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# A Unified Framework for Detection, Isolation and Compensation of Actuator Faults in Uncertain Particulate Processes

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*Abstract*— **This paper presents a methodology for the robust detection, isolation and compensation of control actuator faults in particulate processes described by population balance models with control constraints and time-varying uncertain variables. The main idea is to shape the fault-free closed-loop process response via robust feedback control in a way that enables the derivation of performance-based fault detection and isolation (FDI) rules that are less sensitive to the uncertainty. Initially, an approximate finite-dimensional system that captures the dominant process dynamics is derived and decomposed into interconnected subsystems, each influenced directly by a single manipulated input. A robustly stabilizing bounded feedback controller is then designed for each subsystem leading to (1) an explicit characterization of the fault-free behavior of each subsystem in terms of a time-varying bound on an appropriate Lyapunov function, and (2) an explicit characterization of the robust stability region. Using the fault-free bounds as thresholds for FDI, the detection and isolation of faults in a given actuator is accomplished by monitoring the evolution of the system within the stability region and declaring a fault if the threshold is exceeded. The thresholds are linked to the achievable degree of asymptotic uncertainty attenuation and can therefore be properly tuned by tuning the controllers. The robust FDI scheme is integrated with a controller reconfiguration strategy that preserves closed-loop stability following FDI. Finally, the implementation of the fault-tolerant control architecture on the particulate process is discussed and the proposed methodology is applied to a simulated model of a continuous crystallizer with a fines trap.**

## I. Introduction

Particulate processes are widely used in a number of important processing industries including agricultural, chemical, food, minerals, and pharmaceuticals. It is now well understood that the Particle Size Distribution (PSD) in these processes provides a critical link between the product quality and the process operating variables, and that the ability to effectively manipulate the PSD is essential to controlling the quality of the end product. These realizations have motivated significant research work on the design of model-based feedback control systems for particulate processes to achieve PSDs with desired characteristics (e.g., [1], [2], [3], [4], [5], [6], [7], [8]; see also [9], [10] for surveys of recent results and references). Despite the significant and growing body of research work on this topic, the problems of fault diagnosis and fault-tolerant control of particulate processes have received limited attention. For processes involved in the production of specialty chemicals where the ability to meet stringent product specifications is critical to the product utility, the erosion of control authority caused by control system failures can result in off-spec products and lead to substantial production losses if not properly diagnosed and handled in the control system design.

While an extensive body of literature currently exists on the problems of fault diagnosis and fault-tolerant control (e.g., see [11], [12], [13], [14], [15], [16] and the references therein), the majority of existing methods have been developed for lumped parameter processes described by systems of ordinary differential equations (ODEs). More recently, methods for actuator failure diagnosis and compensation have been developed for distributed parameter systems modeled by partial differential equations (e.g., [17], [18], [19]). The dynamic models of particulate processes, however, are typically obtained through the application of population, material and energy balances and consist of systems of nonlinear partial integro-differential equations that describe the evolution of the PSD, coupled with systems of nonlinear ODEs that describe the evolution of the state variables of the continuous phase. In [20] a methodology for the detection and handling of actuator faults in single-input particulate processes was developed on the basis of appropriate reduced-order models that capture the dominant process dynamics. The fault detection task was addressed by means of a filter that simulates the behavior of the fault-free, reduced-order model and uses the discrepancy from the behavior of the actual process as a residual signal.

For particulate processes with several manipulated inputs, it is important not only to detect that a fault has occurred but also to identify its location in order to avoid the unnecessary shut down of possibly healthy actuators following fault detection. Another important issue that must be accounted for in the design of model-based fault diagnosis and fault-tolerant control systems is the presence of model uncertainty. Population balance models that describe particulate processes are inherently uncertain due to the presence of unknown, or partially known, process parameters as well as time-varying exogenous disturbances. If not properly accounted for, uncertainty can adversely affect all components of the fault-tolerant control architecture by degrading the stability and performance properties of the feedback controller, complicating the design and evaluation of the fault diagnosis residuals, and leading to false alarms and poor supervisory control. One way to decouple the effect of uncertainty on the residual is to re-design the fault detection filters using the unknown-input observer princi-

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ple. This approach, however, is complicated by the strong nonlinear dynamics of particulate processes (e.g., owing to complex growth, nucleation, agglomeration and breakage mechanisms, and the Arrhenius dependence of nucleation laws on solute concentration in crystallizers).

