2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Adaptive motion coordination: Using velocity feedback to achieve parameter convergence

He Bai Murat Arcak John T. Wen

Abstract— **We study a coordination problem where the objective is to steer a group of agents to a formation that translates with a prescribed reference velocity. In [1] we considered the situation where the information of reference velocity is available only to a leader, and developed a decentralized adaptive design with which the other agents reconstruct the reference velocity. Although [1] guaranteed that the desired formation is achieved, it did not ensure parameter convergence. We now propose a new adaptive redesign, which incorporates relative velocity feedback in addition to relative position feedback, to guarantee parameter convergence.**

I. I

Growing research in motion coordination has lead to significant results in formation control, consensus, deep-space flying, swarming, *etc.* [2]–[6]. The main challenge in cooperative control is designing decentralized control laws that depend on relative information, to achieve designed group behaviors. The design techniques developed so far employ Lyapunov analysis and potential function method [7], [8], matrix analysis [5], graph theoretic results [9], *etc.*

Recent study in [10] introduced passivity as a unifying design tool for cooperative control problems, such as consensus and formation control. Reference [10], as well as earlier results such as [2], assumed that the reference velocity is available to each agent. This assumption is further relaxed in [1] to the situation where only the leader possesses the reference velocity while the other agents reconstruct this information by employing a decentralized adaptive design. Although this basic adaptive design guaranteed the convergence to the desired formation, it did not ensure parameter convergence. Lack of parameter convergence means that the reference velocity information is not fully recovered even if the desired formation is achieved.

In this paper, we propose a modified adaptive redesign to guarantee parameter convergence. The main idea in our modified design is to ensure that the relative velocities between agents converge to zero, thereby guaranteeing that all agents converge to the reference velocity. To this end, we include the relative velocity information in our redesign and recover all the convergence results in [10] while achieving parameter convergence in addition. The measurements of relative velocity can be obtained by vision-based techniques or by autonomous formation flying sensors (AFF) [11], [12].

The subsequent sections are organized as follows: Section II reviews the passivity-based nonadaptive and adaptive design in [10] [1]. In Section III, we present a motivating example to show that parameter convergence may fail with the design of [1]. Our modified adaptive redesign is proposed and analyzed in Section IV and the motivating example is revisited in Section V.

He Bai, Murat Arcak and John T. Wen are with Department of Electrical, Computer and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, USA. {baih, arcakm, wenj} @rpi.edu

II. R P - F A D

A. Passivity Framework

In [10], a group of agents is considered, where each agent $i = 1, \cdots N$ is represented by a vector $x_i \in \mathbb{R}^p$. If the ith and jth agents have access to the relative information $x_i - x_j$, then the nodes i and j in the graph representation G are connected by a link. To simplify our notation, we assign an orientation to the graph by denoting one of the nodes of each link to be the positive end. The choice of orientation does not change the results because the information flow is assumed to be bidirectional. We further assume that the information topology is a connected graph. Suppose M is the total number of links, and recall that the $N \times M$ *incidence matrix* D is defined as

$$d_{ik} := \begin{cases} +1 & \text{if the } i\text{th node is the positive end of the } k\text{th link} \\ -1 & \text{if the } i\text{th node is the negative end of the } k\text{th link} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Reference [10] develops coordination laws that guarantee the following group behaviors:

B1) Each member achieves in the limit a common velocity vector $v(t) \in \mathbb{R}^p$ prescribed for the group; that is $\lim_{t\to\infty} |\dot{x}_i - v(t)| = 0$, $i = 1, \cdots, N$.

B2) If ith and jth members are connected by link k, then the difference variable z_k

$$z_k := \sum_{l=1}^{N} d_{lk} x_l = \begin{cases} x_i - x_j & \text{if } i \text{ is the positive end} \\ x_j - x_i & \text{if } j \text{ is the positive end} \end{cases} \tag{2}$$

converges to a prescribed compact set $\mathcal{A}_k \subset \mathbb{R}^p$, $k = 1, \cdots, M$, where d_{ik} is defined in (1).

Examples of such target sets $\mathcal{A}_k$ include the origin if x_i's must reach an agreement within the group, or a sphere in $\mathbb{R}^p$ if x_i's are positions of vehicles that must maintain a prescribed distance.

Reference [10] assumes that, upon a change of variables and a preliminary feedback design, the agent dynamics can be brought to the form

$$\dot{x}_i = y_i + v(t) \qquad i = 1, \cdots, N \tag{3}$$

$$\mathcal{H}_i : \begin{cases} \dot{\xi}_i = f_i(\xi_i, u_i) \\ y_i = h_i(\xi_i) \end{cases} \tag{4}$$

where y_i is the velocity error, $\xi_i \in \mathbb{R}^{n_i}$ is the state variable of subsystem $\mathcal{H}_i$, $f_i(\cdot, \cdot)$ and $h_i(\cdot)$ are C^1 functions such that $h_i(0) = 0$ and

$$f_i(0, u_i) = 0 \Rightarrow u_i = 0. \tag{5}$$

The main restriction in [10] is that the $\mathcal{H}_i$ subsystems in (4) be strictly passive with C^1, positive definite, radially unbounded storage functions $S_i(\xi_i)$ satisfying

$$\dot{S}_i \le -W_i(\xi_i) + u_i^T y_i \quad i = 1, \cdots, N \tag{6}$$

for some positive definite C^1 functions $W_i(\cdot)$.

