

# Dead-Beat and LQ-Optimal Power Control Algorithms in the Uplink of Wireless Systems

D.U. Campos-Delgado, J.M. Luna-Rivera, and F.J. Martínez-López

**Abstract**—Power control is an important problem in today's wireless systems, which is related to the battery utilization in the mobile units. In this work, this problem is addressed using a distributed approach. The uplink of a direct-sequence code-division multiple-access communication (DS-CDMA) system is studied, and through a proper selection of the error function, the nonlinear coupling among the active users is transformed to individual loops, where the power references incorporate the information of the remaining users in the cell. It is concluded that the uplink channel variations do not destroy the stability of the feedback structures. However, the delays in the closed-loop paths can severely affect the stability and performance of the resulting feedback schemes. An LQ-optimal power control strategy is derived and compared to a Dead-Beat approach. In this sense, the Dead-beat algorithm is sensitive to the roundtrip delays estimation, but the LQ-optimal control can be adjusted to be robust to this estimation error. However, there is a severe compromise between robustness and performance. But through an appropriate selection of the LQ weight parameter, a stable closed-loop system can be always guaranteed independently on the uncertainty in the estimation of the roundtrip delay. Simulation results are presented to compare the control algorithms using a standard single-step power correction approach.

## I. INTRODUCTION

The code-division multiple-access (CDMA) strategies are of particular importance in the wireless communication systems due to their efficiency over frequency and time division (FDMA and TDMA) approaches [1]. In a CDMA system, all users transmit information over the same time and frequency, and each user signal is recovered through a particular code sequence [2]. These same properties of CDMA systems limit their capacity and performance in practical scenarios, due to the multiple-access interference (MAI). Power control is a common strategy to alleviate the MAI component, since it can also compensate the near-far effect and optimize the power consumption at the mobile unit [3], [4]. As a result, the objective of the power control law is to reach a desired signal to noise-interference ratio (SNIR) for each user, and consequently, satisfy the requirements for the expected quality of service (QoS) [5], [6]. Now, power control algorithms can be divided into centralized (CPC) or distributed (DPC), according to the information required to make the power adjustments at the mobile units. DPC is more appealing since only local information is needed and the complexity of the control algorithm is largely reduced.

D.U. Campos-Delgado and J.M. Luna-Rivera are with Facultad de Ciencias, UASLP, San Luis Potosí, S.L.P., México, {ducd, mlr}@fciencias.uaslp.mx

F.J. Martínez-López is with Coordinación Académica Región Altiplano, UASLP, Matehuala, S.L.P., México, fjm1@uaslp.mx

On the other hand, DPC has been approached from two different perspectives: multivariable optimization and control theory [5]. The advantage of using a control theory framework is that stability and reference tracking can be jointly studied for each mobile unit. In [7], a cross-layer problem is studied, where the multiuser detection and power control are analyzed using a game-theoretic strategy looking to make an efficient use of the available energy. Meanwhile, a first-order gradient power update is considered in [8] and [9]. This update law possesses passivity properties with respect to the resulting feedback systems, and this property is exploited to guarantee closed-loop stability and robustness against disturbances and time delays. In [10], a decentralized dynamic power control algorithm is developed, which admits a global solution under certain conditions, and looks for the minimum power level that satisfies the service requirement. In addition, an adaptive power control law is suggested in [11], where the adaptation is considered with respect to a quantized feedback signal. Thus, it is pursued to improve the time response of a single-step power correction approach by a dynamic update of the step size.