Motivated by these considerations, we present in this work a unified framework for the design of integrated robust fault detection and isolation (FDI) and fault-tolerant control (FTC) systems for multi-variable particulate processes described by population balance models with control constraints, time-varying uncertain variables and actuator faults. The methodology brings together robust feedback control, robust FDI, and switching between multiple actuator configurations based on an approximate, finite-dimensional system that captures the dominant process dynamics. The main idea is to shape the fault-free closed-loop process response via bounded robust feedback control in a way that facilitates the design of performance-based FDI rules that are practically insensitive to the uncertainty. Uniting the tasks of constrained robust stabilization and FDI allows obtaining an explicit characterization of the state-space regions where robust FDI is feasible under uncertainty and constraints. The rest of the paper is organized as follows. Following some mathematical preliminaries in Section II, the robust FDI-FTC problem is formulated and the solution methodology is highlighted. An approximate, finite-dimensional system is then obtained in Section III using the method of weighted residuals and subsequently used is Section IV to construct and analyze the properties of the FDI-FTC structure. Finally, in Section V the proposed methodology is applied to a continuous crystallizer with a fines trap. Due to space limitations, the proofs of the main results will be omitted, but can be found in the full version of this work [21].

## II. PRELIMINARIES

### A. Particulate processes with uncertainty and constraints

We focus on spatially homogeneous (well-mixed) particulate processes with simultaneous particle growth, nucleation, agglomeration and breakage, and consider the case of a single internal particle coordinate, which is assumed to be the particle size. Applying a population balance to the particle phase, as well as material and energy balances to the continuous phase, we obtain the following general nonlinear system of partial integro-differential equations:

$$\begin{aligned}\frac{\partial n}{\partial t} &= -\frac{\partial (G(x,r)n)}{\partial r} + w(n,x,r,\theta(t)) \\ &+ g_1(n,x,r)[u_1^k(t) + f_{a_1}^k(t)],\ n(0,t) = b(x(t))\end{aligned} \quad (1)$$

$$\dot{x} = f(x) + g_2(x)[u_2^k(t) + f_{a_2}^k(t)] + \mathcal{X}(x, \theta(t), \int_0^{r_{\max}} q(n,x,r)dr) \quad (2)$$

$$\|u_i^k(t)\| \leq u_{i,\max}^k, i = 1,2;\ \|\theta(t)\| \leq \theta_b \quad (3)$$

$$k(t) \in \mathcal{K} := \{1, 2, \cdots, L\},\ L < \infty \quad (4)$$

where $n(r,t) \in \mathcal{L}_2([0, r_{\max}), \mathbb{R})$ is the particle size distribution function which is assumed to be a continuous and sufficiently smooth function of its arguments ($\mathcal{L}_2[0, r_{\max})$ denotes a Hilbert space of continuous functions defined on the interval $[0, r_{\max})$), $r \in [0, r_{\max})$ is the particle size ($r_{\max}$ is the maximum particle size, which may be infinity), $t$ is the time, $x \in \mathbb{R}^n$ is the vector of state variables that describe properties of the continuous phase (e.g., solute concentration, temperature and pH in a crystallizer), $u_1^k \in \mathbb{R}$ is the manipulated input associated with the particulate phase (e.g., fines destruction rate), $u_2^k \in \mathbb{R}$ is the manipulated input associated with the continuous phase (e.g., solute feed concentration in a crystallizer), $u_{i,\max}^k$ is a real number that captures the size of constraints on the magnitude of the $i$-th manipulated input, $f_{a_i}^k \in \mathbb{R}$ denotes a fault in the $i$-th control actuator, $k(t)$ is a discrete variable that denotes which control configuration is active at time $t$, $\theta = [\theta_1\ \theta_2\ \cdots\ \theta_q]^T \in \mathbb{R}^q$ denotes the vector of time-varying (but bounded) uncertain variables (e.g., unknown process parameters and time-varying external disturbances) and $\theta_b$ is a known bound on the size of the uncertainty.

In the population balance of Eq.1, $G(x,r)$ is the growth rate that accounts for particle growth through condensation, and $w(n,x,r,\theta) := w_0(n,x,r) + w_u(n,x,r)\theta$ is a term that accounts for the net rate of introduction of new particles into the system, i.e., it includes all the means by which particles appear or disappear within the system including particle agglomeration, breakage, nucleation, feed and removal. The $x$-subsystem of Eq.2 is derived by applying material and energy balances to the continuous phase. In this subsystem, $f(\cdot)$, $q(\cdot)$, $g_2(\cdot)$, $\mathcal{X}(\cdot)$ are smooth nonlinear functions. The term containing the integral accounts for mass and heat transfer from the continuous phase to all the particles in the population and has the form $\mathcal{X}(\cdot,\cdot,\theta) = \mathcal{X}_0(\cdot,\cdot) + \mathcal{X}_u(\cdot,\cdot)\theta$.