To achieve objectives B1 and B2, [10] employs the feedback law

$$u_i = -\sum_{k=1}^{M} d_{ik}\psi_k(z_k) \tag{7}$$

in which the nonlinearities $\psi_k(z_k)$ are of the form

$$\psi_k(z_k) = \nabla P_k(z_k) \tag{8}$$

where $P_k(z_k)$ is a nonnegative C^2 function

$$P_k : \mathcal{G}_k \to \mathbb{R}_{\ge 0} \tag{9}$$

defined on an open set $\mathcal{G}_k \subseteq \mathbb{R}^p$. As an illustration, if x_i's are positions of vehicles that must maintain a prescribed distance, then the choice $\mathcal{G}_k = \{x_i | x_i \in \mathbb{R}^p \setminus 0\}$ disallows the possibility of collisions.

To steer z_k's into the target sets $\mathcal{A}_k$, we let $P_k(z_k)$ and its gradient $\nabla P_k(z_k)$ vanish on $\mathcal{A}_k$, and let $P_k(z_k)$ grow unbounded as z_k goes to the boundary of $\mathcal{G}_k$:

$$P_k(z_k) \to \infty \text{ as } z_k \to \partial\mathcal{G}_k \tag{10}$$

$$P_k(z_k) = 0 \Leftrightarrow z_k \in \mathcal{A}_k \tag{11}$$

$$\nabla P_k(z_k) = 0 \Leftrightarrow z_k \in \mathcal{A}_k. \tag{12}$$

We introduce the concatenated vectors

$$x := [x_1^T, \cdots, x_N^T]^T \in \mathbb{R}^{p\cdot N} \quad z := [z_1^T, \cdots, z_M^T]^T \in \mathbb{R}^{p\cdot M} \tag{13}$$

$$u := [u_1^T, \cdots, u_N^T]^T \in \mathbb{R}^{p\cdot N} \quad \psi := [\psi_1^T, \cdots, \psi_M^T]^T \in \mathbb{R}^{p\cdot M} \tag{14}$$

$$y = [y_1^T, \cdots, y_N^T]^T \in \mathbb{R}^{p\cdot N}$$

and note from (2) and (7) that

$$z = (D^T \otimes I_p)x \tag{15}$$

$$u = -(D \otimes I_p)\psi(z) \tag{16}$$

where I_p denotes the $p \times p$ identity matrix. For the objective B2 to be feasible, the target sets $\mathcal{A}_k$ must be such that

$$\{\mathcal{A}_1 \times \cdots \times \mathcal{A}_M\} \cap \mathcal{R}(D^T \otimes I_p) \neq \emptyset \tag{17}$$

since, from (15), z is restricted to be in the range space $\mathcal{R}(D^T \otimes I_p)$.

Theorem 1 in [10], proved that the objectives B1 and B2 are achieved with the design (3), (4) and (7). It further showed that the region of attraction is the set

$$\mathcal{G} = \left\{(z,\xi) | \xi \in \mathbb{R}^{n_1} \times \cdots \times \mathbb{R}^{n_N}, z \in \mathcal{G}_1 \times \cdots \times \mathcal{G}_M \cap \mathcal{R}(D^T \otimes I_p)\right\}. \tag{18}$$

when the following additional property holds:

Property 1 $(D \otimes I_p)\psi(z) = 0$ and $z \in \mathcal{R}(D^T \otimes I_p)$ imply $z \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M$. □

B. Basic Adaptive Design for Reference Velocity Recovery

The design (3), (4) and (7) assumes that the reference velocity $v(t)$ is available to each agent. Reference [1] considered the situation where only the leader, say, the first agent $i = 1$, possesses the $v(t)$ information, and developed an adaptive design with which the remaining agents estimate $v(t)$. In [1], $v(t) \in \mathbb{R}^p$ is assumed to be uniformly bounded and piecewise continuous, and parameterized as

$$v(t) = \sum_{j=1}^{r} \phi^j(t)\theta^j \tag{19}$$

where $\phi^j(t)$ are scalar base functions available to each agent and θ^j are column vectors available only to the leader $i = 1$.

