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Awareness-Based Decision Making for Search and Tracking

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***Abstract*— This paper focuses on the management of mobile sensor agents for the search and tracking of multiple objects of interest. In the case where such objects are in greater numbers than available agents, search and tracking are two competing demands since a sensor agent can perform either the tracking mission or the search mission, but not both at the same time. A sensor agent has to decide on whether to continue searching or stop and track once it finds an object of interest. Based on a novel dynamic awareness model and assuming static objects, a decision-making and control strategy is developed to guarantee the full coverage of a domain of interest, and, equivalently, the detection of all objects of interest in the domain with probability one. The strategy also guarantees the tracking of each object's "state" for a minimum guaranteed amount of time τ_c. Centralized and decentralized implementations are described. Numerical simulations are provided to demonstrate the performance of the strategies.**

I. Introduction

This paper focuses on the management of mobile sensor agents for the search and tracking of multiple objects of interest, whose number is unknown beforehand, over a given domain. There are two basic tasks in such a problem. The first task is the *search* task, where the goal is to detect each object of interest and fix its position in space and time for dynamic objects. The second task is the *tracking* task, where the goal is to observe each found object for a desired critical minimum amount of time, after which the desired amount of information about the object has been collected. In the case where such objects are possibly in greater numbers than available sensor agents, search and tracking are two competing demands. This is because a sensor agent can perform either the tracking mission or the search mission, but not both at the same time (search requires mobility and tracking requires neighboring the object). Hence, a sensor agent has to decide whether to continue searching or switch to object tracking once it finds an object of interest. In this work, we define search and tracking metrics that are used for this decision making process.

Inspired by work on particle filtering, in [1] the authors develop a sensor based approach for object tracking. The control goal is to minimize the error in tracking the objects' positions. In [2], a distributed sequential auction scheme is presented for a multi-robot search and destroy operation. Local communications between neighbors are allowed and the shared information is used to make the decision. The control goal is to allocate an agent to an object and complete the mission in minimum time. In [3], the control goal is to maximize the total number of observed objects and the amount of observation time of each.

In this paper, we introduce a novel "awareness-based" formulation that is naturally susceptible to analytic tools from systems and control theory. This model describes how "aware" the agent fleet is of events over a given domain. This formulation can be applied to a wide variety of problems, including large-scale and complex domains, that may be disconnected (surveillance over adversarial pockets in a region), or hybrid discrete and continuous (surveillance over urban environments and inside buildings, where roads and hallways are the continuous part of the domain, and buildings and rooms are discrete nodes).

A predecessor to this awareness-based formulation is the effective coverage control formulation [4], [5], [6]. In effective coverage control, the goal is to survey a given domain such that each point in the domain is satisfactorily sampled using a network of dynamic, limited-range, sensor-equipped agents. In [4], [5], [6], coverage control strategies (that also included flocking and collision avoidance) were developed. In the effective coverage formulation, information to be observed or monitored is assumed static in nature.

The awareness formulation discussed in this paper allows for the analysis of the more general setting where information and events are dynamic in nature. Under appropriate assumptions, in this paper, centralized and decentralized control strategies are proposed that guarantee the detection of all objects in the domain. Each object is also guaranteed to be tracked, and a lower bound on the amount of tracking time is provided for both centralized and decentralized implementations.

The paper is organized as follows. In Section II, we present a novel dynamic model for the notion of "awareness" over a domain, a limited-range sensor model is discussed, and search and tracking metrics are defined. In Section III, we develop centralized and decentralized decision-making strategies. This paper is concluded with a summary of the paper in Section IV.

II. Problem Formulation

A. Setup and Sensor Model

In the search task, all objects of interest in a search domain are required to be found. In the tracking task, each

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found object has to be tracked for an amount of time equal to τ_c, which is the critical minimum information collection time that is needed to characterize the state of an object. Characteristics of interest for immobile objects may be geometric shape, classification and categorization, and/or nature of electromagnetic emissions. For mobile objects, this may also included position and velocity information to predict the motion of the object while it is not being tracked. This paper focuses on the static object case and future research will focus on mobile objects of interest.

Let $\mathcal{D}$ be a domain in which objects are to be found and tracked. Let $N_{\rm o} \geq 0$ be the number of these objects. Both $N_{\rm o}$ and the locations of the objects in $\mathcal{D}$ are unknown beforehand. Let $N_{\rm a}$ be the number of autonomous sensor agents. At time t, let the set $\mathcal{A} = \mathcal{S}(t) \cup \mathcal{T}(t) = \{1, \ldots, N_{\rm a}\}$, which is the set of indices of all the agents in the sensor fleet, and where the set $\mathcal{S}(t)$ contains indices of agents carrying out the search mission, and where the set $\mathcal{T}(t)$ contains indices of agents carrying out an object tracking mission. In this paper, we will assume that agents can either be searching or tracking at any instant time t, but not both simultaneously, and therefore the sets $\mathcal{S}(t)$ and $\mathcal{T}(t)$ are disjoint for all t. Initially, we assume that all agents are in $\mathcal{S}(t)$. When a search agent detects an object and decides to track its state, this search agent turns into a tracking agent and, hence, there is one fewer agent in the set $\mathcal{S}(t)$ and one more agent in the set $\mathcal{T}(t)$.

Assuming some search versus tracking decision making strategy that guarantees coverage of the entire domain and that avoids the assignment of multiple agents for the tracking of a single object, for the case when $N_{\rm o} \leq N_{\rm a}$, after a certain amount of time, each object will be guaranteed to be detected and its state permanently tracked by some agent. However, for the worst case scenario where $N_{\rm o} > N_{\rm a}$ and with a poor choice of decision making strategy, one may end up with $\mathcal{S}(t) = \varnothing$ while there may still exist unfound objects. For example, a strategy where once an object is found it is tracked for all time from that point forward would likely lead to some objects never being detected when there are more objects than agents. In this paper, we investigate strategies that guarantee that each object will be found and tracked, especially for the worst case scenario, while simultaneously providing a lower bound for the amount of tracking time.