### B. Problem formulation and solution methodology

Consider the system of Eqs.1-4 with non-vanishing uncertainty and disturbances. Of the $L$ control actuator configurations available, only one is to be active for control at any given time, while the rest are kept dormant as backup. The backup configurations may contain alternative manipulated variables or redundant actuators of the same manipulated inputs. The problems under consideration include how to suppress the effect of the uncertainty, how to detect and isolate faults in the operating actuator configuration under uncertainty, and how to decide which fall-back actuator configuration should be activated to maintain robust closed-loop stability following FDI. To address these problems, model reduction techniques will be used initially to obtain an approximate, finite-dimensional system that captures the dominant dynamics of the infinite-dimensional system of Eqs.1-4. The approximate system will subsequently be used to: (1) synthesize, for each actuator configuration, a family of bounded robust nonlinear feedback controllers with well-characterized stability regions and robust uncertainty attenuation properties, and (2) derive robust FDI rules that exploit the uncertainty attenuation capabilities of the controllers to detect and isolate destabilizing and/or performance-deteriorating faults. Finally, an actuator reconfiguration law is devised to maintain robust closed-loop stability.

## III. Model reduction

We initially use the method of weighted residuals to derive a set of nonlinear ODEs that accurately reproduce the solutions and dominant dynamics of the distributed parameter system of Eqs.1-4. To this end, we expand the solution of $n(r,t)$ in an infinite series in terms of an orthogonal and complete set of basis functions, $\{\phi_k(r) : r \in [0, r_{\max})\}$, as $n(r,t) = \sum_{k=1}^{\infty} a_k(t)\phi_k(r)$, where $a_k(t)$ are time-varying coefficients. Substituting this expansion into Eqs.1-4, multiplying the population balance with the weighting functions, $\psi_\nu(r)$, integrating over the entire particle size spectrum, and finally truncating the series expansion of $n(r,t)$ up to order $N$ keeping the first $N$ equations, we obtain the following finite-dimensional system:

$$\begin{aligned} \dot{\bar{z}}_1 &= \bar{f}_1(\bar{z}_1, \bar{z}_2) + \bar{g}_1(\bar{z}_1, \bar{z}_2)[u_1^k + f_{a_1}^k] + \bar{w}_1(\bar{z}_1, \bar{z}_2)\theta \\ \dot{\bar{z}}_2 &= \bar{f}_2(\bar{z}_1, \bar{z}_2) + \bar{g}_2(\bar{z}_1, \bar{z}_2)[u_2^k + f_{a_2}^k] + \bar{w}_2(\bar{z}_1, \bar{z}_2)\theta \end{aligned} \tag{5}$$

where $\bar{z}_1 = \bar{a}^N = [\bar{a}_1^N \ \cdots \ \bar{a}_N^N]^T$, $\bar{z}_2 = x_N$, $x_N$ and $\bar{a}_k^N$ are the approximations of $x$ and $a_k$, respectively, obtained by an $N$-th order truncation, $\bar{f}_i(\cdot), \bar{g}_i(\cdot), \bar{w}_i(\cdot)$ are nonlinear functions whose explicit form is omitted brevity, and the bar symbol in $\bar{z}_i$ indicates that this variable is associated with the approximate system. The asymptotic validity of the approximation (i.e., the fact that $\lim_{N\to\infty}\|n(r,t) - \sum_{i=1}^{N} a_i(t)\phi(r)\|_2 = 0$ when $u = \theta = 0$) can be established using results from perturbation theory [6].

## IV. Design of robust FDI-FTC architecture using the reduced-order model

### A. Robust feedback controller synthesis

The objectives of this step are to: (a) synthesize, for each actuator configuration, a family of feedback controllers that enforce constraint satisfaction and robust stability with an arbitrary degree of asymptotic attenuation of the effect of uncertainty on the closed-loop system, and (b) explicitly characterize the robust stability region associated with each configuration in terms of the constraints and the size of uncertainty. While several controller designs can be used to meet the desired objectives, we consider in what follows the bounded robust control law introduced in [22] (inspired by the results in [23]) as a specific example to work with and illustrate the main ideas:

$$u_i^k = -\mathcal{P}_i^k(\bar{z}, u_{i,\max}^k, \theta_b, \chi_i, \rho_i, \bar{\phi}_i) L_{\bar{g}_i}\bar{V}_i, \quad k \in \mathcal{K} \tag{6}$$

where

$$\mathcal{P}_i^k = \frac{\alpha_i(\bar{z}) + \sqrt{\alpha_i^2(\bar{z}) + \left(u_{i,\max}^k \beta_i^k(\bar{z})\right)^4}}{(\beta_i^k(\bar{z}))^2\left[1 + \sqrt{1 + (u_{i,\max}^k \beta_i^k(\bar{z}))^2}\right]} \tag{7}$$