It is important to take into account that the SNIR estimation, and the processing and transmission of the power control commands imply delays in the power adjustments. If these delays are not considered into the control design stage, the performance of the resulting feedback system can be severely deteriorated or even instability could be triggered [12]. Moreover, there is a natural question regarding the effect of the uplink channel variations into the closed-loop stability. As a result, in this work, a control theory perspective is adopted to study the uplink DPC. First, it is shown that the problem of SNIR reference tracking at each mobile unit, which is posed as a nonlinear control problem, can be translated into a linear power reference tracking by a suitable selection of the error function. Furthermore, the resulting linear feedback loops do not depend on the channel gains, and their information is appended into the power references. Next, an LQ-control algorithm is designed, and compared to a Dead-Beat strategy and a standard fixed-step power adjustment. Hence a clear improvement is devised in terms of robustness for the LQ-control algorithm against transmission delays. The rest of the paper is organized as follows. In Section II, the mathematical model of a DS-CDMA system is presented. Section III describes the feedback structure of DPC including uplink and downlink transmission delays. The linear power control laws are synthesized in Section IV. Section V presents a robustness analysis in terms of uncertainty

in the roundtrip transmission delay. Finally, Section VI shows simulation results to illustrate the performance improvements with these control schemes, and Section VII presents some conclusions and describes future work.

## II. MODEL OF A DS-CDMA SYSTEM

### A. Simplified Model of a CDMA System

In a wireless communication CDMA system, the information is sent from the mobile unit (MU) to the base station (BS) by the uplink channel, and from the base station to the mobile unit by the downlink. The model of the communication presented in this section can be considered as a simplified one, since the spreading codes properties are not directly considered. Nevertheless, the main dynamics in the power update interactions among all users are properly addressed.

The uplink communication channel of the  $i$ -th user can be characterized by a power gain  $|h_i[k]|^2$ . This gain is affected by three main components: path loss, shadowing and multipath fading [2]. To achieve a communication link through each channel, which is characterized by gain  $|h_i[k]|^2$ , the information is transmitted using a power level  $p_i[k]$ . Thus, the receiver detects a power signal  $p_i[k]|h_i[k]|^2$  related to the  $i$ -th user, besides the power associated with the MAI component, and lastly the power related to thermal noise at the receiver  $\sigma_i^2$ . Then, the data bits are recovered using detection algorithms that use the spreading code characteristics. Moreover, it is possible to express the QoS of the  $i$ -user in terms of its SNIR  $\gamma_i$  at  $k$ -instant, as proposed in [13]. Now, considering the uplink channel with  $U$  active users, and applying linear detectors at the BS, the SNIR can be calculated by

$$\gamma_i[k] = \frac{\delta_{ii} p_i[k] |h_i[k]|^2}{\sum_{j \neq i} \delta_{ij} p_j[k] |h_j[k]|^2 + \chi_i \sigma_i^2} \quad (1)$$

where  $\delta_{ii}$ ,  $\delta_{ij}$  and  $\chi_i$  are scaling coefficients that depend on the applied linear detector (Matched Filter, Decorrelator, MMSE and Projector) and spreading codes properties [13]. In fact, the SNIR can be expressed using a vector notation

$$\gamma[k] = \begin{bmatrix} \gamma_1[k] \\ \vdots \\ \gamma_U[k] \end{bmatrix} = \phi(\mathbf{x}[k], \mathbf{y}[k]) \triangleq \begin{bmatrix} \frac{x_1[k]}{y_1[k]} \\ \vdots \\ \frac{x_U[k]}{y_U[k]} \end{bmatrix} \quad (2)$$

where  $\mathbf{x}[k], \mathbf{y}[k] \in \mathbb{R}^U$  are given by

$$\mathbf{x}[k] = \mathbf{\Gamma}_{bit} \mathbf{H}[k] \mathbf{p}[k] \quad (3)$$

$$\mathbf{y}[k] = \mathbf{\Gamma}_{MAI} \mathbf{H}[k] \mathbf{p}[k] + \boldsymbol{\eta}, \quad (4)$$

with  $\mathbf{p}[k] = [p_1[k] \dots p_U[k]]^T$  representing the power vector,  $\boldsymbol{\eta}$  the resulting noise vector, and matrix  $\mathbf{\Gamma}_{bit}$  denoting data amplitudes,  $\mathbf{\Gamma}_{MAI}$  multiple access interference, and