Let the position of the static object $\mathcal{O}_j$, $j \in \{1, 2, \ldots, N_{\rm o}\}$, be $\mathbf{p}_j$, which is not known beforehand. Each agent $\mathcal{A}_i$ satisfies the following kinematic equation

$$\dot{\mathbf{q}}_i = \mathbf{u}_i,$$

where $\mathbf{u}_i \in \mathbb{R}^2$ is the velocity of agent $\mathcal{A}_i$. This is a simplified model and the results may be extended to agents with second order nonlinear dynamics evolving on more complex configuration manifolds.

In this work, the word "sensor" refers to the sensor used for the search process. The specific sensor used for tracking the state of an object is not of primary interest in this work. A key feature of the proposed approach is that the sensor may have a limited range. Existing research in the literature on cooperative sensor coverage control for the redeployment problem usually assumes that the sensors have an infinite range. This is especially true for work within the stochastic framework (see, for example, [7]) that assumes Gaussian distributions with infinite sensor ranges. This assumption is not required here. This is very important in applications where $\mathcal{D}$ is large-scale (i.e., too large to be covered by a single set of static sensor agents).

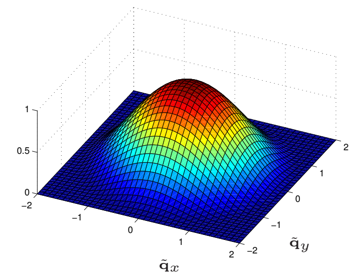

Fig. 1. Instantaneous coverage function A_i with $\mathbf{q}_i = 0$, $M_i = 1$ and $r_i = 2$.

Without loss of generality, here we assume a sensor model that is a fourth order polynomial function of $s = \|\mathbf{q}_i - \tilde{\mathbf{q}}\|$ within the sensor range and zero otherwise:

$$A_i(s) = \begin{cases} \frac{M_i}{r_i^4}\left(s^2 - r_i^2\right)^2 & \text{if } s \leq r_i \\ 0 & \text{if } s > r_i \end{cases}. \quad (1)$$

The function $A_i : \mathcal{D} \times Q \rightarrow \mathbb{R}^+$, where $\mathbb{R}^+ = \{a \in \mathbb{R} : a \geq 0\}$, is the *instantaneous sensor coverage function*. It is a positive semi-definite function that describes the quality of the measurement made at a point $\tilde{\mathbf{q}} \in \mathcal{D}$. Note that each sensor has a limited domain $\mathcal{W}_i(t) = \{\tilde{\mathbf{q}} \in \mathcal{D} : \|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\| \leq r_i\}$ with range r_i, peak sensing capacity M_i exactly at the sensor location $\mathbf{q}_i$, and a sensing capability that degrades with range. We assume that $\mathcal{W}_i(t)$ has a rigid boundary. The instantaneous coverage function in equation (1) is shown in Figure 1. This sensor model is a simplified planar radar sensor model similar to that used in [8]. Other sensor models may be considered. For example, in [9] the authors consider a vision based sensor model that is also similar to the one which combines vision and ultrasonic sensing used in [10]. For a vision-based sensor model applied to a three-dimensional configuration space scenario with 9 viewpoints, see [11]. Such a sensor (which has directionality) may easily be applied in this work.

B. Awareness Model

We first define an individual agent's state of awareness, which is a distribution $\mathbf{x}_i(\tilde{\mathbf{q}}, t)\ \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}$. Below, $\mathbf{x}_i(\tilde{\mathbf{q}}, t) \leq 0$ implies that $\mathbf{x}_i$ is negative for every $\tilde{\mathbf{q}} \in \mathcal{D}$. The state of awareness $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ is defined to be a measure of how "aware" the agent is of events occurring at a specific location $\tilde{\mathbf{q}}$ at time t. Fixing a point $\tilde{\mathbf{q}} \in \mathcal{D}$, the state of awareness of an agent $\mathcal{A}_i$ is assumed to satisfy the following differential equation

$$\begin{aligned}\dot{\mathbf{x}}_i(\tilde{\mathbf{q}},t) &= -\left(A_i(\|\tilde{\mathbf{q}}-\mathbf{q}_i\|)-\alpha\right)\mathbf{x}_i(\tilde{\mathbf{q}},t),\\ \mathbf{x}_i(\tilde{\mathbf{q}},0) &= \mathbf{x}_{i0}=-1,\end{aligned}\qquad (2)$$

where $i \in \mathcal{A}(t)$. The constant parameter $0 < \alpha < A_i$ models an "awareness loss" bath.

Note that under the awareness dynamics (2), the maximum value attainable by $\mathbf{x}_i$ is 0 if the initial awareness level is negative. If $\mathbf{x}_i(\tilde{\mathbf{q}},t) < 0$ then the agent has a less than desired awareness level. The desired awareness level is the equilibrium state $\mathbf{x}_i(\tilde{\mathbf{q}},t) = 0$. If $\mathbf{x}_i(\tilde{\mathbf{q}},t) > 0$ then the agent has an excessive awareness level. The initial "awareness" distribution $\mathbf{x}_{i0}(\tilde{\mathbf{q}})$ is assumed negative, reflecting the fact that at the outset of the surveillance mission the fleet has poor awareness levels. To reflect the difficulty of object detection one simply sets a more negative value for $\mathbf{x}_{i0}(\tilde{\mathbf{q}})$.

The easier the detection process is, the larger the negative value of $\mathbf{x}_{i0}(\tilde{\mathbf{q}})$ can be set. The distribution $\mathbf{x}_{i0}(\tilde{\mathbf{q}})$ may also be nonuniform to reflect regions where objects may be able to camouflage themselves better than in other regions of $\mathcal{D}$ (e.g., dense forests versus open fields).