The other agents estimate the unknown θ^j by $\hat{\theta}_i^j$, and construct $\hat{v}_i(t)$ from

$$\hat{v}_i(t) = \sum_{j=1}^{r} \phi^j(t)\hat{\theta}_i^j = (\Phi(t)^T \otimes I_p)\hat{\theta}_i \quad i = 2, \cdots, N \tag{20}$$

where

$$\Phi(t) := [\phi^1(t), \cdots, \phi^r(t)]^T \tag{21}$$

and

$$\hat{\theta}_i := [(\hat{\theta}_i^1)^T, \cdots, (\hat{\theta}_i^r)^T]^T. \tag{22}$$

The adaptive design in [1] employed the feedback law (7) in Section II-A, and modified (3) as

$$\dot{x}_1 = y_1 + v(t) \tag{23}$$

$$\dot{x}_i = y_i + \hat{v}_i(t) \quad i = 2, \cdots, N \tag{24}$$

where $\hat{v}_i$ is now obtained from (20). The update law for the parameter $\hat{\theta}_i$ is

$$\dot{\hat{\theta}}_i = \Lambda_i(\Phi(t) \otimes I_p)u_i \tag{25}$$

in which $\Lambda_i = \Lambda_i^T > 0$ and u_i is as in (7).

Given the coordination laws in (23), (24) and (25), the main results in [1, Theorem 1] showed that the trajectories of $(z(t), \xi(t))$ starting in $\mathcal{G}$ (18) are bounded and converge to the set $\mathcal{E}$, where

$$\mathcal{E} = \left\{(z,\xi) | \ \xi = 0, \ (D \otimes I_p)\psi(z) = 0 \text{ and } z \in \mathcal{R}(D^T \otimes I_p)\right\}. \tag{26}$$

Moreover, when Property 1 holds, all trajectories $(z(t), \xi(t), \hat{\theta}(t))$ starting in $\mathcal{G} \times \mathbb{R}^{p\cdot r\cdot(N-1)}$ converge to the set $\mathcal{A} \times \mathbb{R}^{p\cdot r\cdot(N-1)}$, where

$$\mathcal{A} = \left\{(z,\xi) | \xi = 0, z \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M \cap \mathcal{R}(D^T \otimes I_p)\right\} \tag{27}$$

Convergence to the set $\mathcal{A} \times \mathbb{R}^{p\cdot r\cdot(N-1)}$ means that the difference variable z_k tends to the target set $\mathcal{A}_k$. Whether objective B1 is achieved or not depends on the convergence of $\hat{\theta}_i$ to θ. When $\hat{\theta}_i$ converges to θ, $v(t)$ is recovered with the adaptive design and, thus, B1 is also achieved.

III. M M D

To illustrate the lack of parameter convergence for the basic adaptive design, we consider the coordination of three vehicles, modeled as fully-actuated point masses

$$\ddot{x}_i = f_i \qquad i = 1,2,3 \tag{28}$$

where $x_i \in \mathbb{R}^2$ is the position of each mass and $f_i \in \mathbb{R}^2$ is the input force.

1) Nonadaptive design of [10]: The nonadaptive design assumes the reference velocity $v(t)$ is available to each agent. Indeed, the following internal feedback law

$$f_i = -K_i(\dot{x}_i(t) - v(t)) + \dot{v}(t) + u_i, \qquad K_i > 0 \tag{29}$$

and the change of variables $\xi_i = \dot{x}_i - v(t)$ bring (28) into the form

$$\begin{aligned} \dot{x}_i &= \xi_i + v(t) \qquad (30)\\ \dot{\xi}_i &= -K_i\xi_i + u_i \end{aligned}$$

which is as in (3) and (4). The ξ_i dynamics act as the subsystem $\mathcal{H}_i$, which is strictly passive from u_i to ξ_i with the storage function $S_i(\xi_i) = \frac{1}{2}\xi_i^T\xi_i$.

To stabilize a formation where the relative distances $|z_i|$, $i = 1,2,3$ are equal to 1, we design $\mathcal{A}_k$ to be the unit circle, $\mathcal{G}_k$ to be $\mathbb{R}^2 \setminus \{0\}$, and let the potential functions be of the form

$$P_k(z_k) = \int_1^{|z_k|} \sigma_k(s)ds \qquad k = 1,2,3 \tag{31}$$

where $\sigma_k : \mathbb{R}_{>0} \to \mathbb{R}$ is a C^1, strictly increasing function such that

$$\sigma_k(1) = 0 \qquad \lim_{s\to\infty}\sigma_k(s) = \infty \qquad \lim_{s\to 0}\sigma_k(s) = -\infty \tag{32}$$

and such that, as $|z_k| \to \infty$, $P_k(z_k) \to \infty$ in (31). Then $P_k(z_k)$ satisfies (8)-(12), and the feedback law (7) with the interaction forces

$$\psi_k(z_k) = \nabla P_k(z_k) = \sigma_k(|z_k|)\frac{1}{|z_k|}z_k \qquad z_k \neq 0 \tag{33}$$

guarantees asymptotic stability of the desired formation from [10, Theorem 1].

