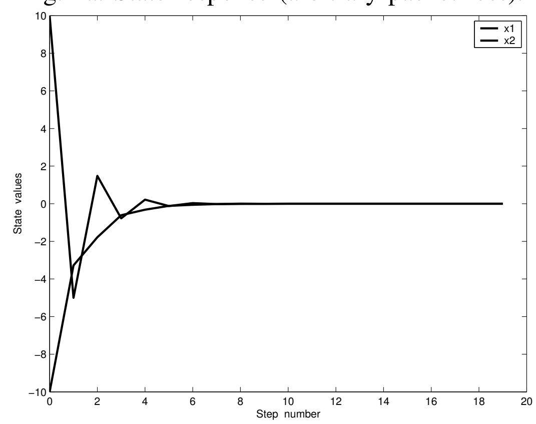

Fig. 1b. State response (no packet loss).

The state feedback gains $F_i, i \in [1, d]$ are to be designed. Denote $C = M \begin{bmatrix} C_0 & 0 \end{bmatrix} N^T$ as the singular value decomposition of C, then it is easy to get

$$M = 1, \ N = \begin{bmatrix} 0.8944 & -0.4472 \\ 0.4472 & 0.8944 \end{bmatrix}, \ C_0 = [2.2361 \ 0].$$

Suppose that the maximum transmission period $d = 3$ which means that the 66% of the packets can be lost, and take the initial state as $x_0 = [-10 \ 10]^T$. When the packet loss of the NCS (25) is arbitrary, we solve the LMIs of Theorem 2 and the use of the Matlab LMI Toolbox yields the following results:

$$X_1 = \begin{bmatrix} 20.1201 & 0 \\ 0 & 20.9190 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 24.2345 & 0 \\ 0 & 23.8506 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 26.0053 & 0 \\ 0 & 25.4020 \end{bmatrix}.$$

$$Y_1 = 2.4811, Y_2 = 2.4811, Y_3 = 2.4811.$$

Based on $\bar{Z}_i = MC_0X_{1i}^{-1}C_0^{-1}M^T$, and $F_i = Y_i\bar{Z}_i$, we obtain the output feedback gains:

$$F_1 = 0.1233, F_2 = 0.1024, F_3 = 0.0954.$$

When the distribution of transmission period is taken as $1, 2, 3, 1, 2, 3, \cdots$, the step response of NCS (25) is shown in

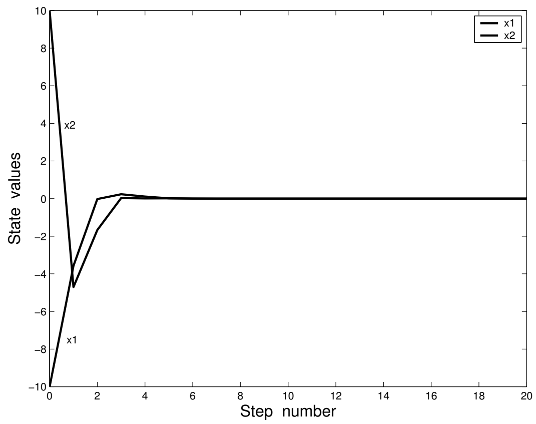

Fig. 2a. State response (Markovian packet losses).

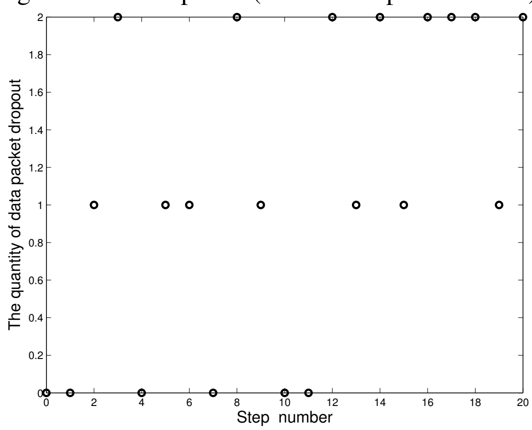

Fig. 2b. Frequency of packet losses (Markovian packet losses).

Fig. 1a. It is clear from the figure that even in such a case, the system can still be effectively stabilized via the switched state feedback given above. Fig. 1b depicts the trajectory of the system when no packet loss occurs.

In addition, we take the transition probability matrix as

$$P = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.3 & 0.1 & 0.3 \\ 0.6 & 0.7 & 0.5 \end{bmatrix}. \quad (26)$$

Suppose the packet loss of the NCS (25) abides by the Markovian process. We solve the LMIs of Theorem 4 and the use of the Matlab LMI Toolbox yields the following results:

$$X_1 = \begin{bmatrix} 45.2926 & 0 \\ 0 & 45.4019 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 43.7725 & 0 \\ 0 & 43.9160 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 39.9295 & 0 \\ 0 & 39.8656 \end{bmatrix}.$$

$$G_1 = \begin{bmatrix} 45.8525 & 0 \\ 0 & 45.9253 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 44.8391 & 0 \\ 0 & 44.9347 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 44.8391 & 0 \\ 0 & 44.9347 \end{bmatrix}.$$

$$Y_1 = 4.2704, Y_2 = 5.0810, Y_3 = 4.7197.$$

Based on $\bar{W}_i = MC_0G_{1i}^{-1}C_0^{-1}M^T$, and $F_i = Y_i\bar{W}_i$, we obtain the output feedback gains:

$$F_1 = 0.0931, F_2 = 0.1133, F_3 = 0.1053.$$

When the initial state is given by $x_0 = [-10\ 10]^T$, the system state trajectory is shown in Fig. 2a under the state feedback above. The small circles in Fig. 2b simulate the time instants when the zero-order hold updates its state.

VI. CONCLUSIONS

This paper has presented the design for the packet-loss dependent output feedback controller of NCSs. Two types of packet-loss processes have been considered: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. For both cases, sufficient conditions for output feedback stabilizability have been derived and stabilizing controllers have been constructed by using the feasible solutions of some LMIs.

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