$\mathbf{H}[k]$  channel gains. These elements are expressed by

$$\mathbf{H}[k] \triangleq \begin{bmatrix} |h_1[k]|^2 & & & \\ & \ddots & & \\ & & |h_U[k]|^2 & \\ & & & \end{bmatrix} \in \mathbb{R}^{U \times U} \quad (5)$$

$$\mathbf{\Gamma}_{bit} \triangleq \begin{bmatrix} \delta_{11} & & & \\ & \ddots & & \\ & & \delta_{UU} & \\ & & & \end{bmatrix} \in \mathbb{R}^{U \times U} \quad (6)$$

$$\mathbf{\Gamma}_{MAI} \triangleq \begin{bmatrix} 0 & \delta_{12} & \dots & \delta_{1U} \\ \delta_{21} & 0 & \dots & \delta_{2U} \\ \delta_{U1} & \delta_{U2} & \dots & 0 \end{bmatrix} \in \mathbb{R}^{U \times U} \quad (7)$$

$$\boldsymbol{\eta} \triangleq [\chi_1 \sigma_1^2 \dots \chi_U \sigma_U^2]^T \in \mathbb{R}^U \quad (8)$$

As a result, the detector properties are embedded into  $\mathbf{\Gamma}_{bit}$ ,  $\mathbf{\Gamma}_{MAI}$  and  $\boldsymbol{\eta}$ .

## III. DISTRIBUTED POWER CONTROL ALGORITHMS

DPC schemes generally involve two feedback loop paths: external and internal loops. The external loop is in charge of estimating the objective SNIR for  $i$ -th user ( $\gamma_i^{obj}$ ) according to the QoS requirements. Furthermore, it has to gradually adjust these reference values for all users in order to achieve an equilibrium state that maximizes the data rates and minimizes the required power. On the other hand, the internal loop has to adjust the user's power to satisfy the objective SNIR and compensate the environment perturbations [5]. In this work, only the internal loop is addressed, assuming a fixed value of  $\gamma_i^{obj}$  and applying control theory to synthesize the power update laws.

### A. Standard Problem Formulation

Next the standard closed-loop power control algorithm is detailed following a vector notation (see Fig. 1):

$$\mathbf{p}[k+1] = \mathbf{p}[k] + \mathbf{u}[k] \quad (9)$$

$$\mathbf{u}[k] = \mathbf{K}(\mathbf{a}[k]) \triangleq \begin{bmatrix} K_1(a_1[k]) \\ \vdots \\ K_U(a_U[k]) \end{bmatrix}$$

$$\mathbf{a}[k] = \mathbf{e}[k - n_p]$$

$$\mathbf{e}[k] = \boldsymbol{\gamma}[k - n_m] - \boldsymbol{\gamma}^{obj}$$

where  $\mathbf{u}[k] \in \mathbb{R}^U$  denotes the power update vector (*control signal*),  $\mathbf{a}[k] \in \mathbb{R}^U$  the error signal vector received at the mobile unit,  $\mathbf{e}[k] \in \mathbb{R}^U$  the SNIR reference error vector calculated at the BS, and  $K_i(\cdot)$  the control algorithm implemented at the  $i$ -th mobile unit. Therefore, the distributed philosophy can be viewed from (9), since the control law  $K_i$  for the  $i$ -th user depends only on the reference error for that user  $a_i[k]$ . Finally,  $n_p$  and  $n_m$  denote the delays involved in the downlink and uplink paths, respectively. Define then the *roundtrip delay* in the system as  $n_{RT} = n_m + n_p + 1$ . Note that the resulting closed-loop systems for all active users are nonlinear coupled-loops by the SNIR computation in (2).

Next, it is presented an important proposition that expresses the equivalence of (9) to  $U$ -decoupled linear closed-loop systems by introducing a new tracking error definition.