The state of awareness of the set of search agents $\mathcal{S}(t)$ in surveying $\tilde{\mathbf{q}}$ then satisfies the differential equation

$$\begin{aligned}\dot{\mathbf{x}}(\tilde{\mathbf{q}},t) &= -\left[\left(\sum_{i\in\mathcal{S}(t)} A_i(\|\tilde{\mathbf{q}}-\mathbf{q}_i\|)\right)-\alpha\right]\mathbf{x}(\tilde{\mathbf{q}},t),\\ \mathbf{x}(\tilde{\mathbf{q}},0) &= \mathbf{x}_0=-1,\end{aligned}\qquad (3)$$

where $\sum_{i\in\mathcal{S}(t)} A_i(\|\tilde{\mathbf{q}}-\mathbf{q}_i\|)$ is the total instantaneous coverage achieved by all the agents in the set $\mathcal{S}(t)$ at time t. If one wishes to consider the state of awareness achieved by a set $\mathcal{K} \subset \mathcal{S}$, then one can use equation (3) but summing only over elements in $\mathcal{K}$. Note that $\mathbf{x}_i \le \mathbf{x}$. That is, the overall awareness of the sensors in a centralized system is better than that of the individual sensors in a decentralized system.

One can define the decentralized awareness error associated with search agent $\mathcal{A}_i$ to be

$$e^i_{\mathcal{W}_i}(t) = \int_{\mathcal{W}_i(t)} \mathbf{x}_i^2(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}}. \qquad (4)$$

This is a decentralized awareness metric associated with agent $\mathcal{A}_i$ that reflects the quality of the state of awareness within $\mathcal{W}_i(t)$ achieved by agent $\mathcal{A}_i$ alone. This metric will be used to develop the control law for the decentralized search and tracking problem. Moreover, define the centralized awareness metric associated with the entire search fleet $\mathcal{S}(t)$ by

$$e_{\mathcal{W}_i}(t) = \int_{\mathcal{W}_i(t)} \mathbf{x}^2(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}}. \qquad (5)$$

This is a centralized awareness metric associated with agent $\mathcal{A}_i$ that reflects the quality of the state of awareness within $\mathcal{W}_i(t)$ achieved by all vehicles in $\mathcal{S}$. This metric will be used to develop the control law for the centralized search and tracking problem.

C. Search and Tracking Metrics under Condition of No Information Loss

For the purpose of this section and the remainder of the paper, we make the following assumption.

Assumption II.1. *There is no information loss where sensors do not exist. That is, assume that $\alpha = 0$.*

This assumption indicates that the awareness states have no inherent/direct time dependence, and are only influenced by the actions of sensing and tracking platforms and target locations in the mission domain $\mathcal{D}$.

The cost associated with a decision not to carry out further searching, $\mathcal{J}_1(t)$, is chosen to be proportional to the size of the un-searched domain. Here, we assume a uniform probability distribution for the locations of objects in $\mathcal{D}$, hence, $\mathcal{J}_1(t)$ is proportional to the probability of finding another object beyond time t. The cost associated with a decision not to track found objects, $\mathcal{J}_2(t)$, is chosen to be proportional to the time spent not tracking the state of a found object.

We define the search cost function to be

$$\mathcal{J}_1(t) = \frac{e(t)}{e_{\max}}, \qquad (6)$$

where

$$e(t) = \int_{\mathcal{D}} \mathbf{x}^2(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}}, \qquad (7)$$

is the global error over the entire mission domain achieved by the whole vehicle fleet. Under Assumption II.1 and considering a uniform probability distribution for the locations of the objects in $\mathcal{D}$, the maximum actual error is given by

$$e_{\max} = e(0) = \text{Area of } \mathcal{D}$$

because $\mathbf{x}_{i0} = -1$. According to this definition, we have $0 \le \mathcal{J}_1(t) \le 1$. Initially, $\mathcal{J}_1(0) = 1$ describes the fact that we know with probability 1 that there exists at least one object which has not been detected. This comes from the assumption that $N_\mathrm{o} > 0$. If N_o happens to be zero, assuming that there exists at least one object of interest over the domain will guarantee verifying that there is none. Under Assumption II.1, when $\mathcal{J}_1(t_s) = 0$ for some time $t_s > 0$, the entire domain has been satisfactorily covered and we know that there are no objects yet to be found. At this point, the search process is said to be *completed*.

For the tracking metric $\mathcal{J}_2(t)$, let $\bar{N}_\mathrm{o}(t) \le N_\mathrm{o}$ be the number of objects found by the sensor fleet up to time t. Define the tracking cost function $\mathcal{J}_2(t)$ to be

$$\mathcal{J}_2(t) = \int_0^t \sum_{j=1}^{\bar{N}_\mathrm{o}(t)} g_j(t)\mathrm{d}\tau, \qquad (8)$$

where

$$g_j(t) = \begin{cases} 1 & \text{if } \mathbf{p}_j(t) \notin \mathcal{W}_i(t) \text{ for all } i \in \mathcal{A} \\ 0 & \text{if } \mathbf{p}_j(t) \in \mathcal{W}_i(t) \text{ for some } i \in \mathcal{A}. \end{cases}$$

If a search agent detects an object $\mathcal{O}_j$ a function $g_j(t)$ is assigned to the object (unless it has already been assigned one if detected in the past). A value of 0 is assigned to

g_j as long as some agent tracks $\mathcal{O}_j$, and the tracking cost associated with $\mathcal{O}_j$ is zero. In this case, $\mathcal{O}_j$ will be labeled as "assigned". Once the search agent decides not to track $\mathcal{O}_j$, $\mathcal{O}_j$ is now labeled "unassigned", and $g_j(t)$ switches its value to 1, implying that a cost is now associated with not tracking the found object $\mathcal{O}_j$. According to Equation (8), this cost is equal to the amount of time during which a found object is not tracked.