2) Basic adaptive design of [1]: We now assume that $v(t)$ is available only to the leader x_1. In the design of [10], f_i is as in (29) for $i = 1$ and thus the x_1 dynamics remain the same as in (30) while for the agents $i = 2,3$, f_i is replaced with

$$f_i = -K_i(\dot{x}_i(t) - \hat{v}_i(t)) + \dot{\hat{v}}_i(t) + u_i \qquad K_i > 0. \tag{34}$$

This feedback together with the change of variables $\hat{\xi}_i = \dot{x}_i - \hat{v}_i$, brings the dynamics of the agents, $i = 2,3$ to the form

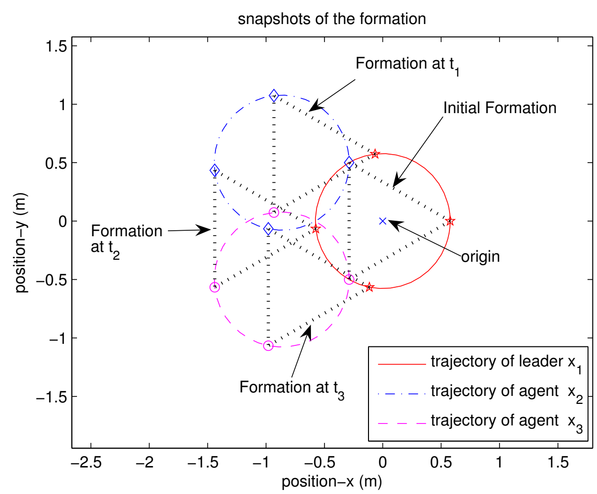
$$\begin{aligned} \dot{x}_i &= \hat{\xi}_i + \hat{v}_i \qquad (35)\\ \dot{\hat{\xi}}_i &= -K_i\hat{\xi}_i + u_i. \qquad (36) \end{aligned}$$

The signal $\hat{v}_i$ is available for implementation in (35) once the parametrization in (20) and the update law in (25) are setup.

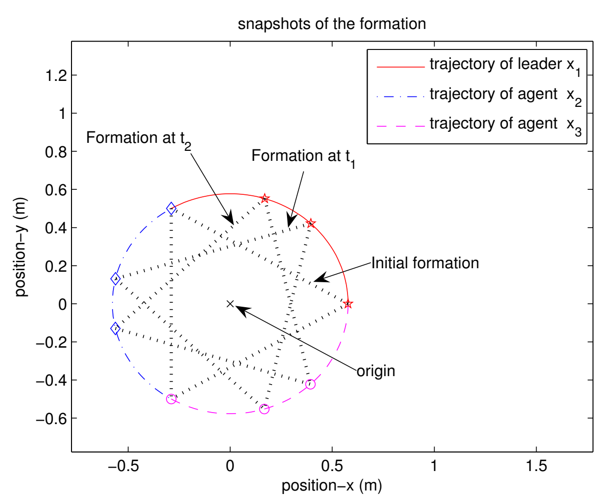
To show the lack of parameter convergence, we suppose that $v(t)$ in the x_1-dynamics (30) is time-varying, and is parameterized as

$$v(t) = ([\sin(t)\ \ \cos(t)] \otimes I_2)\begin{bmatrix}\theta^1\\ \theta^2\end{bmatrix} \tag{37}$$

where $\theta^1, \theta^2 \in \mathbb{R}^2$. Initially the three agents form an equilateral triangle where $x_1(0) = [\frac{\sqrt{3}}{3}\ \ 0]^T$, $x_2(0) = [-\frac{\sqrt{3}}{6}\ \ \frac{1}{2}]^T$ and $x_3(0) = [-\frac{\sqrt{3}}{6}\ \ -\frac{1}{2}]^T$, thus satisfying the desired formation with $|z_k| = 1$, $k = 1,2,3$. We then pick $\theta^1 = [-\frac{\sqrt{3}}{3}\ \ 0]^T$ and $\theta^2 = [0\ \ \frac{\sqrt{3}}{3}]^T$ in (37), which means that the leader x_1 will rotate about the origin with a radius of $\frac{\sqrt{3}}{3}$. The graph G is complete, which means that each agent has the relative information with respect to the other two, and $\sigma_k(\cdot)$ in (31) is chosen as the natural logarithm.



(a) Nonadaptive design with time-varying $v(t)$



(b) Adaptive design with time-varying $v(t)$

Fig. 1. Two group behaviors: (a) The group exhibits a translational motion with x_1 spinning around the origin. (b) The agents x_2 and x_3 exhibit a rotational motion about the leader x_1.

Figure 1(a) shows that in the nonadaptive design where the reference velocity $v(t)$ in (37) is available to each agent, the group exhibits a translational motion with x_1 spinning around the origin. The adaptive case in Figure 1(b) where the initial conditions are set to $\xi_1(0) = \hat{\xi}_2(0) = \hat{\xi}_3(0) = 0$, $\hat{\theta}_2(0) = [\frac{\sqrt{3}}{6}\ -\frac{1}{2}\ -\frac{1}{2}\ -\frac{\sqrt{3}}{6}]^T$ and $\hat{\theta}_3(0) = [\frac{\sqrt{3}}{6}\ \frac{1}{2}\ \frac{1}{2}\ -\frac{\sqrt{3}}{6}]^T$, shows that the agents $i = 2,3$ exhibit a rotational motion about the leader, which means that parameter convergence fails and that the $v(t)$ information is not fully recovered.