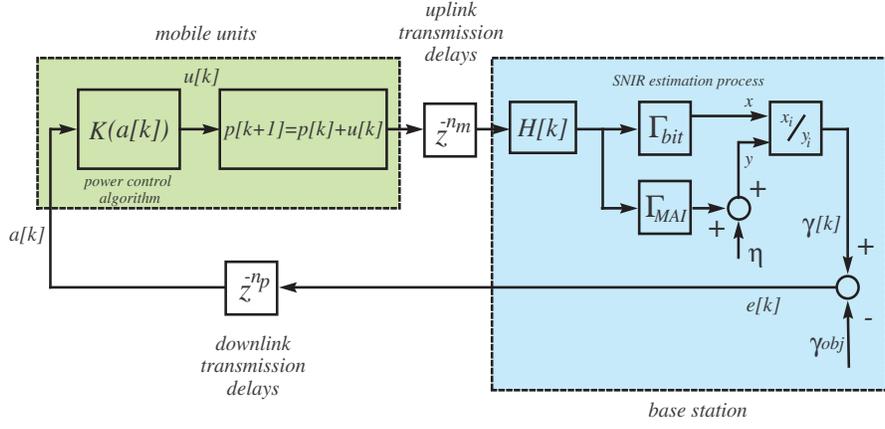


Fig. 1. Standard Formulation of a Distributed Power Control Algorithm for a CDMA Wireless Communication System.

Throughout the paper, due to space limitation, proofs are not given in this version but are available from the authors.

**Proposition 1:** the dynamic systems described by (1) and (9) are equivalent to  $U$  linear-decoupled feedback loops, if the tracking error is computed by

$$e_i[k] = \left[ 1 - \frac{\gamma_i^{obj}}{\gamma_i[k - n_m]} \right] p_i[k - n_m] \quad (10)$$

where the equivalent error terms are given now by

$$e_i[k] = p_i[k - n_m] - \hat{p}_i[k - n_m] \quad (11)$$

with varying power references

$$\hat{p}_i[k] = \gamma_i^{obj} \left[ \sum_{j \neq i} \frac{\delta_{ij} |h_j[k]|^2}{\delta_{ii} |h_i[k]|^2} p_j[k] + \frac{\chi_i \sigma_i^2}{\delta_{ii} |h_i[k]|^2} \right]. \quad (12)$$

■

*Remark 1:* The control problem is now translated into a varying reference tracking problem of a power level that achieves the desired SNIR. This new error definition was first proposed in [14], but its properties with respect to the closed-loop systems were not fully analyzed. Moreover, the  $U$  closed-loops are not coupled, since for the  $i$ -th user, only its power level and estimated SNIR at the BS are involved into the feedback paths, with some delays by the uplink and downlink transmissions. In fact, the power information by the remaining users is employed to emulate a virtual power reference to achieve the desired SNIR value  $\gamma_i^{obj}$ . Besides that, the information of the channel responses are not included inside the feedback paths. As expected, if the  $i$ -th user channel gain  $|h_i[k]|^2$  decreases, its required power reference will be raised to achieve  $\gamma_i^{obj}$ . But if the interference of the remaining users in the cell decreases  $\sum_{j \neq i} |h_j[k]|^2 p_j \rightarrow 0$ , then the power reference will tend to decrease and only the noise contribution will be dominant. The equivalent linear feedback loops are illustrated in Figure 2. Thus, if the channel responses are quickly or slowly changing, then the power reference will vary accordingly but

these changes will not unstabilize the independent loops. On the other hand, the transmission delays will play an important role on performance and closed-loop stability.

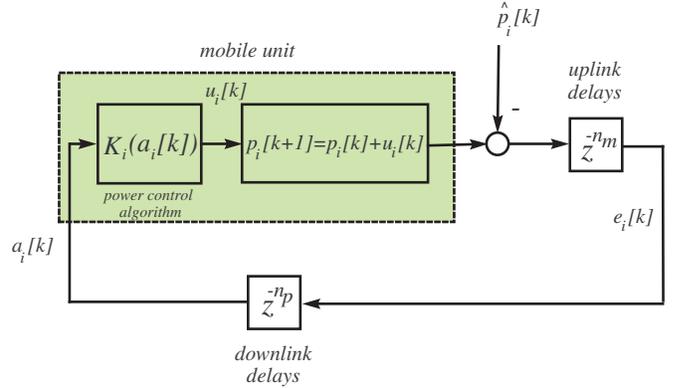


Fig. 2. Equivalent Distributed Power Control Algorithm with Modified Error Function.