$\alpha_i = L_{\bar{f}_i}\bar{V}_i + \left(\rho_i\|\bar{z}_i\| + \chi_i\theta_b\|L_{\bar{w}_i}\bar{V}_i\|\right)(\|\bar{z}_i\|/(\|\bar{z}_i\| + \bar{\phi}_i^k))$, $\beta_i^k = \|L_{\bar{g}_i}\bar{V}_i\|$, $\bar{V}_i$ is a robust control Lyapunov function [24] for the $i$-th subsystem in Eq.5, $L_{\bar{f}_i}\bar{V}_i$, $L_{\bar{g}_i}\bar{V}_i$ and $L_{\bar{w}_i}\bar{V}_i$ are the Lie derivatives of the scalar field $\bar{V}_i$ with respect to the vector fields, $\bar{f}_i$, $\bar{g}_i$ and $\bar{w}_i$, respectively, $\bar{z} = [\bar{z}_1^T \ \bar{z}_2^T]^T$, and $\rho_i > 0$, $\chi_i > 1$, $\bar{\phi}_i > 0$ are adjustable parameters. Let $\Pi_i^k$ be the set defined by:

$$\Pi_i^k := \{\bar{z} \in \mathbb{R}^{n+N} : \alpha_i^k(\bar{z}, \varrho_i, \theta_b) \le u_{i_{\max}}^k \beta_i^k(\bar{z})\} \tag{8}$$

where $\varrho_i = [\rho_i \ \chi_i \ \bar{\phi}_i]'$, and let $\Pi^k := \Pi_1^k \bigcap \Pi_2^k$ be the intersection of the two sets, and consider the subset:

$$\bar{\Omega}_s^k(\theta_b, u_{\max}^k) := \{\bar{z} \in \Pi^k : \bar{V}(\bar{z}) \le c_{\max}^k\} \tag{9}$$

for some $c_{\max}^k > 0$, where $\bar{V} = \sum_{i=1}^{2}\bar{V}_i$ is a composite Lyapunov function for the system of Eq.5. The following proposition characterizes the closed-loop stability properties of the subsystems of Eq.5 under the controllers of Eqs.6-7 in the absence of faults.

**Proposition 1:** *Consider the closed-loop system of Eqs.5-7, for a fixed $k \in \mathcal{K}$, with $f_{a_i}^k \equiv 0$ for a fixed $i$. Then, if $\bar{z}(t) \in \Pi_i^k$, for some $t \ge 0$, there exists positive real numbers, $\gamma_i$, $\tilde{\phi}_i$, and a class $\mathcal{K}$ function $\sigma_i(\cdot)$[1] such that if $\phi_i := \bar{\phi}_i(\chi_i - 1)^{-1} \le \tilde{\phi}_i$, the time-derivative of $\bar{V}_i$ along the trajectories of the $i$-th subsystem in Eq.5 satisfies:*

$$\dot{\bar{V}}_i(t) \le -\gamma_i\bar{V}_i(t) + \sigma_i(\phi_i) \tag{10}$$

*Furthermore, if $\bar{z}(0) \in \bar{\Omega}_s^k(\theta_b, u_{\max}^k)$ and $f_{a_1}^k = f_{a_2}^k \equiv 0$, then for every real number $\bar{\delta}_{d_i} > 0$, there exists $\phi_i^*$ such that if $\phi_i \in (0, \phi_i^*]$, $\limsup_{t\to\infty} \bar{V}_i(t) \le \bar{\delta}_{d_i}$, for $i = 1,2$ and the origin of the closed-loop system is practically stable.*

**Remark 1:** The set $\bar{\Omega}_s^k(u_{\max}^k, \theta_b)$ represents an estimate of the robust stability region for the $k$-th fault-free control configuration in terms of the size of the constraints and the size of the uncertainty. Starting within this region, each controller drives the trajectory of its corresponding subsystem in finite-time to a small terminal neighborhood of the desired steady-state where it remains confined for all future times (residual set). Depending on the desired degree of uncertainty attenuation, the size of the residual sets, $\bar{\delta}_{d_i}$, can be made arbitrarily small provided that the controller tuning parameters are chosen properly.

### B. Performance-based fault detection and isolation

To achieve robust FDI and guard against false alarms due to the presence of uncertainty and disturbances, we follow a performance-based approach. The key idea is to use the characteristic closed-loop behavior obtained in Eq.10 for each subsystem under fault-free actuation as the basis for deriving a set of rules that determine when a fault has occurred in a given controller or actuator. This idea is formalized in Proposition 2.