IV. M A D

Note that in the update law (25), $\hat{\theta}_i$ update stops when u_i in (7) approaches zero. The external feedback u_i, however, only contains the relative distance information and $u_i \to 0$ does not guarantee that all the agents converge to the same reference velocity $v(t)$. In contrast, $\dot{z} \to 0$ would imply that all the agents converge to the reference velocity. Thus, it is plausible that a modified adaptive design that employs $\dot{z}$ can guarantee parameter convergence.

To present our modified design, we introduce another graph G_v representing the information topology for the relative velocity: if the ith and jth agents have access to the relative velocity information $\dot{x}_i - \dot{x}_j$, then the nodes i and j in the graph G_v are connected by a link. This graph may be different from G because the set of agents capable of relative velocity sensing may be different from those that measure relative distance. We further assume that the graph G_v is bidirectional. Suppose there are M_v links in G_v. The corresponding $N \times M_v$ incidence matrix D_v is defined similarly as in (1):

$$d_{ik}^v := \begin{cases} +1 & \text{if the } i\text{th node is the positive end of the } k\text{th link} \\ -1 & \text{if the } i\text{th node is the negative end of the } k\text{th link} \\ 0 & \text{otherwise.} \end{cases} \tag{38}$$

Instead of using the external feedback in (7), we now consider a new external feedback design

$$u_i = -\sum_{k=1}^{M} d_{ik}\psi_k(z_k) - \sum_{k=1}^{M_v} d_{ik}^v \gamma_k(\dot{z}_k) \tag{39}$$

with the update law

$$\dot{\hat{\theta}}_i = \Lambda_i(\Phi(t) \otimes I_p)u_i, \tag{40}$$

where $\gamma(\cdot)$ is a C^1 function satisfying

$$\gamma_k(x)^T x > 0 \quad \forall x \neq 0. \tag{41}$$

We note that the decentralized design (39) employs the relative velocity information, which can be obtained by differentiating the relative distance or by vision-based sensors.

The following theorem requires the regressor $\Phi(t)$ in (21) to be *persistently exciting* (PE), which means that for all $t_o \geq 0$,

$$\int_{t_o}^{t_o+\delta_0} \Phi(t)\Phi(t)^T dt \geq \alpha_0 I \tag{42}$$

for some constants $\delta_0 > 0$ and $\alpha_0 > 0$ that do not depend on t_o. This PE condition ensures the information richness of the time-varying signal $\Phi(t)$, and guarantees parameter convergence as we prove next:

Theorem 1: Suppose that the information topology G_v is time-invariant and connected. Consider the coordination laws in (23), (24) and (25) where $v(t)$ is uniformly bounded and piecewise continuous, parameterized as (19) in which $\phi^j(t)$, $\dot{\phi}^j(t)$, $j = 1, \cdots, r$ are uniformly bounded and the regressor $\Phi(t)$ in (20) is PE as in (21), u_i is defined in (39) and $\mathcal{H}_i$, $i = 1, \cdots, N$, and ψ_k, $k = 1, \cdots, M$ are designed as in (4)-(6) and (8)-(12), respectively. Then, all trajectories $(z(t), \xi(t), \hat{\theta}(t))$ starting in $\mathcal{G} \times \mathbb{R}^{p\cdot r\cdot(N-1)}$ are bounded and converge to the set $\mathcal{E}^*$

$$\mathcal{E}^* = \left\{(z,\xi,\hat{\theta}) | \xi = 0,\ (D \otimes I_p)\psi(z) = 0,\ z \in \mathcal{R}(D^T \otimes I_p),\ \hat{\theta} = \theta^*\right\}, \tag{43}$$

where G is as in (18), $\hat{\theta} = [\hat{\theta}_2^T, \cdots, \hat{\theta}_N^T]^T$, and $\theta^* = 1_{N-1} \otimes \theta$. Moreover, when Property 1 holds, the set $\mathcal{A} \times \theta^*$ is asymptotically stable with region of attraction $\mathcal{G} \times \mathbb{R}^{p\cdot r\cdot(N-1)}$, where A is in (27). □

The proof of Theorem 1 makes use of the following lemma:

Lemma 1: Let

$$\dot{X} = f(X,t), \tag{44}$$

where $X \in \mathbb{R}^n$ and $f(X,t) : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$. If $f(X,t) \to 0$ and $\Omega(t)^T X(t) \to 0$, where $\Omega(t) \in \mathbb{R}^n$ is upper bounded and satisfies the PE property in (42), then $X \to 0$. □