Assume for the rest of the paper the error transformation in (10) for (9), and consider a linear control law for the  $i$ -th loop:

$$K_i(z) = \frac{U_i(z)}{A_i(z)} \triangleq \frac{num_i(z)}{den_i(z)}, \quad (13)$$

where  $U_i(z)$  and  $A_i(z)$  are the  $z$ -transforms of  $u_i[k]$  and  $a_i[k]$ , and  $num_i(z)$  and  $den_i(z)$  denote the numerator and denominator polynomials of the controller in  $z$ -domain, respectively. Therefore, the closed-loop characteristic polynomial is given by

$$\rho(z) \triangleq (1 - z^{-1})den_i(z) - z^{-n_{RT}} num_i(z). \quad (14)$$

As a result, in order to guarantee closed-loop stability, the roots of (14) have to lie inside the unit-circle.

#### IV. POWER CONTROL ALGORITHMS

In this section, two linear control algorithms are suggested, looking to keep closed-loop stability despite transmission delays, and achieve the reference tracking objective with zero error.

### A. Dead-Beat Control, DB-DPC

The Dead-Beat controller guarantees perfect tracking for a constant reference in a finite number of steps [15]. This condition can only be satisfied in discrete systems by locating all the closed-loop poles at the origin.

**Proposition 2:** the linear Dead-Beat controller for each distributed system defined by (9) and (11) is given by

$$K_i^{DB}(z) = \frac{-1}{1 + z^{-1} + z^{-2} + \dots + z^{-(n_{RT}-1)}}. \quad (15)$$

*Remark 2:* Note that (15) shows an auto-regressive structure, so the actual power update depends on the actual reference error and the past power updates. Furthermore, the structure of (15) can be easily adapted according with the estimated delays in the transmission lines ( $n_p, n_m$ ), in order to guarantee closed-loop stability and performance. Moreover, it does not require any tuning during implementations, in contrast with a PID control law [14].

### B. LQ-Optimal Power Update, LQ-DPC

Now, the controller synthesis is carried out by using a linear quadratic (LQ) performance index [16], assuming a regulation control problem. First note that after the error transformation suggested in Proposition 1, the open-loop dynamics are given by

$$a_i[k+1] = a_i[k] + u_i[k - n_{RT} + 1] \quad (16)$$

and define the LQ index

$$J \triangleq \frac{1}{2} \sum_{k=0}^{\infty} \{a_i^2[k] + r u_i^2[k - n_{RT} + 1]\} \quad (17)$$

where  $r > 0$  is a control weight.

**Proposition 3:** the control law that stabilizes the closed-loop system and solves the optimization problem

$$\min_{\substack{u_i[k] \\ \text{stabilizing}}} J \quad (18)$$

such that (16) is satisfied, it is given by

$$u_i[k] = -\Omega \left( a_i[k] + \sum_{l=1}^{n_{RT}-1} u_i[k-l] \right) \quad (19)$$

$$\Rightarrow K_i^{LQ}(z) = \frac{-\Omega}{1 + \Omega [z^{-1} + z^{-2} + \dots + z^{-(n_{RT}-1)]} \quad (20)$$

where the control parameter  $\Omega$  is expressed by

$$\Omega \triangleq \frac{2}{1 + \sqrt{4r + 1}}. \quad (21)$$

*Remark 3:* Note that in (19), if  $r \rightarrow 0$  (no control penalty) then the Dead-Beat control law is obtained, since from (21)

$$\lim_{r \rightarrow 0} \Omega = 1. \quad (22)$$

On the other hand, if  $r \rightarrow \infty$  then  $\Omega \rightarrow 0$ , and a slower response should be expected.

## V. ROBUSTNESS ANALYSIS

It was concluded from Proposition 1 that with the new error function in (10), the channel variations will not destroy the closed-loop stability. However, uncertainty in the uplink and downlink transmission delays can unstabilize the feedback system. Next, two cases are studied with respect to the uncertainty in the estimation of the roundtrip delay:

Case I: the real roundtrip delay  $n_{RT}$  is larger than the estimated  $\hat{n}_{RT}$ , i.e.  $n_{RT} > \hat{n}_{RT}$  (*underestimation*).