**Proposition 2:** *Consider the $i$-th closed-loop system of Eqs.5-7, for a fixed $k \in \mathcal{K}$, with $\phi_i \in (0, \phi_i^*]$, where $\phi_i^*$ was defined in Proposition 1. If, for some $T_d > 0$, $\bar{z}(T_d) \in \bar{\Omega}_s^k(u_{\max}^k, \theta_b)$ and, for some $a \in (0,1)$, either:*
*(a) $\dot{\bar{V}}_i(T_d) > -(1-a)\gamma_i\bar{V}_i(T_d)$ for $\bar{V}_i(T_d^-) > \bar{\delta}_{d_i}$, or*
*(b) $\bar{V}_i(T_d) > \bar{\delta}_{d_i}$ for $\bar{V}_i(T_d^-) \le \bar{\delta}_{d_i}$,*
*where $\bar{\delta}_{d_i}$ was defined in Proposition 1, then $f_{a_i}^k(T_d) \ne 0$ and a fault is declared in the $i$-th control actuator.*

The condition in part (a) of Proposition 2 provides the FDI rule for the case when, immediately prior to a fault in the $i$-th actuator, $\bar{z}_i$ lies outside its residual set. In this case, faults in the $i$-th actuator that cause an increase in $\bar{V}_i$ (destabilizing faults) and faults that slow down the

[1] A continuous real-valued function is said to be of class $\mathcal{K}$ if it is monotonically non-decreasing and is zero at zero.

decay rate of $\bar{V}_i$ beyond the minimum rate enforced by the healthy robust controller (performance-degrading faults) will be detected and isolated. The condition in part (b) gives the FDI rule for the case when, immediately prior to the fault, $\bar{z}_i$ lies within its residual set. In this case, a fault in the $i$-th actuator that causes $\bar{z}_i$ to begin to escape its terminal set gets detected and isolated. Essentially, Eq.10 acts as a dedicated FDI filter for the $i$-th controller – albeit in terms of a differential inequality rather than a differential equation – with either $\dot{\bar{V}}_i$ (as in condition (a)) or $\bar{V}$ (as in condition (b)) serving as a residual to be evaluated and compared against some alarm threshold.

**Remark 2:** Note that the FDI rules in Proposition 2 cannot be used to declare with certainty that a fault exists in the $i$-th actuator unless at the time of FDI $\bar{z}$ lies within $\bar{\Omega}_s^k$ (or, at least, within $\Pi_i^k$) which is a region where $\dot{\bar{V}}_i$ is guaranteed to satisfy Eq.10 under constraints and in the absence of faults in the $i$-th actuator. As such, any observed behavior of the $i$-th subsystem inconsistent with Eq.10, while $\bar{z}$ is within this region, is conclusive indicator of a fault. In this sense, $\bar{\Omega}_s^k$ is not only a stability region but also a region where FDI is feasible. This dual interpretation is a consequence of using robust stabilization as a tool for FDI.

**Remark 3:** The fact that each subsystem in Eq.5 is driven by a different manipulated input is an important structural feature that facilitates the derivation of the desired FDI rules. However, this feature alone is insufficient to uniquely isolate faults in a given actuator due to the inherent coupling between the two subsystems in Eq.5 which implies that the evolution of a given subsystem, say the $\bar{z}_1$-subsystem, will not only be sensitive to $f_{a_1}^k$ but could also be influenced indirectly by $f_{a_2}^k$ through the effect of the latter on $\bar{z}_2$. Decoupling the effect of $f_{a_j}^k$ on $\bar{z}_i$, where $j \neq i$, is achieved by the robust controllers which not only suppress the effect the uncertainty but also cancel the influence of each subsystem on the other and enforce satisfaction of Eq.10. Due to the presence of control constraints, this decoupling is guaranteed only within $\Pi_i^k$ outside of which no conclusions can be drawn regarding stability or FDI.

**Remark 4:** Under the FDI scheme of Proposition 2, faults that do not cause a breach of the expected bounds on $\dot{\bar{V}}_i$ and $\bar{V}$ will go undetected. Such faults, however, are not detrimental to closed-loop stability or the desired performance properties, and thus require no corrective action. The fact that the FDI thresholds can be tightened through proper controller tuning also provides a handle to minimize missed alarms about these faults. Finally, we note that the performance-based FDI scheme can be used to detect and isolate both partial and complete actuator failures, as well as faults that do not necessarily appear in the control actuators, as long as they influence the evolution of the states.

*C. Robust stability-based actuator reconfiguration*

Following FDI, the supervisor needs to determine which backup control configuration can be activated to maintain robust closed-loop stability. To this end, consider the system of Eq.5 where, for each control configuration: (1) a family of robust controllers of the form of Eqs.6-7 have been designed, (2) the robust stability and FDI regions $\bar{\Omega}_s^k(u_{\max}^k, \theta_b)$ have been determined, and (3) given the desired uncertainty attenuation level $\bar{\delta}_d := \min_i\{\bar{\delta}_{d_i}\}$, appropriate values for $\phi_i$ have been determined (e.g., choose $\phi_i \leq \min_i\{\phi_i^*\}$). Theorem 1 below describes how FDI and actuator reconfiguration tasks are integrated to ensure fault-tolerance in the approximate closed-loop system.