Proof of Theorem 1: We denote by $\tilde{\theta}_i$ the error variable

$$\tilde{\theta}_i = \hat{\theta}_i - \theta \qquad i = 2, \cdots, N \tag{45}$$

where $\theta = [(\theta^1)^T, \cdots, (\theta^r)^T]^T$ and note from (40) that

$$\dot{\tilde{\theta}}_i = \Lambda_i(\Phi(t) \otimes I_p)u_i \quad i = 2, \cdots, N. \tag{46}$$

For consistency with (13) and (14), we set $\tilde{\theta}_1^T \equiv 0_{p\cdot r}$, and define

$$\tilde{\theta} = [\tilde{\theta}_1^T, \tilde{\theta}_2^T, \cdots, \tilde{\theta}_N^T]^T$$

where $0_{p\cdot r}$ denotes a column vector with $p \cdot r$ entries equal to zero. Then from (15), we rewrite (23) and (24) in the compact form

$$\dot{z} = (D^T \otimes I_p)(1_N \otimes v(t) + y + \tilde{v}) \tag{47}$$

where 1_N denotes N-vector of ones and

$$\tilde{v} := (I_N \otimes \Phi^T(t) \otimes I_p)\tilde{\theta}. \tag{48}$$

Because 1_N is the null space of D^T, we rewrite (47) as

$$\dot{z} = (D^T \otimes I_p)(y + \tilde{v}). \tag{49}$$

To prove the stability of the closed-loop system described by the adaptive scheme (4), (46) and (47), we exploit the passivity properties of the interconnected system. To this end, we introduce the following storage functions, $V_f(z), V_b(\xi)$ and $V_a(\tilde{\theta})$, for the feedforward, feedback and adaptive subsystems, respectively:

$$V_f(z) := \sum_{k=1}^{M} P_k(z_k) \quad V_b(\xi) := \sum_{i=1}^{N} S_i(\xi_i) \tag{50}$$

$$V_a(\tilde{\theta}) := \frac{1}{2}\sum_{i=2}^{N} \tilde{\theta}_i^T \Lambda_i^{-1} \tilde{\theta}_i.$$

Using the property

$$(D^T \otimes I_p)(1_N \otimes v(t)) = 0 \tag{51}$$

which results from the fact that the sum of the rows of D is zero, and using (47), we get

$$\begin{aligned}\dot{V}_f &= \psi^T(D^T \otimes I_p)\{1_N \otimes v(t) + y + (1_N \otimes \Phi^T(t) \otimes I_p)\tilde{\theta}\} \\ &= \{(D \otimes I_p)\psi\}^T(y + \tilde{v}).\end{aligned} \tag{52}$$

Next, because the feedback blocks $\mathcal{H}_i$ are passive by (6), we get

$$\dot{V}_b \le -\sum_i W_i(\xi_i) + u^T y. \tag{53}$$

Finally, using (46), we obtain

$$\begin{aligned}\dot{V}_a &= \sum_{i=2}^{N} \tilde{\theta}_i^T \Lambda_i^{-1} \dot{\tilde{\theta}}_i \\ &= \sum_{i=2}^{N} \tilde{\theta}_i^T (\Phi(t) \otimes I_p) u_i \\ &= \tilde{v}^T u.\end{aligned} \tag{54}$$

From (52), (53) and (54), the Lyapunov function

$$V(z, \xi, \tilde{\theta}) = V_f(z) + V_b(\xi) + V_a(\tilde{\theta}) \tag{55}$$

yields the negative semidefinite derivative

$$\dot{V}_1 = -\sum_i W_i(\xi_i) - \sum_{k=1}^{M_v} \gamma_k(\dot{z}_k)^T \dot{z}_k \le 0 \tag{56}$$

which implies global stability and boundedness of all the signals $(z(t), \xi(t), \tilde{\theta})$. We further conclude from Barbalat's Lemma that $\xi \to 0$ and $\dot{z}_k \to 0$, $k = 1, \cdots, M^v$. We next show $u \to 0$. To this end we note that

$$\ddot{\xi}_i = \frac{\partial f_i}{\partial u_i}\dot{u}_i + \frac{\partial f_i}{\partial \xi}\dot{\xi}_i \tag{57}$$

is continuous and uniformly bounded because $\dot{u}$ and $\dot{\xi}$ are continuous functions of the bounded signals $(z(t), \xi(t), \tilde{\theta}(t), \Phi(t), \dot{\Phi}(t))$ and because $f_i(\cdot,\cdot)$ and $\gamma_k(\cdot)$ are C^1. Since $\xi_i \to 0$ and $\ddot{\xi}_i$ is continuous and bounded, it follows from [13, Lemma 1] that $\dot{\xi}_i \to 0$, which, from (4) and (5), guarantees $u_i \to 0$. Since $\dot{z}_k \to 0$ also, we conclude from (39) that $(D \otimes I_p)\psi(z) \to 0$.

To conclude the proof of convergence to $\mathcal{E}^*$ in (43), we need to show $\tilde{\theta}_i \to \theta$. We establish this by first showing that $|\hat{v}_i - v(t)| \to 0$ and, next by using the PE property (42) and Lemma 1 to prove that $|\hat{v}_i - v(t)| \to 0$ implies $\tilde{\theta}_i \to \theta$.