A remark on the case with some information loss. If we relax Assumption II.1, the parameter α in the awareness model reflects loss of spatial information over time. It essentially sets a periodicity to how often the entire area must be re-surveyed. On the other hand, g_j reflects loss of information associated with a specific object over time. It is important to realize this distinction between the domain-awareness loss nature of α (and, hence, $\mathcal{J}_1$) and the specific-object awareness loss nature of g_j (and, hence, $\mathcal{J}_2$). The case of $\alpha \neq 0$, will be addressed in future work. •

III. Search versus Tracking Decision-Making

Under Assumption II.1, we will consider a search/tracking decision making strategy that guarantees, in both its centralized and decentralized implementations, finding all objects in $\mathcal{D}$ and tracking each object for some time with a lower bound on tracking time.

A. Centralized Strategy

Since we assume that $N_{\mathrm{o}} > N_{\mathrm{a}}$, whenever an agent detects an object, it has to decide whether to track it or to continue searching. If it does decide to track, it has to decide on how much time it can afford to track before it continues the search process. Before deriving one possible way to determine the amount of tracking time, let us first consider a search strategy. The goal in the search strategy is to attain an awareness level of $\|\mathbf{x}(\tilde{\mathbf{q}}, t)\| \leq \epsilon$ for all $\tilde{\mathbf{q}} \in \mathcal{D}$ and all $t \geq t_s$ for some $t_s > 0$.

For the search process, we use a control law that drives the state of lack of awareness to a neighborhood of zero within the sensory domain. Let us first consider the following condition, whose utility will become obvious shortly.

Condition C1. $\quad \mathbf{x}(\tilde{\mathbf{q}}, t) = 0, \ \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t), \ i \in \mathcal{A}.$

Lemma III.1. If for some $t \geq 0$, Condition **C1** holds, then $e_{\mathcal{W}_i}(t) = 0, i \in \mathcal{A}$. Conversely, if $e_{\mathcal{W}_i}(t) = 0$ for some time $t \geq 0$, then Condition **C1** holds.

Proof. The proof follows directly from the definitions of $e_{\mathcal{W}_i}(t)$ and $\mathbf{x}(\tilde{\mathbf{q}}, t)$. ■

Consider the following centralized search control law

$$\mathbf{u}_i^*(t) = \begin{cases} \bar{\mathbf{u}}_i(t) & \text{if Condition } \mathbf{C1} \text{ doesn't hold for } \mathcal{A}_\mathrm{i} \in \mathcal{S}_\mathrm{i} \\ \bar{\bar{\mathbf{u}}}_i(t) & \text{if Condition } \mathbf{C1} \text{ holds for } \mathcal{A}_\mathrm{i} \in \mathcal{S}_\mathrm{i} \end{cases} \quad (9)$$

where

$$\bar{\mathbf{u}}_i(t) = \bar{k}_i \int_{\mathcal{W}_i(t)} \mathbf{x}^2(\tilde{\mathbf{q}}, t) \underbrace{\left(\int_0^t \frac{\partial A_i(\tilde{\mathbf{q}}, \mathbf{q}_i(\sigma))}{\partial \tilde{\mathbf{q}}} \mathrm{d}\sigma \right)}_{\text{memory term}} \mathrm{d}\tilde{\mathbf{q}} \quad (10)$$

is the *nominal control law*,

$$\bar{\bar{\mathbf{u}}}_i(t) \quad = \quad -\bar{\bar{k}}_i(\mathbf{q}_i(t) - \tilde{\mathbf{q}}_i^*)$$

is the *perturbation control law*, $\bar{k}_i > 0$ and $\bar{\bar{k}}_i > 0$ are controller gains, and $\tilde{\mathbf{q}}_i^* \in \mathcal{D}$ is chosen such that $\|\mathbf{x}(\tilde{\mathbf{q}}, t)\| > \epsilon$. An agent $\mathcal{A}_i$ employs the control law $\bar{\mathbf{u}}_i(t)$ to move in the direction that improves the local (since integration is performed over the sensor domain $\mathcal{W}_i(t)$) awareness level. If Condition **C1** holds, then the linear feedback controller $\bar{\bar{\mathbf{u}}}_i(t)$ is used to drive the agent out of the local minimum of $e_{\mathcal{W}_i}$ to some point $\tilde{\mathbf{q}}_i^* \in \mathcal{D}$ such that $\|\mathbf{x}(\tilde{\mathbf{q}}_i^*, t)\| > \epsilon$ if such a point exist. If such $\tilde{\mathbf{q}}_i^*$ does not exist, then the search mission has been completed. The control law $\mathbf{u}_i^*(t)$ guarantees coverage of the entire domain $\mathcal{D}$ with $\mathcal{J}_1(t)$ converging to a small neighborhood of zero (as will be proven below). The convergence of $\mathcal{J}_1(t)$ to a small neighborhood of zero implies that all agents have been found and the search process is complete. The tracking strategy discussed below will guarantee that all objects will be tracked for a minimum of τ_c amount of time. The search control law (9) and the tracking strategy, together, will guarantee the detection of all objects of interest and their tracking for at least τ_c amount of time.

Let $\mathcal{D}_\epsilon(t) := \{\tilde{\mathbf{q}} \in \mathcal{D} : \|\mathbf{x}(\tilde{\mathbf{q}}, t)\| > \epsilon\}$, which is an open set of all points $\tilde{\mathbf{q}}$ for which $\mathbf{x}(\tilde{\mathbf{q}}, t)$ is smaller than a preset value $-\epsilon$. Let $\overline{\mathcal{D}}_\epsilon(t)$ be the closure of $\mathcal{D}_\epsilon(t)$. Let $\overline{\mathcal{D}}_{\epsilon,i}(t)$ be the set of points in $\overline{\mathcal{D}}_\epsilon(t)$ that minimize the distance between the position vector of agent $\mathcal{A}_i$, $\mathbf{q}_i$, and the set $\overline{\mathcal{D}}_\epsilon(t)$:

$$\overline{\mathcal{D}}_{\epsilon,i}(t) = \left\{ \tilde{\mathbf{q}}^* \in \overline{\mathcal{D}}_\epsilon(t) : \tilde{\mathbf{q}}^* = \mathrm{argmin}_{\tilde{\mathbf{q}} \in \overline{\mathcal{D}}_\epsilon(t)} \|\tilde{\mathbf{q}} - \mathbf{q}_i(t)\| \right\}.$$

If $\overline{\mathcal{D}}_{\epsilon,i}(t)$ is empty, this means that the distribution $\|\mathbf{x}(\tilde{\mathbf{q}}, t)\| < \epsilon$ everywhere over the domain and the search mission is complete. Note that the choice of the point $\tilde{\mathbf{q}}_i^*$ is based on centralized awareness information, which is appropriate in the setting of this section since the control law is centralized. The choice of $\tilde{\mathbf{q}}_i^*$ will be modified for the decentralized implementation.