**Theorem 1:** *Consider the closed-loop system of Eqs.5-7 with $k(0) = j \in \mathcal{K}$, $\bar{z}(0) \in \bar{\Omega}_s^j(u_{\max}^k, \theta_b)$. Let $T_d := \min\{t : f_{a_i}^j(t) \neq 0\}$, for some $i$, then the switching rule:*

$$k(t) = \left.\begin{cases} j, & 0 \leq t < T_d \\ \nu \neq j, & t \geq T_d, \bar{z}(T_d) \in \bar{\Omega}_s^\nu, u_r^\nu = u_r^j, r \neq i \end{cases}\right\} \quad (11)$$

*practically stabilizes the origin of the closed-loop system and $\limsup_{t\to\infty} \bar{V}_i(\bar{z}_i(t)) \leq \bar{\delta}_d$, for $i = 1, 2$.*

The switching law of Eq.11 ensures that (1) the fall-back actuator configuration activated and implemented following FDI is one that guarantees robust closed-loop stability and the desired degree of uncertainty attenuation (this is captured by requiring $\bar{z}(T_d) \in \bar{\Omega}_s^\nu$), and (2) only the faulty actuators of the operating configuration are switched out while the healthy ones remain active in the new configuration (this is captured by the requirement $u_r^\nu = u_r^j, r \neq i$).

**Remark 5:** It can be shown using regular perturbation techniques that the FDI-FTC architecture designed on the basis of the approximate system continues to enforce stability and fault-tolerance in the infinite-dimensional system of Eqs.1-4 if the approximation is of a sufficiently high-order and the FDI rules are appropriately modified to account for approximation errors. Specifically, the closeness of solutions between the approximate and infinite-dimensional systems can be exploited to obtain modified FDI thresholds that are $O(\epsilon(N))$ larger than the bounds obtained for the approximate system, where $\epsilon(N)$ is a small positive real number that satisfies $\lim_{N\to\infty} \epsilon(N) = 0$. By limiting the FDI region to a subset of the stability region (where $z$ can be used to reliably infer $\bar{z}$), a fault in the $i$-th controller is then declared if the modified threshold is breached, since a breach in this case exceeds the maximum possible approximation error (see [21] for the mathematical details).

**Remark 6:** The on-line implementation of the FDI-FTC architecture presented above requires that the values of the state variables $z$ be known. To address the problem when only limited measurements of the principal moments of the PSD and the continuous-phase variables are available, a nonlinear state observer of the following form can be used to estimate $z$ from the measured outputs:

$$\dot{\eta} = \tilde{f}(\eta) + \tilde{g}(\eta)u + \tilde{w}(\eta)\theta_n + L(y - \tilde{h}(\eta)) \quad (12)$$

where $\eta \in \mathbb{R}^{n+N}$ denotes the observer state vector, $y$ is the measured output vector, $\tilde{h}(\eta)$ is the estimated value of the output, $\theta_n$ denotes a nominal value for $\theta(t)$ and $L$ is a matrix chosen so that the eigenvalues of the matrix $C_L = [\frac{\partial \tilde{f}}{\partial \eta} - L\frac{\partial \tilde{h}}{\partial \eta}]_{(\eta=\eta_s)}$, where $\eta_s$ is the operating steady-state, lie in the open left-half of the complex plane. It can be shown (e.g., [25]) that the bounded stability of the closed-loop system resulting from the application of a robust output feedback controller that combines the controller of Eqs.6-7 with the observer of Eq.12 to the particulate process is guaranteed (in the absence of faults), provided that there exists a matrix $L$ such that $C_L = (1/\mu)\bar{A}$ where $\mu$ is a sufficiently small positive parameter (related to the size of the observer gain) and $\bar{A}$ is a Hurwitz matrix. Similar to the handling of approximation errors, state estimation errors can be accounted for in the FDI rules by (1) enlarging the FDI thresholds further to account for the discrepancy between $\eta$ and $\bar{z}$, and (2) limiting the FDI region to a smaller subset of the stability region where $\eta$ can be used to reliably infer $\bar{z}$. Unlike the state feedback case, however, FDI under output feedback is possible only after $\eta$ has converged sufficiently close to $z$ (which occurs fast for sufficiently small $\mu$).