Case II: the opposite scenario, the real roundtrip delay is smaller than the estimated one, i.e.  $n_{RT} < \hat{n}_{RT}$  (*overestimation*).

Note that the control law is designed taking into account  $\hat{n}_{RT}$ , but the real delay in the power control feedback system will be  $n_{RT}$ . Next, robust stability of the Dead-Beat algorithm in (15) and LQ-control in (20) will be analyzed with respect to the roundtrip delay uncertainty.

### A. Case I

Define the uncertainty in the transmission delay as

$$\Delta n \triangleq n_{RT} - \hat{n}_{RT} > 0. \quad (23)$$

The resulting closed-loop characteristic polynomials are:

$$\begin{aligned} \rho_{DB}^I(z) &\triangleq z^{n_{RT}} - z^{\Delta n} + 1, \\ \rho_{LQ}^I(z) &\triangleq z^{n_{RT}} + (\Omega - 1)z^{n_{RT}-1} - \Omega z^{\Delta n} + \Omega, \end{aligned} \quad (24)$$

for the Dead-Beat (DB) and LQ-Controller, respectively. One clear advantage of the LQ-control law is that the synthesis parameter  $r$  (that defines  $\Omega$ ) can be chosen to maximize the robustness of the resulting controller in (20), against the roundtrip delay uncertainty. The next proposition states one of the main contributions of this work.

**Proposition 4:** for any values of  $n_{RT}$  and  $\hat{n}_{RT}$  that satisfy  $n_{RT} > \hat{n}_{RT}$ , there exists a small value of  $\Omega$  ( $0 < \Omega \leq 1$ ) such that the closed-loop poles are always stable. Moreover, a necessary condition for closed-loop stability is  $\Omega < \frac{2}{3}$ .

*Remark 4:* As a result, a small value of  $\Omega$  will improve the closed-loop robustness, but at the expense of a sluggish time response. Thus, this value has to be properly chosen to guarantee closed-loop stability against the possible uncertainty scenarios faced in practice, in order to avoid a severe degradation in the time response.

### B. Case II

Consider now the opposite uncertainty scenario for the transmission delay, and define

$$\Delta d \triangleq \hat{n}_{RT} - n_{RT} > 0. \quad (25)$$

The closed-loop characteristic polynomials are:

$$\begin{aligned} \rho_{DB}^{II}(z) &\triangleq z^{\hat{n}_{RT}} + z^{\Delta d} - 1, \\ \rho_{LQ}^{II}(z) &\triangleq z^{\hat{n}_{RT}} + (\Omega - 1)z^{\hat{n}_{RT}-1} + \Omega z^{\Delta d} - \Omega. \end{aligned} \quad (26)$$

Similarly to Case I, robust stability can be assured for a small  $\Omega$  using classical-control tools, against transmission delay uncertainty.

**Proposition 5:** for any values of  $\hat{n}_{RT}$  and  $n_{RT}$  that satisfy  $\hat{n}_{RT} > n_{RT}$ , there exists a small value of  $\Omega > 0$  such that the closed-loop poles are always stable. Similarly to Proposition 4, a necessary condition for robust stability is  $\Omega < \frac{2}{3}$ . ■

On the other hand, with respect to the characteristic polynomials related to the Dead-Beat controller in (24) and (26), it can be deduced (Chapter 5 in [17]) that the roots  $\tilde{z}$  lie in the interval

$$\frac{1}{\sqrt{3}} \leq |\tilde{z}| \leq \sqrt{3}, \quad (27)$$

for all  $n_{RT}$  and  $\hat{n}_{RT}$ . Thus, it can be expected that some roots could be outside the unit circle. In fact, it is observed that (24) and (26) always exhibit at least one unstable root for any  $n_{RT}$  and  $\hat{n}_{RT}$ . Consequently, the Dead-Beat control law is pretty sensitive to uncertainty in the roundtrip delay, but it provides a fast transient response. Meanwhile, the LQ-control law can be tuned to provide transmission delay robustness, but at the price of sacrificing the transient response.