A remark on the computational advantages of the proposed search approach over possible alternative approaches. The search approach proposed herein requires computations at the order of $\mathcal{O}(\bar{n}^2 + 2)$ at each time step, where $\bar{n}$ is the number of cells in the discretized sensory domain $\mathcal{W}_i$. While alternative approaches, such as Voronoi-partitioning and stochastic-based SLAM methods, are computationally more burdensome. See [5] for more details.

Other Remarks.

- Note that $\bar{\mathbf{u}}_i$ relies on the properties of the sensor coverage function A_i. Hence, the coverage control law relies on the given sensor model to guide the vehicle during the coverage mission.

- In the expression for $\bar{\mathbf{u}}_i(t)$, the time integral term under the spatial integration is an integration of historical data that translates into the reliance on past search history for decision making. Note that the memory term is multiplied by $\mathbf{x}^2(\tilde{\mathbf{q}},t)$ before being integrated over the sensory domain at the current time t. This indicates that historical data as well as up-to-date awareness levels within the vehicle's sensor domain are compounded to decide on motion direction and speed. •

If a search agent finds object(s) within its sensory range, then it will track the object(s) for a T time period from the time of detection, where

$$T = \frac{\tau_c}{\mathcal{J}_1(t_d)}, \tag{11}$$

t_d being the time of object detection, and where $\tau_c > 0$ is the desired critical minimum amount of tracking time. This is the amount of time that is needed to characterize the state of an object. The larger the value of $\mathcal{J}_1(t_d)$ is (i.e., the less aware the agent is of the domain), the less time the agent will spend tracking the state of the object. As the degree of awareness increases at detection time, the more time the agent spends tracking the object. Note that $\mathcal{J}_1(t_d)$ can not be zero unless the mission is completed, at which point there is no need to compute T.

Hence, once an agent detects an object and decides to track this particular object, it becomes a tracking agent and will not carry out any searching for a period of T seconds. Note that while the agent is tracking, other agents may be searching. In the centralized implementation, the amount of centralized system awareness $\mathbf{x}(\tilde{\mathbf{q}},t)$ is available to all vehicles. So is the value of $\mathcal{J}_1(t_d)$. We assume that each object will only be tracked once by only one vehicle during the mission. After a time period of T, the tracking agent will switch back to become a search agent and leave its tracking position to find new objects. At this point in time, the object will be labeled "assigned" and will not be tracked by any other agent if found.

We will now prove that the centralized search and tracking control strategy guarantees the detection of all objects of interest and their tracking for at least a desired amount of time equal to τ_c. Let us first consider the following lemma (see [12] for a detailed exposition), which will be used shortly.

Lemma III.2. For any function $F : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}$ we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{W}_i(t)} F(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}} = \int_{\mathcal{W}_i(t)} \left[(\mathrm{grad}_{\tilde{\mathbf{q}}} F(\tilde{\mathbf{q}},t))\cdot \mathbf{u}_i + \frac{\partial F(\tilde{\mathbf{q}},t)}{\partial t}\right]\mathrm{d}\tilde{\mathbf{q}},$$

where $\mathbf{u}_i$ is the velocity of agent $\mathcal{A}_i$ and $\mathrm{grad}_{\tilde{\mathbf{q}}}$ is the gradient operator with respect to $\tilde{\mathbf{q}}$.

Proof. This is a direct consequence of Equation (3.3) in [12], where we note that $\mathbf{u}_i$ is the velocity of any vector of the rigid domain $\mathcal{W}_i$. ■

Theorem III.1. Under Assumption II.1, the centralized search and tracking control strategy given by equations (9) and (11) will guarantee that $\mathcal{J}_1$ converges asymptotically to zero, which is equivalent to guaranteeing that all objects be found. The minimum amount of time spent tracking any object is given by τ_c.

Proof. Consider the function $\bar{V}_i = e_{\mathcal{W}_i}(t)$. From Lemma III.1, $\bar{V}_i = 0$ if and only if Condition **C1** holds. According to lemma III.2

$$\dot{\bar{V}}_i = \dot{e}_{\mathcal{W},i}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{W}_i(t)} \mathbf{x}^2(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}} \tag{12}$$

$$= \int_{\mathcal{W}_i(t)} \mathrm{grad}(\mathbf{x}^2(\tilde{\mathbf{q}},t))\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} + \int_{\mathcal{W}_i(t)} \frac{\partial(\mathbf{x}^2(\tilde{\mathbf{q}},t))}{\partial t}\mathrm{d}\tilde{\mathbf{q}}.$$

First consider the spatial gradient term in (12):

$$\int_{\mathcal{W}_i(t)} \mathrm{grad}(\mathbf{x}^2(\tilde{\mathbf{q}},t))\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} = \int_{\mathcal{W}_i(t)} \frac{\partial(\mathbf{x}^2(\tilde{\mathbf{q}},t))}{\partial \tilde{\mathbf{q}}}\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}}$$

$$= 2\int_{\mathcal{W}_i(t)} \mathbf{x}(\tilde{\mathbf{q}},t)\frac{\partial(\mathbf{x}(\tilde{\mathbf{q}},t))}{\partial \tilde{\mathbf{q}}}\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}}.$$