## V. Application to a Continuous Crystallizer with Fines Trap

We consider an isothermal continuous crystallizer with a fines trap to demonstrate the implementation of the proposed FDI-FTC strategy. The trap is used to remove small crystals and increase the mean crystal size. In a crystallizer, the precise regulation of the shape of the crystal size distribution (CSD) is important because it significantly influences the necessary liquid-solid separation and the product properties. Under standard modeling assumptions, the following process model can be derived [26]:

$$\begin{aligned} \frac{\partial n}{\partial t} &= -k_1(c-c_s)\frac{\partial n}{\partial r} - \frac{n}{\tau} - \bar{h}(r)\sum_{j=1}^{2}\varepsilon_j(t)\frac{n}{\bar{\tau}_j} \\ &+ \delta(r-0)\bar{\epsilon}k_2\exp\left(-k_3/(c/c_s-1)^2\right) \\ \frac{dc}{dt} &= \frac{\sum_{i=1}^{2}\varphi_i(t)(c_i^0-\rho)}{\bar{\epsilon}\tau} + \frac{(\rho-c)}{\tau} + \frac{(\rho-c)}{\bar{\epsilon}}\frac{d\bar{\epsilon}}{dt} \end{aligned} \tag{13}$$

where $n(r,t)$ is the number of crystals of radius $r \in [0,\infty)$ at time $t$ per unit volume of suspension, $\tau$ is the residence time, $c$ is the solute concentration in the crystallizer, $\rho$ is the particle density, $c_i^0$ is the solute concentration in the $i$-th feed stream, $\varphi_i$ is a binary variable that takes a value of 1 whenever the $i$-th feed stream is used and zero otherwise, $\bar{\epsilon} = 1 - \int_0^\infty n(r,\bar{t})\frac{4}{3}\pi r^3 dr$ is the volume of liquid per unit volume of suspension, $c_s$ is the concentration of solute at saturation, $k_1$, $k_2$, and $k_3$ are constants, and $\delta(r-0)$ is the standard Dirac function. The term containing the Dirac function in Eq.13 accounts for the production of crystals of infinitesimal (zero) size via nucleation. The rate at which crystals are circulated in the fines trap (using the $j$-th flow control actuator) is $1/\bar{\tau}_j = F_{0_j}/V$, ($F_{0_j}$ is the fines recirculation rate and $V$ is the active volume of the crystallizer which is assumed to be constant), $\varepsilon_j$ is a binary variable that takes a value of 1 whenever the $j$-th flow control actuator in the fines trap is used and zero otherwise, and $\bar{h}(r)$ expresses the desired selection curve for fines destruction (classification function). It is desired to remove with the fines trap crystals of size $r_m$ and smaller, and therefore $\bar{h}(r) = 1$ for $r \leq r_m$ and $\bar{h}(r) = 0$ for $r > r_m$. All process parameter values can be found in [6].

The control objective is to stabilize the crystal and solute concentrations at desired set-points, $\nu_1 = 0.015$ and $\nu_2 = 0.5996$, respectively, by manipulating the flow rate of suspension through the fines trap and the inlet solute concentration in the presence of actuator constraints and faults. Measurements of the crystal concentration (e.g., via light scattering techniques) and the solute concentration (e.g., via mass spectrometer) are assumed to be available. Uncertainties in the form of modeling errors in the pre-exponential factor of the nucleation rate, $k_2$, and the density of crystals, $\rho$, are considered. For simulation purposes, we set $k_2 = k_{2,nom} + 0.5k_{2,nom}\sin(0.5t)$ and $\rho = \rho_{nom} + 0.1\rho_{nom}$ where $k_{2,nom}$ and $\rho_{nom}$ are the nominal values. To ensure actuator fault-tolerance, it is assumed that, for each manipulated input, an appropriate redundant actuator is available for use as backup in the event that the primary control actuator fails. At any given time, only one actuator is used for each manipulated variable, while the other is kept dormant (i.e., $\varphi_1(t) + \varphi_2(t) = 1$ and $\varepsilon_1(t) + \varepsilon_2(t) = 1$ for all $t \geq 0$). Using the method of moments and approximating the size distribution function using a Laguerre polynomial expansion to close the set of moment equations [27], an approximate fifth-order nonlinear ODE system describing the evolution of the first four moments of the CSD and the solute concentration is obtained in the form of Eq.5 and used for the synthesis of the appropriate output feedback controllers (by combining the controllers of Eqs.6-7 with the observer of Eq.12) which are then implemented on a sufficiently high-order discretization of the process model of Eq.13 obtained using a finite-difference scheme with 1000 discretization points (higher order discretizations led to identical results), starting from the initial condition $n(r,0) = (2.189\times10^{-3})e^{-1.168r}\ mm^{-4}$, $c(0) = 992.1\ kg\cdot m^{-3}$. The model derivation and output feedback controller synthesis details are omitted due to space limitations.