Since $\xi \to 0$, we conclude $y \to 0$. It then follows from (49) and $\dot{z}_k \to 0$ that $(D_v \otimes I_p)^T \tilde{v} \to 0$. Recall $\tilde{v}_1$ is identically zero and the null space of D_v^T is spanned by 1_N because G_v is connected. We thus conclude $\tilde{v} \to 0$, which implies that $|\hat{v}_i - v(t)| \to 0$. To further show parameter convergence, we note that

$$\tilde{v} = (I_N \otimes \Phi^T(t) \otimes I_p)\tilde{\theta} \to 0 \tag{58}$$

and that

$$\dot{\tilde{\theta}}_i = \Lambda_i(\Phi(t) \otimes I_p)u_i \to 0 \tag{59}$$

since $u_i \to 0$. Because the signal $\Phi^T(t)$ is PE, it then follows from Lemma 1 that $\tilde{\theta}_i \to 0$, which concludes the proof. ■

Convergence to $\mathcal{A} \times \theta^*$ means that the difference variables z_k tend to the target sets $\mathcal{A}_k$. It also implies that $\tilde{\theta} = 0$, $u = 0$ and thus, y in (23) (24) is zero and $\tilde{v}$ in (48) is zero, which means that both objectives B1 and B2 are achieved.

The PE property of $\Phi(t)$ is only used to guarantee the parameter convergence $\hat{\theta}_i \to \theta$. When $\Phi(t)$ is not PE in some situations, Theorem 1 still ensures $\dot{z}_k \to 0$, which means that all the agents reach the same velocity in the limit.

Proof of Lemma 1: We rewrite (44) as

$$\dot{X} = -\Omega(t)\Omega(t)^T X + \zeta(t) \tag{60}$$

where $\zeta(t) := \Omega(t)\Omega(t)^T X + f(X,t)$, and note that $\zeta(t) \to 0$ since $\Omega(t)^T X$ and $f(X,t)$ both converge to zero and $\Omega(t)$ is bounded. Solving for X from the linear time-varying model (60), we obtain

$$X = \Xi(t, t_0)X(t_0) + \int_{t_0}^{t} \Xi(t, \tau)\zeta(\tau)d\tau \tag{61}$$

where $\Xi(t, t_0)$ is the state transition matrix. Because $\Omega(t)$ is PE and because $\zeta(t) \to 0$ as $t \to \infty$, it follows from standard results in adaptive control [14], [15] that $X \to 0$. ■

V. M E R

We now include the relative velocity information in the external feedback u_i in the adaptive design in Section III-.2. We assume that the topology of the relative velocity is the same as that of the relative distance; that is, each agent has the other two agents as neighbors. Following the modified adaptive design in (39)-(40), we obtain

$$u_i = -\sum_{k=1}^{3}\{d_{ik}\psi_k(z_k) + d_{ik}\gamma_k(\dot{z}_k)\} \tag{62}$$

where we take $\psi_k(z_k) = \ln(|z_k|)\frac{z_k}{|z_k|}$ and $\gamma_k(\dot{z}_k) = \dot{z}_k$.

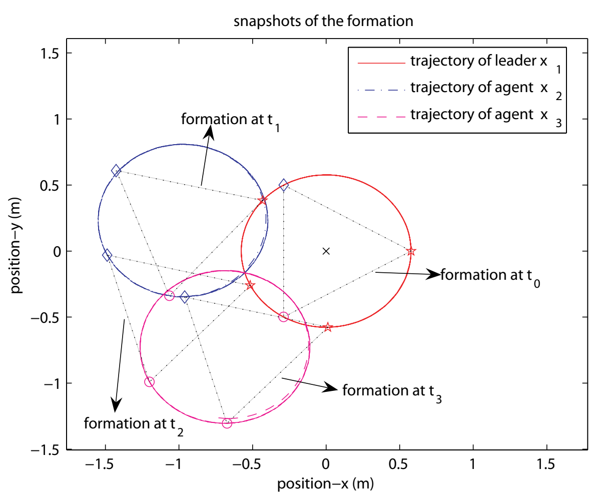
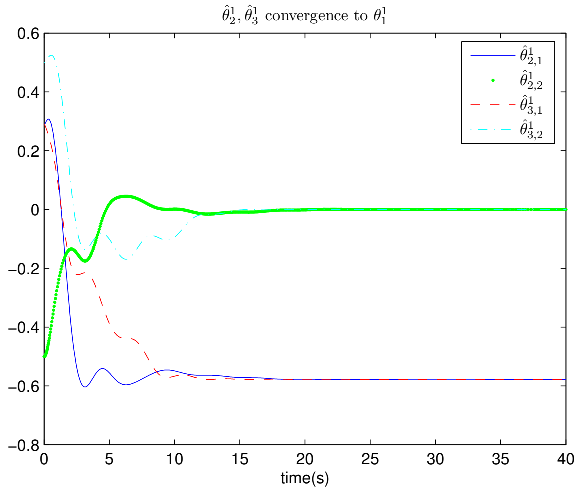
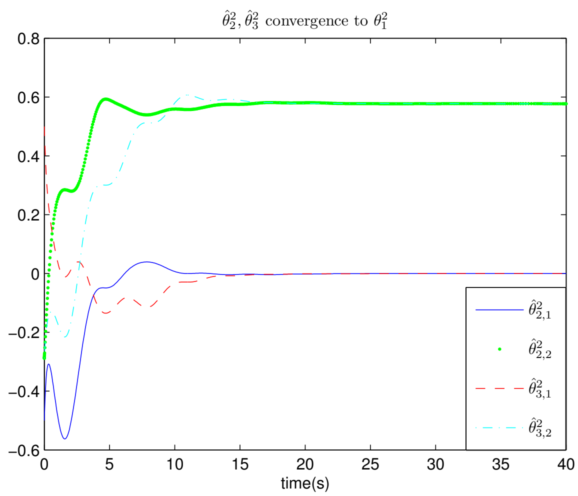