We will derive the expression for $\frac{\partial(\mathbf{x}(\tilde{\mathbf{q}},t))}{\partial \tilde{\mathbf{q}}}$. From Equation (3) and assuming $\alpha = 0$, we have

$$\mathbf{x}(\tilde{\mathbf{q}},t) = e^{-\int_0^t \sum_{i\in\mathcal{S}(t)} A_i(\tilde{\mathbf{q}},\mathbf{q}_i(\sigma))\mathrm{d}\sigma}\mathbf{x}_0.$$

Hence,

$$\frac{\partial(\mathbf{x}(\tilde{\mathbf{q}},t))}{\partial \tilde{\mathbf{q}}} = -\sum_{i\in\mathcal{S}(t)} \mathbf{x}(\tilde{\mathbf{q}},t)\int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}},\mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}}\mathrm{d}\sigma.$$

Therefore, we have

$$\int_{\mathcal{W}_i(t)} \mathrm{grad}(\mathbf{x}^2(\tilde{\mathbf{q}},t))\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} = -2\sum_{i\in\mathcal{S}(t)}\int_{\mathcal{W}_i(t)} \mathbf{x}^2(\tilde{\mathbf{q}},t) \cdot\left(\int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}},\mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}}\mathrm{d}\sigma\right)\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}}.$$

Substitute $\bar{\mathbf{u}}_i(t)$ in Equation (10) into the above equation, we obtain

$$\int_{\mathcal{W}_i(t)} \mathrm{grad}(\mathbf{x}^2(\tilde{\mathbf{q}},t))\cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} = -\bar{k}_i\sum_{i\in\mathcal{S}(t)}\left[\int_{\mathcal{W}_i(t)} \mathbf{x}^2(\tilde{\mathbf{q}},t) \cdot\left(\int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}},\mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}}\mathrm{d}\sigma\right)\mathrm{d}\tilde{\mathbf{q}}\right]^2 \leq 0.$$

Next, let us consider the integral of the time derivation term in Equation (12). According to Equation (3) and assuming no information loss, that is, $\alpha = 0$,

$$\int_{\mathcal{W}_i(t)} \frac{\partial(\mathbf{x}^2(\tilde{\mathbf{q}},t))}{\partial t}\mathrm{d}\tilde{\mathbf{q}} = -2\int_{\mathcal{W}_i(t)} \mathbf{x}^2(\tilde{\mathbf{q}},t)\sum_{i\in\mathcal{S}(t)} A_i(\|\tilde{\mathbf{q}}-\mathbf{q}_i\|^2)\mathrm{d}\tilde{\mathbf{q}} \leq 0.$$

Therefore, $\dot{\bar{V}}_i \leq 0$. One can check that equality holds if and only if Condition **C1** holds.

If Condition **C1** holds, the perturbation control law is applied that moves the sensor vehicle to some point $\tilde{\mathbf{q}}_i^*$ with $\|\mathbf{x}(\tilde{\mathbf{q}}_i^*,t)\| > \epsilon$. If no such point exists, then the mission is completed since $\|\mathbf{x}(\tilde{\mathbf{q}},t)\| < \epsilon$ everywhere inside $\mathcal{D}$.

If $\tilde{\mathbf{q}}_i^*$ exists, by continuity of $\mathbf{x}(\tilde{\mathbf{q}},t)$ as a function of both of its arguments (which results from the fact that

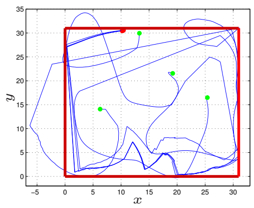

Fig. 2. Centralized Implementation: fleet motion in the plane.

$\mathbf{x}$ satisfies smooth dynamics whose solution are unique, well-defined, and continuous), then in some neighborhood of $\tilde{\mathbf{q}}_i^*$, $\|\mathbf{x}\|$ will be greater than ϵ. Under the perturbation control law, we will have $\|\mathbf{q}_i - \tilde{\mathbf{q}}_i^*\| < r_i$ and, hence, Condition **C1** will not hold. At this point in time, the control is switched back to the nominal control law. This procedure is repeated until whenever Condition **C1** holds and there does not exist $\tilde{\mathbf{q}}_i^*$ according to the criteria given above. The non-existence of such a $\tilde{\mathbf{q}}_i^*$ guarantees that $e_{\mathcal{W}_i}(t)$ is sufficiently close to zero (since $\|\mathbf{x}\|$ is smaller than ϵ everywhere). Hence, by definition, $\mathcal{J}_1$ will be guaranteed to be within a small neighborhood of zero.

The minimum tracking time comes from the fact that once an object is found, it will be tracked for at least $\tau_c/\mathcal{J}_1(t_d)$, t_d being the detection time. $\mathcal{J}_1(t_d)$ assumes a maximum value of 1 if $t_d = 0$. In the extreme scenario where an object is found at $t = 0$, the value of T is exactly τ_c. If an agent is found at a time other than $t = 0$, $\mathcal{J}_1(t_d)$ has to be less than 1 and, hence, T is greater than τ_c. ■

Remark. For the case when N_o is known before hand and $N_\text{o} \leq N_\text{a}$, under the centralized search, and assuming that if some agent finds a target it will track this target for all future time, each object will be guaranteed to be detected and its state permanently tracked by some agent. Proof of complete coverage of the domain, and, hence, detection of each object, follows directly from the proof of Theorem III.1. Since $N_\text{o} \leq N_\text{a}$ and each object can only be tracked by one agent, assigning a unique agent to a single object whenever an object is detected is feasible (i.e., we have enough resources to do so) and every object will be permanently tracked. •

A simulation result is provided in Figures 2, where $N_\text{o} = 6$ and $N_\text{a} = 4$ for some choice of controller gains and coverage sensor parameters. The domain $\mathcal{D}$ is square in shape and discretized into n cells, where $\tilde{\mathbf{q}} \in \mathbb{R}^2$ represents the centroid of each cell. Hence, $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ can be written as a vector of dimension $2n$. Table I shows the tracking time of each object, which is guaranteed to be at least $\tau_c = 5$ seconds.