To verify the robustness of the LQ-strategy, the roots of polynomials  $\rho_{LQ}^I(z)$  and  $\rho_{LQ}^{II}(z)$  in (24) and (26) have been computed numerically, for  $n_{RT}, \hat{n}_{RT} \in [1, 20]$ . The control parameter  $\Omega$  was varied in the interval  $(0, \frac{2}{3})$  to find its maximum value that still maintains stable roots for the studied transmission delays. Figure 3 shows that there is always a value of  $\Omega$  that keeps stable roots independently of  $n_{RT}, \hat{n}_{RT}$ . However, a smaller value of  $\Omega$  is needed as the difference between  $n_{RT}$  and  $\hat{n}_{RT}$  gets larger. From Fig. 3, the maximum value of  $\Omega$  that maintains stability for  $n_{RT}, \hat{n}_{RT} \in [1, 20]$  is  $\Omega = 0.08$ .

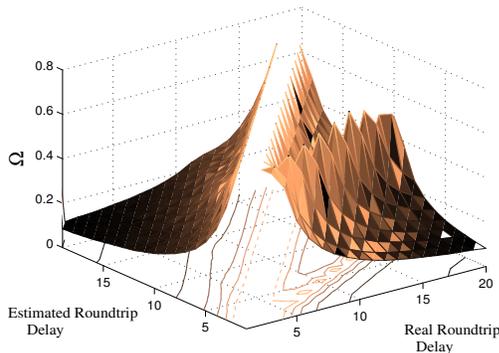


Fig. 3. Maximum Value of  $\Omega$  that Guarantees Closed-Loop Stability for Different Roundtrip Delays Scenarios.

## VI. SIMULATION EVALUATION

The evaluation of the DPC algorithms was carried out using the "Rudimentary Network Emulator" (RUNE) [18]. The studied system considers one cell with radius 1 km,

and an omnidirectional antenna located at the BS. All users have random position, velocity and acceleration. The average velocity and acceleration for each user are 10 m/s and 1 m/s<sup>2</sup>, respectively. The objective SNIR is fixed at  $\gamma^{obj} = 7$  dB with a processing gain  $N = 64$ . The noise power is  $-90$  dBm and the sampling frequency used for power update is 1500 Hz. The maximum power per user is set to 100 mW, meanwhile the minimum is 1  $\mu$ W. Next, the resulting closed-loop performance is evaluated with respect to the SNIR tracking and the required uplink transmission power. Finally, the BS employs a simple Matched-Filter detection strategy for all users considering random spreading codes. The Dead-Beat (DB-DPC) and LQ (LQ-DPC) controllers are compared with a standard fixed-step power update (FS-DPC) given by

$$u_i[k] = \begin{cases} \Delta p_i & a_i[k] < 0 \\ 0 & a_i[k] = 0 \\ -\Delta p_i & a_i[k] > 0 \end{cases} \quad (28)$$

where  $\Delta p_i = 1$  dB. In the LQ strategy, the control weight was selected to obtain  $\Omega = 0.5$ . Two scenarios were simulated according to the roundtrip transmission delay value:

Case A: the real and estimated roundtrip delays are equal and they are given by 3, i.e.  $n_{RT} = 3, \hat{n}_{RT} = 3$ .

Case B: an overestimation scenario is tested, where the real delay is 3 and the estimated is 5, i.e.  $n_{RT} = 3, \hat{n}_{RT} = 5$ .

### A. Evaluation with DPC in One User

The results shown in Figs. 4 and 5 were obtained applying the DPC algorithms to only one random user, for a DS-CDMA system with  $U = 15$  active users, and Cases A and B, respectively. The remaining users transmit at a constant power level of 1 mW, resulting in a constant power reference for the studied user. From these plots, it can be observed that the performance of the FS-DPC is severely affected by the transmission delays, due to the large oscillation around the objective SNIR. Meanwhile, the Dead-Beat shows, as expected, perfect tracking in a finite number of steps for Case A, and an unstable response under delay uncertainty in Case B. Finally, the LQ-control algorithm presents good tracking and robustness on both scenarios. As a result, the LQ algorithm provides the best response for this evaluation.