Fig. 2. The modified adaptive design recovers the convergence properties of the nonadaptive design.



(a)



(b)

Fig. 3. Parameter convergence with the modified adaptive design.

Figure 2 shows the snapshots of the group formation with the same set of initial conditions as in Section III. The group now exhibits a translational motion with x_i circling around the origin, which means that the nonadaptive results are recovered. The parameter convergence is achieved as shown in Figure 3. Note from Theorem 1 that the parameter convergence is guaranteed regardless of the initial conditions.

VI. C

We studied a coordination problem where the reference velocity is available to only one agent while the others estimate this information with an adaptive design. Although the basic adaptive design in [1] guaranteed that the desired formation is achieved, parameter convergence may fail. We proposed a new adaptive redesign, which employed relative velocity feedback, in addition to relative position feedback, to achieve parameter convergence. We illustrated parameter convergence by numerical examples.

ACKNOWLEDGMENTS

This work is supported in part by the Air Force Office of Scientific Research under award No. FA9550-07-1-0308. The work of the third author is supported in part by the Center for Automation Technologies and Systems (CATS) under a block grant from the New York State Office of Science, Technology, and Academic Research (NYSTAR) and by the Outstanding Overseas Chinese Scholars Fund of Chinese Academy of Sciences (No. 2005-1-11).

R

[1] H. Bai, M. Arcak, and J. T. Wen, “Group coordination when the reference velocity is available only to the leader: An adaptive design.” in *Proceedings of the 2007 American Control Conference*, New York City, New York, USA, July 2007.

[2] N. E. Leonard and E. Fiorelli, “Virtual leader, artificial potentials and coordinated control of groups.” in *Proceedings of the 40th IEEE Conference on Decision and Control*, 2001, pp. 2968–2973.

[3] H. Tanner, A. Jadbabaie, and G. Pappas, “Stable flocking of mobile agents, part i: Fixed topology.” in *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, 2003, pp. 2010–2015.

[4] V. Gazi and K. M. Passino, “Stability analysis of social foraging swarms.” *IEEE Transactions on Systems, Man and Cybernetics - Part B: Cybernetics*, vol. 34, no. 1, February 2004.

[5] A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, June 2003.

[6] R. S. Smith and F. Y. Hadaegh, “Closed-loop dynamics of cooperative vehicle formations with parallel estimators and communication,” *IEEE Transactions on Automatic Control*, vol. 52, no. 8, August 2007.

[7] D. A. Paley, N. E. Leonard, R. Sepulchre, D. Grünbaum, and J. K. Parrish, “Oscillator models and collective motion,” *IEEE Control Systems Magazine*, vol. 27, no. 4, pp. 89–105, August 2007.

[8] R. Olfati-Saber and R. M. Murray, “Graph rigidity and distributed formation stabilization of multi-vehicle systems,” in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada, December 2002.

[9] J. A. Fax and R. M. Murray, “Information flow and cooperative control of vehicle formations,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, September 2004.

[10] M. Arcak, “Passivity as a design tool for group coordination.” *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1380–1390, 2007.

[11] K. Lau, S. Lichten, L. Young, and B. Haines, “An innovative deep space application of gps technology for formation flying spacecraft,” *AIAA Paper 96-3819*, July 1996.

[12] R. S. Smith and F. Y. Hadaegh, “Control of deep-space formation-flying spacecraft; relative sensing and switched information,” *Journal of Guidance, Control and Dynamics*, vol. 28, no. 1, January-February 2005.

[13] J. S.-C. Yuan and W. M. Wonham, “Probing signals for model reference identification.” *IEEE Transactions on Automatic Control*, vol. 22, no. 4, August 1977.

[14] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Prentice Hall, 1996.

[15] G. Tao, *Adaptive Control Design and Analysis*. John Wiley & Sons, 2003.