	Object 1	Object 2	Object 3	Object 4	Object 5	Object 6
T, (s)	8.0583	50.2437	7.5215	5.2552	10.3786	6.6144

TABLE I

TRACKING TIME T FOR EACH OBJECT.

B. Decentralized Strategy

We now assume that the sensor fleet is completely decentralized. That is, each agent is aware of coverage achieved by itself alone. Each object it finds will be assumed to be found for the first time. This represents a scenario where communications between agents is not possible (for example, due to security reasons, the sensor agents have to remain "silent" otherwise they themselves may be detected by adversary agents).

We need the following condition and lemma, whose proof is similar to that of Lemma III.1.

Condition **C2.** $\mathbf{x}_i(\tilde{\mathbf{q}}, t) = 0,\ \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t).$

Lemma III.3. If for some $t \geq 0$, Condition **C2** holds, then $e^i_{\mathcal{W}_i}(t) = 0, i \in \mathcal{A}$. Conversely, if $e^i_{\mathcal{W}_i}(t) = 0$ for some time $t \geq 0$, then Condition **C2** holds for agent $\mathcal{A}_i$.

In the decentralized formulation, we employ the search control strategy

$$\mathbf{u}^*_{\text{d}i}(t) = \begin{cases} \bar{\mathbf{u}}_{\text{d}i}(t) & \text{if Condition } \mathbf{C2} \text{ doesn't hold for } \mathcal{A}_\text{i} \in \mathcal{S}_\text{i} \\ \bar{\bar{\mathbf{u}}}_{\text{d}i}(t) & \text{if Condition } \mathbf{C2} \text{ holds for } \mathcal{A}_\text{i} \in \mathcal{S}_\text{i} \end{cases} \tag{13}$$

where

$$\bar{\mathbf{u}}_{\text{d}i}(t) = \bar{k}_i \int_{\mathcal{W}(\mathbf{q}_i(t))} \left(\int_0^t \frac{\partial A_i(\|\tilde{\mathbf{q}} - \mathbf{q}_i\|)}{\partial \mathbf{q}_i} dt \right) \mathbf{x}_i^2(\tilde{\mathbf{q}}, t) d\tilde{\mathbf{q}},$$

and $\bar{\bar{\mathbf{u}}}_{\text{d}i}(t)$ is the same feedback controller as that used in the centralized strategy, except that $\tilde{\mathbf{q}}_i^*$ is chosen based on coverage information available to agent $\mathcal{A}_i$ only. This choice is made as follows.

Let $\mathcal{D}^i_\epsilon(t) := \{\tilde{\mathbf{q}} \in \mathcal{D} : \|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\| > \epsilon\}$, which is an open set of all points $\tilde{\mathbf{q}}$ for which $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ is smaller than a preset value $-\epsilon$. Let $\overline{\mathcal{D}}^i_\epsilon(t)$ be the closure of $\mathcal{D}^i_\epsilon(t)$. Let $\overline{\mathcal{D}}^i_{\epsilon,i}(t)$ be the set of points in $\overline{\mathcal{D}}^i_\epsilon(t)$ that minimize the distance between the position vector of agent $\mathcal{A}_i$, $\mathbf{q}_i$, and the set $\overline{\mathcal{D}}^i_{\epsilon,i}(t) = \left\{ \tilde{\mathbf{q}}^* \in \overline{\mathcal{D}}^i_\epsilon(t) : \tilde{\mathbf{q}}^* = \text{argmin}_{\tilde{\mathbf{q}} \in \overline{\mathcal{D}}^i_\epsilon(t)} \|\tilde{\mathbf{q}} - \mathbf{q}_i(t)\| \right\}$. As done for the centralized case, we will assume that there will exist at most one point in $\overline{\mathcal{D}}^i_{\epsilon,i}(t)$. If $\overline{\mathcal{D}}^i_{\epsilon,i}(t)$ is empty, this means that $\|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\| < \epsilon$ everywhere over the domain and that the search mission is complete. Note that the choice of the point $\tilde{\mathbf{q}}_i^*$ is based on awareness information available only to the vehicle $\mathcal{A}_i$, which is appropriate in the setting of this section since the control law is decentralized.

In the decentralized case, when a search agent $\mathcal{A}_i$ detects object(s) within its sensory range, it tracks the objects for a time period of T, defined by

$$T = \frac{\tau_c}{\mathcal{J}_{1i}(t_d)} \tag{14}$$

where t_d is the time at detection, $\tau_c > 0$ is given, and

$$\mathcal{J}_{1i}(t) = \frac{e^i_{\mathcal{D}}(t)}{e^i_{\mathcal{D},\max}}, \tag{15}$$

and where

$$e^i_{\mathcal{D}}(t) = \int_{\mathcal{D}} \mathbf{x}_i^2(\tilde{\mathbf{q}}, t) d\tilde{\mathbf{q}} \tag{16}$$

	Object 1	Object 2	Object 3	Object 4	Object 5	Object 6
Agent 1	7.0090	5.8723	5.1221	5.3971	6.1709	5.6022
Agent 2	5.5974	7.0428	5.1835	5.0000	5.6022	6.1474
Agent 3	8.7574	7.9469	5.1609	5.6027	5.3634	5.1281
Agent 4	5.7563	5.1835	7.0981	6.3030	5.9109	6.5911

TABLE II

TRACKING TIME OF EACH OBJECT BY EACH AGENT.