### B. Evaluation with DPC in All Users

Figures 6 and 7 illustrate the results applying DPC to each user connected to the BS for Cases A and B, respectively. Now, it is assumed  $U = 15$  active users in the cell, and for simplicity all have the same number of transmission delays. Note that in Case A (see Fig. 6), the Dead-Beat controller has some overshoot in the time response due to a varying reference signal, but under Case B (see Fig. 7), the time response is once more unstable. Meanwhile, the FS-DPC scheme produces large oscillation around the objective SNIR for both cases. On the other hand, the time response for the LQ-control presents also overshoot, but it achieves perfect tracking after some transient time, even under the delay uncertainty scenario in Case B (see Fig. 7). Hence, the best results are also derived for the LQ-strategy.

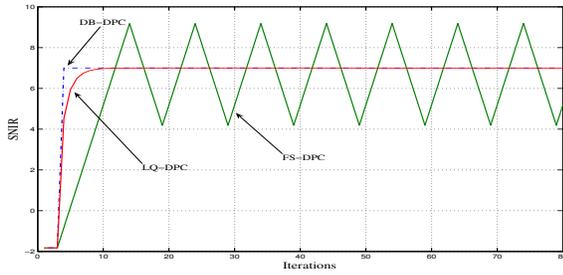


Fig. 4. Evaluation of DPC Applied to One User (Case A).

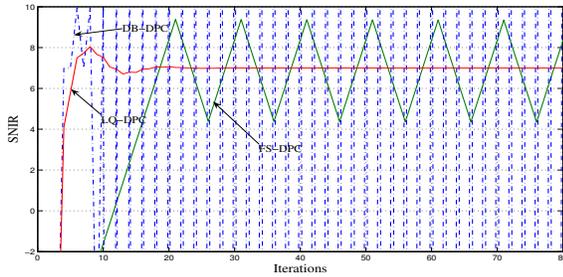


Fig. 5. Evaluation of DPC Applied to One User (Case B).

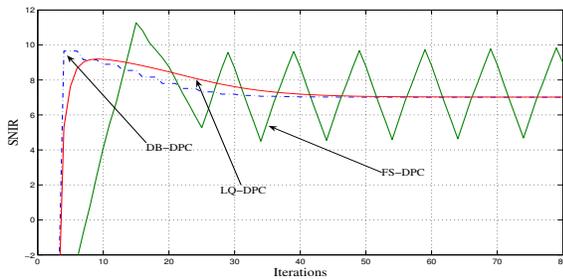


Fig. 6. Evaluation of DPC Applied to All Users (Case A).

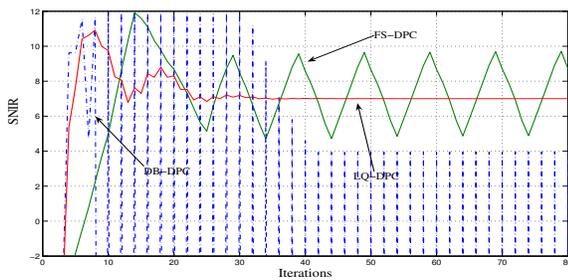


Fig. 7. Evaluation of DPC Applied to All Users (Case B).

## VII. CONCLUSIONS AND FUTURE WORK

In this work, the problem of distributed power control in DS-CDMA systems is studied from a control theory perspective. By a new definition of the error function, the resulting closed-loops for each user are linear and independent from the remaining user's dynamics. Moreover, the channel variations are not affecting the closed-loop stability.

However, the transmission delays play an important role with respect of stability and performance. Dead-Beat and LQ-control laws were proposed for power update. Both control algorithms have a structure that depend on the roundtrip transmission delay. One key advantage of the LQ-control is that it can be tuned to provide robustness against any roundtrip delay uncertainty. In future work,  $H_\infty$  will be also analyzed and compared to these classical control schemes.

## VIII. ACKNOWLEDGMENTS

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