To detect and isolate faults during crystallizer operation, we consider the following two residual signals. The residual, $r_1(t) := \|\eta_1(t) - \bar{w}_1(t)\|$, is dedicated to the first manipulated input (flow rate through fines trap), where $\eta_1$ is an estimate of $z_1$ generated from the observer of Eq.12 using measurements of the crystal concentration, and $\bar{w}_1$ is a copy of $\bar{z}_1$ (when $f_{a_1} = 0$) whose evolution is estimated from Eq.10. The second residual, $r_2(t) := \|\eta_2(t) - \bar{w}_2(t)\|$, is dedicated to the second manipulated input (solute feed concentration), where $\eta_2$ is an estimate of $z_2$ generated from the observer of Eq.12 using measurements of the solute concentration, and $\bar{w}_2$ is a copy of $\bar{z}_2$ (when $f_{a_2} = 0$) whose evolution is also estimated from Eq.10. The following thresholds $\delta_1 = 0.01$ and $\delta_2 = 0.025$ are chosen for $r_1$ and $r_2$, respectively, (by properly tuning the controllers and observer) to account for the combined effects of uncertainties, state estimation and model reduction errors in the absence of faults. To demonstrate how the integrated FDI-FTC scheme works, the process is initialized using the healthy controllers which successfully drive the controlled outputs into their prescribed terminal sets very quickly (see Figs.1(a)-1(d)).

We begin to monitor the evolution of the state estimates at $T_b = 5$ hr (to allow sufficient time for the convergence of the estimation errors; see Remark 6). At $T_{f1} = 15$ hr, failure is introduced in the actuator manipulating the suspension flow rate through the fines trap (see the solid line in Fig.1(c)). Figs.1(e)-1(f) depict how this failure is detected and isolated since it causes $r_1$ to cross its specified threshold at $t = 15.51$ hr, while not affecting $r_2$. Following FDI in the first controller, the supervisor switches to a backup actuator that maintains the closed-loop outputs near their desired set-points. This is shown by the solid lines in Figs.1(a)-1(b). After handling the fault in the first controller, another failure in the actuator manipulating the solute feed concentration occurs at $T_{f2} = 25$ hr as shown by the solid line in Fig.1(d). By examining Figs.1(e)-1(f), it is seen that this failure is detected and isolated immediately since it causes $r_2$ to cross the threshold at $t = 25.07$ hr, while not affecting $r_1$ which remains within its threshold limit. Following the detection and isolation of the second actuator failure, the supervisor switches to a backup actuator to preserve robust closed-loop stability as shown by the solid lines in Figs.1(a)-1)(b). The dashed lines in Figs.1(a)-1(b) show the destabilizing effect of controller failures when no corrective action is taken.

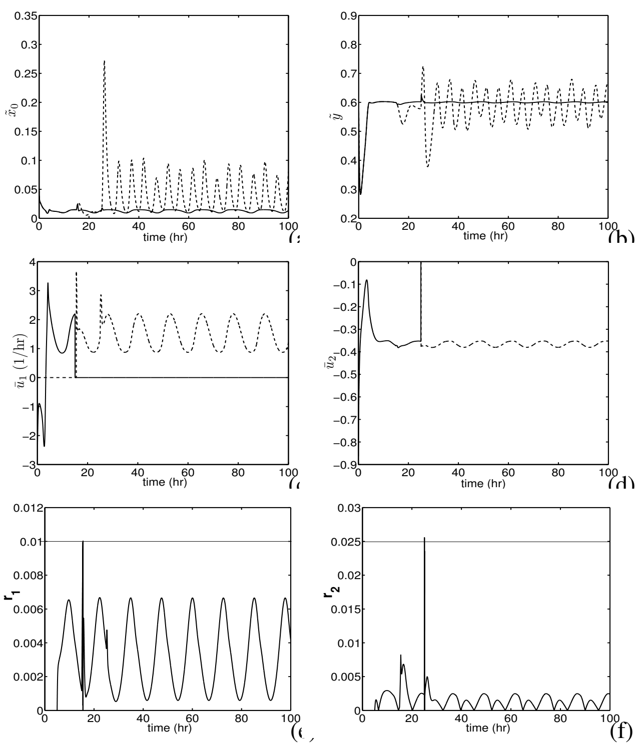


Fig. 1. Evolution of the controlled outputs (a)-(b) (solid), manipulated inputs (c)-(d) and FDI residuals (e)-(f) under two consecutive failures in the control actuators and subsequent actuator reconfiguration. The dashed lines in (a)-(b) show the behavior in the absence of failure compensation, and the dashed lines in (c)-(d) depict the outputs of the backup actuators.

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