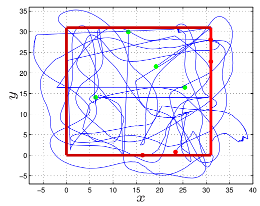

Fig. 3. Decentralized Implementation: fleet motion in the plane.

is the global error over the entire mission domain achieved by the vehicle $\mathcal{A}_i$ only, with $e^i_{\mathcal{D},\max} = e^i_{\mathcal{D}}(0)$ is the area of $\mathcal{D}$ if the initial state $\mathbf{x}_i(\tilde{\mathbf{q}}, t) = -1$ as we had assumed from the outset. Moreover, we define the cost of not tracking an object found by agent $\mathcal{A}_i$ by

$$\mathcal{J}_{2i}(t) = \int_0^t \sum_{j=1}^{\bar{N}^i_{\mathrm{o}}(t)} g_j(\mathbf{p}_j(t))\mathrm{d}\tau, \qquad (17)$$

where $\bar{N}^i_{\mathrm{o}}(t)$ is the number of agents found by agent $\mathcal{A}_i$ up to time t. We assume that each object will only be tracked once by each vehicle during the mission.

Theorem III.2. Under Assumption II.1, the decentralized search and tracking strategy given by equations (13) and (14) will guarantee that $\mathcal{J}_1$ converges asymptotically to zero, which is equivalent to guaranteeing that all agents be found. The minimum amount of time spent tracking any object is given by τ_c.

The proof of this theorem is similar to the proof provided for the centralized case. The only important aspect of the proof that needs highlighting is that, along the same lines as the proof for the centralized case, $\mathcal{J}_{1i}$ is guaranteed to converge to zero for all $i \in \mathcal{A}_i$. It is not immediately clear that the global cost $\mathcal{J}_1$ will also converge to zero as the Theorem III.2 states. However, note that $e^i_{\mathcal{D}}(t) \geq e(t)$ because the more vehicles and sensors available to us, at least the same or higher overall global coverage is achieved by the system. Since $\mathcal{J}_1$ and $\mathcal{J}_{1i}$ (for all $i \in \mathcal{A}_i$) are both initialized to be 1, then $\mathcal{J}_{1i}(t) \geq \mathcal{J}_1(t)$, for all time t, because $e^i_{\mathcal{D}}(t) \geq e(t)$. If $\mathcal{J}_{1i}(t)$ is guaranteed to converge to zero under the control law (13), then so does $\mathcal{J}_1(t)$.

Remark. For the case when $N_{\mathrm{o}} \leq N_{\mathrm{a}}$, under the decentralized search and assuming permanent tracking after target detection, the entire domain will be guaranteed to be searched, and each object detected and its state permanently tracked by some agent. •

A simulation result is provided in Figures 3, where the domain $\mathcal{D}$ is square in shape. Table II shows the tracking time of each object by each vehicle, which is guaranteed to be at least $\tau_c = 5$ seconds.

IV. CONCLUSION

Based on a novel dynamic awareness model, a decision-making and control strategy was developed to guarantee the detection of all objects of interest in a domain. The strategy also guarantees the tracking of each object of a minimum guaranteed amount of time τ_c. These properties are guaranteed under both centralized and decentralized implementations of the strategy. Numerical simulations demonstrated the operation of the two strategies. Future research will focus on locating and tracking dynamic objects. In the dynamic objects case, the novel awareness formulation developed above will prove particularly useful.

REFERENCES

[1] J. R. Spletzer and C. J. Taylor, "Dynamic Sensor Planning and Control for Optimally Tracking Targets," *The International Journal of Robotics Research*, no. 1, pp. 7 – 20, January 2003.

[2] P. B. Sujit and R. Beard, "Distributed Sequential Auctions for Multiple UAV Task Allocation," *Proceedings of 2007 American Control conference*, pp. 3955 – 3960, July 2007.

[3] L. Parker, "Cooperative robotics for multi-target observation," *Intelligent Automation and Soft Computing*, vol. 5, no. 1, 1999.

[4] I. I. Hussein and D. M. Stipanović, "Effective Coverage Control using Dynamic Sensor Networks," *2006 IEEE Conference on Decision and Control*, pp. 2747 – 2752, December 2006. [Online]. Available: http://users.wpi.edu/~ihussein/Publications/HuSt-CDC-06.pdf

[5] ——, "Effective Coverage Control for Mobile Sensor Networks with Guaranteed Collision Avoidance," *IEEE Transactions on Control Systems Technology, Special Issue on Multi-Vehicle Systems Cooperative Control with Applications*, vol. 15, no. 4, pp. 642 – 657, 2007.

[6] ——, "Effective Coverage Control using Dynamic Sensor Networks with Flocking and Guaranteed Collision Avoidance," *2007 American Control Conference*, pp. 3420 – 3425, July 2007. [Online]. Available: http://users.wpi.edu/~ihussein/Publications/HuSt-ACC-07.pdf

[7] B. Grocholsky, "Information-theoretic control of multiple sensor platforms," Ph.D. dissertation, The University of Sydney, 2002.

[8] I. I. Hussein and D. Stipanović, "Effective coverage control for mobile sensor networks with guaranteed collision avoidance," *IEEE Transactions on Control Systems Technology, Special Issue on Multi-Vehicle Systems Cooperative Control with Applications*, vol. 15, no. 4, pp. 642–657, July 2007.

[9] Y. Wang and I. I. Hussein, "Cooperative vision-based multi-vehicle dynamic coverage control for underwater applications," *2007 IEEE Multiconference on Systems and Control*, 2007, invited paper, to appear.

[10] A. Ohya, A. Kosaka, and A. Kak, "Vision-based navigation by a mobile robot with obstacle avoidance using single-camera vision and ultrasonic sensing," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 969 – 978, December 1998.

[11] S. Y. Chen and Y. F. Li, "Automatic sensor placement for model-based robot vision," *IEEE Transactions on Systems, Man, and Cybernetics ł- Part B: Cybernetics*, vol. 34, no. 1, pp. 393 – 408, February 2004.

[12] H. Flanders, "Differentiation Under the Integral Sign," *The American Mathematical Monthly*, vol. 80, no. 6, pp. 615 – 627, 1973.